Two-tier Cellular Random Network Planning for Minimum Deployment Cost

Prodromos-Vasileios Mekikis*, Elli Kartsakli*, Angelos Antonopoulos†, Aris S. Lalos*,
Luis Alonso*, and Christos Verikoukis†
*Signal Theory and Communications Dept., Technical University of Catalonia, Barcelona, Spain
†Telecommunications Technological Centre of Catalonia (CTTC), Castelldefels, Barcelona, Spain
e-mail:{vmekikis, ellik, aristeidis.lalos, luisg}@tsc.upc.edu, {aantonopoulos, cveri}@cttc.es

***Abstract*—Random dense deployment of heterogeneous networks (HetNets), consisting of macro base stations (BS) and small cells (SC), can provide higher quality of service (QoS) while increasing the energy efficiency of the cellular network. In addition, it is possible to achieve lower deployment cost and, therefore, maximize the benefits for the network providers. In this paper, we propose a novel method to determine the minimum deployment cost of a two-tier heterogeneous cellular network using random deployment. After deriving the coverage probability of the two-tier deployment by using stochastic geometry tools, we identify the tier intensities that provide the minimum deployment cost for a given coverage probability. Extensive simulations verify the existence of a unique set of intensities for different coverage constraints.**

***Index Terms*—Heterogeneous networks, Random deployment, Coverage probability, Stochastic geometry, Cost minimization**

I. Introduction

Nowadays that there is an explosion of data traffic, spectral efficiency is a quality of service (QoS) parameter that needs the attention of network providers. Although increasing the density of macro base stations (BS) could provide higher spectral efficiency, it also increases the inter-cell interference that vastly limits the network coverage [1][2]. An alternative approach is the utilization of heterogeneous networks (HetNets) [3], a new paradigm shift in cellular networks technology, which suggest the coexistence, in an area, of multiple network tiers which are different in terms of transmission power, coverage range and data rates [4]. By employing, for example, a combination of macro BSs and small cells (SC), network providers can achieve higher spectral efficiency, higher coverage range and, potentially, lower deployment cost. Network planning is dependent on these three parameters and efficient cell deployment is one of the most crucial problems of cellular network planning.

For many years, the Wyner model [5], which is a one-dimensional downlink model, and other deterministic approaches were used in the network planning, but they were not accurate enough for the inter-cell interference modeling [6]. To overcome these limitations, stochastic geometry [7] is a mathematical tool that has been recently introduced to model wireless networks and analyze their performance. By representing the locations of the wireless nodes with a homogeneous Poisson Point Process (PPP), we can achieve a more realistic interference model [8], in contrast to the classical grid placement of the cells, which is an ideal scheme, not feasible in real life.

Moreover, a deterministic BS topology that requires careful placement of the cells implies big expenses for the network providers. Several works in the literature focus on efficient cell deployment and provide solutions that promise lower annual capital expenditure (CAPEX) and operational expenditure (OPEX). In [9], a hierarchical optimization planning method is utilized for the cost minimization, subject to network coverage and traffic load. The authors suggest a planned network with a fixed placement of cells. In [10], the cost of a self-deployed network with a throughput requirement subject to a coverage constraint is examined and the results are compared to the cost of a traditionally planned network. Both approaches provide efficient solutions, but they do not take under consideration the randomness of the cell deployment to create a more realistic scenario.

During the last years, many papers have studied the coverage probability of randomly deployed cellular networks with the aid of stochastic geometry [8], [11] for single-tier networks. In [8], the authors examine the coverage and mean data rate in cellular networks, but their analysis is based on macro BSs and the model can not be applied for low power tiers with higher intensities. Multiple-tier network scenarios have been also studied in [12], where the authors focus on making the model more realistic regarding the load and network traffic. Considering that the QoS and, more specifically, the network coverage is a crucial factor for the network performance, it should be jointly considered along with the low cost deployment.

