

Irregular Repetition Slotted ALOHA over the Rayleigh Block Fading Channel with Capture

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Abstract—Random access protocols relying on the transmission of packet replicas in multiple slots and exploiting interference cancellation at the receiver have been shown to achieve performance competitive with that of orthogonal schemes. So far the optimization of the repetition degree profile, defining the probability for a user to transmit a given number of replicas, has mainly been performed targeting the collision channel model. In this paper the analysis is extended to a block fading channel model, also assuming capture effect at the receiver. Density evolution equations are developed for the new setting and, based on them, some repetition degree profiles are optimized and analyzed via Monte Carlo simulation in a finite frame length setting. The derived distributions are shown to achieve throughputs largely exceeding 1 [packet/slot].

I. INTRODUCTION

Framed slotted ALOHA (FSA) [1] is a random access scheme in which link-time is divided in frames and frames consist of slots. Users are frame- and slot-synchronized; a user contends by transmitting its packet in a randomly chosen slot of the frame. For the collision channel model, the maximum value of the expected asymptotic throughput of FSA is $1/e$ [packet/slot].

A substantial throughput gain can be achieved by modifying FSA such that (i) the users transmit replicas of their packets in several slots chosen at random, where each replica embeds pointers to slots containing the other replicas, and (ii) when a replica has been decoded at the receiver, all the other replicas are removed through the use of interference cancellation (IC) [2], [3]. Specifically, it has been shown that this enhanced version of FSA, named irregular repetition slotted Aloha (IRSA) [3], bears striking similarities to the erasure-correcting coding framework in which the decoding is performed with iterative belief-propagation. These insights were followed by a strand of works applying various concepts and tools of erasure-coding theory to design slotted ALOHA-based schemes, commonly referred to as coded slotted ALOHA.¹ The overall conclusion is that coded slotted ALOHA schemes can asymptotically achieve the expected throughput of 1 [packet/slot], which is the ultimate limit for the collision channel model.

On the other hand, the collision channel model is a rather simple one, which assumes that (i) noise can be neglected,

such that a transmission can be decoded from a singleton slot by default, and (ii) no transmission can be decoded from a collision slot. This model has a limited practical applicability and does not describe adequately the wireless transmission scenarios where the impact of fading and noise cannot be neglected. In particular, fading may incur power variations among signals observed in collisions slot, allowing for the *capture effect* to occur, when sufficiently strong signals may be decoded. In the context of slotted ALOHA, numerous works assessed the performance of the scheme for different capture effect models [5]–[10]. One of the standardly used models is the threshold-based one, in which a packet is captured, i.e., decoded, if its signal-to-interference-and-noise ratio (SINR) is higher than a predefined threshold, c.f. [8]–[11].

A brief treatment of the capture effect in IRSA framework was made in [3], pointing out the implications related to the asymptotic analysis. In [12], the method for the computation of capture probabilities for the threshold-based model in single-user detection systems with Rayleigh fading was presented and instantiated for the frameless ALOHA framework [13].

In this paper, we extend the treatment of the threshold-based capture effect for IRSA framework. First, we derive the exact expressions of capture probabilities for the threshold-based model and Rayleigh block-fading channel. Next we formulate the asymptotic performance analysis. We then optimize the scheme, in terms of deriving the optimal repetition strategies that maximize throughput given a target packet loss rate (PLR). Finally, the obtained distributions are investigated in the finite frame length scenario via simulations. We show that IRSA exhibits a remarkable throughput performance that is well over 1 [packet/slot], for target PLR, SNR and threshold values that are valid in practical scenarios. This is demonstrated both for asymptotic and finite frame length cases, showing also that the finite-length performance indeed tends to the asymptotic one as the frame length increases.

The paper is organized as follows. Section II introduces the system model. Section III deals with derivation of capture probabilities for Rayleigh block-fading channels, which are used in Section IV to evaluate the asymptotic performance of IRSA. The study of optimal repetition strategies is done in Section V, including their characterization in the finite-frame length case. Section VI concludes the paper.

¹We refer the interested reader to [4] for an overview.

