Physical Layer Security Enhancement in Multi-User Multi-Full-Duplex-Relay Networks

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***Abstract*—We propose a novel joint user and full-duplex (FD) relay selection (JUFDRS) scheme to enhance physical layer security in a multi-user multi-relay network. In this scheme, the user and the FD decode-and-forward relay are selected such that the capacity of the end-to-end channel (i.e., the user-relay-destination channel) is maximized to ensure the highest quality of cooperative transmission. In order to fully examine the benefits of the JUFDRS scheme, we derive a new closed-form expression for the secrecy outage probability. We show that the JUFDRS scheme significantly outperforms the joint user and half-duplex relay selection (JUHDRS) scheme when the self-interference at the FD relay can be reasonably suppressed. This result indicates that adopting the FD technique at relays can effectively enhance the physical layer secrecy performance in the multi-user multi-relay network.**

I. Introduction

The security of wireless communication is a pivotal issue that needs to be addressed in the future ubiquitous wireless world. As a complimentary approach to the traditional cryptographic techniques, physical layer security has been recognized as a key solution to safeguarding wireless data transmissions and therefore, attracted numerous research interest due to its unique advantages [1–3].

Recently, relay-aided physical layer security has been examined to facilitate the secrecy enhancement in cooperative wireless networks, e.g., [4–9]. Among different relay-aided techniques, relay selection [5–7] has been acknowledged as a promising technique in both amplify-and-forward (AF) and decode-and-forward (DF) relay networks. In AF relay networks, relay simultaneously amplifies the information signal and noise as well as interference. In this context, the intercept probability of the multi-relay network was derived in [5] and the opportunistic relaying technique was used to achieve the full diversity gain. Also, the secrecy outage probability (SOP) of the AF relay network with multiple users and multiple half-duplex (HD) relays was examined in [7], in which several joint user and HD relay selection (JUHDRS) schemes were proposed to enhance physical layer security. Differing from AF relay networks, the relay in DF relay networks first decodes the received signals and then retransmits the recovered signals to the destination. The secrecy performance of the DF relay network was studied in [5], where opportunistic relaying was used to exploit the channel fluctuation among relays, which achieves the full diversity gain of the network.

In the aforementioned studies, the relay is operating at the HD mode, which suffers from spectral inefficiency due to the separate listening and retransmitting phases. Fortunately, full-duplex (FD) technique, which was previously considered as impractical due to the limited performance of self-interference cancelation techniques, has now been proved to be feasible in practice to offer spectrum efficiency. Against this background, the secrecy performance of the FD relay network was evaluated in the literature. For example, the secrecy rate achieved by FD relays subject to a total transmit power constraint was derived in [13]. In [10], the secrecy performance of a FD relay network was investigated in terms of the SOP, where a FD relay jamming scheme was proposed to improve the secrecy performance. The studies conducted in [10] and [11] showed that the FD relay can significantly outperform the HD relay in the context of relay networks. However, the secrecy performance achieved by FD relays in multi-user multi-relay networks has not been examined in the literature.

In this work, we exploit the use of FD relays to enhance the secrecy in multi-user multi-relay networks. This exploitation is motivated by the potential benefits offered by multi-user multi-relay networks, e.g., enabling reliable communications among multiple users in cellular networks and wireless sensor networks. Specifically, we propose a novel joint user and FD relay selection (JUFDRS) scheme. In this scheme, the user and the FD relay are selected to perform cooperative transmission such that the end-to-end channel capacity from the user to the destination via the relay is maximized. To disclose the benefits of the JUFDRS scheme relative to the JUHDRS scheme, we derive a new closed-form expression for the SOP of the JUFDRS scheme. We also derive the SOP of the JUHDRS scheme as a benchmark. Our analysis demonstrates that the JUFDRS scheme significantly outperforms the JUHDRS scheme by achieving a lower SOP when the self-interference at the FD relay can be reasonably suppressed. We also determine the self-interference cancellation requirement to guarantee the performance advantage of the JUFDRS scheme relative to the JUHDRS scheme. This provides useful insights into designing a secure multi-user multi-relay network.

II. Proposed JUHDRS Scheme in Multi-User Multi-DF-Relay Network

We consider a secure multi-user multi-relay network, as illustrated in Fig. 1, in which the communication between M users $S_m, m \in \{1, \cdots, M\}$, and the destination D is

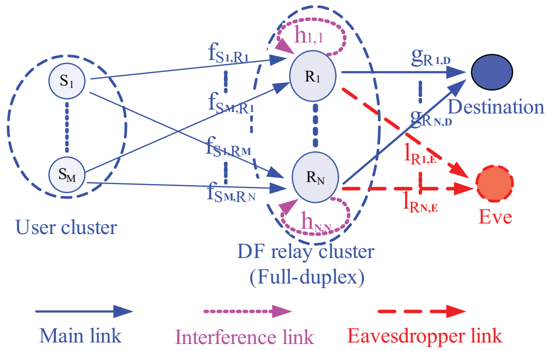

Fig. 1. The model of the multi-user multi-FD-relay network in the presence of an eavesdropper.

assisted by N FD DF relays $R_n, n \in \{1, \cdots, N\}$. In practice, this model represents the uplink of a multi-user cellar system with multiple relays, which assist the user-destination transmission [7, 15]. We assume that an eavesdropper (E) exists in this network and overhears the transmission. Similar to [5], we also assume that the direct links from users to D and E are not available due to the strong path-loss and attenuation between them. We further assume that the relays operate in the FD mode with two antennas, i.e., one antenna used for transmission and the other one used for reception. Each of the other nodes is equipped with a single antenna.

In this work, we denote f_{S_m,R_n}, $g_{R_n,D}$, and $l_{R_n,E}$ as the channel coefficients of the $S_m - R_n$, $R_n - D$, and $R_n - E$ links, respectively. We also denote $h_{n,n}$ as the self-interference channel after imperfect self-interference cancellation. We assume that all channels experience block Rayleigh fading such that the channels remain constant over one block but vary independently from one block to another [15, 16]. We denote P_S and P_R as the transmit power at S_m and R_n respectively. We then denote that $n_{\mathrm{R}} \sim \mathcal{CN}(0, \sigma_{\mathrm{R}}^2)$, $n_{\mathrm{D}} \sim \mathcal{CN}(0, \sigma_{\mathrm{D}}^2)$, and $n_{\mathrm{E}} \sim \mathcal{CN}(0, \sigma_{\mathrm{E}}^2)$ as the complex additive white Gaussian noise components at R_n, D, and E, respectively. Without losing generality, we normalize the transmit powers such that $P_S = P_R = 1$ [16].

