Base-Stations Up in the Air: Multi-UAV Trajectory Control for Min-Rate Maximization in Uplink C-RAN

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Abstract—In this paper we study the impact of unmanned aerial vehicles (UAVs) trajectories on terrestrial users' spectral efficiency (SE). Assuming a strong line of sight path to the users, the distance from all users to all UAVs influence the outcome of an online trajectory optimization. The trajectory should be designed in a way that the fairness rate is maximized over time. That means, the UAVs travel in the directions that maximize the minimum of the users' SE. From the free-space path-loss channel model, a data-rate gradient is calculated and used to direct the UAVs in a long-term perspective towards the local optimal solution on the two-dimensional spatial grid. Therefore, a control system implementation is designed. Thereby, the UAVs follow the data-rate gradient direction while having a more smooth trajectory compared with a gradient method. The system can react to changes of the user locations online; this system design captures the interaction between multiple UAV trajectories by joint processing at the central unit, e.g., a ground base station. Because of the wide spread of user locations, the UAVs end up in optimal locations widely apart from each other. Besides, the SE expectancy is enhancing continuously while moving along this trajectory.

Index Terms—trajectory, unmanned aerial vehicles, MIMO uplink, convex optimization.

I. INTRODUCTION

Future communication systems is expected to be responsive to a plethora of users due to integrating new concepts such as internet of things (IoT) and large-scale sensor networks. Furthermore, the quality of service (QoS) demands of the users is a continuously increasing trend. For improving the QoS, drones also known as unmanned aerial vehicles (UAVs), are proposed to be deployed in communication networks. The authors in [1] study the integration of UAVs into 5G and beyond 5G (B5G) cellular networks as they enable the possibility of an additional line of sight (LoS) connection.

Involving UAVs in communication networks requires addressing some key challenges that have been identified by [2] and [3]. They contain air-to-ground channel modeling [4], resource and trajectory optimization [5], and spectrum management [6]. Moreover, cellular networks need to be planned [7] as a new MAC layer design is required [8] and security issues need to be addressed [9]. In this paper, trajectory and resources are optimized jointly based on the users' location.

Thereby, UAVs can move at the same time as serving terrestrial users. Due to the possibility of drones for being mobile, they should be guided in directions that help the



Fig. 1. UAVs optimize their locations while receiving data from a set of users until an equilibrium is reached. The links towards the fronthaul network have constant high capacity.

users' QoS. The authors in [10], [11] study UAV trajectory in azimuth and altitude for the aim of rate maximization. Interestingly, the coverage area can be controlled by UAV altitude adjustment. This adjustment captures the trade-off between an enlarged coverage area and high power consumption due to high pathloss serving the users [12]. The horizontal placement of a set of UAVs over the ground is optimized in [13]. While moving, the trajectory can be obtimized for either alternating deployment towards multiple users [5] or for maximizing constant data rates as flying base station (FlyBS) [14]. In this paper, the latter one is considered. Having multiple UAVs, the users' signals can either be decoded locally at the supporting UAV, or, in case of C-RAN, jointly processed at a central unit. The former requires allocating the users to UAVs, which is addressed in [15]. However, the latter deals with the latency challenge due to joint processing at the central unit. This challenge can be remedied by having very high capacity links in fronthaul between the UAVs and the central unit.

In this paper, we consider a scenario, where multiple users demand connectivity to the ground base station (BS). Due to the large distance of the users to the ground BS, the achievable spectral efficiency (SE) is considerably low. Here, we exploit multiple UAVs forming a C-RAN, for aiding the communication between the users and the ground BS. For simplicity, we assume that the UAVs fully cooperate by considering an unlimited fronthaul capacity between the UAVs and the ground BS, e.g., optical communication for the fronthaul links [16]. Due to the unlimited fronthaul capacity assumption, and given the location of the users, the optimal travel directions of all UAVs are calculated by the ground BS and then provided to all UAVs through the fronthaul links, for controlled trajectory purposes. The optimized trajectory of a UAV is a function of the optimization utility. That means, a power-efficient trajectory differs from the trajectory which maximizes the SE. As the user locations can be highly dynamic, we investigate UAVs travelling in a way that the data rate is maximized in short time intervals in this paper. Thus, all quadcopter UAVs travel in the direction the maximizes the SE of the users, i.e., traveling in the gradient direction. The scenario is illustrated in Fig. 1.