In this paper, we provide a theoretical framework for finding the minimum deployment cost in a two-tier network for a given QoS. With the suggested method, we can indeed reach a minimum cost for all the different deployment combinations that satisfy a given constraint in the probability of coverage. In particular, our contribution is threefold:

1) First, we derive the coverage probability of a two-tier network consisting of both macro BSs and SCs. For the single-tier probability of coverage we use stochastic geometry by following the mathematical approach of [8].
2) Second, to overcome the limitations that occur by employing the approach adopted in [8] (i.e., the closed

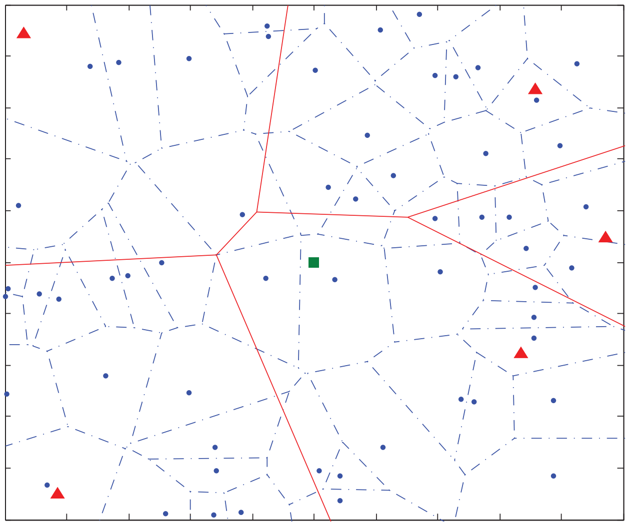


Fig. 1: Voronoi tessellation of the macro BSs PPP, shown in red triangles, and the SCs PPP denoted by the blue dots. The UE under study is located in the center of the network denoted by the green square.

form solution ends up in false values under certain conditions), we evaluate the single integral expressions by using the Gauss quadrature method of [13].

3) Finally, we propose an iterative scheme that extensively searches for a set of minimum intensities, according to a cost model and for a specified level of coverage.

The remaining part of the paper is organized as follows. In section II, we describe the system model. The coverage probability of the two-tier network is analyzed in section III. The cost model and the description of the proposed algorithm for the minimum cost are provided in section IV. The numerical results are presented in section V. Finally, section VI concludes the paper.

II. System Model

We consider a two-tier network deployed in a general flat square territory as the one shown in Fig. 1, consisting of n macro BSs, m SCs and l user equipments (UE).

Both network tiers are assumed to be on the same euclidean plane, represented by two independent homogeneous PPPs. In this way, the cells can be modeled as a set of independently and randomly located points on the plane. The macro BSs are described by the homogeneous PPP $\Phi_{BS} = \{x_1, x_2, \ldots, x_n\}$ where x_i describes the location of the i^{th} macro BS. Φ_{BS} has an intensity λ_{BS} which corresponds to the average number of points per area unit. Accordingly, the SC tier is represented by a homogeneous PPP $\Phi_{SC} = \{y_1, y_2, \ldots, y_m\}$, where y_j describes the location of the j^{th} SC. The intensity of this point process is denoted by λ_{SC}. Finally, the downlink coverage is evaluated at a reference UE that is represented as a single point in the flat square area. Without loss of generality, the location of the UE point will be the origin (i.e., the center of the area), since conditioning on a point at the origin does not affect the statistical properties of the coexisting PPP [14].

In Fig. 1, the topology of the two-tier downlink network is illustrated with a superposition of two Voronoi tessellations[1]. The areas defined by the blue dotted lines correspond to the Voronoi cells of the SCs, while the areas defined by the red solid lines correspond to the macro BS cells. The UE will be associated to either the macro BS or SC of the respective Voronoi cell in which it falls in. However, in our case, we propose a model which is independent of the tier that the UE will eventually associate, since we need to know merely if the desired signal is able to satisfy the UE rate requirements. Furthermore, the two tiers operate in different frequency bands, so that there is no interference from the macro BSs to the SCs and vice versa. It should also be mentioned that network operates under saturation conditions (i.e., all nodes in the network always have a packet to be transmitted).

The average received power at the UE located in a distance R_{BS} from the i^{th} BS is denoted by $P_{UE}^{BS_i} = P_{BS} h R_{BS}^{-\alpha}$, where α is the path loss exponent, P_{BS} is the transmit power of the macro BS and h is the power fast fading coefficient. We assume that $P_{BS_i} = P_{BS}\ \forall\ i$. Also, $\sqrt{h}$ is Rayleigh distributed and, as a result, h is modeled as an exponentially distributed random variable with mean value μ. The Rayleigh fading environment is considered suitable for modeling fast fading in urban environments [15]. Similarly, the received power at the UE located in a distance R_{SC} from the j^{th} SC is given by $P_{UE}^{SC_j} = P_{SC} h R_{SC}^{-\alpha}$, where P_{SC} is the transmit power of the SC. Again, we assume that $P_{SC_i} = P_{SC}\ \forall\ i$. Finally, an additive and constant thermal noise power (N) is assumed that is generated in the receiver.

