

Smart Scheduling and Feedback Allocation over Non-stationary Wireless Channels

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Abstract—It is well known that opportunistic scheduling algorithms are throughput optimal under dynamic channel and network conditions. However, these algorithms achieve a hypothetical rate region which does not take into account the overhead associated with channel probing and feedback required to obtain the full channel state information at every slot. In this work, we design a joint scheduling and channel probing algorithm by considering the overhead of obtaining the channel state information. We adopt a correlated and non-stationary channel model, which is more realistic than those used in the literature. We use concepts from learning and information theory to accurately track channel variations to minimize the number of channels probed at every slot, while scheduling users to maximize the achievable rate region of the network. Simulation results show that with the proposed algorithm, the network can carry higher user traffic.

I. INTRODUCTION

In wireless networks, the channel conditions are time-varying due to the fading and shadowing. Opportunistic scheduling algorithms take advantage of favorable channel conditions in assigning time slots to users. Optimal scheduling in wireless networks has been extensively studied in the literature under various assumptions. The seminal work by Tassiulas and Ephremides have shown that a simple opportunistic algorithm that schedules the user with the highest queue backlog and transmission rate product at every time slot, can stabilize the network, whenever this is possible [1].

A common assumption in the literature on opportunistic algorithms is that the *exact* and *complete* channel state information, (CSI) of all users is available at every time slot. Hence, these algorithms achieve a *hypothetical* rate region by assuming that full channel state information is available without any channel probing or feedback costs. However, in practice acquiring CSI introduces significant overhead to the network, since CSI is obtained either by probing the channel or via feedback from the users. In current wireless communication standards such as WiMax and LTE there is a feedback channel used to relay CSI from the users to base station. Obviously, this feedback channel is bandlimited and it is impossible to obtain CSI from all users at the same slot.

Another common assumption that does not hold in practice is the wireless channel being independent and identically

distributed (iid), and being governed by a *stationary* stochastic process. The most common assumption is that the channel can be modeled by a stationary Markov chain. The measurement study in [2] shows that the wireless channel exhibits time-correlated and non-stationary behavior.

In this work, we develop a joint scheduling and channel probing algorithm for time-correlated and non-stationary channels. The channel probing is based on Gaussian Process Regression (GPR) technique [3], which is used to learn and track the wireless channel. The scheduling part is based on well known Max-Weight algorithm. The joint algorithm dynamically determines the set of channels that must be probed at every time slot based on the information obtained from the previous channel observations, and then schedules a node based on the obtained CSI and queue states. We show that GPR-based probing works well for realistic, time-correlated and non-stationary wireless channels at significantly lower probing cost.

Our contributions are summarized as follows:

- We use information theoretical concepts to quantify the uncertainty in the channel state under finite and infrequent measurements.
- Based on the work in [4], Gaussian Process Regression learning algorithm is proposed to track the channel evolution.
- A joint scheduling and probing algorithm is proposed in which the subset of users probed at every slot is adaptively selected based on the dynamics of the channel processes.
- We implement a realistic network setting where we simulate High Data Rate (HDR) protocol in CDMA cellular networks, and wireless channel is modeled as time-correlated and non-stationary. We show by numerical analysis that when our proposed algorithm is used the network can carry higher user traffic compared to Max-Weight algorithm with full CSI.

II. RELATED WORKS

In [5], the authors propose a throughput-optimal algorithm when channel distribution is known. In [6], the authors present a joint algorithm for multi-channel system with limited feedback bandwidth. The joint scheduling and probing problem is transformed to multi armed bandit problem in [7]. In [8], channel probing is performed at the beginning of transmission by taking a portion of time slot. Then, the problem of finding optimal joint algorithm is transformed into an optimal stopping time problem and is solved by Markov Decision Process

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(MDP). Aforementioned works assume that the underlying stochastic process of the channel evolves according to a fixed stationary process such as ergodic Markov chain. In practice, such an assumption does not hold most of the time. In addition, the authors in [9] proposed a technique to estimate future values of the fading coefficient of a non-stationary channel. The proposed technique is based on the autoregressive (AR) model with order p . According to AR model, the current CSI of a user can only be determined when p previous CSIs of that user are given. Regarding to a joint scheduling and probing problem, the corresponding user must be probed at every previous p slots to obtain p previous CSIs. However, a joint algorithm does not necessarily probe a user at every time slot. Therefore, the proposed technique is not suitable for a scheduling problem with limited feedback.