The trajectory of controlled quadcopters is compared with a more abstract gradient method, where the gradient directly determines the flight direction and velocity, i.e., zero gradient represents zero motion. This comparison has less mathematical complexity, but is not suitable for online applications. Since the travel direction is calculated based on the user locations by the ground BS online, a change in the users' locations results in an instant change in travel direction. The optimization problem is stated in the way that the minimum rate should be maximized in section IV. In section VII, the performance of the control design is compared with an ideal gradient method numerically.

A. Notation

The following notation is used in this paper: Matrices, vectors, and sets are indicated by bold upper-case, bold lowercase, and calligraphy letters, respectively. Determinant, trace, Hermitian and transpose of a matrix \mathbf{A} are represented by $|\mathbf{A}|$, trace $\{\mathbf{A}\}$, \mathbf{A}^H and \mathbf{A}^T respectively. The cardinally of a set \mathcal{A} is indicated by $|\mathcal{A}|$. The L_2 -norm of a vector \mathbf{a} is denoted by $||\mathbf{a}||$. The Kronecker and Hadamard products are represented by \otimes and \circ , respectively. The function phase $\{\mathbf{A}\}$ gives the element-wise complex phases of the matrix.

II. SYSTEM MODEL

A set of users \mathcal{G} intends to transmit data in cellular uplink to a set of UAVs \mathcal{K} , which are considered as flying base stations. This UAVs have a strong fronthaul connection towards a central base station on the ground and can adjust their positions while providing data rates to the users. The number of users and UAVs are given as $G = |\mathcal{G}|$ and $K = |\mathcal{K}|$, respectively. In the following, the channel model and a quadcopter UAV system model are described.

A. Channel Model

Each user is equipped with N_T antennas; each UAV has N_R antennas. We consider a single dominant LoS path between the users and the UAVs. The distance between the *k*th UAV and *i*th user is represented by

$$d_{\mathrm{U}k,i} = \left\| \mathbf{x}_{\mathrm{U}k} - \mathbf{x}_i \right\|,\tag{1}$$

where \mathbf{x}_{Uk} and \mathbf{x}_i indicate the position coordinates of the *k*th UAV, and the *i*th user, respectively.

Based on the free-space path-loss model, the path loss between each antenna of user i and each antenna at UAV k can be considered identical and is approximated by [4]

$$PL(d_{Uk,i}) = 10\alpha \log\left(\frac{d_{Uk,i}}{d_0}\right) dB + PL(d_0) + w_{\sigma ki} dB,$$
(2)

in which, PL(d) is the path-loss at distance d in dB, α is the path loss exponent, and d_0 represents a reference distance. A shadowing component is represented by the Gaussian variable $w_{\sigma ki} \sim \mathcal{N}(0, \sigma^2)$.

Defining $\beta = \exp\left(-\frac{\ln(10)}{10 \text{ dB}} \text{PL}(d_0)\right) d_0^{\alpha}$, the channel matrix between user *i* and UAV *k* is approximated as

$$\mathbf{H}_{\mathrm{U}k,i} \approx \sqrt{\beta} \left(\frac{1}{d_{\mathrm{U}k,i}}\right)^{\alpha/2} \tilde{\mathbf{H}}_{\mathrm{U}k,i}$$
(3)

of dimension $N_R \times N_T$. The entries of the normalized channel matrix without path loss have an absolute value of $|(\tilde{\mathbf{H}}_i)_{kl}| =$ 1; the phase phase $\{(\tilde{\mathbf{H}}_{UK,i})\}_{mn}$ is independent and identically distributed (i.i.d.) on $[0, 2\pi)$ for each $m \in \{1, \ldots, N_R\}$ and $n \in \{1, \ldots, N_T\}$. Moreover it is distributed identically on each possible UAV and user location while changing only moderate with small location changes.