Suppose that the user S_m and the relay R_n are selected for the information transmission, the received signals at R_n, D, and E at the time t are given by $y_{R_n}(t) = f_{S_m,R_n}x(t) + h_{n,n}x(t-L) + n_R(t)$, $y_D(t) = g_{R_n,D}x(t-L) + n_D(t)$, and $y_E(t) = l_{R_n,E}x(t-L) + n_E(t)$, respectively, where L is the transmission delay at R_n, relative to that of S_m in symbol times [16]. Based on the received signals, the channel capacities of the $S_m - R_n$, $R_n - D$, and $R_n - E$ links are given by

$$\begin{aligned} C_{S_m R_n} &= \log_2\left(1 + \frac{|f_{S_m,R_n}|^2}{|h_{n,n}|^2 + \sigma_{\mathrm{R}}^2}\right) \\ &= \log_2\left(1 + \frac{u_{m,n}}{v_{n,n}+1}\right) = \log_2(1 + \gamma_{m,n}), \end{aligned} \tag{1}$$

$$C_{R_n D} = \log_2\left(1 + \frac{|g_{R_n,D}|^2}{\sigma_{\mathrm{D}}^2}\right) = \log_2(1+\gamma_n), \tag{2}$$

and

$$C_{R_n E} = \log_2\left(1 + \frac{|l_{R_n,E}|^2}{\sigma_{\mathrm{E}}^2}\right) = \log_2(1 + w_n), \tag{3}$$

respectively, where $u_{mn} = |f_{S_m,R_n}|^2$, $\gamma_n = |g_{R_n,D}|^2$, and $w_n = |l_{R_n,E}|^2$ are the channel gains of the $S_m - R_n$ link, $R_n - D$ link, and $R_n - E$ link, respectively, $v_{nn} = |h_{n,n}|^2$ is the channel gain of self-interference, and $\gamma_{m,n} - \frac{u_{m,n}}{v_{n,n}+1}$ is the received signal-to-interference-plus-noise ratio (SINR) at R_n. We assume that $u_{m,n}$, γ_n, w_n, and v_{nn} follow the exponential distribution with the mean of π_{SR}, π_{RD}, π_{RD}, and π_{RR}. Therefore, the SOP is given by [5]

$$P_{out,m,n} = \Pr\left[\min(C_{S_m R_n}, C_{R_n D}) - C_{R_n E} < R_S\right], \tag{4}$$

where R_S denotes the secrecy information rate.

If the capacity of $R_n - E$ link is not available, we select the relay and user pair that minimizes the SOP. Mathematically, the index of the optimal relay, m^*, and the index of the optimal user, n^*, can be determined through

$$\begin{aligned} (m^*, n^*) &= \underset{m=1,\cdots,M}{\operatorname{argmax}} \max_{n=1,\cdots,N} \min\left(C_{S_m R_n}, C_{R_n D}\right) \\ &= \underset{m=1,\cdots,M}{\operatorname{argmax}} \max_{n=1,\cdots,N} \min\left(\gamma_{m,n}, \gamma_n\right). \end{aligned} \tag{5}$$

III. Secrecy Performance Analysis of JUFDRS Scheme

In this section we derive the secrecy outage probabilities achieved by the JUFDRS scheme in closed-from expressions, which allow us to examine the benefits it offers. To this end, we first present some preliminary results in the following subsection.

A. Preliminaries

The CDFs of γ_{n^*} and γ_{m^*,n^*} are given by (6) and (7), respectively, shown at the top of next page, where we define the following notations:

$$\sum_i^{\sim} = \sum_{i_1=0}^{N-1} \sum_{i_2=0}^{i_1} \sum_{i_2=0}^{i_2} \cdots \sum_{i_M=0}^{i_M-1},$$

$$f_i = i_1 + i_2 + \cdots + i_M,$$

$$e_i = \frac{f_i}{\pi_{SR}} + \frac{i_1}{\pi_{RD}},$$

$$\begin{aligned} d_i =& (-1)^{f_i} \binom{N-1}{i_1} \binom{i_1}{i_2} \cdots \binom{i_{M-1}}{i_M} \\ &\times \binom{M}{1}^{i_1 - i_2} \binom{M}{2}^{i_2 - i_3} \cdots \binom{M}{M-1}^{i_{M-1} - i_M}, \end{aligned}$$

$$p_{1i} = e_i + \frac{1}{\pi_{RD}} + \frac{m}{\pi_{SR}},$$

$$p_{2i} = e_i + \frac{m}{\pi_{SR}},$$

$$F_{\gamma_{n^*}}(x) = 1 - \sum_{m=1}^{M}\sum_{i}^{\sim}\left[q_{1i}\Gamma\left(-p_{3i}+1, \frac{p_{1i}\pi_{SR}}{\pi_{RR}} + p_{1i}x\right) + q_{2i}\Gamma\left(-p_{3i}, \frac{p_{1i}\pi_{SR}}{\pi_{RR}} + p_{1i}x\right)\right]$$
$$- e^{-\frac{x}{\pi_{RD}}}\sum_{m=1}^{M}\sum_{i}^{\sim}\left[q_{3i}\Gamma\left(-p_{3i}+1, \frac{p_{2i}\pi_{SR}}{\pi_{RR}}\right) - q_{3i}\Gamma\left(-p_{3i}+1, \frac{p_{2i}\pi_{SR}}{\pi_{RR}} + p_{2i}x\right)\right]$$
$$- e^{-\frac{x}{\pi_{RD}}}\sum_{m=1}^{M}\sum_{i}^{\sim}\left[q_{4i}\Gamma\left(-p_{3i}, \frac{p_{2i}\pi_{SR}}{\pi_{RR}}\right) - q_{4i}\Gamma\left(-p_{3i}, \frac{p_{2i}\pi_{SR}}{\pi_{RR}} + p_{2i}x\right)\right]. \quad (6)$$

