

Joint Estimation of the Time Delay and the Clock Drift and Offset Using UWB signals

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Abstract— We consider two transceivers, the first with perfect clock and the second with imperfect clock. We investigate the joint estimation of the delay between the transceivers and the offset and the drift of the imperfect clock. We propose a protocol for the synchronization of the clocks. We derive some empirical estimators for the delay, the offset and the drift, and compute the Cramer-Rao lower bounds and the joint maximum likelihood estimator of the delay and the drift. We study the impact of the protocol parameters and the time-of-arrival estimation variance on the achieved performances. We validate some theoretical results by simulation.

I. INTRODUCTION

Highly accurate positioning can be performed by employing the time-of-arrival (TOA) technique if impulse-radio (IR) ultra wideband (UWB) signals [1]–[3] are transmitted.

However, one of the main challenges facing the realization of UWB-based positioning systems is the need of synchronization among all the network transceivers if the TOA technique is used and among the reference nodes if the time-difference-of-arrival (TDOA) is used. Synchronization can be accomplished by using high-precision clocks which seems to be impractical due to the required high-cost. To overcome this problem two-way ranging strategies can be used as proposed in [4] and adopted in the IEEE802.15.4a standard [5]–[7]. Two-way ranging can mitigate the effects of the offset between clocks. However, the impact of clock drift is still present and causes non-negligible errors when the waited time at the receiver side is relatively long [7]–[10].

The effects of clock drift on TOA estimation accuracy is evaluated in many works where a wide variety of two-way protocols are proposed to reduce as much as possible the impact of the drift [8]–[14]. However the problem of joint delay and clock offset and drift estimation is not investigated, or is investigated but without taking into account the primary impact of TOA estimation errors. Even when TOA estimation errors are considered they are either considered in simulation only, or are considered in the proposed model but the proposed estimators are not optimal.

In this paper we consider two transceivers, one equipped with a perfect clock and one equipped with an imperfect clock. We investigate the joint estimation of the time delay between the two transceivers and the offset and the drift of the imperfect clock. We propose a system model taking into

account the TOA estimation errors at both transceivers. We compute the Cramer-Rao lower bounds (CRLB) for the joint estimation of the delay and the drift and derive the joint maximum likelihood estimator (MLE). Also, we propose some empirical estimators for the delay, the clock offset and the drift. The impact of the different parameters of the protocol and the TOA estimation variance on the proposed estimators is examined. The theoretical results are validated by simulation. The approach followed in this paper can be extended to derive the CRLBs and the joint MLE for many synchronization protocols under different assumptions.

The rest of the paper is organized as follows. In Sec. II, we describe the system model. In Sec. III, we present the estimation protocol. In Sec. IV, we propose an empirical algorithm. In Sec. V, we derive the CRLBs and the MLE. In Sec. VI, we show and discuss some numerical results.

II. SYSTEM MODEL

As mentioned above, we describe in this section our system model. Let us consider two transceivers Tr and Tr' equipped with two clocks Ck and Ck' , respectively, and assume that:

- 1) The clock Ck is perfect whereas the clock Ck' suffers from a drift and an offset.
- 2) The time delay τ between Tr and Tr' (i.e. τ is the time spent by a signal transmitted by Tr to reach Tr') is constant. Therefore, if Tr and Tr' communicate through free space (resp. a cable) then the distance (resp. the cable length) should be constant. In multipath channels, τ is proportional to the length of the detected path (not necessarily the direct one).

The local time t' of Ck' can be written with respect to (w.r.t.) the true time t (local time of Ck) as:

$$t' = \alpha t + \gamma = (1 + \nu)t + \gamma \quad (1)$$

where $\nu = \alpha - 1$ (a coefficient) and γ (in seconds) denote the drift and the offset of Ck' , respectively. The drift is often expressed in terms of parts-per-million (ppm); it is defined as the maximum number of extra or missed clock counts over a total of 10^6 counts. The drift as defined in (1) is obtained from that in ppm ν_{ppm} by $\nu = \nu_{\text{ppm}}10^{-6}$. We assume in this paper that ν can be positive or negative.

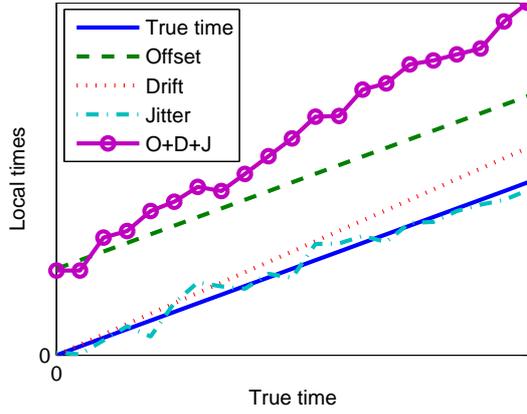


Fig. 1. True time (solid line), local time with offset (dashed line), local time with drift (dotted line), local time with jitter (dash-dotted line), and local time with offset, drift and jitter (line with circles).

Similarly to [7]–[11], [13] the problem of clock jitter is not included in our model for simplicity reasons. The jitter denotes the instantaneous fluctuations around the average local time described in (1). In Fig. 1, we illustrate the lines representing the true time (solid line), a local time with an offset (dashed line), a local time with drift (dotted line), a local with jitter (dash-dotted line), and a local time with all the mentioned imperfections (line with circles).

