

# Eigenvector Precoding for FBMC Modulations under Strong Channel Frequency Selectivity

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**Abstract**—The problem of eigenvector precoding for multi-stream MIMO transmission with filterbank multicarrier (FBMC) modulations is considered. The transmitter is assumed to use a different precoding matrix for each subcarrier, which is constructed by selecting some eigenvectors of the channel matrix. Surprisingly, the phase ambiguity that is inherent to the definition of the eigenvectors turns out to have a crucial effect on the system performance under strong channel frequency selectivity. This is due to the inherent lack of orthogonality of FBMC modulations in these situations. It is shown that by properly selecting the reference phase evolution of each eigenvector across the transmission bandwidth, the global performance of the system can be significantly improved. A phase reference construction strategy is presented that minimizes an upper bound on the total intersymbol/intercarrier interference (ISI/ICI) power.

## I. INTRODUCTION AND SIGNAL MODEL

Filterbank multicarrier (FBMC) modulations have become strong candidates for the physical layer of 5G wireless communication standards. Thanks to the possibility of introducing non-rectangular pulse shapes and the fact that cyclic prefix (CP) can be avoided, these modulations achieve a higher spectral efficiency compared to classical CP-OFDM. Furthermore, by using OQAM modulations at each subcarrier (FBMC/OQAM), one can achieve total orthogonality under frequency flat channel conditions [1].

However, the use of MIMO technologies in combination with FBMC modulations poses important challenges from the technical point of view [2]. The situation is quite different from CP-OFDM, where the inherent orthogonality allows to decompose the MIMO frequency selective channel into a set of  $M$  parallel MIMO frequency flat links (here  $M$  denoting the number of subcarriers). In FBMC modulations, the channel frequency selectivity destroys the system orthogonality, a fact that unavoidably leads to a residual distortion in terms of inter-symbol and inter-carrier interference (ISI/ICI). The effect of this residual ICI/ISI is much more devastating in multi-stream MIMO configurations, essentially because the cumulative effect of the distortion associated with each stream. Traditionally, sophisticated equalization architectures [3][4][5] and interference cancellation methods [6][7][5] have been proposed to combat this effect. More recent approaches [8] have considered the optimization of the transmitter precoder to minimize the ISI/ICI effect. Another interesting approach can be found in [9][10], where a polynomial-based (multi-tap) SVD precoder is applied together with an equivalent multi-tap

equalizer at the receiver. Here, we propose a simpler alternative approach, based on single tap per-subcarrier processing.

Let us consider a MIMO system with  $N_T$  transmit and  $N_R$  receive antennas. Let  $\mathbf{H}(\omega)$  denote an  $N_R \times N_T$  matrix containing the frequency response of the MIMO channels, so that the  $(i, j)$ th entry of  $\mathbf{H}(\omega)$  contains the frequency response between the  $j$ th transmit and the  $i$ th receive antennas. We assume that the MIMO system is used for the transmission of  $N_S$  parallel signal streams,  $1 \leq N_S \leq \min\{N_R, N_T\}$ , which correspond to FBMC modulated signals. We will denote by  $\mathbf{s}(\omega)$  an  $N_S \times 1$  column vector that contains the frequency response of the signal transmitted at each of the  $N_S$  parallel streams.

Assume that the transmitter applies a frequency-dependent linear precoder, which will be denoted by the  $N_T \times N_S$  matrix  $\mathbf{A}(\omega)$ . The signal transmitted through the  $N_T$  transmit antennas can be expressed as  $\mathbf{x}(\omega) = \mathbf{A}(\omega)\mathbf{s}(\omega)$ , where  $\mathbf{x}(\omega)$  is an  $N_T \times 1$  column vector containing the frequency response of the signal transmitted through each of the  $N_T$  antennas. On the other hand, let  $\mathbf{y}(\omega)$  denote an  $N_R \times 1$  column vector containing the frequency response of the received signals in noise, namely

$$\mathbf{y}(\omega) = \mathbf{H}(\omega)\mathbf{x}(\omega) + \mathbf{n}(\omega)$$

where  $\mathbf{n}(\omega)$  is the additive Gaussian white noise. We assume that the receiver estimates the transmitted symbols by linearly transforming the received signal vector  $\mathbf{y}(\omega)$ . More specifically, we consider a certain  $N_R \times N_S$  receive matrix  $\mathbf{B}(\omega)$  so that the symbols are estimated by  $\hat{\mathbf{s}}(\omega) = \mathbf{B}^H(\omega)\mathbf{y}(\omega)$ . For the sake of simplicity, we will generally consider here that the matrices  $\mathbf{A}(\omega)$  and  $\mathbf{B}(\omega)$  are such that

$$\mathbf{B}^H(\omega)\mathbf{H}(\omega)\mathbf{A}(\omega) = \mathbf{I}_{N_S} \quad (1)$$

where  $\mathbf{I}_{N_S}$  is the  $N_S \times N_S$  identity matrix. Indeed, if this condition holds, the output of the receiver contains a noisy version of the transmitted symbol streams, in the sense that  $\hat{\mathbf{s}}(\omega) = \mathbf{s}(\omega) + \mathbf{B}^H(\omega)\mathbf{n}(\omega)$ . Hence, by implementing the transmit and receive filters  $\mathbf{A}(\omega)$  and  $\mathbf{B}(\omega)$  such that (1) holds, we have been able to retrieve the transmitted symbol streams up to a background noise contribution.