Due to C-RAN, all UAVs fully collaborate via the ground BS through the high-capacity fronthaul links. Hence, the aggregate channel matrix from all users to the set of UAVs is given by

$$\mathbf{H}_{i} = \begin{pmatrix} \mathbf{H}_{\mathrm{U1},i}^{T} & \dots & \mathbf{H}_{\mathrm{UK},i}^{T} \end{pmatrix}^{T}.$$
 (4)

Then, the noisy observation vector at the ground BS is given as

$$\mathbf{v} = \sum_{i \in \mathcal{G}} \mathbf{H}_i \mathbf{u}_i + \mathbf{w},\tag{5}$$

where \mathbf{u}_i is the transmit symbol vector of user *i*. \mathbf{w} represents the additive Gaussian noise at the UAV receivers combined with compression noise in fronthaul. This is an independent Gaussian-distributed variables normalized in a way that $\mathbf{w} \sim C\mathcal{N}(\mathbf{0}, \mathbf{I})$. We define the transmit signal covariance matrix as $\mathbf{Q}_i = \mathbb{E}\{\mathbf{u}_i \mathbf{u}_i^H\}$.

B. Quadcopter UAV Model

In control, systems are usually modelled as matrix differential equation with an additional input and a given initial condition

$$\Sigma \begin{cases} \dot{\mathbf{s}}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{s}(0) = \mathbf{s}_0, \end{cases}$$
(6)

where s(t) and u(t) refer to state and input vector at a given time t. The system model is described by A and B describing how each element of input and state vector impact the derivatives of the state vector elements.

In case of a quadcopter UAV, the system model uses the squared speeds of the four rotors as input values $\mathbf{u}_k(t)$. The

state vector is described by location and orientation at a given point of time, as well as the derivatives of both. The location is described by the Cartesian coordinates \mathbf{x}_{Uk} ; the orientation is described as $\mathbf{o}_{Uk} = \begin{pmatrix} -\theta_k & \phi_k & \psi_k \end{pmatrix}^T$, where θ_k , ϕ_k , and ψ_k are the pitch, roll, and yaw angles, respectively. Using a state vector of $\mathbf{s}_k = \begin{pmatrix} \mathbf{x}_{Uk}^T & \mathbf{o}_{Uk}^T & \mathbf{\dot{s}}_{Uk}^T & \mathbf{\dot{o}}_{Uk}^T \end{pmatrix}^T$, a linearized model of a quadcopter can be derived as shown in [17]. It can be phrased as

$$\tilde{\Sigma}_{k}: \begin{cases} \dot{\mathbf{s}}_{k}(t) = \begin{pmatrix} \mathbf{0}_{6\times3} & \mathbf{0}_{6\times1} & \mathbf{0}_{6\times1} & \mathbf{I}_{6\times6} \\ \mathbf{0}_{1\times3} & g & 0 & \mathbf{0} & \mathbf{0}_{1\times6} \\ \mathbf{0}_{1\times3} & \mathbf{0} & g & 0 & \mathbf{0}_{1\times6} \\ \mathbf{0}_{4\times3} & \mathbf{0}_{4\times1} & \mathbf{0}_{4\times1} & \mathbf{0}_{4\times1} & \mathbf{0}_{4\times6} \end{pmatrix} \mathbf{s}_{k}(t) \\ + \begin{pmatrix} \mathbf{0}_{8\times4} \\ \mathbf{\Theta} \end{pmatrix} \mathbf{u}_{k}(t) + \begin{pmatrix} \mathbf{0}_{11\times1} \\ g \end{pmatrix} \\ \mathbf{s}_{k}(0) = \mathbf{s}_{k0}. \end{cases}$$
(7)

Here, Θ is a device-specific full-rank-matrix; g is the gravity constant. Even though it might not be that all states vector elements are given as output value, their values can be accessed through usage of an observer.

While controlling the input signal $\mathbf{u}_k(t)$, there is a chain of four integrators between the input signals and the horizontal parts of the UAV coordinates.

