

# Worst-Case SINR Constrained Robust Coordinated Beamforming for Multicell Wireless Systems

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**Abstract**—Multicell coordinated beamforming (MCBF) has been recognized as a promising approach to enhancing the system throughput and spectrum efficiency of wireless cellular systems. In contrast to the conventional single-cell beamforming (SBF) design, MCBF jointly optimizes the beamforming vectors of cooperative base stations (BSs) (via a central processing unit (CPU)) in order to mitigate the intercell interference. While most of the existing designs assume that the CPU has the perfect knowledge of the channel state information (CSI) of mobile stations (MSs), this paper takes into account the inevitable CSI errors at the CPU, and study the robust MCBF design problem. Specifically, we consider the worst-case robust design formulation that minimizes the weighted sum transmission power of BSs subject to worst-case signal-to-interference-plus-noise ratio (SINR) constraints on MSs. The associated optimization problem is challenging because it involves infinitely many nonconvex SINR constraints. In this paper, we show that the worst-case SINR constraints can be reformulated as linear matrix inequalities, and the approximation method known as semidefinite relation can be used to efficiently handle the worst-case robust MCBF problem. Simulation results show that the proposed robust MCBF design can provide guaranteed SINR performance for the MSs and outperforms the robust SBF design.

## I. INTRODUCTION

Recently, multicell cooperative signal processing has drawn considerable attention since it, when compared with the conventional single-cell processing, can provide significant system throughput gains by exploiting the degrees of freedom provided by multiple multi-antenna base stations (BSs). In contrast to the single-cell transmission design which treats the interference from neighboring cells as noise, in the multicell cooperative system, BSs collaborate with each other to jointly design their transmissions in order to mitigate the intercell interference [1]–[5]. This paper considers the multicell coordinated beamforming (MCBF) design [2], [3] where a set of multiple-antenna BSs jointly design their beamforming vectors aiming at providing desired quality-of-service (QoS) for the mobile stations (MSs). To this end, it is assumed that the BSs are connected with a central processing unit (CPU) (which can be a dedicated control center or a preselected BS), which knows all the channel state information (CSI) of MSs. With the perfect CSI, it has been shown that the MCBF design problem can be efficiently solved via convex optimization theory [2].

In practical systems, however, the CSI available to the CPU may not be perfect. In particular, the CSI may be subject

to channel estimation errors due to finite-length training, and quantization errors owing to limited feedback bandwidth (of the channels from the MSs to BSs). The imperfect CSI may result in performance outage and the QoS requirements of MSs can no longer be guaranteed. In view of this, transmit beamforming designs that take the CSI errors into consideration, also known as robust transmit beamforming, are of great importance to maintain the QoS of MSs.

In this paper, we assume elliptically bounded CSI errors, and study the robust MCBF design problem. Specifically, we consider the worst-case robust design formulation that minimizes the weighted sum transmission powers of the BSs subject to worst-case signal-to-interference-plus-noise ratio (SINR) constraints on the MSs. This robust formulation guarantees the MSs to achieve the desired SINR performance for all possible CSI errors. The worst-case robust design formulation has been studied in the context of single-cell robust transmit beamforming; see [6]–[10]. However, the robust formulation for MCBF is more challenging since the associated SINR constraints involve CSI errors not only in the desired signal and intra-cell interference terms, but also in the intercell interference. In this paper, we show that the worst-case robust MCBF problem can be efficiently handled by semidefinite relaxation (SDR), a convex optimization based approximation method [11]. Specifically, it can be shown that the worst-case SINR constraints can be recast as a finite number of linear matrix inequalities (LMIs), and SDR can be applied to approximate the original nonconvex problem by a convex semidefinite program (SDP), which, thereby, can be efficiently solved [12]. The presented simulation results show that the proposed worst-case robust MCBF design can provide guaranteed SINR performance for the MSs, and is more power efficient and more feasible than the conventional single-cell robust beamforming design.