Assume that the transmitter has perfect channel state information. We define the following channel matrix

$$\Omega_H(\omega) = \mathbf{H}^H(\omega)\mathbf{H}(\omega) \quad (2)$$

and let  $\lambda_1(\omega) > \dots > \lambda_{\min\{N_R, N_T\}}(\omega)$  denote its positive eigenvalues (assumed all simple). Let  $\mathbf{\Lambda}(\omega)$  denote a diagonal matrix containing the  $N_S$  largest eigenvalues and define  $\mathbf{V}(\omega)$

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as the  $N_T \times N_S$  unitary matrix that contains the associated eigenvectors. In this paper, we will consider an implementation of the eigenvector precoder  $\mathbf{V}(\omega)$  at the transmit side, namely

$$\mathbf{A}(\omega) = \mathbf{V}(\omega). \quad (3)$$

It is well known that, when combined with the appropriate power allocation, this precoder is optimal in terms of capacity for the AWGN channel [11]. At the receiver side, we consider the use of the linear transform that inverts the resulting channel according to (1), namely

$$\mathbf{B}(\omega) = \mathbf{H}(\omega)\mathbf{V}(\omega)\mathbf{\Lambda}^{-1}(\omega). \quad (4)$$

**Remark 1** When the channel eigenvalues are all different, the eigenvector matrix  $\mathbf{V}(\omega)$  is defined up to a diagonal orthogonal matrix. Fixing this uncertainty is equivalent to fixing the phase of a selected element of each column  $\mathbf{V}(\omega)$ . To solve this ambiguity, we will assume that a selected entry of each column of  $\mathbf{V}(\omega)$  is constructed to have a predefined phase. More specifically, if we denote by  $\mathbf{v}_\ell(\omega)$  the  $\ell$ th column of  $\mathbf{V}(\omega)$ , we will assume that a certain entry of this vector (denoted by  $k_\ell$ ) is fixed to have a phase  $\phi_{k_\ell\ell}(\omega)$ , which is a predefined continuous function of  $\omega$ . In other words, we will assume that  $\arg\{\mathbf{v}_\ell(\omega)\}_{k_\ell} = \arg\{\mathbf{V}(\omega)\}_{k_\ell\ell} = \phi_{k_\ell\ell}(\omega)$  for  $\ell = 1 \dots N_S$ . Hence, for each column vector of  $\mathbf{V}(\omega)$ , two free parameters ( $k_\ell$  and  $\phi_{k_\ell\ell}(\omega)$ ) need to be fixed in order to fully specify the precoder matrix associated to a  $\mathbf{H}(\omega)$ .

The precoder and receiver matrices  $\mathbf{A}(\omega)$  and  $\mathbf{B}(\omega)$  need to be implemented using real filters, which may prove to be computationally very expensive to implement. For this reason, one typically assumes that their frequency response is sufficiently flat around each subcarrier, so that one might implement them using direct per-subcarrier multiplication by their frequency response at the intended subcarrier. This is equivalent to applying the precoder  $\mathbf{A}(\omega_k)$  and the linear receiver  $\mathbf{B}(\omega_k)$  to the MIMO signal that travels through the  $k$ th subcarrier (see Figure 1). This approximation turns out to perform reasonably well for relatively mild frequency selective channels. However, the performance breaks down as soon as the channel frequency selectivity becomes more severe, mainly due to the ISI/ICI distortion that is generated at the output of the receiver. In the next section, we present an analytical characterization of this effect.

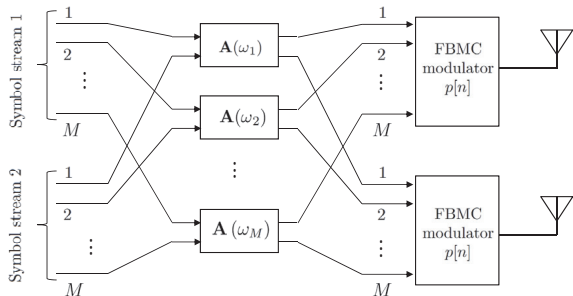


Fig. 1. Classical per-subcarrier precoding in a situation where  $N_S = N_T = 2$ .

## II. RESIDUAL ISI/ICI FOR FBMC/OQAM

In [12]–[13], we characterized the residual distortion in terms of intersymbol and intercarrier interference that is observed at the output of the receiver equalizer of a FBMC/OQAM modulation. More specifically, we considered a FBMC/OQAM modulation with  $M$  subcarriers ( $M$  assumed even) such that transmit and receive filters are constructed by exponentially modulating a certain prototype  $p[n]$ . In order to characterize the distortion generated by the channel frequency selectivity, we need to assume that the prototype pulse has length  $M\kappa$ , where  $\kappa$  is an integer that is typically referred to as the overlapping factor. We will also assume that  $p[n]$  is even symmetric and that this pulse is a discretization of smooth analog waveform. We will denote by  $p'[n]$  the discretization of the derivative of the analog pulse (see [14] for further details).

It is well known that one can construct the prototype pulse  $p[n]$  in order to ensure perfect orthogonality at the receiver under frequency flat channels. These conditions are typically referred to as Perfect Reconstruction (PR) conditions, which can be formulated as follows. Let  $\mathbf{P}$  denote the  $M \times \kappa$  matrix obtained by arranging the original samples of  $p[n]$  in columns, so that the  $k$ th row of  $\mathbf{P}$  contains its Type-I  $k$ th polyphase component. Then, PR conditions hold if [1]

$$[\mathbf{I}_2 \otimes (\mathbf{I}_{M/2} + \mathbf{J}_{M/2})] (\mathbf{P} \circledast \mathbf{J}_M \mathbf{P}) = [\mathbf{0}, \mathbf{1}, \mathbf{0}] \quad (5)$$

where  $\circledast$  indicates row-wise convolution between matrices,  $\mathbf{J}_M$  is the anti-identity matrix,  $\mathbf{1}$  is an all-ones column vector, and  $\mathbf{0}$  is a zero matrix of appropriate dimensions. These conditions are assumed to hold here.