There are very few studies which propose a scheduling algorithm for non-stationary channels. In [10], the authors showed that with Max-Weight algorithm the average queue sizes increases exponentially with the number of users. It was assumed that channel state information of each user is available at the scheduler at every time slot.

III. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a cellular system with a single base station transmitting to N users. Let \mathcal{N} denote the set of users in the cell. Time is slotted, $t \in \{0, 1, 2, \dots\}$, and wireless channel between the base station and a mobile user is modeled as a time-correlated fading process. The gain of the channel is constant over the duration of a time slot but varies between slots. Let $C_n(t)$ denote CSI of user n at time slot t . $C_n(t)$ is a random process which may or may not have a stationary probability distribution. Let $c_n(t)$ represent the realization of $C_n(t)$ at time t , $n \in \{1, 2, \dots, N\}$. In the rest of the paper, we use channel and user interchangeably.

Let $\mu_n(c_n(t))$, or simply $\mu_n(t)$, denote the transmission rate of user n which depends on CSI of that user, and is bounded as $\mu_{min} < \mu_n(t) < \mu_{max}$. We assume that at each time slot at most one user can be scheduled to receive data from the base station. The base station transmits to users at fixed power, so transmission rate of each user only depends on $c_n(t)$.

Let $a_n(t)$ be the amount of data (bits or packets) arriving into the queue of user n at time slot t . We assume that $a_n(t)$ is a stationary process and it is independent across users and time slots. We denote the arrival rate vector as $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_N)$, where $\lambda_n = \mathbb{E}[a_n(t)]$. Let $\mathbf{q}(t) = (q_1(t), q_2(t), \dots, q_N(t))$ denote the vector of queue sizes, where $q_n(t)$ is the queue length of user n at time slot t .

Definition 1: A queue is strongly stable if

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}(q_n(t)) < \infty \quad (1)$$

Moreover, if every queue in the network is stable then the network is called stable.

The operation of the system is as follows. At the beginning of a time slot, CSI of a subset of users is obtained by the base station. Then, the base station schedules a single user

out of this subset for transmission. Here, the only overhead we take into account is the channel bandwidth and the time used for obtaining CSI. We consider *dynamic* feedback channel allocation as follows:

Dynamic feedback channel allocation model: According to this model, there is no dedicated feedback channel, and CSI is relayed over the data channel. Hence, depending on the needs of the algorithm CSI from varying number of users can be obtained. We quantify the overhead of obtaining the CSI of a single user in terms of a time fraction of the time slot. This time duration may include the time spent for pilot signal transmission, measurement of the signal strength of pilot signal and the transmission of CSI to the base station. Assume that β fraction of the time slot is consumed to obtain CSI from a single user. Hence, only $(1 - m\beta) \times T_s$ seconds are available for data transmission when m users are probed. The amount of data that can be transmitted by user n is given by,

$$d_n(t) = (1 - m\beta)T_s \times \mu_n(t). \quad (2)$$

The joint policy π selects the triplet (n, m, S_m) under this model at each time slot t , where n is the scheduled user, S_m is the set of probed users and m is the number of users in S_m . We assume that the scheduled user at slot t is selected among the users probed, i.e., $n \in S_m$. Given $\pi = (n, m, S_m)$, n is determined according to Max-Weight rule, i.e.,

$$n = \operatorname{argmax}_{i \in S_m} q_i(t) d_i(t). \quad (3)$$

Let \mathcal{F} be the set of feasible policies at a given time slot and $\pi \in \mathcal{F}$.

The amount of data that is transmitted by user j at time slot t under the joint scheduling and probing policy π , is given as follows,

$$r_j(\pi, t) = \begin{cases} d_j(t) & ; \text{ if user } j = n \\ 0 & ; \text{ otherwise} \end{cases} \quad (4)$$

The dynamics of the queue of user n under scheduling policy π , is,

$$q_n(t+1) = \max(q_n(t) + a_n(t) - r_n(\pi, t))^+. \quad (5)$$

where $(x)^+ = \max(x, 0)$.

A. Problem Formulation

We present the following definitions before discussing the problem formulation.