When the input signal is chosen to be $\mathbf{u}_k(t) = \mathbf{\Theta}^{-1} \begin{pmatrix} u_{k,1}(t) & u_{k,2}(t) & u_z & 0 \end{pmatrix}^T$, the system in each horizontal direction γ can be described as

$$\Sigma_{k,\gamma} : \begin{cases} \dot{\mathbf{s}}_{Uk,\gamma}(t) \\ = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathbf{s}_{Uk,\gamma}(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u_{k,\gamma}(t) \\ \mathbf{s}_{Uk,\gamma}(0) = \mathbf{s}_{Uk,\gamma,0}, \end{cases}$$
(8)

in which $\gamma \in \{1, 2\}$ and

$$\mathbf{s}_{\mathrm{U}k,\gamma}(t) = \begin{pmatrix} \mathbf{x}_{\mathrm{U}k,\gamma}(t) \\ \dot{\mathbf{x}}_{\mathrm{U}k,\gamma}(t) \\ \mathbf{o}_{\mathrm{U}k,\gamma}(t) \\ \dot{\mathbf{o}}_{\mathrm{U}k,\gamma}(t) \end{pmatrix}.$$
(9)

III. ACHIEVABLE SPECTRAL EFFICIENCY (SE)

A multiple access channel model is considered. The set of achievable rates of multiple users in cellular uplink channels is upper-bounded by [18], [19]

$$\sum_{i \in \mathcal{S}} R_i \leq \frac{1}{2} \log \left| \mathbf{I} + \sum_{i \in \mathcal{S}} \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^H \right|, \ \forall \mathcal{S} \subseteq \mathcal{G},$$
(10)

in which the right-hand side is the capacity. If the limitations of each upper-bound are divided equally among participating users, the minimum rate is upper-bounded by

$$R_{\min} = \min_{\mathcal{S} \subseteq \mathcal{G}} \frac{1}{2|\mathcal{S}|} \log \left| \mathbf{I} + \sum_{i \in \mathcal{S}} \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^H \right|.$$
(11)

Here, the subset actively bounding the optimization problem is given by $S_{\min} = \arg_{S \subset \mathcal{G}} R_{\min}$.

IV. Optimization Problem

The aim is to maximize this fairness rate. It depends on all UAV locations and the users' transmit signal covariance matrix. For a static placement, the optimization problem equals

 \mathbf{S}

$$\max_{\mathbf{Q}_{1}^{1},...,\mathbf{X}_{K}} R_{\min} \left(\mathbf{x}_{1}, \ldots, \mathbf{x}_{K}, \mathbf{Q}_{1}, \ldots, \mathbf{Q}_{G} \right)$$
(12)
ubject to trace $\{\mathbf{Q}_{i}\} \leq P_{i,\max}, \forall i \in \mathcal{G}$
 $\mathbf{Q}_{i} \succeq 0, \forall i \in \mathcal{G}$
 $\mathbf{x}_{k,3} = h, \forall k \in \mathcal{K},$

in which h is the altitude. Due to the change in impact of buildings, there is an optimal UAV altitude for achieving the best coverage as described in [12]. Hence, h is chosen to equal this value.

Since UAV motions will require some time, this is transformed into a dynamic optimization problem considering short intervals of time only. Hence, UAVs are only allowed to have moved a small distance from the previous location. Over this time interval, the channels can be assumed to have negligible changes. Hence, the same covariance matrix is assumed to be optimal over this short time interval.

The trajectory and covariance matrices are optimized alternately, but can be updated in sampled time intervals only. Thus, the covariance matrices remain unchanged on each sampled flight interval. The location at the considered point of time of UAV k is denoted as $\bar{\mathbf{x}}_k$; the optimal covariance matrix of user i on this location is given as $\bar{\mathbf{Q}}_i$. Using those, the optimization problem can be divided into two parts

$$\bar{\mathbf{Q}}_{i} = \underset{\mathbf{Q}_{1},...,\mathbf{Q}_{G}}{\operatorname{arg\,max}} R_{\min}\left(\bar{\mathbf{x}}_{1},...,\bar{\mathbf{x}}_{K},\mathbf{Q}_{1},...,\mathbf{Q}_{G}\right) \quad (13)$$
subject to trace $\{\mathbf{Q}_{i}\} \leq P_{i,\max}, \forall i \in \mathcal{G}$
 $\mathbf{Q}_{i} \succeq 0, \forall i \in \mathcal{G}$
 $\mathbf{x}_{k} = \underset{\mathbf{x}_{1},...,\mathbf{x}_{K}}{\operatorname{arg\,max}} R_{\min}\left(\mathbf{x}_{1},...,\mathbf{x}_{K},\bar{\mathbf{Q}}_{1},...,\bar{\mathbf{Q}}_{G}\right) \quad (14)$
subject to $||\mathbf{x}_{k} - \bar{\mathbf{x}}_{k}|| \leq \varepsilon, \forall k \in \mathcal{K}$
 $\mathbf{x}_{k,3} = h, \forall k \in \mathcal{K},$

where ε is an arbitrarily small number such that R_{\min} can be approximated as an affine function in \mathbf{x}_k for all k. Both problems can be solved in the given order in the interval of the sampling period.