## II. SYSTEM MODEL AND PROBLEM STATEMENT

We consider a multicell wireless system with  $N_c$  cells. Each cell consists of a BS, which is equipped with  $N_t$  antennas, and  $K$  single-antenna MSs; see Fig. 1 for an example of  $N_c = 3$  and  $K = 4$ . The  $N_c$  BSs will collaborate to enhance the strength of the signal of interest for each MS while mitigating the intercell interference. Let  $s_{ik}(t)$  be the information signal

for MS  $k$  in the  $i$ th cell with  $E\{|s_{ik}(t)|^2\}=1$ ; and let  $\mathbf{w}_{ik} \in \mathbb{C}^{N_t}$  be the associated beamforming vector,  $\{\mathbf{w}_{ik}\}$  be the set of all beamforming vectors, i.e.,  $\{\mathbf{w}_{ik}\} \triangleq \{\mathbf{w}_{11}, \dots, \mathbf{w}_{N_c K}\}$ . The transmit signal by the  $i$ th BS is given by  $\sum_{k=1}^K \mathbf{w}_{ik} s_{ik}(t)$  for  $i = 1, \dots, N_c$ . Denote by  $\mathbf{h}_{jik} \in \mathbb{C}^{N_t}$  the channel vector from the  $j$ th BS to the  $k$ th MS in the  $i$ th cell and denote by  $\{\mathbf{h}_{jik}\}_{j=1}^{N_c}$  the set of channel vectors from all BSs to the  $k$ th MS in cell  $i$ . The received signal of MS  $k$  in the  $i$ th cell can be expressed as

$$y_{ik}(t) = \sum_{j=1}^{N_c} \mathbf{h}_{jik}^H \left( \sum_{\ell=1}^K \mathbf{w}_{j\ell} s_{j\ell}(t) \right) + z_{ik}(t) \quad (1a)$$

$$\begin{aligned} &= \mathbf{h}_{iik}^H \mathbf{w}_{ik} s_{ik}(t) + \sum_{\ell \neq k}^K \mathbf{h}_{iik}^H \mathbf{w}_{i\ell} s_{i\ell}(t) \\ &\quad + \sum_{j \neq i}^{N_c} \mathbf{h}_{jik}^H \sum_{\ell=1}^K \mathbf{w}_{j\ell} s_{j\ell}(t) + z_{ik}(t), \end{aligned} \quad (1b)$$

where the first term in (1b) is the signal of interest, the second and third terms are the intra-cell and intercell interference, respectively, and  $z_{ik}(t)$  is the additive noise with zero mean and variance  $\sigma_{ik}^2 > 0$ . From (1), the SINR of the  $k$ th MS in the  $i$ th cell can be shown to be

$$\begin{aligned} \text{SINR}_{ik} &\left( \{\mathbf{w}_{j\ell}\}, \{\mathbf{h}_{jik}\}_{j=1}^{N_c} \right) \\ &= \frac{|\mathbf{h}_{iik}^H \mathbf{w}_{ik}|^2}{\sum_{\ell \neq k}^K |\mathbf{h}_{iik}^H \mathbf{w}_{i\ell}|^2 + \sum_{j \neq i}^{N_c} \sum_{\ell=1}^K |\mathbf{h}_{jik}^H \mathbf{w}_{j\ell}|^2 + \sigma_{ik}^2}. \end{aligned} \quad (2)$$

Using the SINR in (2) as the MSs' QoS measure and under the assumption that the CPU has the perfect knowledge of all the channels  $\{\mathbf{h}_{jik}\}$ , the following design formulation

$$\min_{\{\mathbf{w}_{ik}\}} \sum_{i=1}^{N_c} \alpha_i \left( \sum_{k=1}^K \|\mathbf{w}_{ik}\|^2 \right) \quad (3a)$$

$$\begin{aligned} \text{s.t.} \quad &\text{SINR}_{ik} \left( \{\mathbf{w}_{j\ell}\}, \{\mathbf{h}_{jik}\}_{j=1}^{N_c} \right) \geq \gamma_{ik}, \\ &k = 1, \dots, K, \quad i = 1, \dots, N_c, \end{aligned} \quad (3b)$$

has been considered in [2], where  $\alpha_i > 0$  is the power weight for BS  $i$ , and  $\gamma_{ik} > 0$  is the target SINR for MS  $k$  in cell  $i$ . One can see from (3) that the  $N_c$  BSs jointly design their beamforming vectors such that the weighted sum power of BSs is minimized while each of the MSs can achieve the desired SINR specification  $\gamma_{ik}$ . It has been shown that problem (3) can be reformulated as a convex second-order cone program (SOCP) and can be efficiently solved via standard solvers, e.g., CVX [13].