In the next sessions we propose a protocol and some algorithms to synchronize Tr and Tr' and to estimate the time delay between them.

III. ESTIMATION PROTOCOL

In this section we describe our estimation protocol. “Protocol” stands for the consecutive steps to be followed by Tr and Tr' in order to obtain the observation carrying the information about the unknown parameters. After proposing our protocol we realized that a similar protocol has already been proposed in [8]. The main contribution in this work is the derivation of the CRLBs and the estimation algorithms presented in the next sections rather than the protocol described here.

Let us present the protocol:

- 1) Tr' sends a signal to Tr at the “time of departure” (TOD) t'_D (TOD w.r.t. Ck'); using (1), we can write t'_D w.r.t. the true TOD t_D as:

$$t'_D = \alpha t_D + \gamma.$$

The transmitted signal arrives to Tr at the true “time of arrival” (TOA)

$$t_A = t_D + \tau = \frac{t'_D - \gamma}{\alpha} + \tau.$$

- 2) Tr estimates t_A ; denote by \hat{t}_A the estimated TOA w.r.t. the perfect clock. We can write \hat{t}_A as:

$$\hat{t}_A = t_A + \epsilon_A = \frac{t'_D - \gamma}{\alpha} + \tau + \epsilon_A \quad (2)$$

where ϵ_A denotes the estimation error.

- 3) Tr waits for the durations $\delta_1, \dots, \delta_N$ (known in advance by Tr') before sending N reply signals to Tr'. We will see later in Sec. IV-A that N cannot be lower than two. The n th signal is transmitted at the true “time of departure after waiting” (TOW)

$$t_{W,n} = \hat{t}_A + \delta_n = \frac{t'_D - \gamma}{\alpha} + \tau + \delta_n + \epsilon_A \quad (3)$$

and arrives to Tr' at the true “time of return” (TOR)

$$t_{R,n} = t_{W,n} + \tau = \frac{t'_D - \gamma}{\alpha} + 2\tau + \delta_n + \epsilon_A$$

which corresponds w.r.t. Ck' to

$$\begin{aligned} t'_{R,n} &= \alpha t_{R,n} + \gamma = \alpha(t_{W,n} + \tau) + \gamma \\ &= t'_D + \alpha(2\tau + \delta_n) + \alpha\epsilon_A. \end{aligned}$$

- 4) Tr' estimates $t'_{R,n}$; denote by $\hat{t}'_{R,n}$ the estimated TOR w.r.t. Ck'; $\hat{t}'_{R,n}$ can be written as:

$$\hat{t}'_{R,n} = t'_{R,n} + \epsilon'_{R,n} = \alpha(t_{W,n} + \tau) + \gamma + \epsilon'_{R,n} \quad (4)$$

$$= t'_D + \alpha(2\tau + \delta_n) + \alpha\epsilon_A + \epsilon'_{R,n} \quad (5)$$

where $\epsilon'_{R,n}$ denotes the estimation error w.r.t. Ck'.

- 5) Tr' proceeds to the estimation of the unknown parameters α , γ and τ by making use of the protocol parameters $\delta_1, \dots, \delta_N$, the estimated TOA \hat{t}_A and TORs $\hat{t}'_{R,1}, \dots, \hat{t}'_{R,N}$, and the distributions of the estimation errors $\epsilon_A, \epsilon'_{R,1}, \dots, \epsilon'_{R,N}$ (possible to be estimated jointly with the TOA and the TORs).

To be able to estimate the clock offset γ , the estimated TOA \hat{t}_A should be contained in the reply signals sent by Tr to Tr'. Otherwise, Tr' can only estimate the time delay τ and the clock drift α (from the estimated TORs $\hat{t}'_{R,1}, \dots, \hat{t}'_{R,N}$).

It can be shown [15] that in the presence of an additive white Gaussian noise (AWGN), the MLE of the TOA is unbiased at sufficiently high signal-to-noise ratios (SNR), follows a normal distribution and achieves the CRLB. At low and medium SNRs, the MLE is no longer Gaussian. Let us assume that:

$$\epsilon_A \sim \mathcal{N}(0, \sigma_A^2) \quad (6)$$

$$\epsilon'_{R,n} \sim \mathcal{N}(0, \sigma_{R'}^2 = \alpha^2 \sigma_R^2 \approx \sigma_R^2), \forall n \quad (7)$$

where $\mathcal{N}(\mu_N, \sigma_N^2)$ denotes the normal distribution of mean μ_N and variance σ_N^2 , σ_A^2 the variance of \hat{t}_A w.r.t. a perfect clock, and $\sigma_{R'}^2$ and σ_R^2 the variances of $\hat{t}'_{R,n}$ w.r.t. an imperfect and a perfect clock respectively; $\sigma_{R'}^2$ is approximated by σ_R^2 to make the covariance matrices in Secs. IV, V independent of the unknown parameters to estimate (valid assumption because $\alpha \approx 1$ and because σ_R is much smaller than τ and δ_n). In (7), $\sigma_{R'}^2$ is the same $\forall n$ because the reply signals sent by Tr to Tr' have all the same energy; σ_A^2 is not assumed equal to σ_R^2 because the signals transmitted by Tr and Tr' do not necessarily have the same energy.

