

Error Floor Analysis of Irregular Repetition ALOHA

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Abstract—With the rapid expansion of the Internet of Things, the efficient sharing of the wireless medium by a large amount of simple transmitters is becoming essential. Scheduling-based solutions are inefficient for this setting, where small data units are broadcast sporadically by terminals that most of the time are idle. Modern random access has embraced the challenge and provides suitable slot-synchronous and asynchronous multiple access solutions based on replicating the packets and exploiting successive interference cancellation (SIC) at the receiver. In this work, we focus on asynchronous modern random access. Specifically, we derive an analytical approximation of the performance of irregular repetition ALOHA (IRA) in the so-called error floor region. Numerical results show the tightness of the derived approximation under various scenarios.

I. INTRODUCTION

Under the label of Internet of things (IoT) and machine-to-machine (M2M), a number of services revolutionizing vast industry sectors, from automotive to logistics, from health care to farming, have emerged in the last years [1], [2]. Many of these services are characterized by the sporadic transmission of data units containing few information bits. This apparently small twist in the communication problem poses a number of intriguing challenges. At the physical layer, efficient transmission detection, channel estimation, and error correction for short packets are key. Further, an extensive transmitter population needs to share the common wireless medium as efficiently as possible. Very low duty cycle, sporadic access, and small amount of information are ineffectively handled by classical scheduling-based medium access (MAC) approaches. Random access (RA) inherently provides the sought flexibility with the shortcoming of low efficiency [3].

Modern random access (see [4]–[7]) has shown how the use of packet replication coupled with successive interference cancellation (SIC) is able to drastically boost performance. Tools borrowed from the design of low-density parity-check (LDPC) codes over the erasure channel can be exploited to maximize the system throughput and overcome the low efficiency burden of classical schemes. A random coding bound for the massive random access setting was derived in [8]. In the quest to close the gap emerged with respect to modern random access solutions, the research community has also investigated compressed-sensing inspired schemes [9]–[11].

Slot-synchronous modern RA have been extensively studied both in the asymptotic regime [5], [6], i.e., when the maximum

delay among physical layer packet copies grows very large, and in the finite length regime [7], [12]. Rendering the entire transmitter population synchronized to the slot boundaries entails a certain level of complexity and requires the senders to be able to receive a beacon signal. Such requirements might be undesirable for low-complexity and low-cost IoT applications where terminals can be equipped with transmitter-only hardware and are battery-powered. Asynchronous RA dates back to the original idea of Abramson’s ALOHA protocol [3] and has also been investigated in the context of modern RA. The use of packet copies—referred to as replicas in the following—spaced with a randomized delay and the use of SIC at the receiver has proven to be particularly beneficial [13], [14]. Packet combining can be used to supplement SIC and further improve performance [15]. The extension to a variable number of replicas per user, dubbed irregular repetition ALOHA (IRA), was introduced in [16].

The asymptotic setting of asynchronous modern RA was studied in [16] under the simplified destructive collision channel model. An error floor analysis was provided in [15] for the simplest collision pattern neutralizing SIC with two users and two replicas per user.

In this work, we derive an analytical approximation to the performance of IRA in the error floor region, i.e., for low-to-medium channel loads. Compared to the analysis in [15], the derived approximation is very general and encompasses IRA with different number of replicas per user as well as collision patterns with more than two packets. For the particular case of two replicas per user, it closes the gap to the simulated performance of the prediction in [15]. The derived analytical approximation yields a very accurate prediction of IRA in the error floor region, and can be used for the optimization of the degree distribution according to which users select the replication factor to achieve a target packet loss rate.