$$F_{\gamma_{m^*,n^*}}(x) = 1 - \sum_{m=1}^{M}\sum_{i}^{\sim}\left[q_{1i}\Gamma\left(-p_{3i}+1, \frac{p_{1i}\pi_{SR}}{\pi_{RR}} + p_{1i}x\right) + q_{2i}\Gamma\left(-p_{3i}, \frac{p_{1i}\pi_{SR}}{\pi_{RR}} + p_{1i}x\right)\right]$$
$$- \sum_{m=1}^{M}\sum_{i}^{\sim}\left(\frac{e^{-\frac{x}{\pi_{SR}}}}{\pi_{SR}+\pi_{RR}x}\right)^{m}\left[q_{5i}\Gamma\left(-f_i+1, \frac{p_{4i}\pi_{SR}}{\pi_{RR}}\right) - q_{5i}\Gamma\left(-f_i+1, \frac{p_{4i}\pi_{SR}}{\pi_{RR}} + p_{4i}x\right)\right]. \quad (7)$$

$$p_{3i} = f_i + m,$$

$$p_{4i} = e_i + \frac{1}{\pi_{RD}},$$

$$q_{1i} = N(-1)^{m-1}\binom{M}{m} d_i \pi_{SR}^{p_{3i}}\left(\frac{1}{\pi_{RD}} + \frac{m}{\pi_{SR}}\right)\frac{\pi_{RR}^{-p_{3i}} e^{\frac{p_{1i}\pi_{SR}}{\pi_{RR}}}}{p_{1i}^{-p_{3i}+1}},$$

$$q_{2i} = N(-1)^{m-1}\binom{M}{m} d_i \pi_{SR}^{p_{3i}} m \frac{\pi_{RR}^{-p_{3i}} e^{\frac{p_{1i}\pi_{SR}}{\pi_{RR}}}}{p_{1i}^{-p_{3i}}},$$

$$q_{3i} = N(-1)^{m-1}\binom{M}{m} d_i \pi_{SR}^{p_{3i}-1} m \frac{\pi_{RR}^{-p_{3i}} e^{\frac{p_{2i}\pi_{SR}}{\pi_{RR}}}}{p_{2i}^{-p_{3i}+1}},$$

$$q_{4i} = N(-1)^{m-1}\binom{M}{m} d_i \pi_{SR}^{p_{3i}} m \frac{\pi_{RR}^{-p_{3i}} e^{\frac{p_{2i}\pi_{SR}}{\pi_{RR}}}}{p_{2i}^{-p_{3i}}},$$

$$q_{5i} = N(-1)^{m-1}\binom{M}{m} d_i \pi_{SR}^{p_{3i}} \frac{\pi_{RR}^{-f_i} e^{\frac{p_{4i}\pi_{SR}}{\pi_{RR}}}}{\pi_{RD} p_{4i}^{-f_i+1}},$$

and

$$\Gamma(k+1, x) = \begin{cases} e^{-x}, & k=0, \\ E_1(x), & k=-1, \\ \frac{(-1)^{-k+1}}{(-k-1)!}\left[E_1(x) - e^{-x}\sum_{i=0}^{-k-2}\frac{(-1)^i i!}{x^{i+1}}\right], & k \le -2, \end{cases} \quad (8)$$

Here, $E_1(x)$ denotes the exponential integral of first order [17].

Proof: The proof is presented in Appendix A. ∎

Based on the preliminary results, we obtain the CDF of Z as

$$\begin{aligned} F_Z(z) &= \Pr\left(\min(\gamma_{m^*,n^*}, \gamma_{n^*}) < z\right) \\ &= 1 - \Pr(\gamma_{m^*,n^*} \ge z)\Pr(\gamma_{n^*} \ge z) \\ &= 1 - \left[1 - F_{\gamma_{m^*,n^*}}(z)\right]\left[1 - F_{\gamma_{n^*}}(z)\right]. \end{aligned} \quad (9)$$

Applying the preliminary results into (9), the CDF of Z can be expressed as (10), which shown at the top of next page.

In the following, we provide three lemmas to facilitate our analytical derivations.

Lemma 1: Let $C_1(x) = E_1(\alpha x + \beta)$, the combination of exponential integral with exponentials is given by

$$\begin{aligned} \Pi_1(\alpha, \beta, \mu) &= \int_0^\infty E_1(\alpha x + \beta) e^{-\mu x} dx \\ &= \frac{1}{\mu}\left[E_1(\beta) - e^{\frac{\beta}{\alpha}\mu} E_1\left(\left(1 + \frac{\mu}{\alpha}\right)\beta\right)\right], \end{aligned} \quad (12)$$

where $\alpha > 0$, $\beta > 0$, and $\mu > 0$, respectively.

Proof: We obtain (12) by applying [17, Eq.(5.231.1)] and [17, Eq.(6.224.1)]. ∎

Based on (12), we obtain

$$\Pi_2(k, \alpha, \beta, \mu) = \int_0^\infty \Gamma(k+1, \alpha x + \beta) e^{-\mu x} dx$$
$$= \begin{cases} \frac{1}{\alpha+\beta}\exp(-\beta), & k=0, \\ \Pi_1(\alpha, \beta, \mu), & k=-1, \\ \frac{\Pi_1(\alpha,\beta,\mu)}{(-1)^{k-1}(-k-1)!} - \sum_{i=0}^{-k-2}\frac{(-1)^i i! \Xi_1(i+1, \alpha, \beta, \alpha+\mu)}{(-1)^{k-1}(-k-1)!}, & k \le -2, \end{cases} \quad (13)$$

with

$$\Xi_1(i, \alpha, \beta, \mu) = \int_0^\infty \frac{e^{-\mu x}}{(\alpha x + \beta)^i} dx$$
$$= \begin{cases} \frac{1}{\alpha} e^{\frac{\beta\mu}{\alpha}} E_1\left(\frac{\beta\mu}{\alpha}\right), & i=1, \\ \sum_{n=1}^{i-1}(n-1)!\omega\left(-\frac{\alpha}{\beta\mu}\right)^n + \omega e^{\frac{\beta\mu}{\alpha}} E_1\left(\frac{\beta\mu}{\alpha}\right), & i \ge 2. \end{cases} \quad (14)$$

and $\omega = \frac{(-\mu)^{i-1}}{(i-1)!\alpha^i}$.

