Robust Beamforming with Channel Uncertainty for Two-way Relay Networks

Ahsan Aziz†, Meng Zeng‡, Jianwei Zhou‡, Costas N. Georghiades‡, Shuguang Cui‡
† National Instruments, Austin, Texas
‡ Department of Electrical and Computer Engineering
Texas A&M University, College Station, TX
Email: ahsan.aziz@ni.com, {zengm321, jianwei, georghiades, cui}@tamu.edu

***Abstract*—This paper presents the design of a robust beamforming scheme for a two-way relay network, composed of one multi-antenna relay and two single-antenna terminals, with the consideration of channel estimation errors. Given the assumption that the channel estimation error is within a certain range, we aim to minimize the transmit power at the multi-antenna relay and guarantee that the signal to interference and noise ratios (SINRs) at the two terminals are larger than a predefined value. Such a robust beamforming matrix design problem is formulated as a non-convex optimization problem, which is then converted into a semi-definite programming (SDP) problem by the S-procedure and rank one relaxation. The robust beamforming matrix is then derived from a principle eigenvector based rank-one reconstruction algorithm. We further propose a hybrid approach based on the best-effort principle to improve the outage probability performance, which is defined as the probability that one of two resulting terminal SINRs is less than the predefined value. Simulation results are presented to show that the robust design leads to better outage performance than the traditional non-robust approaches.**

***Index Terms*—Two-way Relay, Robust Beamforming, Semi-definite Programming.**

I. Introduction

In the next-generation wireless communication systems, smart relaying is one of the key technology that is being investigated to extend the cell coverage and to increase system capacity. For example, relaying will be standardized for the Long Term Evolution-Advanced (LTE-A) wireless systems, In perticular two-way relaying has drawn a great amount of attentions where multiple terminal nodes utilize common relays to perform two-way information exchanges [1]–[5]. Since the relays usually operate in the half-duplex mode where they cannot transmit and receive simultaneously, the required physical spectrum resource (in terms of the number of time slots or frequency bands) is usually increased. Fortunately, by mixing the data at the relay and exploiting the underlying self-interference property in two way relay system [1], we could reduce the number of required transmission time slots and thus improve the spectral efficiency.

Specifically, the authors in [4], considered the beamforming design at the relay and characterized the capacity region with the assumption that the relay is equipped with multiple antennas and the terminal nodes are each equipped with a single-antenna. Afterwards, different variations and various system setups were considered. For example, the authors of [5] studied the achievable rate region for the two-way collaborative relay beamforming problem, where multiple collaborative single relays were considered. The authors in [2] studied the AF-based two-way relay system with collaborative beamforming, where the focus is to minimize the total transmit power across the terminal nodes and the relay cluster under a given pair of SINR constraints. In [3], the authors studied the case where all the nodes in the network are equipped with multiple antennas and proposed some suboptimal solutions.

However, all the above results on beamforming design for the two-way relay system are based on the assumption that the channel state information (CSI) at all links are perfectly known. Unfortunately, this assumption may not be true in practice, since CSI can only be obtained by channel estimation, which is usually not perfect. As a result, in this paper we consider the robust two-way relay beamfroming under channel uncertainty where the SINR at each terminal is constrained to be larger than a predefined value. Similar problems for *one-way* relay robust beamforming were studied in [6] and [7] for single-antenna relays was further extended to multiple antenna in [8].

The rest of paper is organized as follows. In Section II, we present the system model and formulate the problem. Afterwards, we transform the original non-convex problem into a SDP problem in Section III with the help of the S-procedure and rank-one relaxation. In Section IV, we propose the principle eigenvector based rank-one reconstruction approach to obtain the beamforming matrix. In Section V, simulation results are presented to show the performance improvement. In Section VI, we conclude the paper.

The notations used in this paper are listed as follows. We define $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$ as the transpose, Hermitian transpose, and conjugate operations, respectively. $\Re(\cdot)$ is the real part of a complex variable. We use $\mathrm{tr}(\cdot)$ and $\mathrm{rank}(\cdot)$ to represent the trace and the rank of a matrix, respectively. A diagonal matrix with the elements of vector $\mathbf{a}$ as the diagonal entries is denoted as $\mathrm{diag}(\mathbf{a})$. $\mathbf{A} \succeq 0$ means that $\mathbf{A}$ is positive semi-definite, $\odot$ stands for the Hadamard (elementwise) multiplication, $\otimes$ denotes the Kronecker product, $\mathrm{vec}(\boldsymbol{A})$ means vectorization of matrix $\boldsymbol{A}$, $\mathrm{ivec}(\boldsymbol{a})$ is the inverse operation of $\mathrm{vec}(\boldsymbol{A})$ to recover $\mathbf{A}$, $\boldsymbol{I}_M$ and $\boldsymbol{O}_M$ are the identity matrix and zero matrix of dimension M, respectively, and $\mathbf{1}_{M\times 1}$ is the all one column vector of dimension M.

II. System Model

We consider a two-way relay system similar to the one given in [4], which includes the relay R and two terminal nodes S1