II. SYSTEM MODEL

We consider an infinitely large user population generating traffic according to a Poisson process of intensity G , called *logical channel load* or simply *load* in the following. G is measured in packet arrivals per packet duration T_p . Further, we assume the IRA asynchronous RA protocol [13], [16]. According to IRA, when a packet is generated, the user samples a degree distribution Λ to compute the repetition degree $d^{(u)}$. The polynomial representation of the user degree

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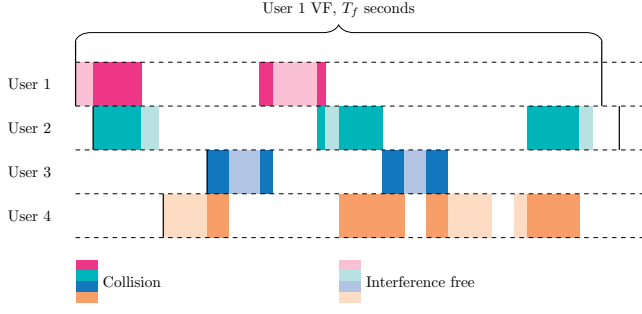


Fig. 1. Four users transmit their replicas according to the IRA protocol. User 1 and 3 send two replicas, User 2 three replicas, and User 4 four replicas.

distribution is of the form

$$\Lambda(w) = \sum_{d=2}^{d_m} \Lambda_d w^d,$$

where Λ_d is the probability that d replicas are transmitted, d_m is the maximum number of packet copies sent, $\sum_{d=2}^{d_m} \Lambda_d = 1$, and $\bar{d} = \sum_{d=2}^{d_m} d \Lambda_d$ is the average number of replicas per data unit transmitted.¹ While the first replica is transmitted immediately upon generation of the data unit of user u , the remaining $(d^{(u)} - 1)$ replicas are sent within a virtual frame (VF) of duration T_f , with transmission times chosen so as to avoid self-interference. Since packets are generated at random with exponential inter-arrival time, the VFs are asynchronous among users. The start time of each replica of a user can be stored in a dedicated portion of the packet header or uniquely determined from the data content via a pseudo-random algorithm known at the receiver [15]. An example of a received signal with four users transmitting two, three, two, and four replicas, respectively, is depicted in Fig. 1.

To withstand the effect of the Gaussian channel and interference, replicas are protected by a channel code \mathcal{C} with Gaussian codebook. Its code rate is $R = k/n_s$, where n_s is the number of packet symbols after channel encoding and modulation. Assuming an ideal estimate of the sampling epoch, the frequency offset, and the phase offset, the discrete-time baseband received signal for the r -th replica of the u -th user, $\mathbf{y}^{(u,r)} = (y_0^{(u,r)}, \dots, y_{n_s-1}^{(u,r)})$, is given by

$$\mathbf{y}^{(u,r)} = \mathbf{x}^{(u)} + \mathbf{z}^{(u,r)} + \mathbf{n}.$$

Here $\mathbf{x}^{(u)} = (x_0^{(u)}, \dots, x_{n_s-1}^{(u)})$ is the sequence of transmitted symbols, $\mathbf{z}^{(u,r)}$ the interference contribution over the user- u replica- r signal and $\mathbf{n} = (n_0, \dots, n_{n_s-1})$ a noise vector sampled from a complex discrete white Gaussian process with $n_i \sim \mathcal{CN}(0, 2\sigma_n^2)$.

For user u , replica r , and symbol i , we define $P_i^{(u)} \triangleq \mathbb{E}[|x_i^{(u)}|^2]$, the received signal power, $N = 2\sigma_n^2$ the

noise power, and $Z_i^{(u,r)} \triangleq \mathbb{E}[|z_i^{(u,r)}|^2]$ the aggregate interference power. Throughout the paper, we assume that all users are received with the same power, i.e. perfect power control is adopted. Thus, $P_i^{(u)} = P$ and $Z_i^{(u,r)} = m_i^{(u,r)} P$, where $m_i^{(u,r)} \in \mathbb{N}$ denotes the number of active interferers over the i -th symbol. The aggregate interference is a discrete Gaussian process, with $z_i \sim \mathcal{CN}(0, m_i^{(u,r)} P)$. Hence, the instantaneous signal-to-interference and noise ratio (SINR) is