At sufficiently high SNRs, both σ_A^2 and σ_R^2 can be computed from the expression of the CRLB for time estimation given by [16], [17]

$$c_T = \frac{1}{\rho\beta^2}$$

where ρ and β^2 denote the SNR and the mean quadratic bandwidth of the transmitted signal (β is also called effective bandwidth) respectively. For a signal occupying the whole UWB band authorized by the US federal commission of communications (FCC) [1] (central frequency of 6.85 GHz and bandwidth of 7.5 GHz so $\beta = 45.14$ GHz) we have $\sqrt{cT} = 7$ ps (resp. 0.7 ps) at $\rho = 10$ dB (resp. 30 ps).

IV. EMPIRICAL ALGORITHM

In this section we propose an empirical algorithm for the estimation of the time delay and the clock offset and drift. We consider in Sec. IV-A the case where the estimation errors ϵ_A in (2) and $\epsilon'_{R,n}$ in (5) are null (i.e. \hat{t}_A and $\hat{t}'_{R,n}$ correctly estimated) and present in Sec. IV-B the proposed algorithm.

Note that an optimal estimator should treat the entire available observation. Accordingly, if \hat{t}_A is known (resp. unknown) by Tr' then α , γ and τ (resp. α and τ) should be jointly estimated by maximizing the likelihood function relative to \hat{t}_A and $\hat{t}'_{R,1}, \dots, \hat{t}'_{R,N}$ (resp. $\hat{t}'_{R,1}, \dots, \hat{t}'_{R,N}$).

A. Error-free case

As mentioned above we assume here that ϵ_A in (2) and $\epsilon'_{R,n}$ in (5) are null.

To find α and τ from (5), we need at least two equations. So by taking $N = 2$ we can write

$$\begin{cases} \hat{t}'_{R,1} = t'_D + \alpha(2\tau + \delta_1) \\ \hat{t}'_{R,2} = t'_D + \alpha(2\tau + \delta_2) \end{cases}$$

so α and τ can be expressed as ($n = 1, 2$):

$$\alpha = \frac{\hat{t}'_{R,2} - \hat{t}'_{R,1}}{\delta_2 - \delta_1} \quad (8)$$

$$\tau = \frac{\hat{t}'_{R,n} - t'_D - \alpha\delta_n}{2\alpha}. \quad (9)$$

Note that τ is also given by

$$\tau = \frac{\delta_2(\hat{t}'_{R,1} - t'_D) - \delta_1(\hat{t}'_{R,2} - t'_D)}{2(\hat{t}'_{R,2} - \hat{t}'_{R,1})}.$$

However, we prefer the expression in (9) because it will be used later in Sec. IV-B in the proposed algorithm.

If we assume that \hat{t}_A is known by Tr' , then γ can be expressed from (2), (3) and (4) as ($n = 1, 2$):

$$\gamma = t'_D - \alpha(\hat{t}_A - \tau) \quad (10)$$

$$= \hat{t}'_{R,n} - \alpha(\hat{t}_A + \delta_n + \tau). \quad (11)$$

Hence, $N = 2$ is sufficient to obtain the exact values of the unknown parameters in the error-free case. In the presence of errors, $N = 2$ is also sufficient to perform the estimation; however, estimation performance can be improved by increasing the number of observations.

B. The proposed algorithm

Many empirical estimators for α , τ and γ can be proposed based on the equations established in Sec. III. However, it will suffice to investigate one estimator only as an example. The main goal is to compare the performances of an empirical estimator with the performances of the optimal estimator considered in Sec. V.

From (8), we can generate the following $N - 1$ estimates of α ($n = 1, \dots, N - 1$):

$$\hat{\alpha}_{n,1} = \frac{\hat{t}'_{R,n+1} - \hat{t}'_{R,1}}{\delta_{n+1} - \delta_1}. \quad (12)$$

Let $\underline{\hat{\alpha}}_1 = (\hat{\alpha}_{1,1}, \dots, \hat{\alpha}_{N-1,1})^T$ with T denoting the transpose operator. By considering $\underline{\hat{\alpha}}_1$ as the observation carrying the information on α , the log-likelihood function for the estimation of α can be written from (7) and (12) as:

$$\Lambda_{\underline{\hat{\alpha}}_1} = -\frac{1}{2} \left(\underline{\hat{\alpha}}_1 - \underline{\mu}_{\underline{\hat{\alpha}}_1} \right)^T \underline{\underline{\Omega}}_{\underline{\hat{\alpha}}_1}^{-1} \left(\underline{\hat{\alpha}}_1 - \underline{\mu}_{\underline{\hat{\alpha}}_1} \right)$$

where

$$\begin{aligned} \underline{\mu}_{\underline{\hat{\alpha}}_1} &= \alpha \underline{1}_{N-1} \\ \underline{\underline{\Omega}}_{\underline{\hat{\alpha}}_1} &= (\omega_{m,n})_{m,n=1,\dots,N-1} \end{aligned}$$

denote the mean and the covariance matrix of $\underline{\hat{\alpha}}_1$ with $\underline{1}_{N-1}$ being a vector of $N - 1$ elements equal to one, and

$$\omega_{m,n} = \begin{cases} \frac{2\sigma_R^2}{(\delta_n - \delta_1)^2} & m = n \\ \frac{\sigma_R^2}{(\delta_m - \delta_1)(\delta_n - \delta_1)} & m \neq n. \end{cases}$$