In order to describe the residual distortion power that is observed at the output of the receiver, we define a second matrix  $\mathbf{P}'$  that is constructed as  $\mathbf{P}$  but using the derivative pulse  $p'[n]$  instead of the original one  $p[n]$ . We also define the two  $M \times (2\kappa - 1)$  matrices  $\mathcal{R}$  and  $\mathcal{S}$  as

$$\mathcal{R} = \mathbf{P} \circledast \mathbf{J}_M \mathbf{P}', \quad \mathcal{S} = (\mathbf{J}_2 \otimes \mathbf{I}_{M/2}) \mathbf{P} \circledast \mathbf{J}_M \mathbf{P}'$$

where  $\otimes$  denotes Kronecker product. We will assume that the channel frequency response  $\mathbf{H}(\omega)$  is a smooth function of  $\omega$  and that the complex symbols are independent and identically distributed (i.i.d.) with i.i.d. real and imaginary parts that have zero mean and variance  $P_s/2$ . It was shown in [13] that under the above technical assumptions and assuming that the number of subcarriers grows large, one can provide a closed form expression for the residual distortion power observed at the output of the receiver. To present this expression, we need to consider the following two functions

$$\alpha_{n,n_s}(\omega) = -j \frac{\sqrt{2}}{M} \{ \mathbf{B}^H(\omega) \mathbf{H}(\omega) \mathbf{A}'(\omega) \}_{n,n_s}$$

$$\beta_{n,n_s}(\omega) = -j \frac{\sqrt{2}}{M} \{ \mathbf{B}'(\omega)^H \mathbf{H}(\omega) \mathbf{A}(\omega) \}_{n,n_s}$$

where  $\mathbf{A}'(\omega)$  and  $\mathbf{B}'(\omega)$  are the frequency domain derivatives of  $\mathbf{A}(\omega)$  and  $\mathbf{B}(\omega)$  respectively. The expression of the asymptotic distortion power provided in [13] was derived under the general assumption that transmit and receive prototype filters are different. For the specific case where the two prototypes are equal, one can express the total aggregated distortion power (ICI/ISI) experimented at the  $k$ th subcarrier as

$$P_e(k) = \sum_{n=1}^{N_S} \sum_{s=1}^{N_S} \eta_{n,n_s}(k) \quad (6)$$

plus a term of order  $o(M^{-2})$ , where  $\eta_{n,n_s}(k)$  is defined as (dropping the dependence on  $\omega_k$  of  $\alpha_{n,n_s}$  and  $\beta_{n,n_s}$ )

$$\eta_{n,n_s}(k) = \frac{P_s}{M} \text{tr} \left[ \mathcal{R} \mathcal{R}^T \mathbf{U}^+ \Phi_R^{(n,n_s)} + \mathcal{S} \mathcal{S}^T \mathbf{U}^- \Psi_R^{(n,n_s)} \right] + \frac{P_s}{M} \text{tr} \left[ \mathcal{R} \mathcal{R}^T \mathbf{U}^- \Phi_I^{(n,n_s)} + \mathcal{S} \mathcal{S}^T \mathbf{U}^+ \Psi_I^{(n,n_s)} \right]$$

where  $\mathbf{U}^\pm = \mathbf{I}_2 \otimes (\mathbf{I}_{M/2} \pm \mathbf{J}_{M/2})$  and where  $\Phi_R^{(n,n_s)}$  and  $\Psi_R^{(n,n_s)}$  are the  $M \times M$  matrices

$$\begin{aligned} \Phi_R^{(n,n_s)} &= [\text{Re}^2 \alpha_{n,n_s} + \text{Re}^2 \beta_{n,n_s}] \mathbf{I}_M + 2 \text{Re} \alpha_{n,n_s} \text{Re} \beta_{n,n_s} \mathbf{J}_M \\ \Psi_R^{(n,n_s)} &= [\text{Re}^2 \alpha_{n,n_s} + \text{Re}^2 \beta_{n,n_s}] \mathbf{I}_M \\ &\quad + 2 \text{Re} \alpha_{n,n_s} \text{Re} \beta_{n,n_s} (\mathbf{I}_2 \otimes \mathbf{J}_{M/2}) \end{aligned}$$

whereas  $\Phi_I^{(n,n_s)}$  and  $\Psi_I^{(n,n_s)}$  are equivalently defined, replacing all instances of  $\text{Re}[\cdot]$  with  $\text{Im}[\cdot]$ .

Let us now illustrate how the system performance critically depends on the definition of the precoder matrix  $\mathbf{V}(\omega)$  in terms of the phase ambiguity mentioned in Remark 1 above. As in Remark 1, let  $\mathbf{v}_\ell(\omega)$  denote the  $\ell$ th column vector of  $\mathbf{V}(\omega)$  and assume that we force the entry  $k_\ell$  to have a specific phase  $\phi_{k_\ell \ell}(\omega)$ , which is a predefined continuous function of  $\omega$ . This function can in principle be arbitrary, although we will see that it can be tuned in order to optimize the system performance. Assume that  $\{\mathbf{V}(\omega)\}_{k_\ell} \neq 0$  and let  $\Omega'_H(\omega)$  and  $\phi'_{k_\ell}(\omega)$  be the derivatives of  $\Omega_H(\omega)$  defined in (2) and  $\phi_{k_\ell}(\omega)$  respectively. Using the steps in [15], one may express the derivative of  $\mathbf{V}(\omega)$  as follows