$$\gamma_i^{(u,r)} = \frac{P}{N + m_i^{(u,r)} P}.$$

A. Modeling of the decoding process

Instead of adopting the well-known destructive collision channel model that is particularly pessimistic in the case of asynchronous access in the presence of error correction, we resort to the *block interference channel model* [17]. Consider again the r -th replica of user u . To ease the notation, we drop the superscript (u, r) . We interpret the n_s symbols as independent Gaussian channels and, leveraging the Gaussian assumption of both the transmitted signals and the noise, we compute the instantaneous mutual information over the i -th symbol, $I(\gamma_i)$, as

$$I(\gamma_i) = \log_2(1 + \gamma_i).$$

The average mutual information \bar{I} over the n_s symbols is²

$$\bar{I} = \frac{1}{n_s} \sum_{i=0}^{n_s-1} I(\gamma_i) = \frac{1}{n_s} \sum_{i=0}^{n_s-1} \log_2(1 + \gamma_i).$$

By comparing the average mutual information over the packet with the rate R , we resort to the decoding condition $D = \mathbb{I}\{R \leq \bar{I}\}$, where $\mathbb{I}\{X\}$ denotes the indicator function. Hence, $D = 1$ if decoding is successful and $D = 0$ otherwise.

The destructive collision channel model is a special case of the block interference channel model, where R is chosen such that only collision-free packets can be successfully decoded, i.e., $R = \log_2(1 + \frac{P}{N})$. In asynchronous RA solutions, to be able to counteract a certain level of interference, a rate $R < \log_2(1 + \frac{P}{N})$ is normally chosen. Additionally, this model allows to take into account features like channel coding, multi-packet reception, and capture effect [19], [20].

In IRA, when the decoding of a replica is successful, its contribution together with the one of all its copies is removed from the received signal via ideal interference cancellation.³

III. ANALYTICAL APPROXIMATION OF THE PACKET LOSS RATE IN THE ERROR FLOOR REGION

In this section, we derive an analytical approximation of the packet loss rate (PLR) of IRA that is tight for low-to-medium channel load conditions. Packet losses are caused by particular

¹The logical channel load G is the measure of innovative packets injected by the user population, equivalent to the channel load of classical RA schemes like ALOHA and slotted ALOHA (SA), and thus does not take into account the user degree distribution. The *physical channel load* instead measures the average number of packets per packet duration effectively transmitted over the channel and thus is computed as $G_p = G\bar{d}$.

²Similar to [18], the idea is to take into account the mutual information carried by each replica symbol, and then compute the average over the entire replica.

³Ideal packet detection is assumed in the following. Through a careful selection of the preamble, the detection algorithm and the detection threshold, misdetections can be minimized.

interference patterns that SIC is not able to resolve. In the slot-synchronous case, these patterns are analogous to the stopping sets of LDPC codes [21], and can be analyzed exploiting tools from coding theory and graph theory. In the asynchronous setting, a graph representation is not straightforward, since no discrete objects as slots are present anymore. Building on the error floor approximation in [7], [22], we derive a tighter PLR approximation than the one in [15] by considering a larger subset of \mathcal{C} -unresolvable collision patterns (\mathcal{C} -UCPs), beyond the two-user case considered in [15]. Enlarging the considered subset not only improves the approximation for regular user degree distributions, but also enables the analysis of irregular user degree distributions for the first time. The following definitions will be useful for the analysis.

Definition 1 (Collision cluster \mathcal{U}). Consider a subset \mathcal{U} of users. Assume that packets of all users in the complement set of \mathcal{U} , denoted by \mathcal{U}^c , have been successfully decoded. The subset \mathcal{U} is referred to as collision cluster if no packet replicas for the users in \mathcal{U} are collision-free.

Under the assumption of a collision channel, none of the users in the collision cluster can be successfully decoded. Conversely, when a channel code \mathcal{C} protects each transmitted packet, a certain level of interference can be tolerated yet allowing correct decoding and thus the collision cluster might be resolvable.