The MLE $\hat{\alpha}_1$ (w.r.t. to the observation $\underline{\hat{\alpha}}_1$) of α consists on maximizing the log-likelihood function $\Lambda_{\underline{\hat{\alpha}}_1}$. The partial derivative of $\Lambda_{\underline{\hat{\alpha}}_1}$ w.r.t. α can be written as:

$$\begin{aligned} \frac{\partial \Lambda_{\underline{\hat{\alpha}}_1}}{\partial \alpha} &= \left(\frac{\partial \underline{\mu}_{\underline{\hat{\alpha}}_1}}{\partial \alpha} \right)^T \underline{\underline{\Omega}}_{\underline{\hat{\alpha}}_1}^{-1} \left(\underline{\hat{\alpha}}_1 - \underline{\mu}_{\underline{\hat{\alpha}}_1} \right) \\ &= \underline{1}_{N-1}^T \underline{\underline{\Omega}}_{\underline{\hat{\alpha}}_1}^{-1} \underline{\hat{\alpha}}_1 - \alpha \underline{1}_{N-1}^T \underline{\underline{\Omega}}_{\underline{\hat{\alpha}}_1}^{-1} \underline{1}_{N-1}. \end{aligned}$$

By equating $\frac{\partial \Lambda_{\underline{\hat{\alpha}}_1}}{\partial \alpha}$ to zero we can express $\hat{\alpha}_1$ as:

$$\hat{\alpha}_1 = \frac{\underline{a}^T \underline{\hat{\alpha}}_1}{A} \quad (13)$$

with

$$\begin{aligned} \underline{a}^T &= \underline{1}_{N-1}^T \underline{\underline{\Omega}}_{\underline{\hat{\alpha}}_1}^{-1} \\ A &= \underline{1}_{N-1}^T \underline{\underline{\Omega}}_{\underline{\hat{\alpha}}_1}^{-1} \underline{1}_{N-1}. \end{aligned}$$

We can see from (13) that $\hat{\alpha}_1$ follows a normal distribution with a mean and a variance respectively given by

$$\begin{aligned} \mu_{\hat{\alpha}_1} &= \frac{\underline{a}^T \underline{\mu}_{\underline{\hat{\alpha}}_1}}{A} = \alpha \\ \sigma_{\hat{\alpha}_1}^2 &= \frac{\underline{a}^T \underline{\underline{\Omega}}_{\underline{\hat{\alpha}}_1} \underline{a}}{A^2} = \frac{1}{A}. \end{aligned} \quad (14)$$

Our estimator is thus unbiased. We have considered $\hat{\alpha}_1$ as empirical because $\underline{\hat{\alpha}}_1$ is not necessarily a sufficient statistic.

From (9) and (13), we can generate the following N estimates of τ ($n = 1, \dots, N$):

$$\hat{\tau}_{n,1} = \frac{\hat{t}'_{R,n} - t'_D - \hat{\alpha}_1 \delta_n}{2\hat{\alpha}_1}. \quad (15)$$

The variance of $\hat{\tau}_{n,1}$ is not the same $\forall n$ due to the term $\hat{\alpha}_1 \delta_n$ (the variance of $\hat{\alpha}_1$ is proportional to σ_R^2); we recall that the variance of $\hat{t}'_{R,n}$ is equal to $\sigma_A^2 + \sigma_R^2, \forall n$. From $\hat{\tau}_{n,1}$ in (15), we propose the following estimator of τ :

$$\hat{\tau}_1 = \frac{\mathbf{1}_N^T \hat{\tau}_1}{N} \quad (16)$$

where $\hat{\tau}_1 = (\hat{\tau}_{1,1}, \dots, \hat{\tau}_{N,1})^T$.

From (11), (13) and (16), we can generate the following N estimates of γ ($n = 1, \dots, N$):

$$\hat{\gamma}_{n,1} = \hat{t}'_{R,n} - \hat{\alpha}_1 (\hat{t}_A + \delta_n + \hat{\tau}_1). \quad (17)$$

From $\hat{\gamma}_{n,1}$ in (17), we propose the following estimator of γ :

$$\hat{\gamma}_{11} = \frac{\mathbf{1}_N^T \hat{\gamma}_1}{N} \quad (18)$$

where $\hat{\gamma}_1 = (\hat{\gamma}_{1,1}, \dots, \hat{\gamma}_{N,1})^T$. Another estimator can be directly proposed from (10) as:

$$\hat{\gamma}_{12} = t'_D - \hat{\alpha}_1 (\hat{t}_A - \hat{\tau}_1). \quad (19)$$

Note that the exact means and variances of $\hat{\tau}_{n,1}$ in (15) and $\hat{\gamma}_{n,1}$ in (17) are not easy to express because $\hat{\tau}_{n,1}$ is the ratio of two random variables. However, the asymptotic statistics are possible to compute. Nevertheless, we did not calculate them here for the sake of conciseness.