II. SYSTEM MODEL

A. Access Protocol

For the sake of simplicity, we focus on a single batch arrival of m users each having a single packet (or *burst*) and contending for the access to the common receiver. The link time is organized in a medium access control (MAC) frame of duration T_F , divided into n slots of equal duration $T_S = T_F/n$, indexed by $j \in \{1, 2, \dots, n\}$. The transmission time of each packet equals the slot duration. The system load G is defined as

$$G = \frac{m}{n} [\text{packet/slot}].$$

According to the IRSA protocol, each user selects a repetition degree d by sampling a probability mass function (p.m.f.) $\{\Lambda_d\}_{d=2}^{d_{\max}}$ and transmits d identical replicas of its burst in d randomly chosen slots of the frame. It is assumed that the header of each burst replica carries information about the locations (i.e., slot indexes) of all d replicas. The p.m.f. $\{\Lambda_d\}$ is the same for all users and is sampled independently by different users, in an uncoordinated fashion. The average burst repetition degree is $\bar{d} = \sum_{d=2}^{d_{\max}} d \Lambda_d$, and its inverse

$$R = \frac{1}{\bar{d}} \quad (1)$$

is called the *rate* of the IRSA scheme. Each user is then unaware of the repetition degree employed by the other users contending for the access. The number of burst replicas colliding in slot j is denoted by $c_j \in \{1, 2, \dots, m\}$. Burst replicas colliding in slot j are indexed by $i \in \{1, 2, \dots, c_j\}$.

B. Received Power and Fading Models

We consider a Rayleigh block fading channel model, i.e., fading is Rayleigh distributed, constant and frequency flat in each block, while it is independent and identically distributed (i.i.d.) on different blocks. Independent fading between different burst replicas is also assumed. In this way, the power of a burst replica $i \in \{1, 2, \dots, c_j\}$ received in slot j , denoted as P_{ij} , is modeled as a random variable (r.v.) with negative exponential distribution

$$f_P(p) = \begin{cases} \frac{1}{\bar{P}} \exp\left[-\frac{p}{\bar{P}}\right], & p \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where \bar{P} is the average received power. This is assumed to be the same for all burst replicas received in the MAC frame by using, e.g., a long-term power control. The r.v.s P_{ij} are i.i.d. for all (i, j) pairs. If we denote by N the noise power, the signal-to-noise ratio (SNR) r.v. $B_{ij} = P_{ij}/N$ is also exponentially distributed as

$$f_B(b) = \begin{cases} \frac{1}{\bar{B}} \exp\left[-\frac{b}{\bar{B}}\right], & b \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where the average SNR is given by

$$\bar{B} = \frac{\bar{P}}{N}.$$

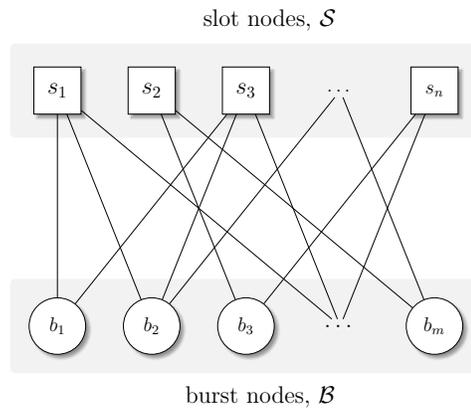


Fig. 1. Graph representation of MAC frame.

C. Graph Representation

In order to analyze the successive interference cancellation (SIC) process, we introduce the graph representation of a MAC frame [3]. As depicted in Fig. 1, a MAC frame is represented as a bipartite graph $\mathcal{G} = (\mathcal{B}, \mathcal{S}, E)$ consisting of a set \mathcal{B} of m burst nodes (or user nodes), one for each user, a set \mathcal{S} of n slot nodes, one per slot, and a set E of edges, one per transmitted burst replica. A burst node $b_k \in \mathcal{B}$ is connected to a slot node $s_j \in \mathcal{S}$ if and only if user k has a burst replica sent in the j -th slot of the frame. The *node degree* represents the number of edges emanating from a node.