This principle can be illustrated as shown in Algorithm 1, where at each sampling point both optimization problems are solved.



Fig. 2. Control circuit for one Cartesian direction using P-controllers for navigating UAVs to a location with maximized minimum rate.

Algorithm 1 Rate and trajectory optimizationloop $\mathbf{Q}_i \leftarrow$ result from (13) $\forall i$ $\mathbf{x}_k \leftarrow$ result from (14) $\forall k$ control the UAV into the direction \mathbf{x}_k seen from thecurrent location of the UAVwait for the sample time intervalend loop

It should be denoted that (13) is a convex problem. However, (14) is non-convex, but since only a small interval is needed, an affine approximation can be used. This is done by calculating a data rate gradient, which is calculated in section V. Algorithm 1 further bases on a UAV direction control, which is explained in section VI.

V. DATA RATE GRADIENT

For any arbitrary $S \subseteq G$, the derivative of the sum of rates from the set S given by (10) w.r.t. the position coordinate of the *k*th UAV is given as

$$\rho_{\mathcal{S}}^{k,\gamma} = \frac{\partial}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}} \sum_{i \in \mathcal{S}} R_i$$
$$= \frac{1}{2} \operatorname{trace} \left\{ \left(\mathbf{I} + \sum_{i \in \mathcal{S}} \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^H \right)^{-1} \sum_{i \in \mathcal{S}} \frac{\partial \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^H}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}} \right\},$$
(15)

which bases on [20]. Due to (4), the individual channel matrices $\mathbf{H}_{Uk,i}$ are implicated in this expression. Since the derivative of all of those is needed, through using (3), this can be calculated as

$$\frac{\partial \mathbf{H}_{\mathrm{U}k,i}}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}} = \sqrt{\beta} \frac{\partial}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}} \frac{1}{d_{\mathrm{U}k,i}^{\alpha/2}} \mathbf{\tilde{H}}_{\mathrm{U}k,i}
= \frac{\sqrt{\beta}\alpha \left(\mathbf{x}_{i,\gamma} - \mathbf{x}_{\mathrm{U}k,\gamma}\right)}{2 \left|\left|\mathbf{x}_{i} - \mathbf{x}_{\mathrm{U}k}\right|\right|^{\alpha/2+2}} \mathbf{\tilde{H}}_{\mathrm{U}k,i} + \frac{\sqrt{\beta}}{\left|\left|\mathbf{x}_{i} - \mathbf{x}_{\mathrm{U}k}\right|\right|^{\alpha/2}} \frac{\partial \mathbf{\tilde{H}}_{\mathrm{U}k,i}}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}}
= \frac{\alpha}{2} \frac{\mathbf{x}_{i,\gamma} - \mathbf{x}_{\mathrm{U}k,\gamma}}{\left|\left|\mathbf{x}_{i} - \mathbf{x}_{\mathrm{U}k}\right|\right|^{2}} \mathbf{H}_{\mathrm{U}k,i} + \frac{\sqrt{\beta}}{\left|\left|\mathbf{x}_{i} - \mathbf{x}_{\mathrm{U}k}\right|\right|^{\alpha/2}} \frac{\partial \mathbf{\tilde{H}}_{\mathrm{U}k,i}}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}}. \quad (16)$$

Please note that (3) has been used to simplify the first term of the result. Inserting this into (15) leads to the derivative of the complete channel matrix of one user.