V. CRLBs AND JOINT MLE

In this section we derive the CRLBs for the joint estimation of the time delay τ and the clock drift α based on the estimated TORs $\hat{t}'_{R,n}$ in (5). We compute the joint MLE of α and τ and propose two empirical estimators for the clock offset γ .

Let:

$$\underline{X} = \hat{t}'_R - t'_D \mathbf{1}_N$$

where $\hat{t}'_R = (\hat{t}'_{R,1}, \dots, \hat{t}'_{R,N})^T$. The log-likelihood function for the joint estimation of α and τ can be written from (5)–(7) as:

$$\Lambda_{\underline{X}} = -\frac{1}{2} (\underline{X} - \underline{\mu}_{\underline{X}})^T \underline{\Omega}_{\underline{X}}^{-1} (\underline{X} - \underline{\mu}_{\underline{X}})$$

where

$$\underline{\mu}_{\underline{X}} = \alpha(2\tau \mathbf{1}_N + \underline{\delta}) \quad (20)$$

$$\underline{\Omega}_{\underline{X}} = (\omega_{m,n})_{m,n=1,\dots,N} \quad (21)$$

respectively denote the mean and the covariance matrix of \underline{X} with $\underline{\delta} = (\delta_1 \dots \delta_N)^T$ and

$$\omega_{m,n} = \begin{cases} \sigma_A^2 + \sigma_R^2 & m = n \\ \sigma_A^2 & m \neq n. \end{cases}$$

A. CRLBs for the joint estimation of α and τ

The CRLB for the estimation of a parameter gives the lowest variance achievable by an unbiased estimator. Denote by \mathbb{E} the expectation operator. The CRLBs of α and τ are [18] the diagonal elements of the inverse of the Fisher information matrix (FIM) given by

$$\underline{\underline{F}}_{\underline{X}} = \begin{pmatrix} f_{\alpha,\alpha} & f_{\alpha,\tau} \\ f_{\tau,\alpha} & f_{\tau,\tau} \end{pmatrix}$$

where

$$f_{\theta,\theta'} = -\mathbb{E} \left\{ \frac{\partial^2 \Lambda_{\underline{X}}}{\partial \theta \partial \theta'} \right\} = \frac{\partial \underline{\mu}_{\underline{X}}^T}{\partial \theta'} \underline{\Omega}_{\underline{X}}^{-1} \frac{\partial \underline{\mu}_{\underline{X}}}{\partial \theta} = f_{\theta',\theta}$$

with $\theta, \theta' \in \{\alpha, \tau\}$ and

$$\frac{\partial \underline{\mu}_{\underline{X}}}{\partial \alpha} = 2\tau \mathbf{1}_N + \underline{\delta} \quad (22)$$

$$\frac{\partial \underline{\mu}_{\underline{X}}}{\partial \tau} = 2\alpha \mathbf{1}_N. \quad (23)$$

Hence,

$$f_{\alpha,\alpha} = 4\tau^2 B + 4\tau D + F$$

$$f_{\tau,\tau} = 4\alpha^2 B$$

$$f_{\alpha,\tau} = 2\alpha(2\tau B + D) = f_{\tau,\alpha}$$

where

$$B = \mathbf{1}_N^T \underline{\Omega}_{\underline{X}}^{-1} \mathbf{1}_N$$

$$D = \mathbf{1}_N^T \underline{\Omega}_{\underline{X}}^{-1} \underline{\delta}$$

$$F = \underline{\delta}^T \underline{\Omega}_{\underline{X}}^{-1} \underline{\delta}.$$

The CRLBs of α and τ can respectively be expressed as:

$$c_{\alpha} = \frac{f_{\tau,\tau}}{f_{\alpha,\alpha} f_{\tau,\tau} - f_{\alpha,\tau}^2} = \frac{B}{BF - D^2} \quad (24)$$

$$c_{\tau} = \frac{f_{\alpha,\alpha}}{f_{\alpha,\alpha} f_{\tau,\tau} - f_{\alpha,\tau}^2} = \frac{4\tau^2 B + 4\tau D + F}{4\alpha^2 (BF - D^2)}. \quad (25)$$

We can show that c_{α} is a function of σ_R^2 , N and the variance of δ_n only. We can show as well that the term $4\tau^2 B + 4\tau D$ can be neglected in the expression of c_{τ} and that α can be approximated by 1 so c_{τ} becomes a function of σ_A^2 , σ_R^2 , N and the mean and the variance of δ_n .

B. Joint MLE of α and τ

The MLE $(\hat{\alpha}_2, \hat{\tau}_2)$ of (α, τ) consists on maximizing the log-likelihood function $\Lambda_{\underline{X}}$. Therefore, $(\hat{\alpha}_2, \hat{\tau}_2)$ can be obtained by equating the partial derivatives of $\Lambda_{\underline{X}}$ to zero:

$$\begin{cases} \left. \frac{\partial \Lambda_{\underline{X}}}{\partial \alpha} \right|_{(\alpha,\tau)=(\hat{\alpha}_2,\hat{\tau}_2)} = 0 \\ \left. \frac{\partial \Lambda_{\underline{X}}}{\partial \tau} \right|_{(\alpha,\tau)=(\hat{\alpha}_2,\hat{\tau}_2)} = 0 \end{cases} \quad (26)$$