In addition to problem (3), we also consider the conventional single-cell beamforming (SBF) design that avoids interfering neighboring cells by per-cell interference control [4], i.e., BS  $i$  designs the beamforming vectors  $\{\mathbf{w}_{ik}\}_{k=1}^K$

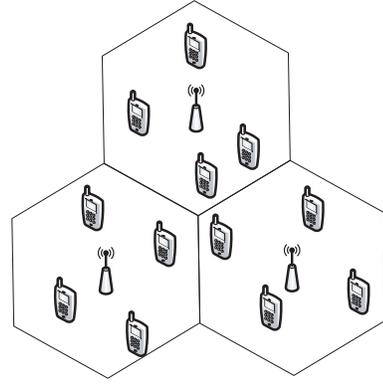


Fig. 1: An example of wireless cellular system with 3 BSs and 4 MSs in each cell.

independently by solving the following problem:

$$\begin{aligned} \min_{\{\mathbf{w}_{ik}\}_{k=1}^K} &\sum_{k=1}^K \|\mathbf{w}_{ik}\|^2 \quad (4) \\ \text{s.t.} &\frac{|\mathbf{h}_{iik}^H \mathbf{w}_{ik}|^2}{\sum_{\ell \neq k}^K |\mathbf{h}_{iik}^H \mathbf{w}_{i\ell}|^2 + \sum_{j \neq i}^{N_c} \xi_{jik} + \sigma_{ik}^2} \geq \gamma_{ik}, k = 1, \dots, K, \\ &\sum_{k=1}^K |\mathbf{h}_{ij\ell}^H \mathbf{w}_{ik}|^2 \leq \xi_{ij\ell}, \ell = 1, \dots, K, j \in \mathcal{N}_c \setminus \{i\}, \end{aligned}$$

for  $i = 1, \dots, N_c$ , where  $\mathcal{N}_c = \{1, \dots, N_c\}$ , and  $\xi_{ij\ell} > 0$  stands for the preset, maximum tolerable interference from BS  $i$  to the  $\ell$ th user in cell  $j$ . As seen from (4), the SBF design conservatively treats the intercell interference upper bound  $\xi_{jik}$  as fixed noise powers, in contrast to the MCBF design in (3) where the  $N_c$  BSs collaborate to dynamically control the intercell interference. It has been shown that the SBF design is less power efficient than the MCBF design in (3) [2]; however the SBF design can inherently be implemented at each BS in a decentralized fashion.

### III. PROPOSED ROBUST COORDINATED BEAMFORMING

#### A. Robust MCBF

The above formulations in (3) and (4) assume that the CPU knows the exact CSI  $\{\mathbf{h}_{jik}\}$ . In the case that the CPU has CSI with errors, the standard formulations in (3) and (4) can no longer guarantee the desired SINR requirements. To resolve this problem, we consider the worst-case robust design formulation [6], [7].

Specifically, we model the true channel vector  $\mathbf{h}_{jik}$  as

$$\mathbf{h}_{jik} = \bar{\mathbf{h}}_{jik} + \mathbf{e}_{jik}, \quad (5)$$

for  $k = 1, \dots, K$ ,  $i, j \in \mathcal{N}_c$ , where  $\mathbf{e}_{jik} \in \mathbb{C}^{N_t}$  represents the channel error vector. Moreover, let us consider the elliptically bounded CSI errors, that is, each  $\mathbf{e}_{jik}$  satisfies

$$\mathbf{e}_{jik}^H \mathbf{C}_{jik} \mathbf{e}_{jik} \leq 1, \quad (6)$$

where  $\mathbf{C}_{jik} \succ 0$  (a positive definite matrix) determines the size and the shape of the error ellipsoid. With (5) and (6), we consider the following worst-case SINR constraint on MS  $k$  in cell  $i$ :

$$\text{SINR}_{ik} \left( \{\mathbf{w}_{j\ell}\}, \{\bar{\mathbf{h}}_{jik} + \mathbf{e}_{jik}\}_{j=1}^{N_c} \right) \geq \gamma_{ik} \\ \forall \mathbf{e}_{jik}^H \mathbf{C}_{jik} \mathbf{e}_{jik} \leq 1, j = 1, \dots, N_c. \quad (7)$$