For the upcoming analysis it is convenient to resort to the concept of *node- and edge-perspective degree distributions*. The burst node degree distribution from a node perspective is identified by the above-defined p.m.f. $\{\Lambda_d\}_{d=2}^{d_{\max}}$. Similarly, the slot node degree distribution from a node perspective is defined as $\{P_c\}_{c=0}^m$, where P_c is the probability that a slot node has c connections (i.e., that c burst replicas have been received in the corresponding slot). The probability P_c may be easily calculated by observing that $(G/R)/m$ is the probability that the generic user transmits a burst replica in a specific slot. Since users behave independently of each other, we obtain

$$P_c = \binom{m}{c} \left(\frac{G/R}{m}\right)^c \left(1 - \frac{G/R}{m}\right)^{m-c}.$$

The polynomial representations for both node-perspective degree distributions are given by

$$\Lambda(x) = \sum_{d=2}^{d_{\max}} \Lambda_d x^d \quad \text{and} \quad P(x) = \sum_{c=0}^m P_c x^c.$$

For $m \rightarrow \infty$ and constant G/R , $P(x) = \exp\left\{-\frac{G}{R}(1-x)\right\}$. Degree distributions can also be defined from an edge-perspective. Adopting a notation similar to the one used for the node-perspective distributions, we define the edge-perspective burst node degree distribution as the p.m.f. $\{\lambda_d\}_{d=2}^{d_{\max}}$, where λ_d is the probability that a given edge is connected to a burst node of degree d . Likewise, we define the edge-perspective slot node degree distribution as the p.m.f. $\{\rho_c\}_{c=0}^m$, where ρ_c is the probability that an edge is connected to a slot node of

degree c . From the definitions we have $\lambda_d = d \Lambda_d / (\sum_t t \Lambda_t)$ and $\rho_c = c P_c / (\sum_t t P_t)$; it can be shown that, for $m \rightarrow \infty$ and constant G/R , $\rho_c = \exp\{-G/R\} (G/R)^{c-1} / (c-1)!$. The corresponding polynomial representation are $\lambda(x) = \sum_{d=2}^{d_{\max}} \lambda_d x^{d-1}$ and $\rho(x) = \sum_{c=0}^m \rho_c x^{c-1}$. Note that $\lambda(x) = \Lambda'(x)/\Lambda'(1)$ and $\rho(x) = P'(x)/P'(1)$.²

D. Receiver Operation

In our model, the receiver is always able to detect burst replicas received in a slot, i.e., to discriminate between an empty slot where only noise samples are present and a slot in which at least one burst replica has been received. We assume that the receiver is able to obtain perfect channel state information, also in the slots undergoing packet collisions. Moreover, a threshold-based capture model for the receiver is assumed, by which the generic burst replica i is successfully decoded (i.e., captured) in slot j if the SINR exceeds a certain threshold b^* , namely,

$$\Pr\{\text{burst replica } i \text{ decoded}\} = \begin{cases} 1, & \frac{P_{ij}}{N+I_{ij}} \geq b^* \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

The quantity I_{ij} in (2) denotes the power of the interference impairing replica i in slot j . In our system model the threshold b^* fulfills $b^* \geq 1$, which corresponds to a conventional narrowband single-antenna system. As we are considering a SIC-based receiver, under the assumption of perfect IC I_{ij} is equal to sum of the powers of those bursts that have not yet been cancelled from slot j in previous iterations (apart from burst i). Specifically,

$$I_{ij} = \sum_{u \in \mathcal{R}_j \setminus \{i\}} P_{uj} \quad (3)$$

where P_{uj} is the power of burst replica u not yet cancelled in slot j and \mathcal{R}_j denotes the set of remaining burst replicas in slot j . Exploiting (3), after simple manipulation we obtain

$$\frac{P_{ij}}{N+I_{ij}} = \frac{B_{ij}}{1 + \sum_{u \in \mathcal{R}_j \setminus \{i\}} B_{uj}}.$$

Hence, in the adopted threshold-based capture model the condition $\frac{P_{ij}}{N+I_{ij}} \geq b^*$ in (2) may be recast as

$$\frac{B_{ij}}{1 + \sum_{u \in \mathcal{R}_j \setminus \{i\}} B_{uj}} \geq b^*. \quad (4)$$

When processing the signal received in some slot j , if burst replica i is successfully decoded due to fulfillment of (4), then (i) its contribution of interference is cancelled from slot j , and (ii) the contributions of interference of all replicas of the same burst are removed from the corresponding slots.³ Hereafter, we refer to the former part of the IC procedure as *intra-slot* IC and to the latter as *inter-slot* IC. Unlike SIC in IRSA protocols over a collision channel, which only rely on inter-slot IC, SIC over a block fading channel with capture takes advantage of intra-slot IC to potentially decode burst replicas interfering each

other in the same slot. In this respect, it effectively enables *multi-user decoding* in the slot.