B. Hypothetical and Functional Rate Regions

The *achievable rate region* (shortly, rate region) of a network is defined as the closure of the set of all arrival rate vectors λ for which there exists an appropriate scheduling policy that stabilizes the network.

Definition 2: Λ_{un} is the hypothetical rate region where full CSI is available (e.g. by an Oracle) without any channel probing or feedback costs.

Definition 3: Λ_{full} is the achievable rate region when probing cost is taken into account and when all users' channels

are probed at every time slot according to *dynamic* feedback model.

Definition 4: Λ is the rate region under *dynamic* feedback model when CSI from a subset of users is available.

C. Optimization Given the Steady-state Channel Distribution

Our aim is to find a joint scheduling and channel probing policy that stabilizes the network for a given set of arrival rates within achievable rate region Λ by dynamically determining a subset of channels probed, and by scheduling a user from this subset at every time slot. Given the queue state $\mathbf{q}(t)$, we consider the following optimization problem:

$$\max_{\pi \in \mathcal{F}} \left\{ \mathbb{E} \left[\sum_{n=1}^N q_n(t) r_n(\pi, t) | \mathbf{q}(t) \right] \right\}, \quad (6)$$

D. Tracking the Instantaneous Channel States

In practice, it is not possible to accurately determine the exact channel distributions a priori to system operation. Hence, we propose to use a learning algorithm to track the channel evolution. Let $\hat{c}_n(t)$ denote the estimated CSI of user n at the beginning of time t . Let $\hat{\mu}_n(t)$ denote the estimated transmission rate of user n at time t . One can replace the actual rates $\mu_n(t)$ by $\hat{\mu}_n(t)$ to obtain a new set of policies $\hat{\pi} = (\hat{\mu}, \hat{m}, \hat{S}_m)$ according to *dynamic* feedback model. Also let $\hat{d}_n(t)$, denote the amount of data that can be transmitted by user n by using $\hat{c}_n(t)$ at time t . Similarly, the estimated service rate $\hat{r}_n(\hat{\pi}, t)$ is defined according to (4) by replacing $d_n(t)$ with $\hat{d}_n(t)$.

The quality of the estimate of an instantaneous channel state depends on which users are probed at each slot, i.e., \hat{S}_m . Here, we design a joint algorithm that takes past observations of the channels as an input and determines a subset of users to be probed at time t so that the channel estimation error is minimized and the rate region is maximized.

E. Multi-objective Dynamic Network Control

Note that channel estimation is inherently error-prone. The degree of uncertainty in the estimate of the current channel state depends on the previous channel observations, and the dynamics of the channel. In this context, we define *information of an unexplored channel* as the uncertainty in the channel state given its past observations. This information can be exactly quantified by using the entropy definition given by Shannon. Accordingly, the scalar quantity $I_n(t)$ denotes the *information* of channel state of user n at the beginning of time slot t given past observations of the channel. For instance, the information about a channel whose state was observed recently and many times before is less than the channel which has not been probed for a long time, since the uncertainty in the state of the latter is higher.

Hence, we have two objectives. First one is to schedule users so that stability of the network is preserved. The second closely related objective is to probe users to acquire as much information about their current channel state as possible.

- objective 1: $\max \sum_{n=1}^N q_n(t) \hat{d}_n(t)$

- objective 2: $\max \sum_{n=1}^N I_n(t)$

We seek a joint feasible policy $\hat{\pi}$ which determines a subset of users probed by considering both objectives, and schedules a user out of this subset according to Max-Weight algorithm. The most common approach to find the solution of multi-objective optimization problems is the weighted sum method. The problem under *dynamic* feedback model is given with a constraint which ensures at most M channels are probed at a given time slot:

Problem :

$$\begin{aligned} \max_{\hat{\pi} \in \mathcal{F}} \quad & \sum_{n=1}^N \alpha_1 q_n(t) \hat{r}_n(\hat{\pi}, t) + \alpha_2 I_n(t) \\ \text{s.t.} \quad & m \leq M, \end{aligned} \quad (7)$$

Note that the scheduling and probing decision depends not only on the queue sizes and the estimated channel rates as in the original Max-Weight algorithm, but also on the uncertainty in each channel state given its past observations. Also, (7) exhibits the well-known ‘‘Exploration vs. Exploitation’’ trade-off, since the first term in the summation aims to stabilize the network while the second term aims to maximize the information collected about the channel states. In the following sections, we deal with a modified version of this problem, where we divide the objective function in (7) by α_1 , and define a single weight $\xi = \frac{\alpha_2}{\alpha_1}$. Note that when ξ is tuned to higher (lower) values, the channels are probed more (less) frequently.