$$\mathbf{V}'(\omega) = \mathbf{V}(\omega) \mathbf{D}(\omega) + \mathbf{W}(\omega) \quad (7)$$

where  $\mathbf{W}(\omega)$  is a matrix with the  $\ell$ th column defined as

$$\mathbf{w}_\ell(\omega) = \sum_{\substack{r=1 \\ r \neq \ell}}^{N_T} \frac{\mathbf{v}_r^H(\omega) \Omega'_H(\omega) \mathbf{v}_\ell(\omega)}{\lambda_\ell(\omega) - \lambda_r(\omega)} \mathbf{v}_r(\omega)$$

where  $\mathbf{D}(\omega)$  is a diagonal matrix with its  $\ell$ th entry given by

$$\{\mathbf{D}(\omega)\}_{\ell\ell} = j \left( \phi'_{k_\ell \ell}(\omega) - \text{Im} \left[ \frac{\{\mathbf{w}_\ell(\omega)\}_{k_\ell}}{\{\mathbf{v}_\ell(\omega)\}_{k_\ell}} \right] \right).$$

From this derivation, we can readily see that the derivative of the eigenvector matrix  $\mathbf{V}(\omega)$  inherently depends on the choice of the entries  $k_\ell$  and the phase functions  $\phi_{k_\ell \ell}(\omega)$ . Different choices of these two parameters will result in different values of  $\mathbf{D}(\omega)$ , which will in turn lead to a distinct value of  $\mathbf{V}'(\omega)$ , and thus a different performance. Let us now examine with more detail some particular ways of constructing the precoder matrix  $\mathbf{V}(\omega)$  without ambiguities, and their associated performance.

### III. PROPOSED APPROACH

The objective of this section is to propose a method to fix, for each column vector of the precoding matrix  $\mathbf{v}_\ell(\omega)$ , the phase reference index  $k_\ell$  and the associated phase response  $\phi_{k_\ell \ell}(\omega)$ . We will assume that the function  $\phi_{k_\ell \ell}(\omega)$  is differentiable with derivative  $\phi'_{k_\ell \ell}(\omega)$ . Direct optimization of the above expression of  $P_e(k)$  in terms of these two parameters appears to be extremely complicated. For this reason, we propose to fix the pair  $\{k_\ell, \phi_{k_\ell \ell}\}$ ,  $\ell = 1 \dots N_S$ , by minimizing an upper bound of  $P_e(k)$ , which is presented next (the proof can be found in the Appendix).

**Proposition 1** *The asymptotic distortion power  $P_e(k)$  accepts the following upper bound*

$$P_e(k) \leq \frac{4P_s}{M^3} \zeta_p \left[ 2\Psi_H(\omega_k) + 3\vartheta_H(\omega_k) + 6 \sum_{\ell=1}^{N_S} \left( |\phi'_{k_\ell \ell}(\omega_k)|^2 + \frac{\vartheta_H(\omega_k)}{|\{\mathbf{v}_\ell(\omega_k)\}_{k_\ell}|^2} \right) \right] \quad (8)$$

where  $\zeta$  is the following pulse-specific quantity

$$\zeta_p = \frac{1}{M} \text{tr} [\mathcal{R} \mathcal{R}^T \mathbf{U}^+ + \mathcal{S} \mathcal{S}^T \mathbf{U}^-] \quad (9)$$

whereas  $\Psi_H(\omega)$  and  $\vartheta_H(\omega)$  are two positive functions that depend on the channel but are independent of both  $k_\ell$  and  $\phi_{k_\ell \ell}(\omega)$ :

$$\Psi_H(\omega) = \text{tr} \left[ (\mathbf{H}(\omega) \mathbf{H}^H(\omega))^{\#} (\mathbf{H}'(\omega) \mathbf{H}'(\omega)^H) \right] \quad (10)$$

$$\vartheta_H(\omega) = \sum_{\ell=1}^{N_S} \sum_{\substack{m=1 \\ m \neq \ell}}^{N_S} \left| \frac{\mathbf{v}_m^H(\omega) \Omega'_H(\omega) \mathbf{v}_\ell(\omega)}{\lambda_\ell(\omega) - \lambda_m(\omega)} \right|^2 \quad (11)$$

where  $(\cdot)^{\#}$  denotes Moore-Penrose pseudo inverse.

This upper bound provides some intuitive insights on the behavior of the distortion power caused by the channel frequency selectivity. First of all, by observing the form of  $\Psi_H(\omega)$  and  $\vartheta_H(\omega)$  one can intuitively conclude that the distortion observed at the output of the receiver does not roughly depend on the channel magnitude, but on the channel variation with respect to this magnitude (a similar conclusion was drawn in [14] for the SISO channel). On the other hand, from the actual form of  $\vartheta_H(\omega)$  we see that the channel distortion is quickly aggravated when two channel eigenvalues become close to one another. This will impose the ultimate performance limit of the FMBC system with eigenvector precoding. Finally, from the last term in (8) we can also roughly conclude that the residual distortion will significantly increase when either the phase reference entry of the precoder  $\{\mathbf{v}_\ell(\omega_k)\}_{k_\ell}$  approaches zero, or the corresponding phase  $\phi_{k_\ell \ell}(\omega_k)$  experiences rapid variations. Now, given the nature of the radio channel, one typically observes that all the entries of  $\mathbf{V}(\omega)$  are strongly attenuated for at least some subcarrier (spectrum notch). This implies that if we fix the phase reference entry  $k_\ell$  to be constant for the whole spectrum, the performance will consistently drop at the subcarriers for which  $\{\mathbf{v}_\ell(\omega)\}_{k_\ell}$  becomes heavily attenuated (this will be illustrated in simulations).