III. Two-tier Coverage Probability

In this section, we derive the probability that a user is in the coverage of any of the two tiers that exist in the area. This is a necessary step towards fulfilling the goal of minimizing the deployment cost, since the cost model is directly connected with the tier intensities and, consequently, the coverage probability.

A UE can successfully decode the received signal when the signal to interference plus noise ratio (SINR) is higher than a target SINR denoted by γ, which depends on the application requirements of the UE. The interference is the sum of the power received by the UE due to the transmissions of other cells of the same tier, since the tiers operate in different bands.

According to the system model described in section II, the SINR of a UE located at the origin and associated with the macro BS is given by:

$$\mathrm{SINR}_{BS} = \frac{P_{BS} h x_0^{-\alpha}}{\sum_{x_i \in \Phi / x_0} P_{BS} h \|x_i\|^{-\alpha} + N} \tag{1}$$

[1]The Voronoi tessellation is a decomposition of the space into cells that are obtained from the intersection of half-spaces.

whereas the respective expression for the SC associated UE is:

$$\text{SINR}_{SC} = \frac{P_{SC} h y_0^{-\alpha}}{\sum_{y_j \in \Phi / y_0} P_{SC} h \|y_j\|^{-\alpha} + N} \tag{2}$$

where x_0, y_0 denote the respective locations of the BS or SC cells which the user is associated with, and $\|.\|$ denotes the euclidean norm. The summations in the denominators of Eq.(1) and Eq.(2) correspond to the interference caused by all cells except for the associated one.

In a two-tier network, there are four different possibilities to determine whether the UE can successfully decode the received signal or not. In particular, there is: i) the possibility that both SINRs measured by the UE from the two tiers are over the threshold, ii) the possibility that only the measured SINR of the macro BS tier is over the threshold, iii) the possibility that only the measured SINR of the SC tier is over the threshold, and iv) the possibility that none of the two tiers is able to provide connectivity to the UE. The first three possibilities correspond to the coverage probability of a two-tier network, which can be written as:

$$\begin{aligned} Pr_{\text{cov}} &= Pr(\max(\text{SINR}_{BS}, \text{SINR}_{SC}) > \gamma) \\ &= 1 - Pr(\max(\text{SINR}_{BS}, \text{SINR}_{SC}) < \gamma) \\ &= 1 - Pr(\text{SINR}_{BS} < \gamma, \text{SINR}_{SC} < \gamma) \end{aligned} \tag{3}$$

Since the events of the probability in Eq.(3) are independent and not mutually exclusive, we can write the coverage probability as:

$$Pr_{\text{cov}} = 1 - Pr(\text{SINR}_{BS} < \gamma) Pr(\text{SINR}_{SC} < \gamma). \tag{4}$$

Eq.(4) suggests that the coverage probability of a two-tier network depends exclusively on the coverage probability of each individual tier.

Following the approach of [8], the probability of coverage for a single-tier network is given by:

$$\begin{aligned} Pr(\text{SINR} < \gamma) &= 1 - Pr(\text{SINR} > \gamma) \\ &= 1 - \pi\lambda \int_0^\infty \mathrm{e}^{-\pi\lambda\upsilon(1+\rho(\gamma,\alpha)) - \frac{\mu\gamma N \upsilon^{\alpha/2}}{P}} \,\mathrm{d}\upsilon \end{aligned} \tag{5}$$

where $\rho(\gamma, \alpha) = \gamma^{2/\alpha} \int_{\gamma^{-2/\alpha}}^{\infty} \frac{1}{1+\upsilon^{\alpha/2}} \,\mathrm{d}\upsilon$, γ is the threshold, N denotes the thermal noise power, α corresponds to the path loss exponent, λ represents the intensity of the PPP, P denotes the transmit power of the cell, μ is the mean value of the power fast fading coefficient h and υ denotes the square of the distance from the associated cell.