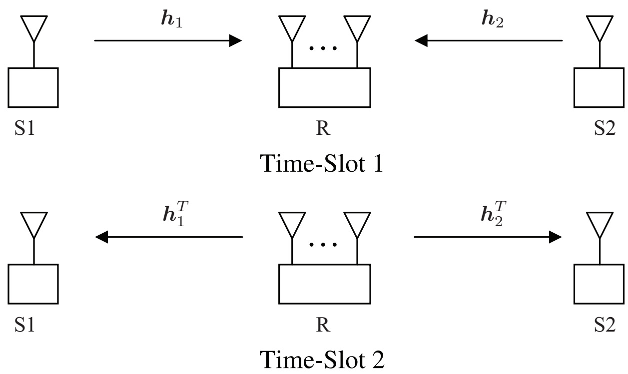

Fig. 1. System model

and S2 as shown on Fig. 1. The relay is equipped with M antennas and terminal nodes are each equipped with a single antenna. Based on the principle of analog network coding (ANC) [1], the two terminal nodes exchange information in two consecutive time slots via the help of relay R. In the first time slot, S1 and S2 send message s_1 and s_2 with power p_1 and p_2 respectively to R, and the received signal at R is given as

$$\boldsymbol{y}_R = \boldsymbol{h}_1\sqrt{p_1}s_1 + \boldsymbol{h}_2\sqrt{p_2}s_2 + \boldsymbol{z}_R, \tag{1}$$

where $\boldsymbol{h}_1, \boldsymbol{h}_2 \in \mathcal{C}^{M\times 1}$ are complex channel gains from the terminal nodes S1 and S2 to the relay respectively, and $\boldsymbol{z}_R$ is the criculant symmetric complex Gaussian (CSCG) noise with covariance $\sigma_R^2\boldsymbol{I}$. In the second time slot, the relay R multiplies a beamforming matrix $\boldsymbol{A}$ to the received signal $\boldsymbol{y}_R$ and transmits the resulting vector $\boldsymbol{A}\boldsymbol{y}_R$ to the terminal nodes. Based on the assumption of channel reciprocity [5], the received signals at S1 and S2 are given as

$$\begin{aligned} y_1 &= \boldsymbol{h}_1^T\boldsymbol{A}\boldsymbol{h}_1\sqrt{p_1}s_1 + \boldsymbol{h}_1^T\boldsymbol{A}\boldsymbol{h}_2\sqrt{p_2}s_2 \\ &\quad + \boldsymbol{h}_1^T\boldsymbol{A}\boldsymbol{z}_R + z_1, \end{aligned} \tag{2}$$

$$\begin{aligned} y_2 &= \boldsymbol{h}_2^T\boldsymbol{A}\boldsymbol{h}_2\sqrt{p_2}s_2 + \boldsymbol{h}_2^T\boldsymbol{A}\boldsymbol{h}_1\sqrt{p_1}s_1 \\ &\quad + \boldsymbol{h}_2^T\boldsymbol{A}\boldsymbol{z}_R + z_2, \end{aligned} \tag{3}$$

where z_1 and z_2 are the CSCG noises at S1 and S2 with variances σ_1^2 and σ_2^2, respectively. Ideally as in [4], now S1 and S2 need to cancel out the self-interference terms $\boldsymbol{h}_1^T\boldsymbol{A}\boldsymbol{h}_1\sqrt{p_1}s_1$ from y_1 and $\boldsymbol{h}_2^T\boldsymbol{A}\boldsymbol{h}_2\sqrt{p_2}s_2$ from y_2, respectively. However, here they only have access to an estimated versions of $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$, which can be modeled as $\hat{\boldsymbol{h}_1} = \boldsymbol{h}_1 + \Delta\boldsymbol{h}_1$ and $\hat{\boldsymbol{h}_2} = \boldsymbol{h}_2 + \Delta\boldsymbol{h}_2$, where $\boldsymbol{\Delta}h_1$ and $\boldsymbol{\Delta}h_2$ are the channel estimation errors with the assumption that $\|\boldsymbol{\Delta}h_1\| \le \epsilon_1$ and $\|\boldsymbol{\Delta}h_2\| \le \epsilon_2$, with ϵ_1 and ϵ_2 some small positive constants. Since the channel estimation error is assumed isomorphic, for the convenience of later derivations, we also represent the true channel gains as $\boldsymbol{h}_1 = \hat{\boldsymbol{h}}_1 + \Delta\boldsymbol{h}_1$ and $\boldsymbol{h}_2 = \hat{\boldsymbol{h}}_2 + \Delta\boldsymbol{h}_2$. By substituting $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ into (2) and (3) and using the fact that the amplitude of channel estimation error is usually much smaller than the estimated channel gain, the second order terms of the channel estimation errors can be neglected. Therefore, $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ can be simplified as (4) and (5). The corresponding transmit power at the relay R is given by

$$\begin{aligned} p_R(\boldsymbol{A}) &= \|\boldsymbol{A}\boldsymbol{h}_1\|^2p_1 + \|\boldsymbol{A}\boldsymbol{h}_2\|^2p_2 + \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A})\sigma_R^2 \\ &= p_1\hat{\boldsymbol{h}}_1^H\boldsymbol{A}^H\boldsymbol{A}\hat{\boldsymbol{h}}_1 + p_2\hat{\boldsymbol{h}}_2^H\boldsymbol{A}^H\boldsymbol{A}\hat{\boldsymbol{h}}_2 + \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A})\sigma_R^2 \\ &\quad + 2\Re(p_1\hat{\boldsymbol{h}}_1^H\boldsymbol{A}^H\boldsymbol{A}\Delta\boldsymbol{h}_1) + 2\Re(p_2\hat{\boldsymbol{h}}_2^H\boldsymbol{A}^H\boldsymbol{A}\Delta\boldsymbol{h}_2) \\ &\quad + p_1\Delta\boldsymbol{h}_1^H\boldsymbol{A}^H\boldsymbol{A}\Delta\boldsymbol{h}_1 + p_2\Delta\boldsymbol{h}_2^H\boldsymbol{A}^H\boldsymbol{A}\Delta\boldsymbol{h}_2. \end{aligned} \tag{6}$$

In addition, the SINRs at S1 and S2 are given by (7). In order to guarantee the SINR requirements at both S1 and S2 in the presence of channel uncertainty, the SINRs are desired to be greater than or equal to γ_1 and γ_2, respectively, with all possible estimation errors. Since we assume the estimation errors are bounded, i.e., $\|\Delta\boldsymbol{h}_1\| \le \epsilon_1, \|\Delta\boldsymbol{h}_2\| \le \epsilon_2$, the problem can be formulated as follows.