In view of all this, and noting that the derived upper bound in (8) only depends on  $k_\ell$  through the last term, we propose to select this parameter entry for each  $\omega$  such that

$$k_\ell(\omega) = \arg \max_k |\{\mathbf{v}_\ell(\omega)\}_k|. \quad (12)$$

In plain words, we will take  $k_\ell(\omega)$  to be the index of the entry of the column vector  $\mathbf{v}_\ell(\omega)$  that presents highest modulus. This is an obvious way of avoiding the performance drop whenever a specific entry of  $\mathbf{v}_\ell(\omega)$  becomes small, since the fact that  $\mathbf{v}_\ell(\omega)$  has unit norm will always ensure that  $\{\mathbf{v}_\ell(\omega)\}_{k_\ell(\omega)}$  stays bounded away from zero. Having selected this entry, it now remains to determine the function  $\phi_{k_\ell\ell}(\omega)$  for every  $\omega$  so as to guarantee that all the phase functions  $\phi_{k_\ell\ell}(\omega)$ ,  $k_\ell = 1 \dots N_S$ , are continuous/differentiable for the whole range of  $\omega$ . In view of (8), our objective will be the minimization of the total mean squared variation of the reference phase over the total transmitted bandwidth, namely

$$\kappa_\ell = \int |\phi'_{k_\ell\ell}(\omega)|^2 d\omega.$$

We next explore some possible designs for the phase functions  $\phi_{k_\ell\ell}(\omega)$  in order to comply with the above objectives. We will choose to fix  $\phi_{k_\ell\ell}(\omega)$  as polynomial functions of  $\omega$  and then impose some constraints on the coefficients of these polynomials to guarantee global differentiability across the whole bandwidth. The remaining degrees of function will be optimized to achieve a minimum value of  $\kappa_\ell$  as defined above.

In order to propose a specific design for the function  $\phi_{k_\ell\ell}(\omega)$ , we assume that the index  $k_\ell(\omega)$  changes  $N$  times on the whole spectrum domain, at frequencies  $\bar{\omega}_1 < \dots < \bar{\omega}_N$ , and assume that  $\bar{\omega}_1 > 0$  (the case  $\bar{\omega}_1 = 0$  can be similarly handled, but we omit the derivations here for the sake of clarity). We can divide the interval  $[0, 2\pi]$  into  $N + 1$  non-overlapping subintervals  $I_0, \dots, I_N$  where  $I_n = [\bar{\omega}_n, \bar{\omega}_{n+1})$  for  $0 \leq n \leq N$  and where we will use the convention  $\bar{\omega}_0 = 0$  and  $\bar{\omega}_{N+1} = 2\pi$ . The index  $k_\ell(\omega)$  is constant within each interval  $I_n$ , and we will denote  $k_\ell[n] = k_\ell(\omega)$  for  $\omega \in I_n$ . We will also define, for any  $n \in \mathbb{N}$ , the  $(N + 1) \times (N + 1)$  matrix  $\Psi_n$  as

$$\Psi_n = \begin{bmatrix} (\bar{\omega}_0)^n & & & -(\bar{\omega}_{N+1})^n \\ -(\bar{\omega}_1)^n & (\bar{\omega}_1)^n & & \\ & & \ddots & \\ & & & -(\bar{\omega}_N)^n & (\bar{\omega}_N)^n \end{bmatrix} \quad (13)$$

where we understand  $(\bar{\omega}_0)^0 = 1$ . This matrix will play a fundamental role in the definition of the polynomial coefficients that specify  $\phi_{k_\ell\ell}(\omega)$ . Our objective is to determine the form of the phase function  $\phi_{k_\ell\ell}(\omega)$  for each of these intervals, while guaranteeing a certain degree of global smoothness. We will write  $\phi_{k_\ell[n]\ell}(\omega) = \arg \left( \{\mathbf{v}_\ell(\omega)\}_{k_\ell[n]} \right)$ . Observe that there is a very strong connection between  $\phi_{k_\ell[0]\ell}$  and  $\phi_{k_\ell[N+1]\ell}$ , since the phase reference entry is the same in the two associated intervals ( $I_0$  and  $I_{N+1}$ ), i.e.  $k_\ell[0] = k_\ell[N+1]$ . We will design the two associated functions  $\phi_{k_\ell[0]\ell}$ ,  $\phi_{k_\ell[N+1]\ell}$  so that they are restrictions of a more general function to these two intervals, up to a constant multiple of  $2\pi$ .

### A. Linear phase design

We consider here the simplest form of  $\phi_{k_\ell\ell}(\omega)$  that guarantees continuity of this function on the whole domain and differentiability inside the intervals  $I_n$ . We will assume that the phase ambiguity functions  $\phi_{k_\ell\ell}(\omega)$  follow a linear model within each interval  $I_n$ , namely

$$\phi_{k_\ell[n]\ell}(\omega) = a_n + b_n\omega, \quad \omega \in I_n \quad (14)$$

where  $a_n$  and  $b_n$  are real-valued parameters to be tuned for  $n = 0, \dots, N$ . Additionally, as mentioned above, we will ensure by design that  $\phi_{k_\ell[0]\ell}$  and  $\phi_{k_\ell[N+1]\ell}$  are restrictions of a general linear function of the form in (14) to the intervals  $I_0, I_{N+1}$  (up to a constant multiple of  $2\pi$ ). Assuming that  $\phi_{k_\ell[0]\ell}(\bar{\omega}_0) = \phi_{k_\ell[N+1]\ell}(\bar{\omega}_N) \pmod{2\pi}$ , this is equivalent to fixing  $b_0 = b_{N+1}$ . Now, the problem here is how to fix these parameters in order to guarantee that all the phases of  $\mathbf{v}_\ell(\omega)$  are, at least, continuous and almost everywhere differentiable.