Defining $\Phi_i = \text{phase}\{\mathbf{H}_i\}$ as the real-valued matrix of phases, the relation $\frac{\partial \tilde{\mathbf{H}}_i}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}} = j \frac{\partial \Phi_i}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}} \circ \tilde{\mathbf{H}}_i$ holds, since, from (3), the absolute value of each element of the normalized

channel matrices is constant one. This means, the added multiplication factor $j \frac{\partial \Phi_i}{\partial \mathbf{x}_{Uk,\gamma}}$ is imaginary solely. Since the same expression appears with the negative sign for the Hermitian matrix with identical absolute values and opposite sign, the sum of the traces of this two derivatives has little impact and vanishes for the SISO case. In the MIMO case, we assume that the phase of the channel does have a random, not-significant impact only. Hence, we ignore this term for the trajectory.

Using the channel definition in (3) and the distance between UAV and users, the last part of (15) can be reformulated as

$$\sum_{i \in \mathcal{S}} \frac{\partial \mathbf{H}_{i} \mathbf{Q}_{i} \mathbf{H}_{i}^{H}}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}} \approx \frac{\alpha}{2} \sum_{i \in \mathcal{S}} \frac{\mathbf{x}_{i,\gamma} - \mathbf{x}_{\mathrm{U}k,\gamma}}{||\mathbf{x}_{i} - \mathbf{x}_{\mathrm{U}k}||^{2}} \begin{pmatrix} \mathbf{0}_{(k-1)N_{R} \times N_{T}} \\ \mathbf{H}_{Uki} \\ \mathbf{0}_{(K-k)N_{R} \times N_{T}} \end{pmatrix} \mathbf{Q}_{i} \mathbf{H}_{i}^{H} + \mathbf{H}_{i} \mathbf{Q}_{i} \begin{pmatrix} \mathbf{0}_{(k-1)N_{R} \times N_{T}} \\ \mathbf{H}_{Uki} \\ \mathbf{0}_{(K-k)N_{R} \times N_{T}} \end{pmatrix} \begin{pmatrix} \mathbf{0}_{(K-k)N_{R} \times N_{T}} \end{pmatrix} \begin{pmatrix} \mathbf{0}_{(K-k)N_{R} \times N_{T}} \\ \mathbf{0}_{(K-k)N_{R} \times N_{T}} \end{pmatrix}$$
(17)

Note that the channel \mathbf{H}_i depends on the location of the UAV at the specific moment of time. Because the transmit signal covariance matrix \mathbf{Q}_i is constant within the sampling interval, it does not appear in above expression.

Using (11), the gradient of the achievable minimum rate (15) can be calculated as

$$\frac{\partial R_{\min}}{\partial \mathbf{x}_{\mathrm{U}k,\gamma}} = \frac{\rho_{\mathcal{S}_{\min}}^{k,\gamma}}{|\mathcal{S}_{\min}|} =: \rho_{\min}^{k,\gamma}.$$
(18)

Using this, the gradient can be phrased as a three-dimensional vector $\nabla_k R_{\min}$ containing the elements

$$\left(\nabla_k R_{\min}\right)_{\gamma} = \rho_{\min}^{k,\gamma}.$$
(19)

VI. QUADCOPTER UAV CONTROL

The UAV locations should remain unchanged when a rate maximum is obtained. Thus, the static end value is supposed to fulfill the expression $\nabla_k R_{\min} = 0 \Rightarrow \dot{\mathbf{x}}_{Uk} = \mathbf{0}$. While the rate gradient does not equate to zero, the UAV should be navigated into the direction of the gradient.

In order to fulfill this, the input values $u_{k,\gamma}(t)$ should be controlled in a way such that the static end values of $\dot{\mathbf{x}}_{Uk,\gamma}$ match for constant rate derivatives their corresponding entries of $\nabla_k R_{\min}$ for all $\gamma \in \{1, 2\}$. By using a P-controller, the input signal is chosen to be

$$u_{k,\gamma} = p\left(\nabla_k R_{\min}\right)_{\gamma} - \mathbf{k}^T \begin{pmatrix} \dot{\mathbf{x}}_{Uk,\gamma}(t) \\ \mathbf{o}_{Uk,\gamma}(t) \\ \dot{\mathbf{o}}_{Uk,\gamma}(t) \end{pmatrix}, \forall \gamma \in \{1,2\}, \quad (20)$$



Fig. 3. Trajectory and minimum rate for controlled UAVs ideal gradient method. The UAVs start in the corners and move then with decreasing velocity into the direction of the users. Receiving UAVs have eight antennas, transmitting users one. UAV altitude is 50 m.

in which $\mathbf{k} = \begin{pmatrix} k_1 & k_2 & k_3 \end{pmatrix}^T$ is the controller gain; *p* refers to the prefilter. In the vector used for feedback, $\mathbf{x}_{Uk,\gamma}$ does not occur. This is because the velocity should be controlled to zero, not the location itself. An individual, combined control circuit is shown in Fig. 2 on top of the previous page.