Note from (7) that the SINR specification  $\gamma_{ik}$  is satisfied for all possible CSI errors. Taking the worst-case SINR constraints in (7) into consideration, we obtain the following design formulation

$$\min_{\{\mathbf{w}_{ik}\}} \sum_{i=1}^{N_c} \alpha_i \left( \sum_{k=1}^K \|\mathbf{w}_{ik}\|^2 \right) \quad (8a)$$

$$\text{s.t. SINR}_{ik} \left( \{\mathbf{w}_{j\ell}\}, \{\bar{\mathbf{h}}_{jik} + \mathbf{e}_{jik}\}_{j=1}^{N_c} \right) \geq \gamma_{ik} \\ \forall \mathbf{e}_{jik}^H \mathbf{C}_{jik} \mathbf{e}_{jik} \leq 1, j = 1, \dots, N_c, \quad (8b) \\ i = 1, \dots, N_c, k = 1, \dots, K,$$

as a worst-case robust counterpart of problem (3). Solving the optimization problem (8) is challenging due to the infinitely many nonconvex SINR constraints in (8b). To handle this problem, let us present a suboptimal method via SDR and S-procedure [12] in the next subsection.

### B. Solving (8) by SDR and S-Procedure

Let us express the objective function of problem (8) as  $\sum_{i=1}^{N_c} \alpha_i \sum_{k=1}^K \text{Tr}(\mathbf{w}_{ik} \mathbf{w}_{ik}^H)$ , where  $\text{Tr}(\cdot)$  denotes the trace of a matrix, and express the worst-case SINR constraint of the  $k$ th MS in the  $i$ th cell [in (7)] as

$$\left( \bar{\mathbf{h}}_{iik}^H + \mathbf{e}_{iik}^H \right) \left( \frac{1}{\gamma_{ik}} \mathbf{w}_{ik} \mathbf{w}_{ik}^H - \sum_{\ell \neq k} \mathbf{w}_{i\ell} \mathbf{w}_{i\ell}^H \right) \left( \bar{\mathbf{h}}_{iik} + \mathbf{e}_{iik} \right) \\ \geq \sum_{j \neq i}^{N_c} \left( \bar{\mathbf{h}}_{jik}^H + \mathbf{e}_{jik}^H \right) \left( \sum_{\ell=1}^K \mathbf{w}_{j\ell} \mathbf{w}_{j\ell}^H \right) \left( \bar{\mathbf{h}}_{jik} + \mathbf{e}_{jik} \right) + \sigma_{ik}^2 \\ \forall \mathbf{e}_{jik}^H \mathbf{C}_{jik} \mathbf{e}_{jik} \leq 1, j = 1, \dots, N_c. \quad (9)$$

The idea of SDR is to replace each rank-one matrix  $\mathbf{w}_{ik} \mathbf{w}_{ik}^H$  by a general-rank positive semidefinite matrix  $\mathbf{W}_{ik}$ , i.e.,  $\mathbf{W}_{ik} \succeq \mathbf{0}$  [11]. By applying SDR to (8), we obtain the following problem

$$\min_{\{\mathbf{W}_{ik} \succeq \mathbf{0}\}} \sum_{i=1}^{N_c} \alpha_i \left( \sum_{k=1}^K \text{Tr}(\mathbf{W}_{ik}) \right) \quad (10a)$$

$$\text{s.t. } \left( \bar{\mathbf{h}}_{iik}^H + \mathbf{e}_{iik}^H \right) \left( \frac{1}{\gamma_{ik}} \mathbf{W}_{ik} - \sum_{\ell \neq k} \mathbf{W}_{i\ell} \right) \left( \bar{\mathbf{h}}_{iik} + \mathbf{e}_{iik} \right) \\ \geq \sum_{j \neq i}^{N_c} \left( \bar{\mathbf{h}}_{jik}^H + \mathbf{e}_{jik}^H \right) \left( \sum_{\ell=1}^K \mathbf{W}_{j\ell} \right) \left( \bar{\mathbf{h}}_{jik} + \mathbf{e}_{jik} \right) + \sigma_{ik}^2, \\ \forall \mathbf{e}_{jik}^H \mathbf{C}_{jik} \mathbf{e}_{jik} \leq 1, j = 1, \dots, N_c, \quad (10b) \\ i = 1, \dots, N_c, k = 1, \dots, K.$$