where

$$\frac{\partial \Lambda_{\underline{X}}}{\partial \theta} = \frac{\partial \underline{\mu}_{\underline{X}}^T}{\partial \theta} \underline{\Omega}_{\underline{X}}^{-1} (\underline{X} - \underline{\mu}_{\underline{X}})$$

with $\theta \in \{\alpha, \tau\}$. Using (20)–(23), we can write from (26):

$$(2\hat{\tau}_2\mathbf{1}_N + \underline{\delta})^T \underline{\underline{\Omega}}_X^{-1} [\underline{X} - \hat{\alpha}_2(2\hat{\tau}_2\mathbf{1}_N + \underline{\delta})] = 0 \quad (27)$$

$$2\hat{\alpha}_2\mathbf{1}_N^T \underline{\underline{\Omega}}_X^{-1} [\underline{X} - \hat{\alpha}_2(2\hat{\tau}_2\mathbf{1}_N + \underline{\delta})] = 0. \quad (28)$$

By taking account of (28), (27) becomes:

$$\underline{\delta}^T \underline{\underline{\Omega}}_X^{-1} [\underline{X} - \hat{\alpha}_2(2\hat{\tau}_2\mathbf{1}_N + \underline{\delta})] = 0. \quad (29)$$

After some manipulations, we can write (28) and (29) as:

$$C - 2\hat{\alpha}_2\hat{\tau}_2B - \hat{\alpha}_2D = 0 \quad (30)$$

$$E - 2\hat{\alpha}_2\hat{\tau}_2D - \hat{\alpha}_2F = 0 \quad (31)$$

where

$$C = \mathbf{1}_N^T \underline{\underline{\Omega}}_X^{-1} \underline{X}$$

$$E = \underline{\delta}^T \underline{\underline{\Omega}}_X^{-1} \underline{X}.$$

By solving the equation system in (30) and (31) we obtain the following expressions of $\hat{\alpha}_2$ and $\hat{\tau}_2$:

$$\hat{\alpha}_2 = \frac{BE - CD}{BF - D^2} = \underline{g}^T \underline{X} \quad (32)$$

$$\hat{\tau}_2 = \frac{CF - DE}{2(BE - CD)} = \frac{\underline{k}^T \underline{X}}{\underline{l}^T \underline{X}} \quad (33)$$

where

$$\underline{g}^T = \frac{(B\underline{\delta}^T - D\mathbf{1}_N^T) \underline{\underline{\Omega}}_X^{-1}}{BF - D^2}$$

$$\underline{k}^T = (F\mathbf{1}_N^T - D\underline{\delta}^T) \underline{\underline{\Omega}}_X^{-1}$$

$$\underline{l}^T = 2(B\underline{\delta}^T - D\mathbf{1}_N^T) \underline{\underline{\Omega}}_X^{-1}.$$

In order to compute the statistics of our estimators we write \underline{X} , using (5), in the expressions of $\hat{\alpha}_2$ and $\hat{\tau}_2$ as:

$$\underline{X} = \alpha(2\tau\mathbf{1}_N + \underline{\delta}) + \underline{\epsilon}_R \quad (34)$$

where $\underline{\epsilon}_R = \alpha\epsilon_A\mathbf{1}_N + (\epsilon'_{R,1} \cdots \epsilon'_{R,N})^T$; $\underline{\epsilon}_R$ is zero-mean and has the same covariance matrix as \underline{X} . Then,

$$\hat{\alpha}_2 = \underline{g}^T [\alpha(2\tau\mathbf{1}_N + \underline{\delta}) + \underline{\epsilon}_R] = \alpha + \underline{g}^T \underline{\epsilon}_R \quad (35)$$

$$\hat{\tau}_2 = \frac{\underline{k}^T [\alpha(2\tau\mathbf{1}_N + \underline{\delta}) + \underline{\epsilon}_R]}{\underline{l}^T [\alpha(2\tau\mathbf{1}_N + \underline{\delta}) + \underline{\epsilon}_R]} = \frac{\tau + \frac{\underline{k}^T \underline{\epsilon}_R}{2\alpha(BF - D^2)}}{1 + \frac{\underline{l}^T \underline{\epsilon}_R}{2\alpha(BF - D^2)}} \quad (36)$$

$$\approx \left[\tau + \frac{\underline{k}^T \underline{\epsilon}_R}{2\alpha(BF - D^2)} \right] \left[1 - \frac{\underline{l}^T \underline{\epsilon}_R}{2\alpha(BF - D^2)} \right] \quad (37)$$

$$\approx \tau + \frac{(\underline{k}^T - \tau\underline{l}^T) \underline{\epsilon}_R}{2\alpha(BF - D^2)}. \quad (38)$$

We have obtained (37) from (36) by using the approximation $(1 + \xi)^m \approx 1 + m\xi$ for $\xi \ll 1$, and (38) from (37) by neglecting the noise product (i.e. the noise of second order).

We can see from (35) that $\hat{\alpha}_2$ is unbiased and follows a normal distribution with a variance given by:

$$\sigma_{\hat{\alpha}_2}^2 = \underline{g}^T \underline{\underline{\Omega}}_X \underline{g} = \frac{B}{BF - D^2} = c_\alpha. \quad (39)$$

This result is very interesting because it shows that $\hat{\alpha}_2$ is efficient; it always achieves the CRLB.