**Remark 2** For each  $\omega \in [0, 2\pi]$ , continuity and differentiability of the phases of all the entries of the column vector  $\mathbf{v}_\ell(\omega)$  is guaranteed by continuity and differentiability of any of its entries. Indeed, for any two entries  $k, k'$ , the quantity  $\arg(\{\mathbf{v}_\ell(\omega)\}_k) - \arg(\{\mathbf{v}_\ell(\omega)\}_{k'})$  does not depend on the choice of the reference phase  $\phi_{k_\ell\ell}(\omega)$  and is therefore a continuously differentiable function of  $\omega$  [15].

Observe that the linear choice of the functions  $\phi_{k_\ell[n]\ell}(\omega)$  in (14) guarantees continuous differentiability of the phases  $\phi_{k_\ell\ell}(\omega)$  inside each of the intervals  $I_n$ ,  $n = 0, \dots, N$ . Hence, in order to ensure that the phases of all the entries of  $\mathbf{v}_\ell(\omega)$  in  $[0, 2\pi]$  it remains to impose continuity at the transition points  $\bar{\omega}_0, \dots, \bar{\omega}_N$ . We can ensure continuity of the phase reference function  $\phi_{k_\ell\ell}(\omega)$  at the transition points  $\bar{\omega}_0, \dots, \bar{\omega}_N$  by guaranteeing that

$$\lim_{\omega \rightarrow (\bar{\omega}_n)^-} \phi_{k_\ell[n]\ell}(\omega) = \lim_{\omega \rightarrow (\bar{\omega}_n)^+} \phi_{k_\ell[n]\ell}(\omega) \quad (15)$$

for<sup>2</sup>  $n = 0, \dots, N$ . Now, observe that the function  $\phi_{k_\ell[n]\ell}(\omega)$  is well defined for  $\omega \geq \bar{\omega}_n$ , because it takes the form in (14). However, for  $\omega < \bar{\omega}_n$ , this function does not need to take a linear form. In order to reformulate (15) in terms of known quantities, we point out that, for any  $\omega \in I_{n-1}$ ,

$$\phi_{k_\ell[n]\ell}(\omega) = \phi_{k_\ell[n-1]\ell}(\omega) + \arg \frac{\{\mathbf{v}_\ell(\omega)\}_{k_\ell[n]}}{\{\mathbf{v}_\ell(\omega)\}_{k_\ell[n-1]}}.$$

Observe that the second term on the right hand side does not depend on the phase normalization and can therefore be computed from  $\mathbf{H}(\omega)$ . On the other hand, the first term takes a linear form according to (15). By taking  $\omega \rightarrow (\bar{\omega}_n)^-$  in the above expression, we obtain

$$\lim_{\omega \rightarrow (\bar{\omega}_n)^-} \phi_{k_\ell[n]\ell}(\omega) = a_{n-1} + b_{n-1}\bar{\omega}_n + \arg \frac{\{\mathbf{v}_\ell(\bar{\omega}_n)\}_{k_\ell[n]}}{\{\mathbf{v}_\ell(\bar{\omega}_n)\}_{k_\ell[n-1]}}$$

for  $n = 1 \dots N + 1$ . By gathering the polynomial coefficients in (15) into two column vectors  $\mathbf{a} = [a_0, \dots, a_N]^T$ ,

<sup>2</sup>For the case  $n = 0$ , the function  $\phi_{k_\ell[0]\ell}$  should be understood as the periodic extension of itself beyond the interval  $(0, 2\pi)$ .

$\mathbf{b} = [b_0, \dots, b_N]^T$  we are able to express the conditions in (15) together with  $b_0 = b_{N+1}$  in compact form as

$$\begin{bmatrix} \Psi_0 & \Psi_1 \\ \mathbf{0} & \mathbf{u} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = \phi \bmod 2\pi \quad (16)$$

where  $\Psi_0, \Psi_1$  are defined in (13),  $\mathbf{u} = [1, 0, \dots, 0, -1]$  and where  $\phi$  is a  $(N+2) \times 1$  column vector defined as  $\{\phi\}_1 = \{\phi\}_{N+2} = 0$ , and  $\{\phi\}_{i+1} = \arg(\{\mathbf{v}_\ell(\bar{\omega}_i)\}_{k_\ell[i]}) - \arg\{\mathbf{v}_\ell(\bar{\omega}_i)\}_{k_\ell[i-1]}$ ,  $i = 1 \dots N$ . Let  $\Psi$  denote the matrix on the left hand side of (16) and write  $\mathbf{x} = [\mathbf{a}^T, \mathbf{b}^T]^T$ , so that (16) is expressed as  $\Psi\mathbf{x} = \phi \bmod 2\pi$ . The set of solutions to this underdetermined set of equations can be expressed as

$$\mathbf{x}(\mathbf{m}, \xi) = \Psi^\# (\phi + 2\pi\mathbf{m}) + \mathbf{P}_\Psi^\perp \xi \quad (17)$$

where  $\Psi^\#$  is the Moore-Penrose pseudo-inverse of  $\Psi$ ,  $\mathbf{P}_\Psi^\perp = \mathbf{I}_{2N+2} - \Psi^\# \Psi$ , and where  $\xi$  (respectively  $\mathbf{m}$ ) is any  $2(N+1)$  column vector with real-valued (resp. integer-valued) entries. The two column vectors  $\mathbf{m}$  and  $\xi$  are free parameters that can be chosen as desired in order to optimize the system performance. As mentioned above, our approach is to fix these two free vectors so as to minimize the total mean squared variation of the reference phase, which is defined as