Upon reception of a new MAC frame, slots are processed sequentially by the receiver. By definition, one *SIC iteration* consists of the sequential processing of all n slots. In each slot, intra-slot IC is performed repeatedly, until no burst replicas exist for which (4) is fulfilled. When all burst replicas in slot j have been successfully decoded, or when intra-slot IC in slot j stops prematurely, inter-slot IC is performed for all burst replicas successfully decoded in slot j and the receiver proceeds to process slot $j+1$. When all n slots in the MAC frame have been processed there are three possible cases: (1) a success is declared if all user packets have been successfully received; (2) a new iteration is started if at least one user packet has been recovered during the last iteration, its replicas removed via inter-slot IC, and there still are slots with interfering burst replicas; (3) a failure is declared if no user packets have been recovered during the last iteration and there still are slots with interfering burst replicas, or if a maximum number of SIC iterations has been reached and there still are slots with interfering burst replicas.

Exploiting the graphical representation reviewed in Section II-C, the SIC procedure performed at the receiver may be described as a successive removal of graph edges. Whenever a burst replica is successfully decoded in a slot, the corresponding edge is removed from the bipartite graph as well as, due to inter-slot IC, all edges connected to the same burst node. A success in decoding the MAC frame occurs when all edges are removed from the bipartite graph. Two important features pertaining to the receiver operation, when casted into the graph terms, should be remarked. The first one is that, due to capture effect, an edge may be removed from the graph when it is connected to a slot node with residual degree larger than one. The second one is that an edge connected to a slot node with residual degree one may not be removed due to poor SNR, when (4), with $\mathcal{R}_j \setminus \{i\} = \emptyset$, is not fulfilled.

III. DECODING PROBABILITIES

Consider the generic slot node j at some point during the decoding of the MAC frame and assume it has degree r under the current graph state. This means that r could be the original slot node degree c_j or the residual degree after some inter-slot and intra-slot IC processing. Note that, as we assume perfect IC, the two cases $r = c_j$ and $r < c_j$ are indistinguishable.

Among the r burst replicas not yet decoded in slot j , we choose one randomly and call it the reference burst replica. Moreover, we denote by $D(r)$ the probability that the reference burst replica is decoded starting from the current slot setting and only running intra-slot IC within the slot. As we are considering system with $b^* \geq 1$, the threshold based criterion (4) can be satisfied only for one single burst replica at a time. Therefore there may potentially be r decoding steps (and $r-1$ intra-slot IC steps), in order to decode the reference burst replica. Letting $D(r, t)$ be the probability that the reference burst replica is successfully decoded in step t and not in any

²Notation $f'(x)$ denotes the derivative of $f(x)$.

³We assume that the receiver is able to estimate the channel coefficients required for the removal of the replicas.

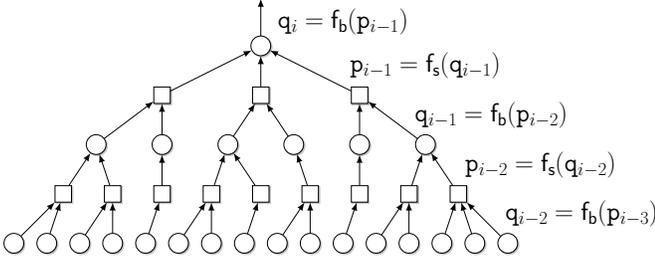


Fig. 2. Tree representation of the MAC frame.

step prior to step t , we may write

$$D(r) = \sum_{t=1}^r D(r, t).$$

Now, with a slight abuse of the notation, label the r burst replicas in the slot from 1 to r , arranged such that: (i) the first $t-1$ are arranged by their SNRs in the descending order (i.e., $B_1 \geq B_2 \geq \dots \geq B_{t-1}$), (ii) the rest have SNR lower than B_{t-1} but do not feature any particular SNR arrangement among them, (iii) the reference burst is labeled by t , i.e., its SNR by B_t , and (iv) the remaining $r-t$ bursts are labeled arbitrarily. The probability of having at least t successful burst decodings through successive intra-slot IC for such an arrangement is