Lemma 2: Let $C_2(x) = E_1(\alpha x + \beta)$, the combination of

$$
\begin{aligned}
F_Z(z) = 1 - \sum_{m_1=1}^{M}\sum_{m_2=1}^{M}\sum_{i}^{\sim}\sum_{j}^{\sim}\Bigg[& q_{1i}q_{1j}\Gamma_1(-p_{3i},-p_{3j},p_{1i},p_{1j},\phi_{1i},\phi_{1j}) + q_{1i}q_{2j}\Gamma_2(-p_{3i},-p_{3j}-1,p_{1i},p_{1j},\phi_{1i},\phi_{1j}) \\
& + q_{2i}q_{2j}\Gamma_3(-p_{3i}-1,-p_{3i}-1,p_{1i},p_{1j},\phi_{1i},\phi_{1j}) + q_{2i}q_{2j}\Gamma_4(-p_{3i}-1,-p_{3i},p_{1i},p_{1j},\phi_{1i},\phi_{1j}) \\
& + \left[q_{1i}q_{5j}\Gamma_5(-p_{3i},-f_j,p_{1i},p_{4j},\phi_{1i},\phi_{4j}) + q_{2i}q_{5j}\Gamma_6(-p_{3i}-1,-f_j,p_{1i},p_{4j},\phi_{1i},\phi_{4j})\right]\left(\frac{e^{-\frac{x}{\pi_{SR}}}}{\pi_{SR}+\pi_{RR}x}\right)^{m_2} \\
& + \left[q_{3i}q_{1j}\Gamma_7(-p_{3i},-p_{3j},p_{2i},p_{1j},\phi_{2i},\phi_{1j}) + q_{3i}q_{2j}\Gamma_8(-p_{3i},-p_{3j}-1,p_{2i},p_{1j},\phi_{2i},\phi_{1j})\right]e^{-\frac{x}{\pi_{RD}}} \\
& + \left[q_{4i}q_{2j}\Gamma_9(-p_{3i}-1,-p_{3j}-1,p_{2i},p_{1j},\phi_{2i},\phi_{1j}) + q_{4i}q_{1j}\Gamma_{10}(-p_{3i}-1,-p_{3j},p_{2i},p_{1j},\phi_{2i},\phi_{1j})\right]e^{-\frac{x}{\pi_{RD}}} \\
& + \left[q_{3i}q_{5j}\Gamma_{11}(-p_{3i},-f_j,p_{2i},p_{4j},\phi_{2i},\phi_{4j}) + q_{4i}q_{5j}\Gamma_{12}(-p_{3i}-1,-f_j,p_{2i},p_{4j},\phi_{2i},\phi_{4j})\right]\frac{e^{-(\frac{m_2 x}{\pi_{SR}}+\frac{x}{\pi_{RD}})}}{(\pi_{SR}+\pi_{RR}x)^{m_2}}\Bigg], \quad (10)
\end{aligned}
$$

with $\Gamma_i(k_1,k_2,\alpha_1,\alpha_2,\beta_1,\beta_2) =$

$$
\begin{cases}
\Gamma(k_1+1,\alpha_1 x+\beta_1)\Gamma(k_2+1,\alpha_2 x+\beta_2), & i=1,2,3,4,\\
\Gamma(k_1+1,\alpha_1 x+\beta_1)\left[\Gamma(k_2+1,\beta_2)-\Gamma(k_2+1,\alpha_2 x+\beta_2)\right], & i=5,6,\\
\left[\Gamma(k_1+1,\beta_1)-\Gamma(k_1+1,\alpha_1 x+\beta_1)\right]\Gamma(k_2+1,\alpha_2 x+\beta_2), & i=7,8,9,10,\\
\left[\Gamma(k_1+1,\beta_1)-\Gamma(k_1+1,\alpha_1 x+\beta_1)\right]\left[\Gamma(k_2+1,\beta_2)-\Gamma(k_2+1,\alpha_2 x+\beta_2)\right], & i=11,12.
\end{cases} \quad (11)
$$

---

exponential integral with exponentials is given by

$$
\begin{aligned}
\Pi_3(k,\alpha,\beta,\mu) &= \int_0^\infty \frac{e^{-\mu x}}{(\alpha x+\beta)^k}E_1(\alpha x+\beta)\,dx \\
&= \sum_{l=1}^{M} w_l f(y_l), \quad (15)
\end{aligned}
$$

where

$$
f(y) = \frac{2y}{\mu}E_1\left(\frac{\alpha y^2}{\mu}+\beta\right)\left(\frac{\alpha y^2}{\mu}+\beta\right)^{-k},
$$

the weights w_l and the abscissas y_l are real numbers given in [14] for $M = 2,\cdots,15$, $\alpha > 0$, $\beta > 0$, $\mu > 0$, and $k > 0$, respectively.

Proof: The proof is presented in Appendix B. ∎

Based on (15), we obtain $\Pi_4(k_1,k_2,\alpha,\beta,\mu)$ as (16), shown on the next page.

***Lemma** 3:* Let $C_3(x) = E_1(\alpha_1 x+\beta_1)E_1(\alpha_2 x+\beta_2)$, the combination of exponential integral with exponentials is given by

$$
\begin{aligned}
\Pi_5(k,\alpha_1,\alpha_2,\beta_1,\beta_2,\mu) &= \int_0^\infty \frac{C_3(x)e^{-\mu x}}{(\alpha_1 x+\beta_1)^k}dx \\
&= \sum_{l=1}^{M} w_l f(y_l), \quad (17)
\end{aligned}
$$

where

$$
\begin{aligned}
f(y) =& \frac{2y}{\mu}E_1\left(\frac{\alpha_1 y^2}{\mu}+\beta_1\right)E_1\left(\frac{\alpha_2 y^2}{\mu}+\beta_2\right) \\
&\times\left(\frac{\alpha_1 y^2}{\mu}+\beta_1\right)^{-k}. \quad (18)
\end{aligned}
$$

Proof: The proof is similar to that of ***Lemma** 2*. ∎

Based on ***Lemma** 3*, we obtain (19), which is shown in the next page.