Setting $\alpha = 4$, which is a frequently used value for the considered dense urban environment, Eq.(5) becomes an integral of the form:

$$\int_0^\infty \mathrm{e}^{-ax^2} \mathrm{e}^{-bx} \,\mathrm{d}x. \tag{6}$$

In spite of the fact that there is a closed form solution for the integral in Eq.(6), which is:

$$\int_0^\infty \mathrm{e}^{-ax^2} \mathrm{e}^{-bx} \,\mathrm{d}x = \sqrt{\frac{\pi}{b}} \exp\left(\frac{a^2}{4b}\right) Q\left(\frac{a}{\sqrt{2b}}\right), \tag{7}$$

we have noticed that it does not behave properly for high λ values, which are essential in HetNets for the low power tiers. In this situation, the Q-function[2] takes very high values while the exponential term takes very low values, leading to a zero coverage[3], which is not correct. Hence, address this issue we propose an alternative way to handle Eq.(5).

By using a modified Gauss-Hermite quadrature [13], we can accurately approximate the value of the integral, which is given by:

$$\int_0^\infty \mathrm{e}^{-x^2} f(x) \,\mathrm{d}x = \sum_{i=1}^{n} w_i f(x_i) \tag{8}$$

where w_i and x_i are the weights and the roots, respectively, given in [13, Table II], while n denotes the degree of the Gauss-Hermite polynomial. To reach the form of Eq.(6) we first need to apply the substitution $-\frac{\mu\gamma N}{P}\upsilon^2 \to x^2$ in Eq.(5) which results in the following equation for the probability of coverage:

$$\begin{aligned} Pr(\text{SINR} < \gamma) =& 1 - q \int_0^\infty \mathrm{e}^{-qx(1+\rho(\gamma,4))} \mathrm{e}^{-x^2} \,\mathrm{d}x \\ =& 1 - q \sum_{i=1}^{n} w_i \mathrm{e}^{-qx_i(1+\rho(\gamma,4))} \end{aligned} \tag{9}$$

where

$$q = \pi\lambda \sqrt{\frac{P}{\mu\gamma N}} \tag{10}$$

and

$$\rho(\gamma, 4) = \sqrt{\gamma}\left(\frac{\pi}{2} - \arctan\left(\frac{1}{\sqrt{\gamma}}\right)\right). \tag{11}$$

By replacing Eq.(9) to Eq.(4) appropriately for each tier, we can find the probability of coverage in a two-tier network. Hence, in the following section, we introduce the cost model, which enables us to estimate the minimum deployment cost.

IV. Cost Model

The total cost $C_{\text{tot},x}$, including CAPEX and OPEX, of the x tier in area A is:

$$C_{\text{tot},\, x} = \lambda_x C_x A \tag{12}$$

where $\lambda_x A$ is the intensity measure of tier x in a total area A, while C_x denotes the total cost of one cell. In the two-tier case, we have a total cost for a specific area A that consists of the CAPEX and OPEX of two sets of cells with different intensities. This can be modeled as:

$$C_{\text{total}} = \lambda_{BS} C_{BS} A + \lambda_{SC} C_{SC} A. \tag{13}$$

Dividing Eq.(13) by A, we get on the left-hand side of the equation the term $C_{\text{total}}/A = C_{\text{norm}}$ which denotes the normalized cost per square unit.

Having derived the cost model and the two-tier probability of coverage, we can proceed to the formulation of the cost

[2] The Q-function is the tail probability of the standard normal distribution, defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{u^2}{2}) \mathrm{d}u$.

[3] MATLAB and Mathematica software packages were used for this purpose.

minimization problem which will provide us with the optimal tier intensities.

$$\begin{aligned}\min_{\lambda_{BS},\lambda_{SC}} \quad & \lambda_{BS}C_{BS} + \lambda_{SC}C_{SC} \\ \text{s.t.} \quad & Pr_{\text{cov}} = \epsilon, \text{ where } 0 \le \epsilon \le 1.\end{aligned}$$

Two different approaches can be followed for the solution of this problem. First, using Eq.(4) and Eq.(13) we can formulate the following equation system:

$$\begin{cases} C_{\text{norm}} = \lambda_{BS}C_{BS} + \lambda_{SC}C_{SC} \\ 1 - Pr(\text{SINR}_{BS} < \gamma)Pr(\text{SINR}_{SC} < \gamma) = \epsilon \end{cases} \tag{14}$$

where ϵ is the coverage probability threshold that is selected according to the preferences of the network provider. In the equation system, there are only two unknown parameters which are the tier intensities. Since the probability of coverage depends on the cost, through the tier intensities λ_{BS} and λ_{SC}, the minimum cost results from the minimum intensities that satisfy the coverage constraint. Therefore, by substituting the tier intensities of Eq.(13) to Eq.(4), we end up in Eq.(15). Differentiating this equation with respect to C_{norm} and setting it equal to zero, we can find the set of intensities that provide the minimum total cost per square unit. However, providing a closed form solution for this problem using the aforementioned approach is a difficult procedure because of the multiple-solutions of the derived system.