$$\begin{aligned} & \Pr \left\{ \frac{B_1}{1 + \sum_{i=2}^r B_i} \geq b^*, \dots, \frac{B_t}{1 + \sum_{i=t+1}^r B_i} \geq b^* \right\} \\ &= \frac{1}{B^r} \int_0^\infty db_r \cdots \int_0^\infty db_{t+1} \\ & \times \int_{b^*(1 + \sum_{i=t+1}^r b_i)}^\infty db_t \cdots \int_{b^*(1 + \sum_{i=2}^r b_i)}^\infty db_1 e^{-\frac{b_r}{B}} \cdots e^{-\frac{b_1}{B}} \\ &= \frac{e^{-\frac{b^*}{B} \sum_{i=0}^{t-1} (1+b^*)^i}}{(1+b^*)^{t(r-\frac{t+1}{2})}} = \frac{e^{-\frac{1}{B}((1+b^*)^t - 1)}}{(1+b^*)^{t(r-\frac{t+1}{2})}}. \end{aligned}$$

Further, the number of arrangements in which the SNR of the reference replica is not among first $(t-1)$ largest is $\frac{(r-1)!}{(r-t)!}$, where it is assumed that all arrangements are a priori equally likely. Thus, the probability that the reference burst is decoded exactly in the t -th step is

$$D(r, t) = \frac{(r-1)!}{(r-t)!} \frac{e^{-\frac{1}{B}((1+b^*)^t - 1)}}{(1+b^*)^{t(r-\frac{t+1}{2})}}, \quad 1 \leq t \leq r. \quad (5)$$

We conclude this section by noting that $D(1) = e^{-\frac{b^*}{B}} \leq 1$, i.e., a slot of degree 1 is decodable with probability that may be less than 1 and that depends on the ratio of the capture threshold and the expected SNR. Again, this holds both for slots whose original degree was 1 and for slots whose degree was reduced to 1 via IC, as these two cases are indistinguishable when the IC is perfect.

IV. DENSITY EVOLUTION ANALYSIS AND DECODING THRESHOLD DEFINITION

In this section, we apply the technique of density evolution (DE) in order to evaluate asymptotic performance of the proposed technique, i.e., when $m \rightarrow \infty$ and $n \propto m$. For this

purpose, we unfold the graph representation of the MAC frame (Fig. 1) into a tree, choosing a random burst node as its root, as depicted in Fig. 2. The evaluation is performed in terms of probabilities that erasure messages are exchanged over the edges of the graph, where the erasure message denotes that the associated burst is not decoded.⁴ The message exchanges are modeled as successive (i.e., iterative) process, corresponding to the decoding algorithm described in Section II-D in the asymptotic case, when the lengths of the loops in the graph tends to infinity. Specifically, the i -th iteration consists of the update of the probability q_i that an edge carries an erasure message from a burst node to a slot node, followed by the update of the probability p_i that an edge carries an erasure message from a slot node to a burst node. These probabilities are averaged over all edges in the graph. We proceed by outlining the details.

The probability that an edge carries an erasure message from burst nodes to slot nodes in i -th iteration is

$$q_i = \sum_{d=1}^{d_{\max}} \lambda_d q_i^{(d)} = \sum_{d=1}^{d_{\max}} \lambda_d p_{i-1}^{d-1} =: f_b(p_{i-1}) \quad (6)$$

where λ_d is the probability that an edge is connected to a burst node of degree d (see Section II-C) and $q_i^{(d)}$ is the probability that an edge carries an erasure message given that it is connected to a burst node of degree d . In the second equality we used the fact that the an outgoing message from a burst node carries an erasure only if all incoming edges carry an erasure, i.e., $q_i^{(d)} = p_{i-1}^{d-1}$.

Similarly, the probability that an edge carries an erasure message from SNs to BNs in i -th iteration is

$$p_i = \sum_{c=1}^{+\infty} \rho_c p_i^{(c)} \quad (7)$$

where ρ_c is the probability that an edge is connected to a slot node of degree c , and where $p_i^{(c)}$ is the probability an edge carries an erasure message given that it is connected to a slot node of degree c . This probability may be expressed as

$$p_i^{(c)} = 1 - \sum_{r=1}^c D(r) \binom{c-1}{r-1} q_i^{r-1} (1-q_i)^{c-r}, \quad (8)$$

where summation is done over all possible values of the reduced degree r , i.e., $1 \leq r \leq c$, and where the term $\binom{c-1}{r-1} q_i^{r-1} (1-q_i)^{c-r}$ corresponds to the probability that the degree of the slot node is reduced to r and $D(r)$ is the probability that the burst corresponding to the outgoing edge is decoded when the (reduced) degree of the slot node is r .⁵

⁴For a more detailed introduction to the DE, we refer the interested reader to [14].