B. Secrecy Outage Probability

We present the SOP of the JUFDRS scheme in the following theorem.

***Theorem** 1:* The SOP of the JUFDRS scheme is given by (20), where $\psi_{ni} = \frac{p_{ni}\pi_{SR}}{\pi_{RR}} + p_{ni}(2^{R_S}-1)$, $\phi_{ni} = p_{ni}2^{R_S}$, $\varphi_{ni} = \frac{p_{ni}\pi_{SR}}{\pi_{RR}}$, $\eta_1 = \frac{m_2(2^{R_s}-1)}{\pi_{SR}}$, $\eta_2 = \frac{m_2 2^{R_s}}{\pi_{SR}} + \frac{1}{\pi_{RE}}$, $\eta_3 = \frac{(2^{R_s}-1)}{\pi_{RD}}$, $\eta_4 = \frac{2^{R_S}}{\pi_{RD}} + \frac{1}{\pi_{RE}}$, $\eta_5 = \frac{2^{R_S}-1}{\pi_{RD}} + \frac{m_2(2^{R_s}-1)}{\pi_{SR}}$, and $\eta_6 = \frac{2^{R_S}}{\pi_{RD}} + \frac{m_2 2^{R_s}}{\pi_{SR}} + \frac{1}{\pi_{RE}}$.

Proof: Based on (1), (2), (3), and (4), the SOP of the JUFDRS scheme can be expressed as

$$
\begin{aligned}
P_{out} &= \Pr[\min(C_{S_{m^*}R_{n^*}}, C_{R_{n^*}D}) - C_{R_{n^*}E} < R_S] \\
&= \Pr[\min(\gamma_{m^*,n^*},\gamma_{n^*}) < 2^{R_S}(1+w_{n^*})-1]. \quad (21)
\end{aligned}
$$

Based on the preliminary results and ***Lemmas** 1–3*, and performing some mathematical manipulations, we obtain (20), which completes the proof of Theorem 1. ∎

Following a similar procedure to derive the SOP of the proposed JUFDRS scheme, we can obtain SOP of the JUHDRS scheme, which is used as a baseline to compare with the proposed JUFDRS scheme. Specifically, the SOP of the JUHDRS scheme is given by

$$
\begin{aligned}
P_{out}^{HRF} =& 1 - \frac{1}{\pi_{RE}}\sum_{m_1=1}^{M}\sum_{m_2=1}^{M}\sum_{i}^{\sim}\sum_{j}^{\sim}\Bigg[\frac{\tau_{3i}\tau_{4j}e^{-(\frac{m_2}{\pi_{SR}}+\frac{1}{\pi_{RD}})t}}{\Phi(\frac{m_2}{\pi_{SR}}+\frac{1}{\pi_{RD}})} \\
&+ \frac{\tau_{3i}\tau_{2j}e^{-(\frac{1}{\pi_{RD}}+p_{1j})t}}{\Phi(\frac{1}{\pi_{RD}}+p_{1j})} + \frac{\tau_{1i}\tau_{4j}e^{-(p_{1i}+\frac{m_2}{\pi_{SR}})t}}{\Phi(p_{1i}+\frac{m_2}{\pi_{SR}})} \\
&+ \frac{\tau_{1i}\tau_{2j}e^{-(p_{1i}+p_{1j})t}}{\Phi(p_{1i}+p_{1j})}\Bigg], \quad (22)
\end{aligned}
$$

where

$$
\tau_{1i} = N(-1)^{m-1}\binom{M}{m}\frac{e_i}{\pi_{RD}p_{2i}p_{1i}},
$$

$$\Pi_4(k_1,k_2,\alpha,\beta,\mu)=\int_0^{\infty}\Gamma\left(k_1+1,\alpha x+\beta\right)\frac{e^{-\mu x}}{(\alpha x+\beta)^{k_2}}dx$$
$$=\begin{cases}\Pi_2(k_1,\alpha,\beta,\mu), & k_2=0\\ e^{-\beta}\Xi_1(k_2,\alpha,\beta,\alpha+\mu), & k_2\neq 0, k_1=0,\\ \Pi_3(k_2,\alpha,\beta,\mu), & k_2\neq 0, k_1=-1,\\ \frac{\Pi_3(k_2,\alpha,\beta,\mu)}{(-1)^{k-1}(-k-1)!}-\sum\limits_{i=0}^{-k-2}\frac{i!\Xi_1(k_2+i+1,\alpha,\beta,\alpha+\mu)}{(-1)^{k-i-1}(-k-1)!}, & k_2\neq 0, k_1\leq -2.\end{cases}\tag{16}$$