As an alternative, we propose a scheme that exhaustively searches for the set of intensities that provide the minimum cost while satisfying at the same time a given QoS. The proposed algorithm is presented in Algorithm 1. First, using the equation of the coverage probability, we extensively search the intensities of the two tiers that satisfy the coverage probability constrains. Once we find the satisfying intensity sets, we replace them to the cost model to estimate the minimum total cost out of a set of possible solutions.

Algorithm 1

Extensive search method for finding optimum tier intensities

Input: sets of intensity values
  $S_1 = \{\lambda_{BS_{min}}, \dots, \lambda_{BS_{max}}\}$ and
  $S_2 = \{\lambda_{SC_{min}}, \dots, \lambda_{SC_{max}}\}$,
  coverage probability constraint ϵ
  and vectors $IntBS = \mathbf{0}_M$, $IntSC = \mathbf{0}_M$, $Cost = \mathbf{0}_M$,
  where $\mathbf{0}_M$ denotes an $M \times 1$ vector with zeros.
  for all $\lambda_{BS} \in S_1$ **do**
    for all $\lambda_{SC} \in S_2$ **do**
      Calculate coverage probability Pr_{cov} by Eq.(4)
      if Pr_{cov} satisfies the constraint ϵ **then**
        Calculate the cost with current intensities
        using Eq.(13);
        Store $Cost[i] = \lambda_{BS_i}C_{BS} + \lambda_{SC_i}C_{SC}$
        and the intensities of the current loop
        $IntBS[i] = \lambda_{BS_i}$ and $IntSC[i] = \lambda_{SC_i}$;
      else
        Continue;
      end if
    end for
  end for
  Sort vector $Cost[i]$ such that:
  $Cost[j_1] \ge Cost[j_2] \ge \dots \ge Cost[j_N]$;
Output: Intensities $IntBS[j_1]$ and $IntSC[j_1]$
  that provide the minimum cost

TABLE I:
SIMULATION PARAMETERS

Simulation Parameter	Symbol	Value
BS transmission power	P_{BS}	48 dBm
SC transmission power	P_{SC}	12 dBm
Path loss exponent	α	4
Threshold ratio	γ	-3 dB
Power fading coef. mean	μ	0.5
Thermal noise power	N	-100 dBm
Simulation area	A	1 km^2
Gauss-Hermite coefficients	n	15

V. NUMERICAL RESULTS

In order to validate our analysis and present the results of the proposed method, we have developed a MATLAB simulator that creates PPPs on the plane and measures the coverage probability after a set of iterations. Using the cost model and the proposed Algorithm 1, the simulator provides the minimum tier intensities. In the following subsections we present the simulation setup along with the results of our experiments.

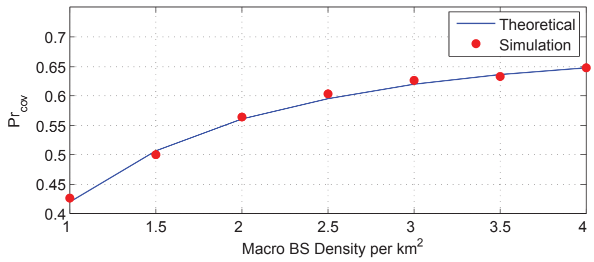
A. Simulation setup

The network under simulation consists of two coexisting realizations of homogeneous and independent PPPs which represent the macro BS and SC tiers with intensity λ_{BS} and λ_{SC}, respectively, as it is depicted in Fig. 1. A point in the center of the area under examination represents the reference UE.

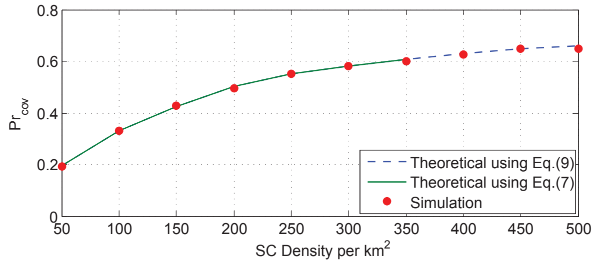
The network is assumed to be under saturated conditions and, therefore, all the cells of the same tier contribute to the interference measured at the UE. To measure the coverage probability, we run a set of 10^4 iterations of the proposed network and check whether the SINR of the UE exceeds the given threshold for the given set of intensities. The intensity of the tiers is the parameter that is varying in every set of iterations.