⁵As in the asymptotic case the loops in the graph are assumed to be of infinite length, such that the tree representation in Fig. 2 holds, the reduction of the slot degree happens only via inter-slot IC, which is implicitly assumed in the term $\binom{c-1}{r-1} q_i^{r-1} (1-q_i)^{c-r}$. On the other hand, $D(r)$ expresses the probability that an outgoing edge from the slot node is decoded using intra-slot IC (see Section III). In other words, inter- and intra-slot IC are in the asymptotic evaluation separated over DE iterations.

Combining (8) and the expression for the edge-oriented slot-node degree distribution (see Section II-C) into (7) yields

$$p_i = 1 - e^{-\frac{G}{\bar{R}}} \sum_{c=1}^{\infty} \left(\frac{G}{\bar{R}}\right)^{c-1} \sum_{r=1}^c \frac{D(r)}{(r-1)!} q_i^{r-1} (1-q_i)^{c-r}. \quad (9)$$

It can be shown that in case of perfect IC, (9) becomes

$$\begin{aligned} p_i &= 1 - e^{-\frac{G}{\bar{R}}} \sum_{r=1}^{+\infty} \frac{D(r)}{(r-1)!} \left(\frac{G}{\bar{R}} q_i\right)^{r-1} \sum_{c=0}^{+\infty} \frac{\left(\frac{G}{\bar{R}}(1-q_i)\right)^c}{c!} \\ &= 1 - e^{-\frac{G}{\bar{R}} q_i} \sum_{r=1}^{+\infty} \frac{D(r)}{(r-1)!} \left(\frac{G}{\bar{R}} q_i\right)^{r-1}, \end{aligned} \quad (10)$$

where $D(r) = \sum_{t=1}^r D(r, t)$, see (5). Further, defining $z_t = (1 + b^*)^t$, (10) becomes

$$\begin{aligned} p_i &= 1 - e^{-\frac{G}{\bar{R}} q_i} \sum_{r=1}^{+\infty} \left(\frac{G}{\bar{R}} q_i\right)^{r-1} \sum_{t=1}^r \frac{e^{-\frac{1}{\bar{B}}(z_t-1)}}{(r-t)! z_t^{r-\frac{t+1}{2}}} \\ &= 1 - e^{-\frac{G}{\bar{R}} q_i} \sum_{t=1}^{+\infty} \frac{\left(\frac{G}{\bar{R}} q_i\right)^{t-1}}{z_t^{\frac{t-1}{2}}} e^{-\frac{1}{\bar{B}}(z_t-1)} \sum_{r=0}^{+\infty} \frac{\left(\frac{G}{\bar{R}} q_i\right)^r}{r! z_t^r} \\ &= 1 - \sum_{t=1}^{+\infty} \frac{\left(\frac{G}{\bar{R}} q_i\right)^{t-1}}{z_t^{\frac{t-1}{2}}} e^{-\frac{1}{\bar{B}}(z_t-1)} \left(\frac{1}{\bar{B}} + \frac{G}{z_t} q_i\right) =: f_5(q_i). \end{aligned} \quad (11)$$

A DE recursion is obtained combining (6) with (11), consisting of one recursion for q_i and one for p_i . In the former case, the recursion assumes the form $q_i = (f_b \circ f_s)(q_{i-1})$ for $i \geq 1$, with initial value $q_0 = 1$. In the latter case, it assumes the form $p_i = (f_s \circ f_b)(p_{i-1})$ for $i \geq 1$, with initial value $p_0 = f_s(1)$. Note that the DE recursion for p_i allows expressing the asymptotic PLR of an IRSA scheme in a very simple way. More specifically, let $p_\infty(G, \{\Lambda_d\}, \bar{B}, b^*) = \lim_{i \rightarrow \infty} p_i$ be the limit of the DE recursion, where we have explicitly indicated that the limit depends on the system load, on the burst node degree distribution, on the average SNR, and on the threshold for successful intra-slot decoding. Since $[p_\infty(G, \{\Lambda_d\}, \bar{B}, b^*)]^d$ represents the probability that a user packet associated with a burst node of degree d is not successfully received at the end of the decoding process, the asymptotic PLR is given by

$$\text{PLR}(G, \{\Lambda_d\}, \bar{B}, b^*) = \sum_{d=2}^{d_{\max}} \Lambda_d [p_\infty(G, \{\Lambda_d\}, \bar{B}, b^*)]^d.$$