Definition 2 (\mathcal{C} -unresolvable collision pattern). Assuming that each packet is encoded by a code \mathcal{C} , a \mathcal{C} -unresolvable collision pattern (\mathcal{C} -UCP) \mathcal{S} is a collision cluster where no user in the set can be successfully decoded.

A \mathcal{C} -UCP is also a collision cluster, but not vice versa. For low-to-medium channel loads, decoding failures are caused by \mathcal{C} -UCPs involving few users and correspond to the minimal stopping sets in the slot-synchronous scenario [22], [23].

Definition 3 (Dominant \mathcal{C} -unresolvable collision pattern). A dominant \mathcal{C} -UCP does not contain a nonempty \mathcal{C} -UCP of smaller size.

In order to evaluate the probability of \mathcal{C} -UCPs, we extend the definition of vulnerable period [24] as follows:

Definition 4 (\mathcal{C} -vulnerable period for $|\mathcal{S}| = 2$). Consider the transmission of a packet protected by a code \mathcal{C} between time τ and $\tau + T_p$. The packet's \mathcal{C} -vulnerable period is the interval of time $[\tau - \tau_l^*, \tau + \tau_r^*]$ in which the presence of a single interferer leads to a decoding failure. Hence, the vulnerable period duration T_v is defined as $T_v = \tau_l^* + \tau_r^*$.

Under the collision channel model, $\tau_l^* = \tau_r^* = T_p$, so $T_v = 2T_p$, as known from the literature [24]. Instead, when packets are protected by a code \mathcal{C} characterized by a rate R [b/symb], some interference can be sustained before decoding may fail, effectively reducing the vulnerable period to

$$T_v = 2\varphi T_p, \quad 0 \leq \varphi \leq 1.$$

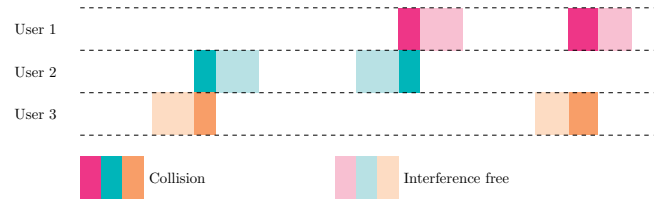


Fig. 2. Example of a \mathcal{C} -UCP characterized by $\mu^{\mathcal{S}} = 3$ replica-collision sets. From left to right, the first set involves user 2 and 3, the second user 1 and 2 and the last one user 1 and 3. The number of involved users in the \mathcal{C} -UCP is $\nu^{\mathcal{S}} = 3$ and the user profile follows $\nu^{\mathcal{S}} = [0, 3, 0, 0]$, i.e. all three users transmitted 2 replicas.

Leveraging on the Gaussian assumption of both the signals and noise, φ is the unique solution of the equation

$$\varphi \log_2 \left(1 + \frac{P}{N} \right) + (1 - \varphi) \log_2 \left(1 + \frac{P}{N + P} \right) = R \quad (1)$$

when it exists. Hence,

$$\varphi = \max \left(0, \frac{R - \log_2 \left(1 + \frac{P}{N + P} \right)}{\log_2 \left(1 + \frac{P}{N} \right) - \log_2 \left(1 + \frac{P}{N + P} \right)} \right), \quad (2)$$

with $0 \leq \varphi \leq 1$.⁴

An \mathcal{C} -UCP is characterized by a number of replica-collision sets where the users following the user profile $\nu^{\mathcal{S}}$ transmitted their replicas. Both the replica-collision set and $\nu^{\mathcal{S}}$ are defined as follows.

Definition 5 (Replica-collision set). Consider the \mathcal{C} -UCP \mathcal{S} . A replica-collision set within \mathcal{C} -UCP collects all packets that cause reciprocal interference, i.e., no packet within a replica-collision set is interference-free and its collision is with one or more data units of the replica-collision set.

We denote by $\mu^{\mathcal{S}}$ the number of replica-collision sets in \mathcal{C} -UCP \mathcal{S} .