$$\Pi_6(k,k_1,k_2,\alpha_1,\alpha_2,\beta_1,\beta_2,\mu)=\int_0^{\infty}\Gamma(k_1+1,\alpha_1x+\beta_1)\Gamma(k_2+1,\alpha_2x+\beta_2)\frac{e^{-\mu x}}{(\alpha_2x+\beta_2)^k}dx$$
$$=\begin{cases}e^{-\beta_1}\Pi_4(k_2,k,\alpha_2,\beta_2,\alpha_1+\mu), & k_1=0,\\ e^{-\beta_2}\left(\frac{\alpha_1}{\alpha_2}\right)^k\Pi_3(k,\alpha_1,\beta_1,\alpha_2+\mu), & k_1=-1,k_2=0,\\ \Pi_5(k,\alpha_2,\beta_2,\alpha_1,\beta_1,\mu), & k_1=-1,k_2=-1,\\ \frac{(-1)^{-k_2+1}}{(-k_2-1)!}\Big[\Pi_5(k,\alpha_2,\beta_2,\alpha_1,\beta_1,\mu) & \\ \quad-\sum\limits_{i=0}^{-k_2-2}(-1)^i i! e^{-\beta_2}\left(\frac{\alpha_1}{\alpha_2}\right)^{k+i+1}\Pi_3(k+i+1,\alpha_1,\beta_1,\alpha_2+\mu)\Big], & k_1=-1,k_2<-1,\\ \frac{(-1)^{-k_1+1}}{(-k_1-1)!}\Big[e^{-\beta_2}\left(\frac{\alpha_1}{\alpha_2}\right)^k\Pi_3(k,\alpha_1,\beta_1,\alpha_2+\mu) & \\ \quad-\sum\limits_{i=0}^{-k_1-2}(-1)^i i! e^{-(\beta_1+\beta_2)}\left(\frac{\alpha_1}{\alpha_2}\right)^{k}\Pi_3(k+i+1,\alpha_1,\beta_1,\alpha_1+\alpha_2+\mu)\Big], & k_1<-1,k_2=0,\\ \frac{(-1)^{-k_1+1}}{(-k_1-1)!}\Big[\Pi_5(k,\alpha_2,\beta_2,\alpha_1,\beta_1,\mu) & \\ \quad-\sum\limits_{i=0}^{-k_1-2}(-1)^i i! e^{-\beta_1}\left(\frac{\alpha_2}{\alpha_1}\right)^{i+1}\Pi_3(k+i+1,\alpha_2,\beta_2,\alpha_1+\mu)\Big], & k_1<-1,k_2=-1,\\ \frac{(-1)^{-k_1-k_2+2}}{(-k_1-1)!(-k_2-1)!}\Big[(\Pi_5(k,\alpha_2,\alpha_1,\beta_2,\beta_1,\mu) & \\ \quad+\sum\limits_{i=0}^{-k_1-2}\sum\limits_{j=0}^{-k_2-2}(-1)^{i+j}i!j!e^{-(\beta_1+\beta_2)}\left(\frac{\alpha_2}{\alpha_1}\right)^{i+1}\Xi_1(k+i+j+2,\alpha_2,\beta_2,\alpha_1+\alpha_2+\mu) & \\ \quad-\sum\limits_{i=0}^{-k_1-2}(-1)^i i! e^{-\beta_1}\left(\frac{\alpha_2}{\alpha_1}\right)^{i+1}\Pi_3(k+i+1,\alpha_2,\beta_2,\alpha_1+\mu) & \\ \quad-\sum\limits_{j=0}^{-k_2-2}(-1)^j j! e^{-\beta_2}\left(\frac{\alpha_1}{\alpha_2}\right)^{k+j+1}\Pi_3(k+j+1,\alpha_1,\beta_1,\alpha_2+\mu))\Big], & k_1<-1,k_2<-1.\end{cases}\tag{19}$$

$$\tau_{2i}=N(-1)^{m-1}\binom{M}{m}\frac{me_i}{\pi_{SR}p_{4i}p_{1i}},$$

$$\tau_{3i}=N(-1)^{m-1}\binom{M}{m}\frac{m}{m+e_i\pi_{SR}},$$

$$\tau_{4i}=N(-1)^{m-1}\binom{M}{m}\frac{1}{1+\pi_{RD}e_i},$$

$\Phi(x)=x2^{2R_s}+\frac{1}{\pi_{RE}}$ and $t=2^{2R_s}-1$.

We note that HD relaying is known to suffer from a spectral efficiency loss compared to FD relaying due to its time-orthogonal relay listening/forwarding suffering, so half-duplex suffer from 50% loss in data rate, there is 1/2 factor in both data transmission and eavesdropping capacities [11].

IV. Numerical Results

In this section, we provide numerical results to examine the secrecy performance of the JUFDRS scheme. The JUHDRS scheme is also shown as benchmarks in the figures.

Fig. 2 plots the SOP versus SNR of the R-D link. We set the average SNR of the R-E link as 5 dB, i.e., $\pi_{\mathrm{RE}}=5$ dB, and the residual self-inference as 2 dB, i.e., $\pi_{RR}=2$ dB. The average SNR of the S-R link is the same as that of the R-D link. It is observed from Fig. 2 that, both the SOP of the JUFDRS scheme and the SOP of the JUHDRS scheme decrease significantly as N increases. This indicates that increasing the number of cooperative relays enhances the physical layer security against eavesdropping attack. It is also observed from Fig. 2 that the JUFDRS scheme significantly outperforms the JUHDRS scheme, illustrating the security benefits of exploiting the FD mode to prevent eavesdropping attacks. Furthermore, we observe from the figure that the SOP decreases as M increases. In addition, we note that this improvement becomes marginal when the SNR is high, since the R_n-D link becomes the bottleneck of the end-to-end cooperative transmission.

Fig. 3 plots the SOP versus R_s with the residual self-inference $\pi_{RR}=2$ dB and $\pi_{RR}=4$ dB. First, we see that