$$\begin{aligned} \min_{\boldsymbol{A}} \quad & \max_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} p_R \\ \text{s.t.} \quad & \min_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} \mathrm{SINR}_1 \ge \gamma_1 \\ & \min_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} \mathrm{SINR}_2 \ge \gamma_2, \end{aligned}$$

which could be equivalently recast as

$$\mathcal{Q}_1: \quad \min_{\boldsymbol{A},t} \; t \tag{8}$$

$$\text{s.t.} \quad \min_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} \mathrm{SINR}_1 \ge \gamma_1 \tag{9}$$

$$\min_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} \mathrm{SINR}_2 \ge \gamma_2 \tag{10}$$

$$\max_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} p_R \le t. \tag{11}$$

Definition 1: Given the SINR thresholds γ_1 and γ_2, define the system outage as the event that the resulting $\mathrm{SINR}_1 < \gamma_1$ or/and $\mathrm{SINR}_2 < \gamma_2$.

III. SDP Formulation with Rank-one Relaxation

In the optimization problem $\mathcal{Q}_1$, the number of constraints is essentially infinite, since $\Delta\boldsymbol{h}_1$ and $\Delta\boldsymbol{h}_2$ are of continuous complex values. As a result, problem $\mathcal{Q}_1$ cannot be solved directly. Therefore, we need to transform the original problem into an effectively solvable problem of finite dimensions by applying the following theorem [9].

Theorem 1 (S-procedure): Given Hermitian matrices $\boldsymbol{A}_j \in \mathbb{C}^{n\times n}$, vectors $\boldsymbol{b}_j \in \mathbb{C}^n$, and numbers $c_j \in \mathbb{R}$, define the functions $f_j(\boldsymbol{x}) = \boldsymbol{x}^H\boldsymbol{A}_j\boldsymbol{x} + 2\Re(\boldsymbol{b}_j^H\boldsymbol{x}) + c_j$ with $\boldsymbol{x} \in \mathbb{C}^n$, $j = 0, 1, 2$. The following two conditions are equivalent.

1) $f_0(\boldsymbol{x}) \ge 0$ for every $\boldsymbol{x} \in \mathbb{C}^n$ such that $f_1(\boldsymbol{x}) \ge 0$ and $f_2(\boldsymbol{x}) \ge 0$;
2) There exist $\lambda_1, \lambda_2 \ge 0$ such that $\begin{pmatrix} \boldsymbol{A}_0 & \boldsymbol{b}_0 \\ \boldsymbol{b}_0^H & c_0 \end{pmatrix} \succeq \lambda_1\begin{pmatrix} \boldsymbol{A}_1 & \boldsymbol{b}_1 \\ \boldsymbol{b}_1^H & c_1 \end{pmatrix} + \lambda_2\begin{pmatrix} \boldsymbol{A}_2 & \boldsymbol{b}_2 \\ \boldsymbol{b}_2^H & c_2 \end{pmatrix}$.

Some details about S-procedure can be found in [10] and its applications can be found in [6].

Based on the S-procedure, we first need to transform the constraints of problem $\mathcal{Q}_1$ into the quadratic forms in terms of $\Delta\boldsymbol{h}_1$ and $\Delta\boldsymbol{h}_2$. We then transform the constraints into linear matrix inequalities (LMIs). Let us define following notations: $\tilde{\boldsymbol{h}}_i \triangleq \hat{\boldsymbol{h}}_i \otimes \boldsymbol{1}_{M\times 1}$, $\check{\boldsymbol{h}}_i \triangleq \boldsymbol{1}_{M\times 1} \otimes \hat{\boldsymbol{h}}_i$, $\Delta\tilde{\boldsymbol{h}}_i \triangleq \Delta\boldsymbol{h}_i \otimes \boldsymbol{1}_{M\times 1}$,

$$\tilde{y}_1 \approx \underbrace{(\hat{\boldsymbol{h}}_1^T \boldsymbol{A}\Delta\boldsymbol{h}_1 + \Delta\boldsymbol{h}_1^T \boldsymbol{A}\hat{\boldsymbol{h}}_1)\sqrt{p_1}s_1}_{\text{remaining self-interference}} + \underbrace{(\hat{\boldsymbol{h}}_1^T \boldsymbol{A}\hat{\boldsymbol{h}}_2 + \hat{\boldsymbol{h}}_1^T \boldsymbol{A}\Delta\boldsymbol{h}_2 + \Delta\boldsymbol{h}_1^T \boldsymbol{A}\hat{\boldsymbol{h}}_2)\sqrt{p_2}s_2}_{\text{desired signal}} + \underbrace{(\hat{\boldsymbol{h}}_1^T \boldsymbol{A} + \Delta\boldsymbol{h}_1^T \boldsymbol{A})\boldsymbol{z}_R + z_1}_{\text{noise}}, \quad (4)$$

$$\tilde{y}_2 \approx \underbrace{(\hat{\boldsymbol{h}}_2^T \boldsymbol{A}\Delta\boldsymbol{h}_2 + \Delta\boldsymbol{h}_2^T \boldsymbol{A}\hat{\boldsymbol{h}}_2)\sqrt{p_2}s_2}_{\text{remaining self-interference}} + \underbrace{(\hat{\boldsymbol{h}}_2^T \boldsymbol{A}\hat{\boldsymbol{h}}_1 + \hat{\boldsymbol{h}}_2^T \boldsymbol{A}\Delta\boldsymbol{h}_1 + \Delta\boldsymbol{h}_2^T \boldsymbol{A}\hat{\boldsymbol{h}}_1)\sqrt{p_1}s_1}_{\text{desired signal}} + \underbrace{(\hat{\boldsymbol{h}}_2^T \boldsymbol{A} + \Delta\boldsymbol{h}_2^T \boldsymbol{A})\boldsymbol{z}_R + z_2}_{\text{noise}}. \quad (5)$$