For all the numerical results, we use a path loss exponent $\alpha = 4$ which is a reasonable value for dense urban environments. Furthermore, the threshold ratio is fixed at $\gamma = -3\ dB$ and the thermal noise power at $N = -100\ dBm$. Also, the mean value of the power fading coefficient h is set to $\mu = 0.5$. The transmission power is $48\ dBm$ for the macro BSs and $12\ dBm$ for the SCs . For our analysis, the degree of the Gauss-Hermite polynomial is $n = 15$, in order to achieve a highly accurate approximation.The system parameters are summarized in Table I.

$$Pr_{\text{cov}} = 1 - (1 - \pi\lambda_{BS}\sqrt{\frac{P_{BS}}{\mu\gamma N}}\sum_{i=1}^{n} w_i \mathrm{e}^{-\pi\lambda_{BS}\sqrt{\frac{P_{BS}}{\mu\gamma N}}x_i(1+\sqrt{\gamma}(\frac{\pi}{2}-\arctan(\frac{1}{\sqrt{\gamma}})))}) \times$$
$$(1 - \pi(\frac{C_{\text{norm}}}{C_{SC}} - \lambda_{BS}\frac{C_{BS}}{C_{SC}})\sqrt{\frac{P_{SC}}{\mu\gamma N}}\sum_{i=1}^{n} w_i \mathrm{e}^{-\pi(\frac{C_{\text{norm}}}{C_{SC}}-\lambda_{BS}\frac{C_{BS}}{C_{SC}})\sqrt{\frac{P_{SC}}{\mu\gamma N}}x_i(1+\sqrt{\gamma}(\frac{\pi}{2}-\arctan(\frac{1}{\sqrt{\gamma}})))}) \quad (15)$$



(a)



(b)

Fig. 2: (a) Single-tier macro BS simulation and theoretical model comparison. (b) Single-tier small cell simulation and theoretical model comparison.

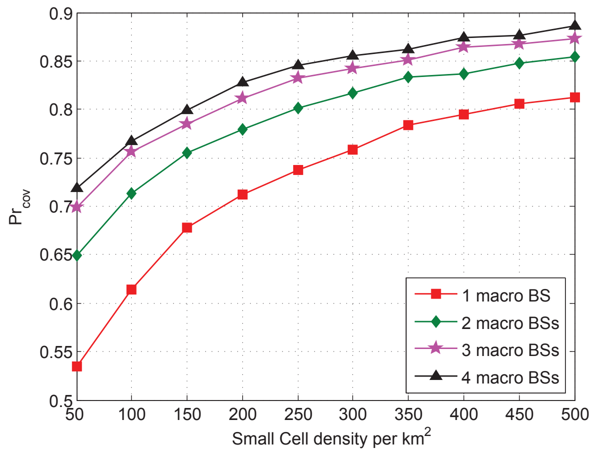


Fig. 3: Two-tier simulation for coverage probability using fixed number of macro BSs and variable number of SCs.

B. Results

First, we provide simulation results for a single-tier network to prove that the model in Eq.(5) is behaving as expected. In Fig. 2a, it is illustrated that the simulations for the coverage probability in a single-tier macro BS network match the theoretical model. Intuitively, we could assume that by introducing more cells into the network, the coverage would increase. However, as the number of macro BSs increases, the coverage probability tends to saturate to a maximum value. This is due to the fact that increasing the number of cells increases at the same time the interference at the UE.

In Fig. 2b, the coverage probability for a single-tier SC network is presented. In this case, we use both Eq.(7) and Eq.(9) for the theoretical model. As we have already mentioned, for high λ values, the theoretical model using the Eq.(7) cannot provide analytical results. However, using the Gauss-Hermite quadrature approach, we can achieve accurate results for a larger range of intensities.

In Fig. 3, we plot the probability of coverage versus the SC intensity in a two-tier network for different values of macro BS (i.e., one to four). As we can see, the probability of coverage is showing similar performance as in Fig. 2, while it is increasing by adding more macro BSs into the HetNet. Again, increasing the number of macro BSs results in gradually decreasing the slope of the coverage probability due to the interference raise.