In a final system, the controller matrices depends on aspects such as altitude control and disturbance cancellation, and thus, i.e., they can not be chosen completely freely. In the numeric results given here, they are determined using a linear-quadratic regulator (LQR).

The overall system can be described by combining (8) and (20). Then, the system parameters can be derived as

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -k_1 & -k_2 & -k_3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ p \end{pmatrix}, \qquad (21)$$

the state vectors are given in (9).

From this, the trajectory of each controlled UAV can be described using the state equation

$$\begin{pmatrix} \dot{\mathbf{s}}_{\mathrm{U}k,1} \\ \dot{\mathbf{s}}_{\mathrm{U}k,2} \end{pmatrix} = (\mathbf{I}_2 \otimes \mathbf{A}) \begin{pmatrix} \mathbf{s}_{\mathrm{U}k,1} \\ \mathbf{s}_{\mathrm{U}k,2} \end{pmatrix} + \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \otimes \mathbf{b} \right) \nabla_k R_{\min}.$$
(22)

Since the UAV altitude is supposed to remain constant, the vertical dimension of the gradient is multiplied with zero only in above expression.

Each individual UAV $k \in \{1, \ldots, K\}$ can be described by this equation. Combined, this describes the full system of UAVs, which are used for rate maximization on locations determined by control methods.

VII. NUMERICAL RESULTS

For the numerical results here, the controllers are determined using a LQR. This way, their values are set to $\mathbf{k}^T = (0.5477 \,\mathrm{m\,s} \ 23.9683 \,\mathrm{s}^2 \ 6.9308 \,\mathrm{s})$. The initial states are set to $\mathbf{s}_{Uk,\gamma,0} = (\mathbf{x}_{Uk,\gamma,0} \ 0 \ 0 \ 0)^T$, where $\mathbf{x}_{Uk,\gamma,0} = \pm 40 \,\mathrm{m}$ as shown in the corners of Fig. 3. The overall control system is discretized, such that the rate gradients only need to be known in short time intervals.

For comparison, additional simulations are done with a more simple gradient method. There, the velocities are set directly proportional to the rate gradient. The integration of those over time equals the location of the UAV. This system does not contain any feedback paths, and there is no controller to be designed. This is a mathematical idealized model for the trajectory. Thereby, the UAV trajectory can perform fast direction and velocity changes. This comparison system is without practical relevance for real-time applications, but might be relevant in scenarios where full trajectories are planned beforehand. The trajectories and data rates over time are compared for both methods in Fig. 3.

In both simulations, the different UAVs are placed on locations apart from each other leading to having a higher coverage area. The control system adapts slower to gradient changes than the gradient method. This comes with a rather slowly growth of the data rate at the beginning, but leads to a more smooth trajectory with less noise near the optimal points.

VIII. CONCLUSION AND FUTURE WORK

The presented algorithm optimizes the UAV placements and trajectories online while serving dynamically located users with data rates. For the static case, the UAVs reach a local optimal set of positions and remain there until the user locations change. This has been achieved throug applying control methods on the minimum rate maximization problem. This methods only require information about channels and locations for the given moment of time. Additional Information are not required. The approach presented here is directly suitable for real-time applications, as it can adapt to changes of user locations and channel coefficients instantaneously. There is no exhaustive search in use. Nevertheless, the UAVs find a good location to be placed for a permanent rate transfer. From the numerical results, this location is even more stable with the control model than with the mathematical gradient method.

This concept works for homogeneous areas, such as rural areas or concert places. In future work, this will be extended by considering heterogeneous building maps, where the channel amplitude is variable due to non-constant LoS connections. A combination of statistical approaches basing on user densities and this type of real-time adaption might be useful to achieve further enhancements in dense urban environments.

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