$$\mathrm{SINR_i} = \frac{|\hat{\boldsymbol{h}}_i^T \boldsymbol{A}\hat{\boldsymbol{h}}_j + \hat{\boldsymbol{h}}_i^T \boldsymbol{A}\Delta\boldsymbol{h}_j + \Delta\boldsymbol{h}_i^T \boldsymbol{A}\hat{\boldsymbol{h}}_j|^2 p_j}{|\hat{\boldsymbol{h}}_i^T \boldsymbol{A}\Delta\boldsymbol{h}_i + \Delta\boldsymbol{h}_i^T \boldsymbol{A}\hat{\boldsymbol{h}}_i|^2 p_i + \|(\hat{\boldsymbol{h}}_i^T + \Delta\boldsymbol{h}_i^T)\boldsymbol{A}\|^2\sigma_R^2 + \sigma_i^2}, \quad i = 1,2, \quad j = 3-i. \quad (7)$$

and $\Delta\breve{\boldsymbol{h}}_i = \boldsymbol{1}_{M\times 1} \otimes \Delta\boldsymbol{h}_i$, for $i = 1,2$, with $\boldsymbol{1}_{M\times 1}$ an all-one M-dimensional column vector. In addition, $\boldsymbol{a} \triangleq \mathrm{vec}(\boldsymbol{A})$, $\bar{\boldsymbol{A}} \triangleq \boldsymbol{a}\boldsymbol{a}^H$, $\boldsymbol{h} = \mathrm{vec}(\boldsymbol{h}_1, \boldsymbol{h}_2)$, $\Delta\boldsymbol{h} = \mathrm{vec}(\Delta\boldsymbol{h}_1, \Delta\boldsymbol{h}_2)$, $\boldsymbol{G}_1 = [\boldsymbol{I}_M, \boldsymbol{O}_M]$, $\boldsymbol{G}_2 = [\boldsymbol{O}_M, \boldsymbol{I}_M]$, $\boldsymbol{D}_R = \boldsymbol{I}_M \otimes \boldsymbol{1}_{M\times 1}$, and $\boldsymbol{D}_L = \boldsymbol{1}_{M\times 1} \otimes \boldsymbol{I}_M$, with $\boldsymbol{O}_M$ and $\boldsymbol{I}_M$ as the $\boldsymbol{M}\times\boldsymbol{M}$ all-zero and identity matrices respectively. It is easy to see that $\Delta\tilde{\boldsymbol{h}}_i = \boldsymbol{D}_R\boldsymbol{G}_i\Delta\boldsymbol{h}$ and $\Delta\breve{\boldsymbol{h}}_i = \boldsymbol{D}_L\boldsymbol{G}_i\Delta\boldsymbol{h}$, for $i = 1,2$.

Let α_1 be the numerator of SINR_1 (as given in (7)). Note that $\boldsymbol{g}^T\boldsymbol{A}\boldsymbol{h} = [(\boldsymbol{h} \otimes \boldsymbol{1}_{M\times 1}) \odot (\boldsymbol{1}_{M\times 1} \otimes \boldsymbol{g})]^T \mathrm{vec}(\boldsymbol{A})$, we could write α_1 as

$$\alpha_1 = |\big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1 + \Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1 + \tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \boldsymbol{a}|^2 p_2. \quad (12)$$

After some mathematical manipulations α_1 can be rewritten as

$$\alpha_1 = \Delta\boldsymbol{h}^T\boldsymbol{Q_1}\Delta\boldsymbol{h}^* + 2\Re(\boldsymbol{q}_1^H\Delta\boldsymbol{h}^*) + c_1,$$

where

$$\begin{aligned}
\boldsymbol{Q}_1 &= p_2\boldsymbol{G}_2^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_1)\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_2^*)\boldsymbol{D}_L\boldsymbol{G}_1 \\
&\quad + p_2\boldsymbol{G}_1^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_1^*)\boldsymbol{D}_R\boldsymbol{G}_2 \\
&\quad + p_2\boldsymbol{G}_2^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_1)\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_1^*)\boldsymbol{D}_R\boldsymbol{G}_2 \\
&\quad + p_2\boldsymbol{G}_1^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_2^*)\boldsymbol{D}_L\boldsymbol{G}_1, \\
\boldsymbol{q}_1^H &= p_2(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1)^T\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_1^*)\boldsymbol{D}_R\boldsymbol{G}_2 \\
&\quad + p_2\big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_2^*)\boldsymbol{D}_L\boldsymbol{G}_1, \\
c_1 &= p_2\big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T\bar{\boldsymbol{A}}\big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^*,
\end{aligned}$$

Similarly, the denominator of SINR_1, denoted by β_1, can be written as

$$\beta_1 = \Delta\boldsymbol{h}^T\boldsymbol{Q}_2\Delta\boldsymbol{h}^* + 2\Re(\boldsymbol{q}_2^H\Delta\boldsymbol{h}^*) + c_2,$$

where

$$\begin{aligned}
\boldsymbol{Q}_2 &= p_1\boldsymbol{G}_1^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_1)\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_1^*)\boldsymbol{D}_R\boldsymbol{G}_1 \\
&\quad + p_1\boldsymbol{G}_1^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_1)\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_1^*)\boldsymbol{D}_L\boldsymbol{G}_1 \\
&\quad + p_1\boldsymbol{G}_1^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_1)\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_1^*)\boldsymbol{D}_R\boldsymbol{G}_1 \\
&\quad + p_1\boldsymbol{G}_1^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_1)\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_1^*)\boldsymbol{D}_L\boldsymbol{G}_1 \\
&\quad + \sigma_R^2\boldsymbol{G}_1^T\boldsymbol{D}_L^T\boldsymbol{E} \odot \bar{\boldsymbol{A}}\boldsymbol{D}_L\boldsymbol{G}_1,
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{q}_2^H &= \sigma_R^2\breve{\boldsymbol{h}}_1^T\boldsymbol{E} \odot \bar{\boldsymbol{A}}\boldsymbol{D}_L\boldsymbol{G}_1, \\
c_2 &= \sigma_R^2\breve{\boldsymbol{h}}_1^T\boldsymbol{E} \odot \bar{\boldsymbol{A}}\breve{\boldsymbol{h}}_1^* + \sigma_1^2.
\end{aligned}$$