By applying Algorithm 1 in the two-tier coverage probability model, we can reach a unique set of intensities that satisfies the coverage probability constraint and provide the lowest cost under this constraint. Since the cost of the cells is normalized, it is sufficient to define a cost relation between the two tiers in the cost model, i.e., $C_{BS} = zC_{SC}$, where z is the cost ratio. According to current market values [16] and considering that the technology evolves (e.g., part of future macro BSs could be implemented in the cloud, thus reducing drastically the OPEX), we have adopted two different values for the cost ratio z, i.e., $z = 1000$ and $z = 100$.

For the first case ($z = 1000$), the results of the minimization method are shown in Fig. 4. In the double y-axis figure, the total number of cells of each tier (left blue for BS and right green for SC) that could be placed randomly in an area of $1\ km^2$ are shown, in order to achieve a certain coverage probability. For example, deploying randomly 1 BS and around 190 SCs in an area of $1km^2$, we can achieve a coverage of at least 70% with the minimum deployment cost. Also, it is interesting to notice that while the probability of coverage increases, the number of needed SCs does not follow an upward trend. Instead, the addition of macro BSs results in a decrease of SCs, which is reasonable because by adding a macro BS into the network we can meet the coverage

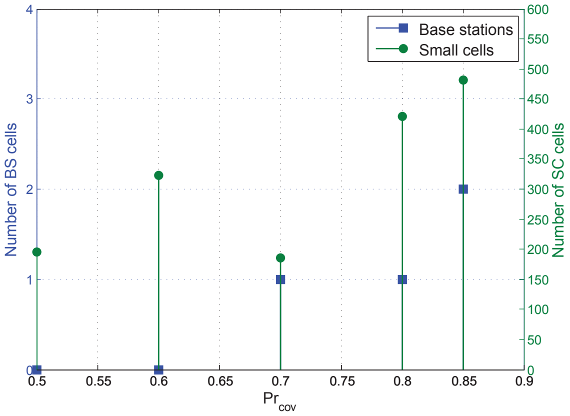


Fig. 4: Number of cells needed to satisfy the coverage constraint in a 1 km^2 area. The cost relation between the macro BSs and the SCs is $z = 1000$.

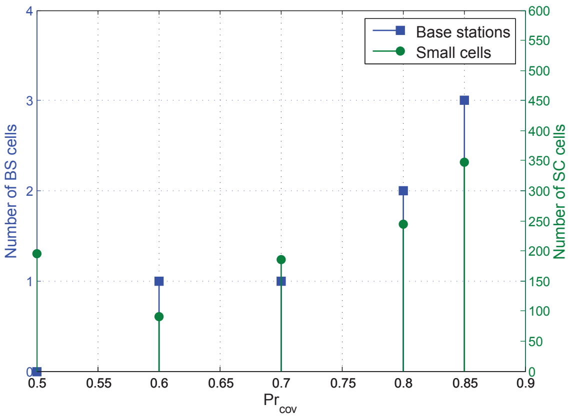


Fig. 5: Number of cells needed to satisfy the coverage constraint in a 1 km^2 area. The cost relation between the macro BSs and the SCs is $z = 100$.

constraint with less SCs.

In the second case, presented in Fig. 5, we use a cost relation between the tiers of $z = 100$. We can notice that for low values of coverage, z does not affect vastly the result of the algorithm. However, for higher values of P_{cov} and as z decreases, the number of macro BSs increases. This is due to the impact that the deployment of 100 more SCs (i.e., the cost of one macro BS in this case) causes to the interference, which is greater to the one caused by a single low-cost macro BS. Thus, we can achieve a higher coverage with the same cost.

Finally, in case that the coverage probability constraint is not hard, the algorithm could provide different solutions for a similar coverage probability, almost at the same cost. For instance, the network operator could have to decide between the minimum cost sets $\{2$ BSs, 200 SCs$\}$ and $\{3$ BSs, 100 SCs$\}$. In this situation, the decision should be made according to the environment that the network operates. In a dense urban environment, a solution with more SCs would be more beneficial, because it can provide higher spectral efficiency and serve more users with higher QoS. On the contrary, in a rural area the solution with more BSs and less SCs is more appropriate.