Next, we introduce the concept of *asymptotic decoding threshold* for an IRSA scheme over the considered block fading channel model and under the decoding algorithm described in Section II-D. Let $\bar{\text{PLR}}$ be a target PLR. Then, the asymptotic decoding threshold, denoted by $G^* = G^*(\{\Lambda_d\}, \bar{B}, b^*, \bar{\text{PLR}})$, is defined as the supremum system load value for which the target PLR is achieved in the asymptotic setting:

$$G^* = \sup_{G \geq 0} \{G : \text{PLR}(G, \{\Lambda_d\}, \bar{B}, b^*) < \bar{\text{PLR}}\}.$$

V. NUMERICAL RESULTS

Table I shows some degree distributions designed combining the DE analysis developed in Section IV with the differential evolution optimization algorithm proposed in [15]. For each design we set $\bar{\text{PLR}} = 10^{-2}$, $\bar{B} = 20$ dB, and $b^* = 3$ dB,

TABLE I
OPTIMIZED USER NODE DEGREE DISTRIBUTION AND CORRESPONDING THRESHOLD G^* FOR $\bar{\text{PLR}} = 10^{-2}$.

\bar{d}	Distribution $\Lambda(x)$	G^*
4	$\Lambda_1(x) = 0.59x^2 + 0.27x^3 + 0.02x^5 + 0.12x^{16}$	1.863
3	$\Lambda_2(x) = 0.61x^2 + 0.25x^3 + 0.03x^6 + 0.02x^7 + 0.07x^8 + 0.02x^{10}$	1.820
2.5	$\Lambda_3(x) = 0.66x^2 + 0.16x^3 + 0.18x^4$	1.703
2.25	$\Lambda_4(x) = 0.65x^2 + 0.33x^3 + 0.02x^4$	1.644
4	$\Lambda_5(x) = 0.49x^2 + 0.25x^3 + 0.01x^4 + 0.03x^5 + 0.13x^6 + 0.01x^{13} + 0.02x^{14} + 0.06x^{16}$	1.734

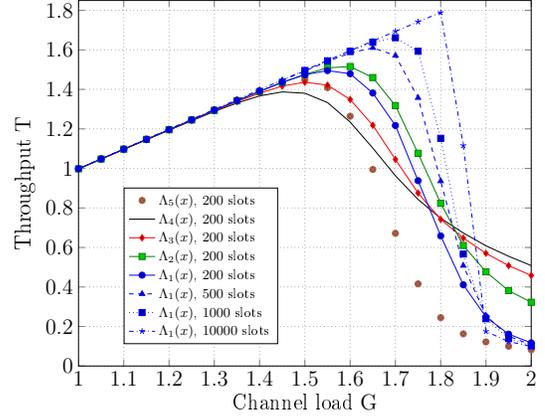


Fig. 3. Throughput values achieved by the burst node distributions in Table I, versus the channel load. Various frame sizes n , $\bar{B} = 20$ dB, $b^* = 3$ dB.

and we constrained the optimization algorithm to find the distribution $\{\Lambda_d\}$ with the largest threshold G^* subject to a given average degree \bar{d} and maximum degree $d_{\max} = 16$.

For all chosen average degrees, the G^* threshold of the optimized distribution largely exceeds the value 1 [packet/slot], the theoretical limit under a collision channel model. In general, the higher the average degree \bar{d} , the larger is the load threshold G^* . However, as \bar{d} increases, more complex burst node distributions are obtained. For instance, under a $\bar{d} = 4$ constraint, the maximum degree is $d_{\max} = 16$ (i.e., a user may transmit up to 16 copies of its packet); when reducing \bar{d} , the optimization converges to degree distributions with a lower maximum degree and degree-2 nodes become increasingly dominant.

To assess the effectiveness of the proposed design approach, tailored to the block fading channel with capture, we optimized a distribution $\Lambda_5(x)$ using the DE recursion over the collision channel [3] and again constraining the optimization to $\bar{d} = 4$ and $d_{\max} = 16$. As from Table I, due to the mismatched channel model, a 7% loss in terms of G^* threshold is observed w.r.t. the distribution $\Lambda_1(x)$ that fulfills the same constraints but was obtained with the DE developed in this paper.