Now, we rewrite constraint (9) as

$$\begin{cases}
\Delta\boldsymbol{h}^T(\boldsymbol{Q}_1 - \gamma_1\boldsymbol{Q}_2)\Delta\boldsymbol{h}^* \\
+2\Re\big((\boldsymbol{q}_1 - \gamma_1\boldsymbol{q}_2)^H\Delta\boldsymbol{h}^*)\big) + c_1 - \gamma_1 c_2 \geq 0 \\
\Delta\boldsymbol{h}^T\boldsymbol{G}_1^H\boldsymbol{G}_1\Delta\boldsymbol{h}^* \leq \epsilon_1^2 \\
\Delta\boldsymbol{h}^T\boldsymbol{G}_2^H\boldsymbol{G}_2\Delta\boldsymbol{h}^* \leq \epsilon_2^2.
\end{cases}$$

According to the S-procedure in Theorem 1, the above quadratic three-inequality system in equivalent to the following LMI:

$$\begin{pmatrix} \boldsymbol{Q}_1 - \gamma_1\boldsymbol{Q}_2 + \lambda_1\boldsymbol{G}_1^H\boldsymbol{G}_1 + \lambda_2\boldsymbol{G}_2^H\boldsymbol{G}_2 & \boldsymbol{q}_1 - \gamma_1\boldsymbol{q}_2 \\ \boldsymbol{q}_1^H - \gamma_1\boldsymbol{q}_2^H & c_1 - \gamma_1 c_2 - \lambda_1\epsilon_1^2 - \lambda_2\epsilon_2^2 \end{pmatrix} \succeq 0$$

Similarly, for SINR_2, we have

$$\begin{aligned}
\alpha_2 &= \Delta\boldsymbol{h}^T\boldsymbol{Q_3}\Delta\boldsymbol{h}^* + 2\Re(\boldsymbol{q}_3^H\Delta\boldsymbol{h}^*) + c_3 \\
\beta_2 &= \Delta\boldsymbol{h}^T\boldsymbol{Q}_4\Delta\boldsymbol{h}^* + 2\Re(\boldsymbol{q}_4^H\Delta\boldsymbol{h}^*) + c_4,
\end{aligned}$$

where

$$\begin{aligned}
\boldsymbol{Q}_3 &= p_1\boldsymbol{G}_1^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_1^*)\boldsymbol{D}_L\boldsymbol{G}_2 \\
&\quad + p_1\boldsymbol{G}_2^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_1)\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_2^*)\boldsymbol{D}_R\boldsymbol{G}_1 \\
&\quad + p_1\boldsymbol{G}_1^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_2^*)\boldsymbol{D}_R\boldsymbol{G}_1 \\
&\quad + p_1\boldsymbol{G}_2^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_1)\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_1^*)\boldsymbol{D}_L\boldsymbol{G}_2 \\
\boldsymbol{q}_3^H &= p_1(\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2)^T\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_2^*)\boldsymbol{D}_R\boldsymbol{G}_1, \\
&\quad + p_1\big(\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2\big)^T\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_1^*)\boldsymbol{D}_L\boldsymbol{G}_2, \\
c_3 &= p_1\big(\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2\big)^T\bar{\boldsymbol{A}}\big(\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2\big)^*,
\end{aligned}$$

and

$$\begin{aligned}
\boldsymbol{Q}_4 &= p_2\boldsymbol{G}_2^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_2^*)\boldsymbol{D}_R\boldsymbol{G}_2 \\
&\quad + p_2\boldsymbol{G}_2^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_2^*)\boldsymbol{D}_L\boldsymbol{G}_2 \\
&\quad + p_2\boldsymbol{G}_2^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}\mathrm{diag}(\breve{\boldsymbol{h}}_2^*)\boldsymbol{D}_R\boldsymbol{G}_2 \\
&\quad + p_2\boldsymbol{G}_2^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}\mathrm{diag}(\tilde{\boldsymbol{h}}_2^*)\boldsymbol{D}_L\boldsymbol{G}_2 \\
&\quad + \sigma_R^2\boldsymbol{G}_2^T\boldsymbol{D}_L^T\boldsymbol{E} \odot \bar{\boldsymbol{A}}\boldsymbol{D}_L\boldsymbol{G}_2, \\
\boldsymbol{q}_4^H &= \sigma_R^2\breve{\boldsymbol{h}}_2^T\boldsymbol{E} \odot \bar{\boldsymbol{A}}\boldsymbol{D}_L\boldsymbol{G}_2, \\
c_4 &= \sigma_R^2\breve{\boldsymbol{h}}_2^T\boldsymbol{E} \odot \bar{\boldsymbol{A}}\breve{\boldsymbol{h}}_2^* + \sigma_2^2.
\end{aligned}$$

In a similar way, constraint (10) could be shown equivalent to

$$\begin{pmatrix} \boldsymbol{Q}_3 - \gamma_2\boldsymbol{Q}_4 + \lambda_3\boldsymbol{G}_1^H\boldsymbol{G}_1 + \lambda_4\boldsymbol{G}_2^H\boldsymbol{G}_2 & \boldsymbol{q}_3 - \gamma_2\boldsymbol{q}_4 \\ \boldsymbol{q}_3^H - \gamma_2\boldsymbol{q}_4^H & c_3 - \gamma_2 c_4 - \lambda_3\epsilon_1^2 - \lambda_4\epsilon_2^2 \end{pmatrix} \succeq 0$$