VI. Conclusion

In this paper, we propose a theoretical framework for finding the minimum deployment cost in a two-tier heterogeneous cellular network when using a random location deployment of both BSs and SCs. Employing stochastic geometry, we have derived the probability of coverage of a two-tier network. By using a cost model and an extensive search minimization method, we were able to find a set of PPP intensities that provide the minimum deployment cost, while fulfilling a given coverage probability constraint.

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References

[1] A. Damnjanovic, J. Montojo, W. Yongbin, J. Tingfang, L. Tao, M. Vajapeyam, Y. Taesang, S. Osok and D. Malladi, “A survey on 3GPP heterogeneous networks,” *IEEE Trans. Wireless Commun.*, vol. 18, no. 3, pp. 10-21, Jun. 2011.

[2] E.Z. Tragos et al, “The impact of interference on the performance of a multi-path metropolitan wireless mesh network,” in *Proc. 2011 IEEE Symp. Comp. and Comm.*, Kerkyra, ISCC. 2011, pp. 199-204.

[3] J.G. Andrews, H. Claussen, M. Dohler, S. Rangan and M.C. Reed, “Femtocells: Past, Present, and Future,” *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 497-508, Apr. 2012.

[4] Arc chart, “HetNet Market Summary & Forecasts: Macro Cells, Small Cells & Wi-Fi Offload,” Oct. 2012.

[5] A. D. Wyner, “Shannon-Theoretic Approach to a Gaussian Cellular Multiple-Access Channel,” *IEEE Trans. Inf. Theory*, vol. 40, no. 11, pp. 1713-1727, Nov. 1994.

[6] Xu JiaMing, Zhang Jun and J.G. Andrews, “On the Accuracy of the Wyner Model in Cellular Networks,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 9, pp. 3098-3109, Sep. 2011.

[7] M. Haenggi, J.G. Andrews, F. Baccelli, O. Dousse and M. Franceschetti, “Stochastic Geometry and Random Graphs for the Analysis and Design of Wireless Networks,” *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029-1046, Sep. 2009.

[8] J.G. Andrews, F. Baccelli and R.K. Ganti, “A Tractable Approach to Coverage and Rate in Cellular Networks,” *IEEE Trans. Commun.*, vol. 59 , no. 11, pp. 3122 - 3134 , Nov. 2011.

[9] Q. Hao, B. Soong, E. Gunawan, J. Ong, C. Soh and Z. Li, “A Low-Cost Cellular Mobile Communication System: A Hierarchical Optimization Network Resource Planning Approach,” *IEEE J. Sel. Areas Commun.*, vol. 15, no. 7, pp. 1315-1326, Sep. 1997.

[10] D. H. Kang, K. W. Sung and J. Zander, “Cost and Feasibility Analysis of Self-deployed Cellular Network,” in *Proc. 2011 22nd IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications,* Toronto, PIMRC. 2011, pp. 248-252.

[11] T.T. Vu, L. Decreusefond and P. Martins, “An analytical model for evaluating outage and handover probability of cellular wireless networks,” in *Proc. 2012 15th Int. Symp. on Wireless Personal Multimedia Communications,* Taipei, WPMC. 2012, pp. 643-647.

[12] H.S. Dhillon, R.K. Ganti, F. Baccelli and J.G. Andrews, “Modeling and Analysis of K-Tier Downlink Heterogeneous Cellular Networks,” *IEEE J. Sel. Areas Commun.,* vol. 30, no. 3, pp. 550-560, Apr. 2012.

[13] N. M. Steen, G. D. Byrne and E. M. Gelbard, “Gaussian Quadratures for the integral $\int_0^\infty \mathrm{e}^{-x^2} f(x)\,\mathrm{d}x$,” *Journal Math. Comp.,* no. 23, pp. 661-667, 1969.

[14] D. Stoyan, W. Kendall and J. Mecke, “Stochastic Geometry and Its Applications,” 2nd ed. John Wiley and Sons, 1996.

[15] B. Sklar, “Rayleigh Fading Channels in Mobile Digital Communication Systems, Part I: Characterization,” *IEEE Commun. Mag.,* vol. 35, no. 7, pp. 90-100, Jul. 1997.

[16] S. Tombaz, A. Vastberg and J. Zander, “Energy- and cost-efficient ultra-high-capacity wireless access,” *IEEE Wireless Commun. Mag.,* vol. 18, no. 5, pp. 18-24, Oct. 2011.