While the SDR problem (10) is convex, it is still difficult to handle owing to an infinite number of linear inequality constraints. To resolve this, we observe that the left-hand side and right-hand side of the first inequality in (10b) involve independent CSI errors. Hence, the constraint (10b) can be equivalently decoupled into the following  $N_c$  constraints:

$$\left( \bar{\mathbf{h}}_{iik}^H + \mathbf{e}_{iik}^H \right) \left( \frac{1}{\gamma_{ik}} \mathbf{W}_{ik} - \sum_{\ell \neq k} \mathbf{W}_{i\ell} \right) \left( \bar{\mathbf{h}}_{iik} + \mathbf{e}_{iik} \right) \\ \geq \sum_{j \neq i}^{N_c} t_{jik} + \sigma_{ik}^2 \quad \forall \mathbf{e}_{iik}^H \mathbf{C}_{iik} \mathbf{e}_{iik} \leq 1, \quad (11)$$

$$\left( \bar{\mathbf{h}}_{jik}^H + \mathbf{e}_{jik}^H \right) \left( \sum_{\ell=1}^K \mathbf{W}_{j\ell} \right) \left( \bar{\mathbf{h}}_{jik} + \mathbf{e}_{jik} \right) \leq t_{jik} \\ \forall \mathbf{e}_{jik}^H \mathbf{C}_{jik} \mathbf{e}_{jik} \leq 1, j \in \mathcal{N}_c \setminus \{i\}, \quad (12)$$

where  $\{t_{jik}\}_{j \neq i}$  are slack variables. Note that equation (11) involves only the CSI error  $\mathbf{e}_{iik}$  and each of the constraints in (12) involves only one CSI error  $\mathbf{e}_{jik}$ . Furthermore, (11) and (12) can be reformulated as finite LMIs, by applying the following S-procedure:

**Lemma 1** [12, S-procedure] Let  $\mathbf{A}, \mathbf{C} \in \mathbb{C}^{N_t \times N_t}$  be complex Hermitian matrices,  $\mathbf{e} \in \mathbb{C}^{N_t}$  and  $c \in \mathbb{R}$ . The following condition

$$\mathbf{e}^H \mathbf{A} \mathbf{e} + \mathbf{b}^H \mathbf{e} + \mathbf{e}^H \mathbf{b} + c \geq 0 \quad \forall \mathbf{e}^H \mathbf{C} \mathbf{e} \leq 1$$

holds true if and only if there exists a  $\lambda \geq 0$  such that

$$\begin{bmatrix} \mathbf{A} + \lambda \mathbf{C} & \mathbf{b} \\ \mathbf{b}^H & c - \lambda \end{bmatrix} \succeq \mathbf{0}.$$

By applying Lemma 1, one can recast (11) as

$$\Phi_{ik} \left( \{\mathbf{W}_{i\ell}\}_{\ell=1}^K, \{t_{jik}\}_{j \neq i}, \lambda_{iik} \right) \triangleq \\ \begin{bmatrix} \mathbf{I} \\ \bar{\mathbf{h}}_{iik}^H \end{bmatrix} \left( \frac{1}{\gamma_{ik}} \mathbf{W}_{ik} - \sum_{\ell \neq k} \mathbf{W}_{i\ell} \right) \begin{bmatrix} \mathbf{I} \\ \bar{\mathbf{h}}_{iik}^H \end{bmatrix} \\ + \begin{bmatrix} \lambda_{iik} \mathbf{C}_{iik} & \mathbf{0} \\ \mathbf{0} & -\sum_{j \neq i}^{N_c} t_{jik} - \sigma_{ik}^2 - \lambda_{iik} \end{bmatrix} \succeq \mathbf{0}, \quad (13)$$

where  $\mathbf{I}$  is the  $N_t \times N_t$  identity matrix, and recast (12), for each  $j \in \mathcal{N}_c \setminus \{i\}$ , as

$$\Psi_{jik} \left( \{\mathbf{W}_{j\ell}\}_{\ell=1}^K, t_{jik}, \lambda_{jik} \right) \triangleq \begin{bmatrix} \mathbf{I} \\ \bar{\mathbf{h}}_{jik}^H \end{bmatrix} \left( -\sum_{\ell=1}^K \mathbf{W}_{j\ell} \right) \begin{bmatrix} \mathbf{I} \\ \bar{\mathbf{h}}_{jik}^H \end{bmatrix} \\ + \begin{bmatrix} \lambda_{jik} \mathbf{C}_{jik} & \mathbf{0} \\ \mathbf{0} & t_{jik} - \lambda_{jik} \end{bmatrix} \succeq \mathbf{0}, \quad (14)$$

where  $\lambda_{jik} \geq 0$  for all  $i, j = 1, \dots, N_c$ , and  $k = 1, \dots, K$ .