Definition 6 (User profile $\nu^{\mathcal{S}}$ for \mathcal{C} -UCP \mathcal{S}). Consider the \mathcal{C} -UCP \mathcal{S} . The user profile $\nu^{\mathcal{S}}$ is defined as the vector $\nu^{\mathcal{S}} = [\nu_1^{\mathcal{S}}, \dots, \nu_m^{\mathcal{S}}]$, where $\nu_l^{\mathcal{S}}$ is the number of users in \mathcal{S} transmitting l replicas. We denote by $\|\nu^{\mathcal{S}}\|_1$ the L^1 -norm of $\nu^{\mathcal{S}}$, i.e., $\|\nu^{\mathcal{S}}\|_1 = |\mathcal{S}|$.

An example of a \mathcal{C} -UCP is depicted in Fig. 2. For this example, the \mathcal{C} -UCP contains $\mu^{\mathcal{S}} = 3$ replica-collision sets, the number of involved users is $\nu^{\mathcal{S}} = 3$, and the user profile is $\nu^{\mathcal{S}} = [0, 3, 0, 0]$. This set will be denoted by \mathcal{S}_3 in the following (see Table I).

A. Packet loss rate approximation

We are now ready to derive an approximation to the PLR for IRA. Consider a generic user u . Denote by \mathcal{A} the set of all \mathcal{C} -UCPs, with $\mathcal{A}^* \subset \mathcal{A}$ the set of dominant \mathcal{C} -UCPs, and by

⁴The present analysis can be easily extended to the block fading channel. Conditioning on the received power for the replica under investigation and the power of the interfering data unit, (1) still holds. By removing the conditioning over the two power levels, one can retrieve the density of φ and hence its mean. This value can then be used for the computation of the vulnerable period required in the packet loss rate approximation.

$n_p = T_f/T_p$ the VF time span measured in packet durations. Similar to [7], [15], [22], the PLR p can be approximated as

$$\begin{aligned} p &= \Pr\left(\bigcup_{\mathcal{S} \in \mathcal{A}} u \in \mathcal{S}\right) \stackrel{(a)}{\leq} \sum_{\mathcal{S} \in \mathcal{A}} \Pr(u \in \mathcal{S}) \\ &\stackrel{(b)}{\approx} \sum_{\mathcal{S} \in \mathcal{A}^*} \Pr(u \in \mathcal{S}) \\ &\stackrel{(c)}{=} \sum_{m=2}^{\infty} \sum_{\mathcal{S} \in \mathcal{A}^*} \Pr(u \in \mathcal{S}|m) \Pr(M=m) \\ &= \sum_{m=2}^{\infty} \sum_{\mathcal{S} \in \mathcal{A}^*} \Pr(u \in \mathcal{S}|m) \frac{e^{-n_p G} (n_p G)^m}{m!}, \end{aligned} \quad (3)$$

where (a) is the well-known union bound, (b) stems from approximating the PLR by considering only the dominant \mathcal{C} -unresolvable collision patterns, and (c) follows by conditioning on having m users transmitting in a VF time span. Note that the truncation of the union bound is by construction an approximation. Further, focusing on the \mathcal{C} -UCP contributing the most to the PLR in the low-to-moderate channel load region is a natural choice since enumerating all possible \mathcal{C} -UCPs is infeasible. Additionally, for low channel load values it is clear that \mathcal{C} -UCPs with a small number of users involved are more likely to occur, hence \mathcal{A}^* will comprise only the dominant \mathcal{C} -UCPs with a small number of transmitters involved.