At last, we address the power constraint in (11). Let $\boldsymbol{K}$ be the commutation matrix such that $\text{vec}(\boldsymbol{A^T}) = \boldsymbol{K}\text{vec}(\boldsymbol{A})$. Then according to (6), p_R can be rewritten as

$$p_R = c_0 + 2\Re(\boldsymbol{q}_0^H \Delta\boldsymbol{h}) + \Delta\boldsymbol{h}^H \boldsymbol{Q}_0 \Delta\boldsymbol{h},$$

where

$$\begin{aligned}
\boldsymbol{Q}_0 &= p_1 \boldsymbol{G}_1^T \boldsymbol{D}_L^T \boldsymbol{E} \odot [\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^T]\boldsymbol{D}_L \boldsymbol{G}_1 \\
&\quad + p_2 \boldsymbol{G}_2^T \boldsymbol{D}_L^T \boldsymbol{E} \odot [\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^T]\boldsymbol{D}_L \boldsymbol{G}_2, \\
\boldsymbol{q}_0^H &= p_1 \breve{\boldsymbol{h}}_1^H \boldsymbol{E} \odot [\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^T]\boldsymbol{D}_L \boldsymbol{G}_1 \\
&\quad + p_2 \breve{\boldsymbol{h}}_2^H \boldsymbol{E} \odot [\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^T]\boldsymbol{D}_L \boldsymbol{G}_2, \\
c_0 &= p_1 \breve{\boldsymbol{h}}_1^T \boldsymbol{E} \odot [\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^T]\breve{\boldsymbol{h}}_1^* \\
&\quad + p_2 \breve{\boldsymbol{h}}_2^T \boldsymbol{E} \odot [\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^T]\breve{\boldsymbol{h}}_1^* + \text{tr}(\bar{\boldsymbol{A}}).
\end{aligned}$$

As such, (11) is equivalent to

$$\begin{pmatrix} -\boldsymbol{Q}_0 + \kappa_1 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \kappa_2 \boldsymbol{G}_2^H \boldsymbol{G}_2 & -\boldsymbol{q}_0 \\ -\boldsymbol{q}_0^H & t - c_0 - \kappa_1 \epsilon_1^2 - \kappa_2 \epsilon_2^2 \end{pmatrix} \succeq 0.$$

Therefore, the original problem $\mathcal{Q}_1$ can be transformed into the following form.

$$\begin{aligned}
\mathcal{Q}_2 : \quad & \min_{\bar{\boldsymbol{A}}, t, \boldsymbol{\lambda}, \boldsymbol{\kappa}} \quad t \\
\text{s.t.} \quad & \begin{pmatrix} \boldsymbol{Q}_1 - \gamma_1 \boldsymbol{Q}_2 + \lambda_1 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \lambda_2 \boldsymbol{G}_2^H \boldsymbol{G}_1 & \boldsymbol{q}_1 - \gamma_1 \boldsymbol{q}_2 \\ \boldsymbol{q}_1^H - \gamma_1 \boldsymbol{q}_2^H & c_1 - \gamma_1 c_2 - \lambda_1 \epsilon_1^2 - \lambda_2 \epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \begin{pmatrix} \boldsymbol{Q}_3 - \gamma_2 \boldsymbol{Q}_4 + \lambda_3 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \lambda_4 \boldsymbol{G}_2^H \boldsymbol{G}_2 & \boldsymbol{q}_3 - \gamma_2 \boldsymbol{q}_4 \\ \boldsymbol{q}_1^H - \gamma_2 \boldsymbol{q}_2^H & c_3 - \gamma_2 c_4 - \lambda_3 \epsilon_1^2 - \lambda_4 \epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \begin{pmatrix} -\boldsymbol{Q}_0 + \kappa_1 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \kappa_2 \boldsymbol{G}_2^H \boldsymbol{G}_2 & -\boldsymbol{q}_0 \\ -\boldsymbol{q}_0^H & t - c_0 - \kappa_1 \epsilon_1^2 - \kappa_2 \epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \text{Rank}(\bar{\boldsymbol{A}}) = 1, \qquad (13)
\end{aligned}$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$, and $\boldsymbol{\kappa} = (\kappa_1, \kappa_2)$. Since constraint (13) is not convex, problem $\mathcal{Q}_2$ is still not a convex problem. Note that, however, if we ignore the rank-one constraint in (13), this problem becomes

$$\begin{aligned}
\mathcal{Q}_3 : \quad & \min_{\bar{\boldsymbol{A}}, t, \boldsymbol{\lambda}, \boldsymbol{\kappa}} \quad t \\
\text{s.t.} \quad & \begin{pmatrix} \boldsymbol{Q}_1 - \gamma_1 \boldsymbol{Q}_2 + \lambda_1 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \lambda_2 \boldsymbol{G}_2^H \boldsymbol{G}_1 & \boldsymbol{q}_1 - \gamma_1 \boldsymbol{q}_2 \\ \boldsymbol{q}_1^H - \gamma_1 \boldsymbol{q}_2^H & c_1 - \gamma_1 c_2 - \lambda_1 \epsilon_1^2 - \lambda_2 \epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \begin{pmatrix} \boldsymbol{Q}_3 - \gamma_2 \boldsymbol{Q}_4 + \lambda_3 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \lambda_4 \boldsymbol{G}_2^H \boldsymbol{G}_2 & \boldsymbol{q}_3 - \gamma_2 \boldsymbol{q}_4 \\ \boldsymbol{q}_1^H - \gamma_2 \boldsymbol{q}_2^H & c_3 - \gamma_2 c_4 - \lambda_3 \epsilon_1^2 - \lambda_4 \epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \begin{pmatrix} -\boldsymbol{Q}_0 + \kappa_1 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \kappa_2 \boldsymbol{G}_2^H \boldsymbol{G}_2 & -\boldsymbol{q}_0 \\ -\boldsymbol{q}_0^H & t - c_0 - \kappa_1 \epsilon_1^2 - \kappa_2 \epsilon_2^2 \end{pmatrix} \succeq 0,
\end{aligned}$$

which is convex and actually a semi-definite programming (SDP) problem. Such a relaxation procedure is called SDP relaxation. In general, the relaxed solution may not be of rank-one. In the next section, we will discuss how to obtain an approximate rank-one solution from the relaxed problem.