We tested the optimized distributions performance through Monte Carlo simulations for finite frame lengths, setting $n = 200$ (unless otherwise stated), $\bar{B} = 20$ dB, $b^* = 3$

⁶A constraint on the average degree \bar{d} can be turned into a constraint on the rate R as there is a direct relation between the two; see also equation (1).

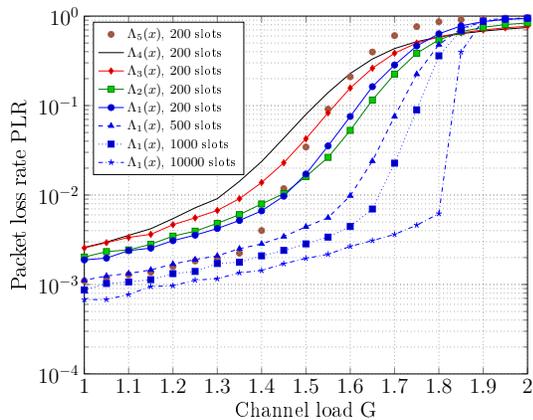


Fig. 4. PLR values achieved by the burst node distributions in Table I, versus the channel load. Various frame sizes n , $\bar{B} = 20$ dB, $b^* = 3$ dB.

dB, and the maximum number of IC iterations to 20. Fig. 3 illustrates the throughput T , defined as the average number of successfully decoded packets per slot, versus the channel load. For relatively short frames of $n = 200$, all distributions exhibit peak throughputs exceeding 1 [packet/slot]. The distribution $\Lambda_2(x)$ is the one achieving the highest throughput of 1.52 [packet/slot] although $\Lambda_1(x)$ is the distribution with the highest threshold G^* . It is important to recall that, the threshold is computed for a target PLR of $\text{PLR} = 10^{-2}$ while considering finite frame lengths, the threshold effect on the PLR tends to vanish and a more graceful degradation of the PLR curve as the channel load increases is expected. Moreover, as there are user nodes transmitting as high as 16 replicas, the distribution $\Lambda_1(x)$ is more penalized for short frame sizes w.r.t. to $\Lambda_2(x)$ where at most 10 replicas per user node are sent.⁷ This effect, coupled with the fact that the threshold G^* of $\Lambda_1(x)$ is only slightly better than the one of $\Lambda_2(x)$, explains the peak throughput behavior. To investigate the benefit of larger frames, we selected $\Lambda_1(x)$ and we increased the frame size up to 10000 slots. As expected, the peak throughput is greatly improved from 1.49 to 1.79 [packet/slot], i.e., 20% of gain. The PLR performance is illustrated in Fig. 4. Coherently with the optimization results, the $\Lambda_1(x)$ distribution achieves $\text{PLR} = 10^{-2}$ for values of the channel load slightly larger than the ones required by $\Lambda_2(x)$. Indeed, the steeper PLR curve of $\Lambda_1(x)$ is the reason for the slightly larger peak throughput of $\Lambda_2(x)$ observed in Fig. 3. Finally, as expected, an increase of the number of slots per frame yields an increase of the channel load for which the target PLR is achieved.

VI. CONCLUSIONS

The asymptotic analysis of IRSA access schemes, assuming both a Rayleigh block fading channel and capture effect, was presented in the paper. We derived the decoding probability of a burst replica in presence of intra-slot IC. The DE analysis is modified considering the Rayleigh block fading channel model, and the user/slot nodes updates of the iterative

⁷Results for frame size of 500 slots, not presented in the figures, show that the peak throughput for $\Lambda_1(x)$ is 1.61 while for $\Lambda_2(x)$ is 1.60.

procedure are explicitly derived. Due to the presence of fading, the optimization procedure target has been modified as well. The distribution able to achieve the highest channel load value without exceeding a properly defined PLR target is selected. We designed some degree distributions with different values of average degree. Remarkably, all of them present a load threshold that guarantees PLR below 10^{-2} for values well above 1 [packet/slot]. The best distribution exceeds 1.8 [packet/slot]. The derived distributions were shown to perform well also for finite frame durations. In a frame with 200 slots, the peak throughput exceeds 1.5 [packet/slot] and up to 1.45 [packet/slot] the PLR remains below 10^{-2} .

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