$$
\begin{aligned}
P_{out} =& 1 - \sum_{m_1=1}^{M}\sum_{m_2=1}^{M}\sum_{i}^{\sim}\sum_{j}^{\sim}\Big[\frac{q_{1i}}{\pi_{RE}}\Big(q_{1j}\Pi_6(0,-p_{3i},-p_{3j},\phi_{1i},\phi_{1j},\psi_{1i},\psi_{1j},\frac{1}{\pi_{RE}}) + q_{2j}\Pi_6(0,-p_{3i},-p_{3j}-1,\phi_{1i},\phi_{1j},\psi_{1i},\psi_{1j},\frac{1}{\pi_{RE}})\\
&- q_{5j}e^{-\eta_1}(\frac{p_{4j}}{\pi_{RR}})^{m_2}\big[\Pi_6(m_2,-p_{3i},-f_j,\phi_{1i},\phi_{4j},\psi_{1i},\psi_{4j},\eta_2) - \Gamma(-f_j+1,\varphi_{4j})(\frac{\phi_{1i}}{\phi_{4j}})^{m_2}\Pi_4(-p_{3i},m_2,\phi_{1i},\psi_{1i},\eta_2)\big]\Big)\\
&+ \frac{q_{2i}}{\pi_{RE}}\big[q_{2j}\Pi_6(0,-p_{3i}-1,-p_{3j}-1,\phi_{1i},\phi_{1j},\psi_{1i},\psi_{1j},\frac{1}{\pi_{RE}}) + q_{1j}\Pi_6(0,-p_{3i}-1,-p_{3j},\phi_{1i},\phi_{1j},\psi_{1i},\psi_{1j},\frac{1}{\pi_{RE}})\\
&- q_{5j}e^{-\eta_1}(\frac{p_{4j}}{\pi_{RR}})^{m_2}(\Pi_6(m_2,-p_{3i}-1,-f_j,\phi_{1i},\phi_{4j},\psi_{1i},\psi_{4j},\eta_2) - \Gamma(-f_j+1,\varphi_{4j})(\frac{\phi_{1i}}{\phi_{4j}})^{m_2}\Pi_4(-p_{3i}-1,m_2,\phi_{1i},\psi_{1i},\eta_2))\big]\\
&- \frac{q_{3i}}{\pi_{RE}}\big(q_{1j}e^{-\eta_3}\big[\Pi_6(0,-p_{3i},-p_{3j},\phi_{2i},\phi_{1j},\psi_{2i},\psi_{1j},\eta_4) - \Gamma(-p_{3j}+1,\varphi_{2i})\Pi_4(-p_{3j},0,\phi_{1j},\psi_{1j},\eta_4)\big]\\
&+ q_{2j}e^{-\eta_3}\big[\Pi_6(0,-p_{3i},-p_{3j}-1,\phi_{2i},\phi_{1j},\psi_{2i},\psi_{1j},\eta_4) - \Gamma(-p_{3j}+1,\varphi_{2i})\Pi_4(-p_{3j}-1,0,\phi_{1j},\psi_{1j},\eta_4)\big]\\
&- q_{5j}(\frac{p_{4j}}{\pi_{RR}})^{m_2}e^{-\eta_5}\big[\Pi_6(m_2,-p_{3i},-f_j,\phi_{2i},\phi_{4j},\psi_{2i},\psi_{4j},\eta_6) + \Gamma(-p_{3i}+1,\varphi_{2j})\Gamma(-f_j+1,\varphi_{4j})\Xi_1(m_2,\phi_{4j},\psi_{4j},\eta_6)\\
&- \Gamma(-p_{3i}+1,\varphi_{2j})\Pi_4(-f_j,m_2,\phi_{4j},\psi_{4j},\eta_6) - \Gamma(-f_j+1,\varphi_{4j})(\frac{\phi_{2i}}{\phi_{4j}})^{m_2}\Xi_4(-p_{3i},m_2,\phi_{2j},\psi_{2i},\eta_6)\big]\big)\\
&- \frac{q_{4i}}{\pi_{RE}}\Big(q_{2j}e^{-\eta_3}\big[\Pi_6(0,-p_{3i}-1,-p_{3j}-1,\phi_{2i},\phi_{1j},\psi_{2i},\psi_{1j},\eta_4) - \Gamma(-p_{3j}+1,\varphi_{2i})\Pi_4(-p_{3j}-1,0,\phi_{1j},\psi_{1j},\eta_4)\big]\\
&+ q_{1j}e^{-\eta_3}\big[\Pi_6(0,-p_{3i}-1,-p_{3j},\phi_{2i},\phi_{1j},\psi_{2i},\psi_{1j},\eta_4) - \Gamma(-p_{3j},\varphi_{2i})\Pi_4(-p_{3j},0,\phi_{1j},\psi_{1j},\eta_4)\\
&- q_{5j}(\frac{p_{4j}}{\pi_{RR}})^{m_2}e^{-\eta_5}\Pi_6(m_2,-p_{3i}-1,-f_j,\phi_{2i},\phi_{4j},\psi_{2i},\psi_{4j},\eta_6) + \Gamma(-p_{3i},\varphi_{2j})\Gamma(-f_j+1,\varphi_{4j})\Xi_1(m_2,\phi_{4j},\psi_{4j},\eta_6)\\
&- \Gamma(-p_{3i},\varphi_{2j})\Pi_4(-f_j,m_2,\phi_{4j},\psi_{4j},\eta_6) - \Gamma(-f_j+1,\varphi_{4j})(\frac{\phi_{2i}}{\phi_{4j}})^{m_2}\Pi_4(-p_{3i}-1,m_2,\phi_{2i},\psi_{2j},\eta_6)\big]\Big)\Big].
\end{aligned} \tag{20}
$$

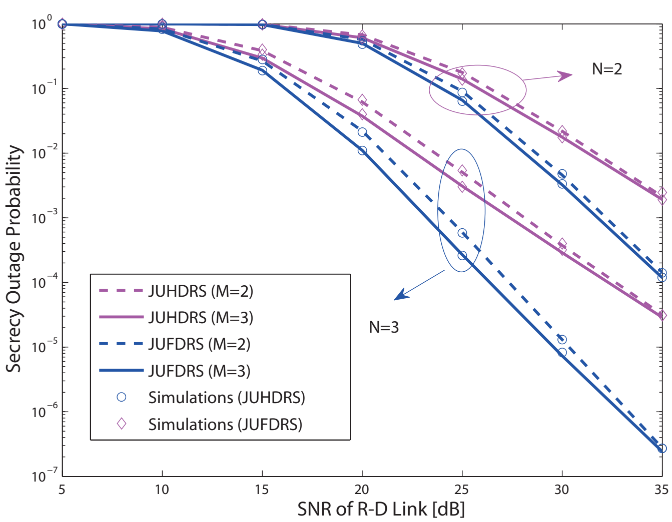

Fig. 2. The secrecy outage probability versus SNR of R-D with $M = 2,\ 3$, $N = 2,\ 3$, and $R_s = 2$.