IV. ESTIMATION OF CSI WITH GPR

The problems given in (7) involves estimating $\hat{d}_n(t)$ from a set of past channel observations. The problem of predicting or forecasting the value of a variable from observations of other dependent variables is called regression. There is a plethora of work for carrying out regression analysis. In this work, we employ Gaussian Process Regression (GPR) as the technique for channel estimation. Before explaining how channel state is estimated with GPR in detail, we first give the main reasons behind this choice.

- One of the well known methods is autoregressive (AR) model-based techniques or linear regression. AR is parametric, in that the channel function is defined in terms of a finite number of unknown parameters. However, determining these parameters is a difficult task especially when collecting data is costly and the function varies over time. GPR is a nonparametric regression method model. Thus, it can offer a more flexible framework for unknown nonlinearities.
- In contrast to other regression models, GPR provides a simple way to measure the uncertainty in the estimation for any given any set of CSI observations. AR model is lack of providing an analytical way to measure the uncertainty of the estimation which is important for our scheduling algorithm and we will mention next.
- The most attractive reason is that GPR can give decisions with only using the most recent channel observations. This is especially important for non-stationary channels,

since previous channel observations may become outdated and may not give much information about current condition.

Let $\mathcal{D}_n(t) = (\mathbf{c}_n, \boldsymbol{\tau}_n)$ denote the set of observations for channel n at the beginning of time slot t , where $\mathbf{c}_n = \{c_n^1, c_n^2, \dots, c_n^w\}$ denotes the set of latest w CSI values taken at times, $\boldsymbol{\tau}_n = \{\tau_n^1, \tau_n^2, \dots, \tau_n^w\}$, and $\tau_n^i < t, \forall \tau_n^i \in \boldsymbol{\tau}_n, i \in \{1, 2, \dots, w\}$. We use GPR to predict the value of CSI, i.e., $\hat{c}_n(t)$ at the beginning of time slot t , given $\mathcal{D}_n(t)$.

Let $p(c_n(t)|t, \mathcal{D}_n(t))$ be a posterior distribution of channel n . According to GPR, a posterior distribution is Gaussian with mean $\hat{c}_n(t)$ and variance $v_n(t)$. Specifically, Gaussian process is specified by the kernel function, $k_n(\tau_n^i, \tau_n^j)$, that describes the correlation of channel n between two of its measurements taken at times τ_n^i and τ_n^j . It is possible to choose any positive definite kernel function. However, the most widely used is the squared exponential, i.e., Gaussian, kernel:

$$k_n(\tau_n^i, \tau_n^j) = \exp \left[-\frac{1}{2}(\tau_n^i - \tau_n^j)^2 \right]. \quad (8)$$

Given $\mathcal{D}_n(t)$, $\hat{c}_n(t)$ and variance $v_n(t)$ are determined as follows:

$$\hat{c}_n(t) = \mathbf{k}_n^T(t) \mathbf{K}_n^{-1} \mathbf{c}_n, \quad (9)$$

$$v_n(t) = k_n(t, t) - \mathbf{k}_n^T(t) \mathbf{K}_n^{-1} \mathbf{k}_n(t), \quad (10)$$

where \mathbf{K}_n is a $w \times w$ matrix composed of elements $k_n(\tau_n^i, \tau_n^j)$ for $1 \leq i, j \leq w$ and $\mathbf{k}_n(t)$ is a vector with elements $k(\tau_n^i, t)$ for $\forall \tau_n^i \in \boldsymbol{\tau}_n$. Hence, the network scheduler can easily predict the CSI of users at time t by using (9). Furthermore, the variance $v_n(t)$ is used to measure the level of uncertainty in the estimations, i.e., $I_n(t)$ as discussed next.