$$\kappa_\ell = \sum_{n=0}^N \int_{\bar{\omega}_n}^{\bar{\omega}_{n+1}} \left| \phi'_{k_\ell[n]\ell}(\omega) \right|^2 d\omega = \mathbf{x}^T(\mathbf{m}, \xi) \mathbf{M} \mathbf{x}(\mathbf{m}, \xi) \quad (18)$$

where matrix  $\mathbf{M}$  is defined as

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_1 \end{bmatrix}$$

and  $\mathbf{M}_i = \text{diag}\{\bar{\omega}_n^i - \bar{\omega}_{n-1}^i, n = 0 \dots N+1\}$ . We can express the associated cost function  $\kappa_\ell$  as

$$\kappa_\ell = \left( \Psi^\# (\phi + 2\pi\mathbf{m}) + \mathbf{P}_\Psi^\perp \xi \right)^H \mathbf{M} \left( \Psi^\# (\phi + 2\pi\mathbf{m}) + \mathbf{P}_\Psi^\perp \xi \right).$$

By computing the gradient with respect to  $\xi$ , we can clearly see that a possible solution is given by

$$\xi_{\text{opt}} = - \left( \mathbf{P}_\Psi^\perp \mathbf{M} \mathbf{P}_\Psi^\perp \right)^\# \mathbf{P}_\Psi^\perp \mathbf{M} \Psi^\# (\phi + 2\pi\mathbf{m}). \quad (19)$$

Inserting this into the above expression of  $\kappa_\ell$  we end up with a cost function that only depends on the integer vector  $\mathbf{m}$ . However, optimization of this resulting cost function in terms of  $\mathbf{m}$  requires a multidimensional exhaustive search that quickly becomes computationally unfeasible as the number of phase reference transitions  $N$  increases. For this reason, we propose to use an approximate solution for  $\mathbf{m}$  that instead minimizes the norm of  $\phi + 2\pi\mathbf{m}$ . This is equivalent to choosing  $\mathbf{m}$  such that the entries of  $\phi + 2\pi\mathbf{m}$  belong to the interval  $[-\pi, \pi]$ . The resulting set of coefficients is fixed as in (17) with  $\xi$  replaced by (19).

#### B. Generalization to higher order regularity

The approach presented in above can be extended in order to achieve smoother phase reference functions  $\phi_{k_\ell\ell}(\omega)$  beyond simple continuity while increasing the total number of degrees of freedom that can be tuned to optimize performance. For example, instead of imposing continuity on the channel eigenvector phase functions, we could generate continuously

differentiable phases. To achieve this, we need to consider at least a second order model for the reference phase  $\phi_{k_\ell[n]\ell}(\omega)$ , namely  $\phi_{k_\ell\ell}(\omega) = a_n + b_n\omega + c_n\omega^2$ ,  $\omega \in I_n$ , where  $a_n, b_n$  and  $c_n$ ,  $n = 0, \dots, N$ , need to be designed to guarantee continuity of the phase derivatives at the transition points  $\bar{\omega}_1, \dots, \bar{\omega}_N$ , together with the equivalence of  $\phi_{k_\ell[0]\ell}(\omega)$  and  $\phi_{k_\ell[N]\ell}(\omega)$  up to a constant multiple of  $2\pi$ . Repeating the reasoning above, one can formulate these conditions as

$$\begin{bmatrix} \Psi_0 & \Psi_1 & \Psi_2 \\ \mathbf{0} & \Psi_0 & 2\Psi_1 \\ \mathbf{0} & \mathbf{0} & \mathbf{u} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{bmatrix} = \tilde{\phi} \bmod 2\pi$$

where  $\mathbf{c} = [c_0, \dots, c_N]^T$ ,  $\tilde{\phi} = [\phi_{1:N+1}^T, (\phi')^T, 0]^T$ , and where the  $n$ th entry of  $\phi'$  is defined as

$$\{\phi'\}_n = \phi'_{k_\ell[n+1]\ell}(\bar{\omega}_{n+1}) - \phi'_{k_\ell[n]\ell}(\bar{\omega}_{n+1}).$$

This is an underdetermined system of equations which can be solved using the same approach as in Section III-A. In particular, we choose to fix the remaining degrees of freedom in order to minimize the total variation of the squared derivative of the reference phase, i.e.  $\kappa_\ell$ . The problem is solved by fixing two free vectors  $\mathbf{m}$  (integer-valued) and  $\xi$  (real-valued) in order to minimize the cost function in (18), where in this case matrix  $\mathbf{M}$  takes the form

$$\mathbf{M} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{0} & \mathbf{M}_2 & \frac{4}{3}\mathbf{M}_3 \end{bmatrix}.$$

The problem can thus be solved as in Section III-A.

#### IV. SIMULATION RESULTS AND CONCLUSIONS

In order to evaluate the performance of the proposed method for reference phase determination, we considered here multiple random independent realizations of an Extended Vehicular A (EVA) channel model with  $N_T = 4$ ,  $N_R = 2$  and  $N_S = 1$ . The total number of subcarriers was  $M = 512$  and the inter-carrier frequency separation was fixed to 15kHz as in LTE. The symbols transmitted through each subcarrier were drawn from QPSK and 16-QAM modulations, and no channel coding was implemented. Finally, the eigenvector-based precoder was implemented following the approach outlined in Section III-B, namely the phase reference antenna was selected according to the criterion in (12) and the actual phase was constructed using second order polynomials with coefficients fixed in order to ensure the continuity of its first order derivatives. Figure 2 represents the raw symbol error rate as a function of the real signal to noise power ratio measured at each subcarrier for QPSK and 16-QAM modulations. Apart from the performance obtained with the proposed method for reference phase selection (“optimized phase”), we also represent the performance corresponding to the precoding construction that sets the first entry of the eigenvectors to be positive real-valued (“constant phase”).