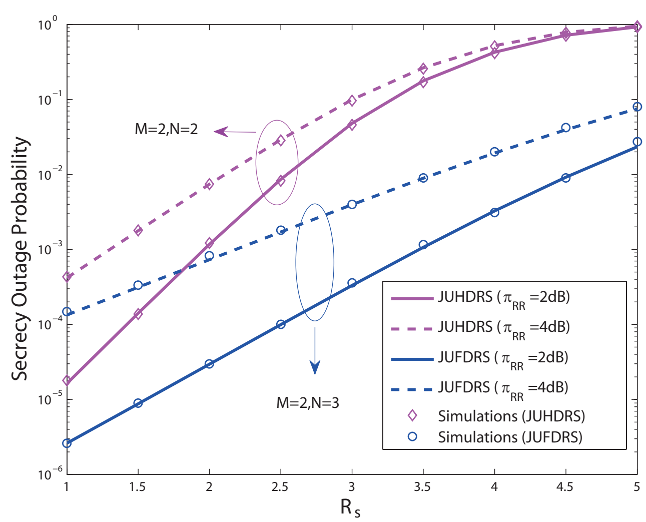

Fig. 3. The secrecy outage probability versus R_s with $\pi_{SR} = \pi_{RD} = 30$ dB and $\pi_{RE} = 5$ dB.

the SOP of the two schemes tend to increase with R_s, but the proposed JUFDRS scheme achieves a better performance. Second, we see that the SOPs of both schemes decrease when N increases for any fixed R_s. Finally, we see that the secrecy performance of the JUFDRS scheme relative to JUHDRS scheme becomes more prominent as π_{RR} decreases, e.g., from $\pi_{RR} = 4$ dB to $\pi_{RR} = 2$ dB. This can be explained by the fact that the JUFDRS scheme has a better secrecy performance than the JUFHRS scheme when the self-interference is well suppressed.

V. Conclusions

In this paper, we proposed a new JUFDRS scheme in a cooperative relay network in the presence of an eavesdropper. To analyze the benefits of the JUFDRS scheme, we derive its SOP in closed form. It was shown that the JUFDRS scheme significantly outperforms the JUHDRS scheme by achieving a lower SOP when the self-interference can be reasonably suppressed. This result indicates that adopting the FD technique at relays can greatly improve the physical layer security in the multi-user multi-relay network.

APPENDIX A
PROOF OF PRELIMINARY RESULTS

The best user $S_{m_n^*}$ conditioned on a given R_n should be selected to maximize $\gamma_{m,n}$, i.e., $m_n^* = \mathrm{argmax}_{m=1,\cdots,M}\, \gamma_{m,n}$. For exponentially distributed link gains, the CDF of $\gamma_{m,n}$ is given by [15]

$$F_{\gamma_{m,n}}(x) = 1 - \frac{\pi_{SR} e^{-\frac{x}{\pi_{SR}}}}{x\pi_{RR} + \pi_{SR}}. \tag{23}$$

Thus the CDF of $\gamma_{m_n^*,n}$ is given by $F_{\gamma_{m_n^*,n}}(x) = F_{\gamma_{m,n}}(x)^M$.

The CDF of γ_{n^*} is defined as

$$\begin{aligned} F_{\gamma_{n^*}}(x) =& \Pr\left[\gamma_{n^*} < x\right] \\ =& N \Pr\left[\gamma_1 < x, \min\left(\gamma_1, \gamma_{m_1^*,1}\right) > \theta\right], \end{aligned} \tag{24}$$

where $\theta = \max_{n_1=1,\cdots,N, n_1 \neq n} \min\left(\gamma_{n_1}, \gamma_{m_{n_1}^*, n_1}\right)$. Based on (23), we obtain the CDF and the PDF of θ as

$$\begin{aligned} &F_\theta(\theta) \\ &= \left[1 - e^{-\frac{\theta}{\pi_{RD}}} \sum_{m=1}^{M} (-1)^{m-1} \binom{M}{m} \left(\frac{\pi_{SR} e^{-\frac{\theta}{\pi_{SR}}}}{\pi_{SR} + \theta\pi_{RR}}\right)^m\right]^{N-1} \\ &= \sum_i^{\sim} \frac{d_i \pi_{SR}^{f_i} e^{-e_i\theta},}{\left(\pi_{SR} + \theta\pi_{RR}\right)^{f_i}} \end{aligned} \tag{25}$$

and

$$f_\theta(\theta) = -\sum_i^{\sim} d_i \frac{\left[\left(\pi_{RR}\theta + \pi_{SR}\right) e_i + \pi_{RR} f_i\right] e^{-e_i\theta}}{\left(\pi_{SR} + \theta\pi_{RR}\right)^{f_i+1}}, \tag{26}$$

respectively, where $\sum_i^{\sim}$, d_i, f_i, and e_i are defined in Section III.

To proceed with our analysis, we further express (24) as

$$\begin{aligned} F_{\gamma_{n^*}}(x) &= N \Pr\left(\theta < \gamma_1 < x, \gamma_{m_1^*,1} > \theta, 0 < \theta < x\right) \\ &= N \int_0^x \int_\theta^x \int_\theta^\infty f_\theta(\theta) f_{\gamma_1}(\gamma_1) f_{\gamma_{m_1^*,1}}(\gamma_{m_1^*,1}) d\gamma_{m_1^*,1} d\gamma_1 d\theta. \end{aligned} \tag{27}$$

By substituting (23) and (26) into (27), and using [17, Eq. (3.462.17)], we derive the CDF of $F_{\gamma_{n^*}}(x)$ as in (6), where we have calculated the resultant integral using [17, Eq.(3.462.17)]. Similarly, we derive the CDF of γ_{m^*,n^*} as in (7).

APPENDIX B
PROOF OF LEMMA 2

In order to derive the integral of Π_3, we first define $y^2 = \mu x$. Thus, we express (15) as

$$\begin{aligned} \Pi_3(k,\alpha,\beta,\mu) &= \int_0^\infty E_1\left(\alpha x + \beta\right) \frac{e^{-\mu x}}{(\alpha x + \beta)^k} dx \\ &= \int_0^\infty E_1\left(\frac{\alpha}{\mu} y^2 + \beta\right) \frac{2y \exp\left(-y^2\right)}{\mu\left(\alpha \frac{y^2}{\mu} + \beta\right)^k} dy. \end{aligned} \tag{28}$$

We note that a simple closed form of the analytical result of (28) is difficult to obtain. Fortunately, the infinite integral term involved in (28) has the structure, given by $\int_0^\infty f(y)\exp(-y^2)dy$, which can be evaluated accurately via the one-sided Gauss-Hermite quadrature (GHQ) rule [14], which completes the proof.

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