IV. Rank-one Approximation and Outage Condition

The SDP problem $\mathcal{Q}_3$ can be efficiently solved by numerical methods, such as the interior point method [11]. If the resulting solution $\bar{\boldsymbol{A}}^*$ is of rank-one, the optimal beamforming matrix at relay $\boldsymbol{A}^*$ can be easily obtained as $\boldsymbol{A}^* = \text{ivec}(\boldsymbol{a}_{opt})$, where $\boldsymbol{a}_{opt}\boldsymbol{a}_{opt}^H = \bar{\boldsymbol{A}}^*$. However, given the SINR thresholds γ_1 and γ_2, problem $\mathcal{Q}_3$ may not be feasible, which leads to outage. In addition, even when the problem $\mathcal{Q}_3$ is feasible, the resulting matrix $\bar{\boldsymbol{A}}^*$ may not be of rank-one. Next, we first propose the rank-one reconstruction approach for the case when $\mathcal{Q}_3$ is feasible.

A. Principle Eigenvector Rank-one Approximation

Since matrix $\bar{\boldsymbol{A}}^*$ is Hermitian, we have $\bar{\boldsymbol{A}}^* = \boldsymbol{U}\boldsymbol{D}\boldsymbol{U}^H$. Let $\boldsymbol{A}^* = \text{ivec}(\boldsymbol{u}_1)$, where $\boldsymbol{u}_1$ is the column vector in $\boldsymbol{U}$ that corresponds to the largest eigenvalue. In general, this approach is suboptimal unless $\text{Rank}(\bar{\boldsymbol{A}}^*) = 1$. However, this approach is easy to implement with reasonably good performance, which will be shown in Section V. In the journal version of this paper, we will further consider the randomization based approach given in [12], which may lead to better averaged performance.

B. Outage Consideration: A Hybrid Approach

Besides the feasibility-caused outage, we have another source of outage if the above principle eigenvector based rank-one approximation leads to a solution that does not satisfy the SINR constraints in (13). This happens more often when the largest eigenvalue is not dominating the others. To reduce the overall outage probability, we propose the following hybrid approach :

1) Solve problem $\mathcal{Q}_3$, if it is feasible, obtain $\boldsymbol{A}^* = \text{ivec}(\boldsymbol{u}_1)$.
2) If problem $\mathcal{Q}_3$ is not feasible, adopt the non-robust approach given in [4].

The hybrid inclusion of the routine in 2) is motivated by the observation that the feasibility of the non-robust formulation is easier to satisfy given the less number of constraints.

V. Simulation Results

In the simulation, we make the following assumptions: 1) All nodes have unit noise power; 2) than power values p_1 and p_2 are 10 W; 3) $\hat{\boldsymbol{h}}_i \sim \mathcal{CN}(0, \boldsymbol{I})$, $\Delta\boldsymbol{h}_i \sim \frac{a_i}{\sqrt{M}} e^{j\boldsymbol{\theta}}$, $i = 1, 2$, where a_i is uniformly distributed in $[0, \epsilon_i]$ and $\boldsymbol{\theta}$ is uniformly distributed over $[0, 2\pi]$; and 4) to set $\epsilon_1 = \epsilon_2$. We compute the outage probability over 1000 channel realizations.

First, we show the relationship between the overall outage probability and the error bound. We compare the hybrid approach against the non-robust approach by assuming that the number of relay antennas is four. As we see in Fig. 2, a significantly smaller outage probability is achieved with the robust approach. In particular, when the error bound ϵ_1 and ϵ_2 are large, the outage probability almost remains the same as the error bound changes. When the error bound decreases to a certain critical point, the outage probability decreases significantly with the error bound.

To further understand this phenomenon, we decompose the outage probability into two parts. One is the probability that problem $\mathcal{Q}_3$ is infeasible, denoted as P_1 and shown by the solid curve with squares in Fig. 3. The other is the conditional outage probability P_2 given that problem $\mathcal{Q}_3$ is feasible, shown by the solid curve in Fig. 3. As we see, P_1 increases when error bound increases, since it is more difficult for the robust approach to find feasible solutions given larger channel uncertainty. When the problem is feasible, the conditional outage probability P_2 is low compared to the non-robust approach. Interestingly, we see that P_2 first increases and then decreases when error bound decreases. The overall outage probability is given as $P_1 + P_2 - P_1 P_2 \approx P_1 + P_2$.

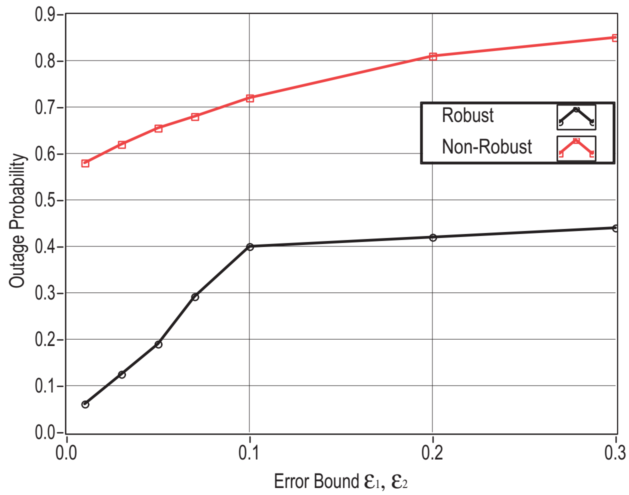

Fig. 2. Outage Probability vs. Error Bound.