Unlike $\hat{\alpha}_2$, $\hat{\tau}_2$ is biased and follows the distribution of the ratio of two correlated normal variables. The PDF of $\hat{\tau}_2$ can be computed by making use of the work in [19], [20] about the ratio of normal variables. For sufficiently high SNRs, $\hat{\tau}_2$ becomes, as can be observed from (38) unbiased and follows a normal distribution with a variance given by:

$$\sigma_{\hat{\tau}_2}^2 = \frac{(\underline{k}^T - \tau\underline{l}^T) \underline{\underline{\Omega}}_X (\underline{k} - \tau\underline{l})}{4\alpha^2(BF - D^2)^2} = \frac{4\tau^2B + 4\tau D + F}{4\alpha^2(BF - D^2)} = c_\tau. \quad (40)$$

This result is very interesting as well because it shows that $\hat{\tau}_2$ is asymptotically efficient.

C. Empirical estimators of γ

Assume now that the TOA \hat{t}_A is known by Tr' . The joint MLE of α , γ and τ consists in this case on maximizing the log-likelihood function corresponding to \hat{t}_A and all $\hat{t}'_{R,n}$. This estimator is not investigated in this paper. In this subsection we propose two empirical estimators of γ by making use of $\hat{\alpha}_2$ and $\hat{\tau}_2$ derived in the last subsection.

Similarly to the estimators in (18) and (19), we propose the following two estimators:

$$\hat{\gamma}_{21} = \frac{\mathbf{1}_N^T \hat{\gamma}_2}{N} \quad (41)$$

$$\hat{\gamma}_{22} = t'_D - \hat{\alpha}_2(\hat{t}_A - \hat{\tau}_2). \quad (42)$$

where $\hat{\gamma}_2 = (\hat{\gamma}_{1,2}, \dots, \hat{\gamma}_{N,2})^T$ with

$$\hat{\gamma}_{n,2} = \hat{t}'_{R,n} - \hat{\alpha}_2(\hat{t}_A + \delta_n + \hat{\tau}_2).$$

VI. NUMERICAL RESULTS AND DISCUSSION

In this section we discuss some numerical results. The main two goals are to evaluate our estimators and to study the impact of some parameters (σ_A , σ_R , δ_N and N) on the achieved performances. Unfortunately, we cannot show all our results due to the lack of space.

Unless mentioned otherwise, we consider the following values in our simulations: $\nu_{\text{ppm}} = 20$ ppm, $\gamma = 1$ μs , $\tau = 100$ ns (which corresponds to a distance of 30 m), $\sigma_A = \sigma_R = 0.1$ ns, $\delta_N = 1$ ms, and $N = 4$; δ_n is given by $\delta_n = \frac{n\delta_N}{N}$. In our simulations the variances are obtained based on 10^4 noise samples.

We denote by $\sigma_{\alpha,1}$, $\sigma_{\alpha,2}$, $\sigma_{\tau,1}$, $\sigma_{\tau,2}$, $\sigma_{\gamma,11}$, $\sigma_{\gamma,12}$, $\sigma_{\gamma,21}$ and $\sigma_{\gamma,22}$ the standard deviations (Stds) obtained by simulation of the estimators $\hat{\alpha}_1$ in (13), $\hat{\alpha}_2$ in (32), $\hat{\tau}_1$ in (16), $\hat{\tau}_2$ in (33), $\hat{\gamma}_{11}$ in (18), $\hat{\gamma}_{12}$ in (19), $\hat{\gamma}_{21}$ in (41) and $\hat{\gamma}_{22}$ in (42), respectively, by $\kappa_{\alpha,1}$ the Std of $\hat{\alpha}_1$ (square root of $\sigma_{\hat{\alpha}_1}^2$ in (14)), and by $\kappa_{\alpha,2}$ and $\kappa_{\tau,2}$ the square roots of the CRLBs c_α in (24) and c_τ in (25), respectively.

In Figs. 2(a)–2(l) we show the Stds for drift, offset and delay estimation, respectively, w.r.t. σ_A , σ_R , δ_N and N , respectively.

A. Impact of σ_A

Fig. 2(a) shows that $\hat{\alpha}_1$ and $\hat{\alpha}_2$ achieve the same performance; they both achieve the CRLB which is independent of σ_A . The variance of an unbiased estimator can never be lower than the CRLB. However, $\sigma_{\alpha,1}$, $\sigma_{\alpha,2}$ are sometimes lower than $\kappa_{\alpha,2}$ because they are obtained by simulation. Fig. 2(b) shows that $\hat{\gamma}_1$ and $\hat{\gamma}_2$ approximately achieve the same performance. The achieved variances increase with σ_A . Fig. 2(c) shows that $\hat{\tau}_1$ and $\hat{\tau}_2$ approximately achieve the same performance. They both achieve the CRLB that increases with σ_A .

B. Impact of σ_R

For the estimation of γ and τ , we observe in Figs. 2(e) and 2(f) the same results discussed in Sec. VI-A. However, the variance achieved by the estimators of α increases now with σ_R as can be observed in Fig. 2(d).

C. Impact of δ_N

The variance achieved by the estimators of α decreases as δ_N increases as can be seen in Fig. 2(g). This result can be expected from (12).