Recall that the **entropy** of a random variable A is defined as $H(A) = \sum_s p_s \log_s(\frac{1}{p_s})$, where $p(\cdot)$ is the probability distribution function of A . In our context, the current realization of CSI, i.e., $c_n(t)$, is a random variable. Accordingly, let $H_n^0(c_n(t)|t, \mathcal{D}_n(t))$ and $H_n^1(c_n(t)|t, \mathcal{D}_n(t))$ denote the entropy of the random variable $c_n(t)$ before and after the probing, respectively when $\mathcal{D}_n(t)$ is given. If channel n is probed at time t , then $H_n^1(c_n(t)|t, \mathcal{D}_n(t))$ will be zero since the channel state is known exactly. Otherwise, the uncertainty increases, i.e., $H_n^1(c_n(t)|t, \mathcal{D}_n(t)) > H_n^0(c_n(t)|t, \mathcal{D}_n(t))$. Hence, the information acquired by probing channel n is the reduction in its uncertainty, which is simply the difference between its entropies before and after the probing:

$$I_n(t) = H_n^0(c_n(t)|t, \mathcal{D}_n(t)) - H_n^1(c_n(t)|t, \mathcal{D}_n(t)).$$

The following Proposition is similar to the one given in [4], and establishes that information obtained by probing a channel is equal to the variance of the estimate of the state of that channel.

Proposition 5: Given $\mathcal{D}_n(t), \forall n = 1, \dots, N$, finding the channel that has the highest information at time slot t is equal to finding the channel which has the highest variance at that time slot, i.e.,

$$i^* = \operatorname{argmax}_{n \in \mathcal{N}} I_n(t) = \operatorname{argmax}_{n \in \mathcal{N}} v_n(t). \quad (11)$$

Proof: Since $H_n^1(c_n(t)|t, \mathcal{D}_n(t)) = 0$ after probing, $I_n(t)$ is simply

$$I_n(t) = H_n^0(c_n(t)|t, \mathcal{D}_n(t)), \quad (12)$$

Note that according to GPR a posterior distribution of state of channel given \mathcal{D}_n is

$$p(c_n(t)|t, \mathcal{D}_n) \sim \mathcal{N}(\hat{c}_n(t); v_n(t)). \quad (13)$$

Then, the entropy of this Gaussian distribution is given by,

$$H_n^0(c_n(t)|t, \mathcal{D}_n) = \frac{1}{2} \log(2\pi e v_n(t)). \quad (14)$$

Hence,

$$i^* = \operatorname{argmax}_{n \in \mathcal{N}} I_n(t) = \operatorname{argmax}_{n \in \mathcal{N}} v_n(t).$$

■

V. JOINT SCHEDULING AND PROBING ALGORITHMS

Here, we define an algorithm for solving problem (7) when $\hat{c}_n(t)$ and $v_n(t)$ are calculated as described in the previous section.

A. Joint Algorithm Under dynamic feedback model

For given, $M, \xi, \mathbf{q}(t), \beta$, and $\hat{c}_n(t)$ and $v_n(t)$ determined by GPR for each user at every time slot t , Algorithm gives $\hat{\pi}_1^* = (\hat{n}^*, \hat{m}^*, \hat{S}_{\hat{m}^*}^*)$:

Algorithm :

(1) probing decision:

For each value of $m, m = \{1, 2, \dots, M\}$, the scheduler calculates the following weights for ever user n ,

$$J_m^n \triangleq q_n(t) \hat{d}_n(t) + \xi I_n(t).$$

Then, the scheduler sorts J_m^n in a descending order and sums the first m weights. The maximum of the sums is the maximum of (7). Then, the corresponding m and the first m users in the order gives \hat{m}^* and $\hat{S}_{\hat{m}^*}^*$, respectively.

(2) scheduling decision:

The base station acquires CSI of each user in $\hat{S}_{\hat{m}^*}^*$ and user $n^* \in \hat{S}_{\hat{m}^*}^*$ is scheduled to transmit,

$$\hat{n}^* = \operatorname{argmax}_{n \in \hat{S}_{\hat{m}^*}^*} q_n(t) d_n(t).$$

Proposition 6: Algorithm solves (7).