We can conclude that the performance can be extraordinarily improved by carefully selecting the phase function associated with the eigenvector entry with highest modulus. In fact, the performance becomes very close to the optimum one (corresponding to the AWGN channel) for reasonable values of the SER. On the other hand, it should be pointed out that the proposed method is also affected by an error floor,

although this effect typically occurs at much lower values of the SER. This error floor is inherent to the eigenvector precoding and occurs due to the bad performance in scenarios for which two of the channel eigenvalues become close to one another, which causes the term  $\vartheta_H(\omega_k)$  in Proposition 1 to grow without bound. This effect could be avoided by proper power allocation, or by avoiding to transmit through subcarriers that present two or more similar eigenvalues.

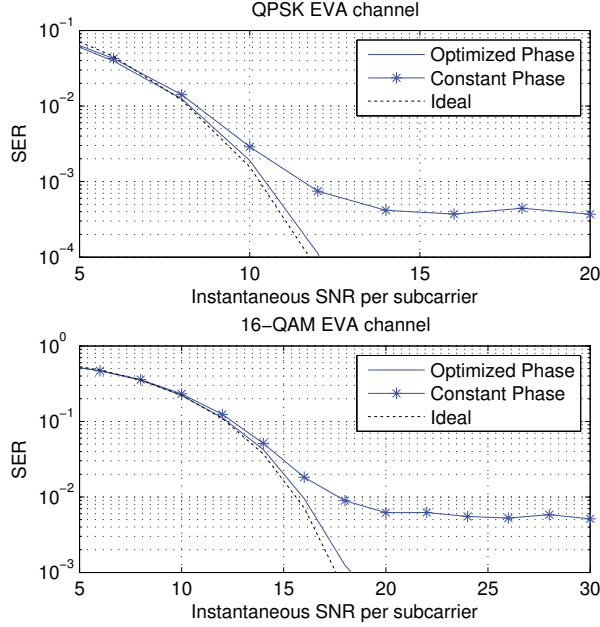


Fig. 2. Symbol error rate of the proposed eigenvector-based precoding for QPSK (up) and 16-QAM (down) with two different choices of reference phase construction: a constant phase reference with respect to the first antenna (“constant phase”) and a maximum modulus reference with second order polynomial phase (“optimized phase”).

## APPENDIX

To derive the upper bound in (8), we first observe that for any two Hermitian positive semidefinite matrices  $\mathbf{A}_1, \mathbf{A}_2$

$$\text{tr}[\mathbf{A}_1 \mathbf{A}_2] \leq \text{tr}[\mathbf{A}_1] \lambda_{\max}(\mathbf{A}_2), \quad (20)$$

where  $\lambda_{\max}(\mathbf{A}_2)$  denotes the maximum eigenvalue of  $\mathbf{B}$ . Note also that the eigenvalues of  $\{\Phi_R, \Psi_R\}$  and  $\{\Phi_I, \Psi_I\}$  are  $\text{Re}^2(\alpha_{n,n_s} \pm \beta_{n,n_s})$  and  $\text{Im}^2(\alpha_{n,n_s} \pm \beta_{n,n_s})$  respectively. Using this, together with Jensen’s inequality and the fact that  $\text{tr}[\mathcal{R}\mathcal{R}^T \mathbf{U}^+ + \mathcal{S}\mathcal{S}^T \mathbf{U}^-] = \text{tr}[\mathcal{R}\mathcal{R}^T \mathbf{U}^- + \mathcal{S}\mathcal{S}^T \mathbf{U}^+]$  for symmetric pulses [14], we can readily obtain

$$\eta_{n,n_s}(k) \leq 2P_s \left( |\alpha_{n,n_s}|^2 + |\beta_{n,n_s}|^2 \right) \zeta_p \quad (21)$$

where  $\zeta_p$  is defined in (9). The upper bound in (21) can be simplified by noting that, using the definition of  $\mathbf{A}(\omega)$  and  $\mathbf{B}(\omega)$  above together with (1), (20) and Jensen’s inequality, we may write

$$\sum_{n=1}^{N_s} \sum_{n_s=1}^{N_s} |\beta_{n,n_s}|^2 \leq \frac{4}{M^2} \Psi_H(\omega_k) + 2 \sum_{n=1}^{N_s} \sum_{n_s=1}^{N_s} \alpha_{n,n_s}^2 \quad (22)$$

where  $\Psi_H(\omega)$  is defined in (10). On the other hand, using the expression of  $\alpha_{n,n_s}$  together with (7) we obtain

$$\sum_{n=1}^{N_s} \sum_{n_s=1}^{N_s} |\alpha_{n,n_s}|^2 = \frac{2}{M^2} \left( \vartheta_H(\omega_k) + \sum_{\ell=1}^{N_s} |\{\mathbf{D}(\omega_k)\}_{\ell\ell}|^2 \right) \quad (23)$$

where  $\vartheta_H(\omega)$  is defined in (11). Finally, the second term of the above equation can easily be bounded using, once again, Jensen and Cauchy-Schwarz inequalities so that

$$|\{\mathbf{D}(\omega_k)\}_{\ell\ell}|^2 \leq 2 |\phi'_{k\ell}(\omega_k)|^2 + 2 \frac{\vartheta_H(\omega_k)}{|\{\mathbf{v}_\ell(\omega_k)\}_{k\ell}|^2}. \quad (24)$$

Inserting all these bounds in the definition of  $P_e(k)$ , we obtain the result of this proposition.

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