We further define $n_v = \lfloor T_f/T_v \rfloor$, the number of disjoint vulnerable periods within one VF. Exploiting the parallelism between \mathcal{C} -UCPs for the asynchronous case and stopping sets for the slot-synchronous case, following [7], $\Pr(u \in \mathcal{S}|m)$ can be written as

$$\Pr(u \in \mathcal{S}|m) = \frac{a(m, \nu^{\mathcal{S}}, \Lambda) b(n_v, \mu^{\mathcal{S}}) c(\mathcal{S})}{d(n_v, \nu^{\mathcal{S}})} \cdot \frac{\nu^{\mathcal{S}}}{m}, \quad (4)$$

where $a(m, \nu^{\mathcal{S}}, \Lambda)$ is the number of ways to select $\nu^{\mathcal{S}}$ users with degree profile $\nu^{\mathcal{S}}$ from a set of m users with degree distribution $\Lambda(w)$, $b(n_v, \mu^{\mathcal{S}})$ is the number of ways to select the vulnerable periods of \mathcal{S} such that $u \in \mathcal{S}$, $c(\mathcal{S})$ is the number of graph-isomorphisms of \mathcal{S} , and $d(n_v, \nu^{\mathcal{S}})$ is the total number of ways in which $\nu^{\mathcal{S}}$ users (including u) with degree profile $\nu^{\mathcal{S}}$ can connect edges to the n_v (disjoint) vulnerable periods in their VFs. These terms can be easily obtained via combinatorial arguments as [7]

$$a(m, \nu^{\mathcal{S}}, \Lambda) = \binom{m}{\nu^{\mathcal{S}}} \nu^{\mathcal{S}}! \prod_{l=2}^{d_m} \frac{(\Lambda_l)^{\nu_l^{\mathcal{S}}}}{\nu_l^{\mathcal{S}}!}, \quad (5)$$

$$b(n_v, \mu^{\mathcal{S}}) \approx \binom{n_v-1}{\mu^{\mathcal{S}}-1}, \quad (6)$$

$$d(n_v, \nu^{\mathcal{S}}) \approx \frac{1}{n_v} \prod_{l=2}^{d_m} \left(n_v \binom{n_v-1}{l-1} \right)^{\nu_l^{\mathcal{S}}}. \quad (7)$$

Using (5), (6) and (7) in (4) and substituting (4) in (3), we get the sought PLR approximation. Note that when computing $b(n_v, \mu^{\mathcal{S}})$ and $d(n_v, \nu^{\mathcal{S}})$, we leverage the definition of disjoint vulnerable periods within a VF by $n_v = \lfloor T_f/T_v \rfloor$. These are the equivalent of time slots in slot-synchronous RA. The

TABLE I
PARAMETERS OF DOMINANT \mathcal{C} -UCPs WITH $\mu^{\mathcal{S}} \leq 4$

| \mathcal{A}^* | $\nu^{\mathcal{S}}$ | $\nu^{\mathcal{S}}$ | $\mu^{\mathcal{S}}$ | $c(\mathcal{S})$ |
|--------------------|---------------------|---------------------|---------------------|------------------|
| \mathcal{S}_1 | [0, 2, 0, 0] | 2 | 2 | 1 |
| \mathcal{S}_2 | [0, 0, 2, 0] | 2 | 3 | 1 |
| \mathcal{S}_3 | [0, 3, 0, 0] | 3 | 3 | 6 |
| \mathcal{S}_4 | [0, 2, 1, 0] | 3 | 3 | 6 |
| \mathcal{S}_5 | [0, 0, 0, 2] | 2 | 4 | 1 |
| \mathcal{S}_6 | [0, 2, 0, 1] | 3 | 4 | 6 |
| \mathcal{S}_7 | [0, 1, 2, 0] | 3 | 4 | 12 |
| \mathcal{S}_8 | [0, 1, 1, 1] | 3 | 4 | 12 |
| \mathcal{S}_9 | [0, 0, 3, 0] | 3 | 4 | 24 |
| \mathcal{S}_{10} | [0, 0, 2, 1] | 3 | 4 | 12 |
| \mathcal{S}_{11} | [0, 3, 0, 1] | 4 | 4 | 24 |
| \mathcal{S}_{12} | [0, 4, 0, 0] | 4 | 4 | 72 |

duration is computed according to (2), which assumes the collision among only two packets and therefore is exact only for \mathcal{C} -UCPs with $\nu^{\mathcal{S}} = 2$. For \mathcal{C} -UCPs involving more than two users there is a nonzero probability that collisions among more than two replicas appear. Nonetheless, we conjecture that considering only the case of collisions among two packets for the computation of n_v provides a good approximation, and we provide quantitative proof via the numerical results presented in the next section.