Replacing (10b) with (13) and (14) leads to the following SDR problem

$$\begin{aligned} \min_{\{\mathbf{W}_{ik}\}, \{\lambda_{jik}\}, \{t_{jik}, j \neq i\}} \quad & \sum_{i=1}^{N_c} \alpha_i \left( \sum_{k=1}^K \text{Tr}(\mathbf{W}_{ik}) \right) \\ \text{s.t.} \quad & \Phi_{ik}(\{\mathbf{W}_{i\ell}\}_{\ell=1}^K, \{t_{jik}\}_{j \neq i}, \lambda_{iik}) \succeq \mathbf{0}, \\ & \Psi_{jik}(\{\mathbf{W}_{j\ell}\}_{\ell=1}^K, t_{jik}, \lambda_{jik}) \succeq \mathbf{0}, j \in \mathcal{N}_c \setminus \{i\}, \\ & t_{jik} \geq 0, j \in \mathcal{N}_c \setminus \{i\}, \\ & \mathbf{W}_{ik} \succeq \mathbf{0}, \lambda_{jik} \geq 0, j \in \mathcal{N}_c, \\ & i = 1, \dots, N_c, k = 1, \dots, K. \end{aligned} \quad (15)$$

Problem (15) is a convex semidefinite program (SDP); hence it can be efficiently solved [13].

Similarly, one can also consider a worst-case robust design for the SBF design in (4), which is given by

$$\begin{aligned} \min_{\{\mathbf{w}_{ik}\}_{k=1}^K} \quad & \sum_{k=1}^K \|\mathbf{w}_{ik}\|^2 \\ \text{s.t.} \quad & \frac{|\bar{\mathbf{h}}_{iik} + \mathbf{e}_{iik}\|^H \mathbf{w}_{ik}|^2}{\sum_{\ell \neq k}^{N_c} |(\bar{\mathbf{h}}_{iik} + \mathbf{e}_{iik})^H \mathbf{w}_{i\ell}|^2 + \sum_{j \neq i}^{N_c} \xi_{jik} + \sigma_{ik}^2} \geq \gamma_{ik} \\ & \forall \mathbf{e}_{iik}^H \mathbf{C}_{iik} \mathbf{e}_{iik} \leq 1, k = 1, \dots, K, \\ & \sum_{k=1}^K |(\bar{\mathbf{h}}_{ij\ell} + \mathbf{e}_{ij\ell})^H \mathbf{w}_{ik}|^2 \leq \xi_{ij\ell} \forall \mathbf{e}_{ij\ell}^H \mathbf{C}_{ij\ell} \mathbf{e}_{ij\ell} \leq 1, \\ & \ell = 1, \dots, K, j \in \mathcal{N}_c \setminus \{i\}, \end{aligned} \quad (16)$$

for  $i = 1, \dots, N_c$ . By using similar techniques of S-procedure and SDR, one can obtain an SDR problem for (16) which can also be efficiently handled.

Since the SDR problem (15) is obtained by rank relaxation of problem (3), the obtained optimal  $\{\mathbf{W}_{ik}\}$  of (15) may not be of rank one. If the obtained optimal  $\{\mathbf{W}_{ik}\}$  happens to be of rank one, i.e.,  $\mathbf{W}_{ik} = \mathbf{w}_{ik} \mathbf{w}_{ik}^H$  for all  $i, k$ , then  $\{\mathbf{w}_{ik}\}$  is an optimal solution of the original problem (8); otherwise additional solution approximation procedure is needed; see [11] for the details. Interestingly, it is observed in our simulations that problem (15) always yields rank-one optimal  $\{\mathbf{W}_{ik}\}$ , which implies that an optimal solution of problem (8) can always be obtained for the problem instances in our simulations. The same rank-one optimality results are also observed for the SDR problem of problem (16).

#### IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, some simulation results are presented to demonstrate the performance of the proposed robust MCBF design. We consider a multicell system with three cells ( $N_c = 3$ ) and two MSs ( $K = 2$ ) in each cell. Assume that each BS has five antennas ( $N_t = 5$ ) and the inter-BS distance is 500 meters. In the simulations, we incorporate both large-scale and small-scale channel fadings. Specifically, we define the true channel  $\{\mathbf{h}_{jik}\}$  with parameters taken from the 3GPP Long

Term Evolution (LTE) channel model [14], as follows:

$$\mathbf{h}_{jik} = 10^{\frac{34.6 + 35 \log_{10}(d_{jik})}{-20}} \cdot \psi_{jik} \cdot \varphi_{jik} \cdot (\bar{\mathbf{h}}_{jik} + \mathbf{e}_{jik}), \quad (17)$$

where the exponential term is due to the path loss depending on the distance between the  $j$ th BS and the  $k$ th MS in cell  $i$  (denoted by  $d_{jik}$  in meters),  $\psi_{jik}$  reflects the shadowing effect,  $\varphi_{jik}$  represents the transmit-receive antenna gain, and the term inside the parenthesis denotes the small-scale fading which is composed of the channel estimate  $\bar{\mathbf{h}}_{jik}$  and the CSI error  $\mathbf{e}_{jik}$ . As seen from (17), it is assumed that the CPU can accurately track the large-scale fading with CSI errors only in the small-scale fading. In the simulations, the locations of the two MSs in each cell are randomly determined (with distance to the associated BS at least 35 meters, i.e.,  $d_{iik} \geq 35$  for all  $i, k$ ), and thereby the distances to neighboring BSs, i.e.,  $\{d_{jik}\}_{j \neq i}$ , can be determined. The shadowing coefficient  $\psi_{jik}$  follows the log-normal distribution with zero mean and standard deviation equal to 8. The elements of the channel estimate  $\{\bar{\mathbf{h}}_{jik}\}$  are independent and identically distributed complex Gaussian random variables with zero mean and unit variance. For simplicity, we assume the spherically bounded CSI errors, i.e.,  $\mathbf{C}_{jik} = 1/\epsilon^2 \mathbf{I}$ , with the uncertainty radius  $\epsilon$  set to 0.1. We also assume that all the MSs have the same noise power equal to  $\sigma_{jik}^2 \triangleq \sigma^2 = -106.27$  dBm [2], the same target SINRs, i.e.,  $\gamma_{jik} \triangleq \gamma$ , and the same antenna gains, i.e.,  $\varphi_{jik} = 5$  dBi for all  $j, i$  and  $k$ . We consider the total sum power minimization problem for formulations in (3), (4), (8) and (16) by setting  $\alpha_i = 1$  for all  $i = 1, \dots, N_c$ . For problems (4) and (16), we set all the intercell interference constraints  $\{\xi_{jik}\}_{j \neq i}$  equal to the noise power  $\sigma^2$  [4]. The robust formulations in (8) and (16) are handled by the proposed SDR method described in Sec. III-B, and CVX [13] is used to solve the associated SDPs.

In the first example, we investigate the minimal achievable SINRs of the four formulations, namely, the non-robust SBF design in (4), the non-robust MCBF design in (3) and their robust counterparts in (16) and (8), in the presence of CSI errors. We generated 5,000 sets of channel estimates  $\{\bar{\mathbf{h}}_{jik}\}$ , and, for each set of  $\{\bar{\mathbf{h}}_{jik}\}$ , 100 sets of CSI errors  $\{\mathbf{e}_{jik}\}$  satisfying  $\|\mathbf{e}_{jik}\|^2 \leq \epsilon^2$  were uniformly generated to evaluate the achievable SINRs [in (2)] by the four formulations. Figure 2 shows the simulation results of the minimal achievable SINR among all the MSs, by averaging over the channel estimates for which the four formulations under test are all feasible. It can be observed from this figure that both the robust designs in (16) and (8) can guarantee the minimal SINR of MSs no less than the target SINR  $\gamma$ ; whereas the non-robust designs can have SINR far below  $\gamma$  due to the CSI errors. In particular, one can see from Fig. 2 that, for  $\gamma = 9$  dB, the minimal SINR achieved by non-robust MCBF is more than 10 dB lower than that achieved by robust MCBF.