Fig. 2(h) shows that the variances achieved by the estimators of γ decrease as δ_N increases, then increase to reach a given ceil. The convergence to a constant value is due to the fact that the lowest variance achieved by an estimator of γ should be a function of σ_A , σ_R and N . However, to understand the non-monotonous behavior of the achieved variance we need a closed-form expression of the variance or the CRLB.

We can see in Fig. 2(i) that the variances achieved by the estimators of τ decrease as δ_N increases until they converge to a constant value. This result is expected like for the estimation of γ .

D. Impact of N

We can observe in Figs. 2(j)–2(l) that the variances achieved by the estimators of α , γ and τ decrease as N increases; in fact, by increasing N we increase the total SNR because $\hat{t}'_{R,1}, \dots, \hat{t}'_{R,N}$ are independent.

VII. CONCLUSION

We have considered the joint estimation of the time delay between two transceivers and the offset and the drift of an imperfect clock. We have proposed a protocol for the synchronization of the transceivers. We have proposed some empirical estimators for the delay, the offset and the drift. Also, we have derived the CRLBs and the joint MLE of the delay and the drift. We have studied the impact of the parameters of the protocol and the TOA estimation variance on the achieved performances. Some theoretical results are validated by simulation.

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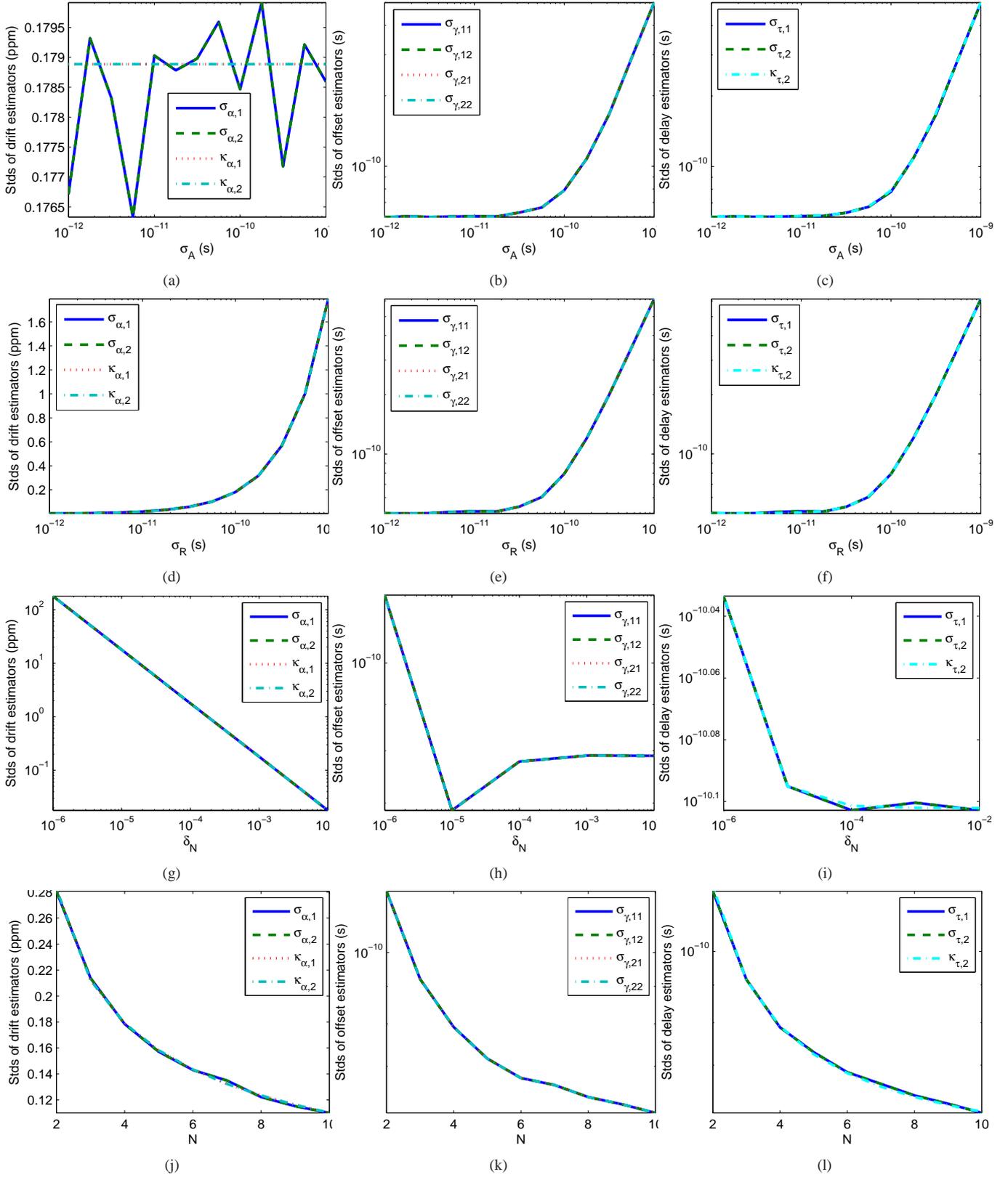


Fig. 2. (a)–(l) Stds for drift, offset and delay estimation, respectively, w.r.t. σ_A , σ_R , δ_N and N , respectively.