Proof: The proof is straightforward and it is omitted here due to lack of space. ■

VI. NUMERICAL ANALYSIS

In our simulations, we model a single cell CDMA downlink transmission utilizing high data rate (HDR). The base station serves keeps a separate queue for each user. Time is slotted with length $T_s = 1.67$ ms as defined in HDR specifications. Packets arrive at each slot according to Bernoulli distribution. For all simulations, the wireless channel is modeled as *correlated* Rayleigh fading according to Jakes's model. Each user

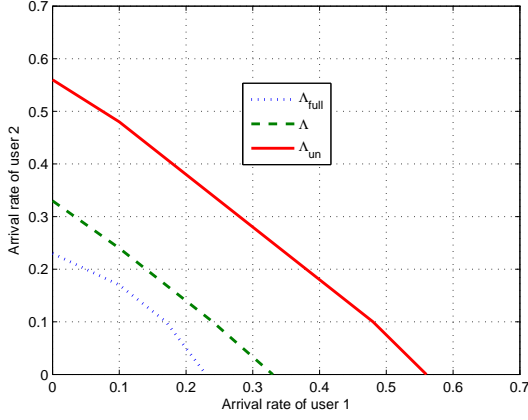


Fig. 1: Rate regions under dynamic feedback model.

has different Doppler frequency roughly characterizing how fast its channel changes. The sampling rate for the simulations is 600Hz which also corresponds to the slot size of HDR. Finally, the channel process is non-stationary, i.e, the mean of the channel gain changes over time. The transmission rate and the number of bits of a user transmits is given as,

$$\begin{aligned}\mu_n(t) &= \text{BW} \log_2(1 + \text{SNR} \times c_n(t)), \\ R_n(t) &= T_s \times \mu_n(t),\end{aligned}$$

where BW is the channel bandwidth set to $\text{BW} = 1.25$ MHz. The base station has power control to set Signal-to-Noise-Ratio $\text{SNR} = 10$ dB.

In the first simulation, we demonstrate the rate region of both algorithms. There are only two users and the probing cost is $\beta = 0.3$. The arrival process for each user is again assumed to be Bernoulli with a packet size of 631 Bytes. As depicted in Figure 1, Λ_{un} represents the hypothetical rate region which is obtained when Max-Weight with full CSI is used for scheduling. The boundary of this region cannot be achieved in practice since β is never zero. On the other extreme, when all channels are probed at every slot without neglecting the cost of probing, we obtain a rate region given as Λ_{full} . Meanwhile, Algorithm achieves the rate region Λ . Clearly, by predicting the channel states by employing Algorithm, we can increase the achievable rate region beyond Λ_{full} . This is because by reducing the number of channels probed at every slot, we can use a larger portion of time slot for transmission of data.

Next, we show the performance of Algorithm when there are 20 users in the network. The size of a packet is set to 128 bytes which corresponds to the size of an HDR packet. Figure 2 depicts the sum of the queue lengths vs. the overall arrival rate when $\beta = 0.02$. Clearly, as the overall arrival rate exceeds 10 packets/slot queue sizes suddenly increase within full CSI case and the network becomes unstable. However, Algorithm improves over Max Weight with full CSI by supporting the overall arrival rate of up to 12 packets/slot. Therefore, the proposed algorithm can achieve larger rate region.

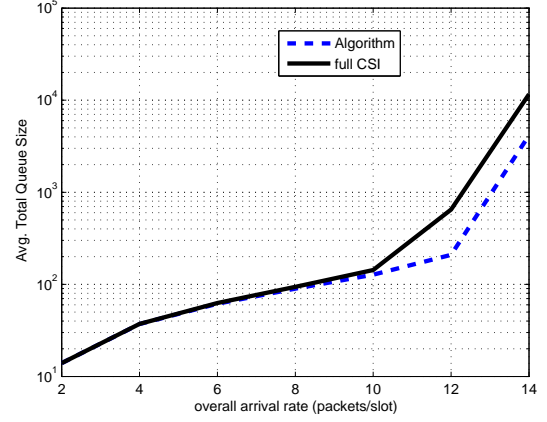


Fig. 2: Average total queue sizes vs. overall mean arrival rate.

VII. CONCLUSION

We have developed joint scheduling channel probing algorithms for time-correlated and stationary/non-stationary wireless channels. The proposed algorithm has been designed for the channel probing model where the acquiring CSI of a user requires β fraction of the time slot. The proposed algorithm first decides the set of channels that must be probed at the beginning of each time slot. The set of channels is determined by considering not only the queue sizes and estimated transmission rate but also the information on each channel. We apply Gaussian Process technique to predict CSI at each time slot based on the previous actual CSI observed. In simulation results, we show that by applying GPR with the proposed algorithm, the network can carry higher user traffic.

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