To show the power efficiency of MCBF, we present in Fig. 3 the corresponding average transmission powers of the four methods under test. As a price for worst-case performance guarantee, one can observe from this figure that the robust designs in (16) and (8) require more transmission powers

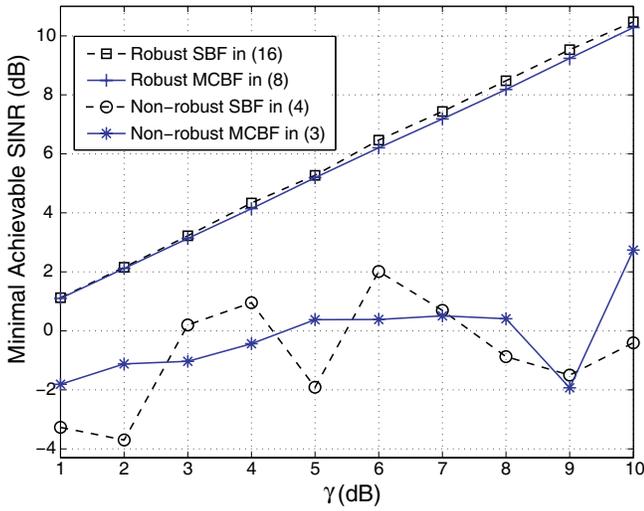


Fig. 2: Minimal achievable SINR of MSs versus target SINR  $\gamma$ .

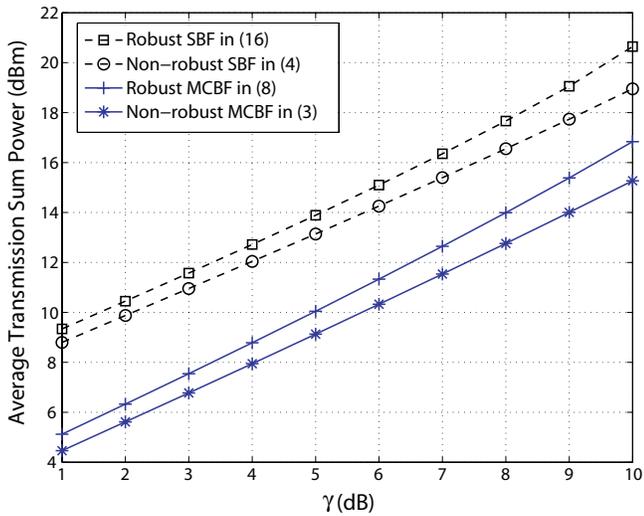


Fig. 3: Average transmission sum power versus target SINR  $\gamma$ .

than their non-robust counterparts in (4) and (3), respectively. However, the robust MCBF design (+) has an average sum power which is around 4 dBm less than that of the robust SBF design ( $\square$ ). Comparing Fig. 3 with Fig. 2, one can see that the robust MCBF is more power efficient than the robust SBF in achieving the same SINR performance. Finally, we show the feasibility rates of the four formulations under test in Fig. 4. As seen, the robust designs have lower feasibility rates compared to their non-robust counterparts; whereas, the proposed robust MCBF design (+) exhibits a significantly higher feasibility rate than the robust SBF design ( $\square$ ) since the former design makes use of the full degrees of freedom of the multicell system in intercell interference suppression.

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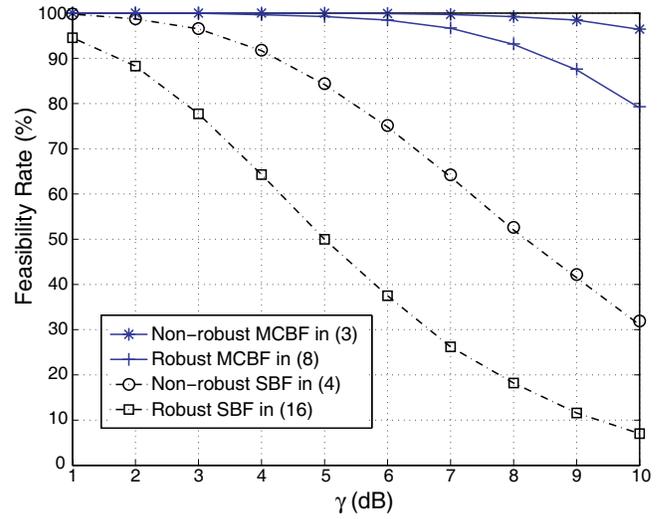


Fig. 4: Feasibility rate (%) versus target SINR  $\gamma$ .

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