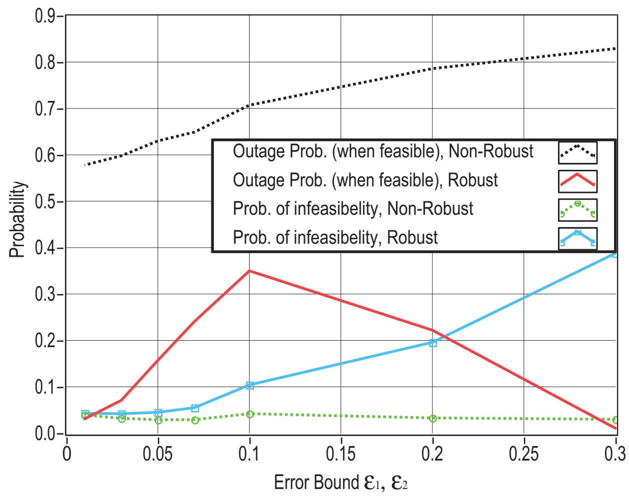

Fig. 3. Decomposed View of Outage Probability.

As we see from Fig. 3, the sum of P_1 and P_2 almost remains constant when the error bound is larger than 0.1. As a result, the overall outage probability almost remains unchanged till the error bound decreases to some critical point, after which the outage probability decreases significantly with the error bound. In our future work, we will further investigate this interesting behavior of P_2 and find out the conditions under which the rank-one solution exists. In addition, in Fig. 4, we show the relationship between the outage probability and the number of relay antennas. The error bound is now fixed to be 0.01. As we see, the outage probabilities for both the robust and non-robust approaches decrease. This is due to the fact that higher spatial dimension offers a higher degree-of-freedom.

VI. Conclusion

In this paper we proposed the robust beamforming approach under realistic channel estimation conditions (i.e., with estimation errors) for a two-way relay system with analog network coding. By using the S-procedure, the original constraints with infinite dimensions are converted to several LMIs and then

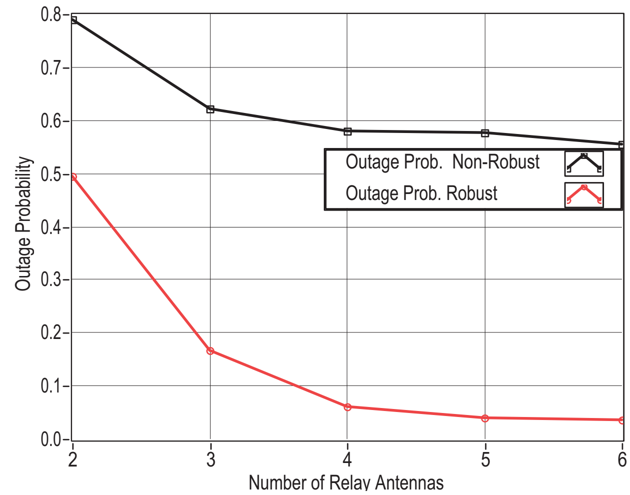

Fig. 4. Outage Probability vs. Number of Relay Antennas.

the relaxed SDP is solved by applying rank-one relaxation. A principle eigenvector based rank-one reconstruction approach is proposed to reconstruct the solution. To reduce the outage probability, we further proposed a hybrid approach that incorporates the non-robust approach when robust problem formulation is infeasible. Simulations are conducted to show that the proposed hybrid approach leads to significant improvement in terms of outage probability over the non-robust one.

References

[1] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog networking coding," Computer Science and Artificial Intelligence Laboratory Technical Report, MIT-CSAIL-TR-2007-012, Feb. 2007.
[2] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Optimal distributed beamforming for two-way relay networks," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1238 – 1250, Mar. 2010.
[3] K. Lee, H. Sung, E. Park, and I. Lee, "Joint optimization for one and two-way MIMO AF multiple-relay systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3671 – 3681, Dec. 2010.
[4] R. Zhang, Y.-C. Liang, C. Choy, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 699–712, Jun. 2009.
[5] M. Zeng, R. Zhang, and S. Cui, "On design of distributed beamforming for two-way relay networks," *IEEE Trans. Signal Process.*, vol. 59, no. 5, pp. 2284–2295, May 2011.
[6] G. Zheng, K. Wong, A. Paulraj, and B. Ottersten, "Robust collaborative relay beamforming," *IEEE Trans. Signal Process.*, vol. 57, no. 8, pp. 3130–3143, 2009.
[7] B. K. Chalise and L. Vandendorpe, "MIMO relay design for multipoint-to-multipoint communications with imperfect channel state information," *IEEE Trans. on Signal Process.*, vol. 57, no. 7, pp. 2785–2796, 2009.
[8] P. Ubaidulla and A. Chockalingam, "MIMO relay precoder optimization in MIMO-relay networks with imperfect CSI," *IEEE Trans. on Signal Process.*, vol. 59, no. 11, pp. 5473–5484, 2011.
[9] D. P. Palomar and Y. C. Eldar, *Convex Optimization in Signal Processing and Communications*. Cambridge University Press, 2009.
[10] A. Beck and Y. Eldar, "Strong duality in nonconvex quadratic optimization with two quadratic constraints," *SIAM J. Opt.*, vol. 17, no. 3, pp. 844–860, 2006.
[11] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2003.
[12] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Trans. Signal Process.*, vol. 54, no. 6, pp. 2239 – 2252, Jun. 2006.