For a regular user degree distribution with degree d , $a(m, \nu^{\mathcal{S}}, \Lambda)$ and $d(n_v, \nu^{\mathcal{S}})$ simplify to

$$\begin{aligned} a(m, \nu^{\mathcal{S}}, \Lambda) &= \binom{m}{\nu^{\mathcal{S}}} \\ d(n_v, \nu^{\mathcal{S}}) &\approx \frac{1}{n_v} \left(n_v \binom{n_v-1}{d-1} \right)^{\nu^{\mathcal{S}}}, \end{aligned}$$

leading to the approximation of the PLR

$$p \approx \sum_{m=2}^{\infty} \sum_{\mathcal{S} \in \mathcal{A}^*} \frac{\nu^{\mathcal{S}} \binom{m}{\nu^{\mathcal{S}}} \binom{n_v-1}{\mu^{\mathcal{S}}-1} c(\mathcal{S})}{\frac{m}{n_v} \left(n_v \binom{n_v-1}{d-1} \right)^{\nu^{\mathcal{S}}}} \frac{e^{-n_p G} (n_p G)^m}{m!}. \quad (8)$$

For the particular case where we consider only the single unresolvable collision pattern consisting of two users, \mathcal{S} , it follows $\nu^{\mathcal{S}} = 2$, $\mu^{\mathcal{S}} = d$, and $c(\mathcal{S}) = 1$, and (8) further simplifies to

$$p \approx \sum_{m=2}^{\infty} \frac{\binom{m}{2}}{n_v \binom{n_v-1}{d-1}} \cdot \frac{2}{m} \cdot \frac{e^{-n_p G} (n_p G)^m}{m!}, \quad (9)$$

which is the expression in [15, eq. (18)].

IV. NUMERICAL RESULTS

In this section, numerical results for the PLR approximation derived in (3) are provided. The set of twelve dominant \mathcal{C} -UCPs \mathcal{A}^* taken into account for the approximation is listed in Table I.⁵ We consider perfect power control, i.e., all users are

⁵Depending on the user degree distribution, \mathcal{A}^* may include only the subset of \mathcal{C} -UCPs that are viable. As an example, for $\Lambda = x^2$, only the subset $\{\mathcal{S}_1, \mathcal{S}_3, \mathcal{S}_{12}\}$ contributes to the PLR approximation.

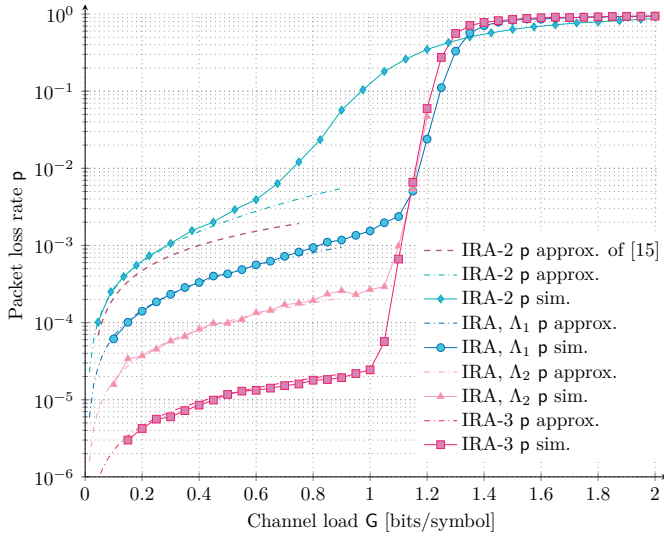


Fig. 3. PLR performance simulated (marked solid lines) compared to the derived approximation (dash-dotted lines), for $P/N = 6$ dB, $R = 1.5$ [b/sym] and $T_f = 200 T_p$. Regular user degree distributions with $\Lambda = x^2$ and $\Lambda = x^3$ are denoted with IRA-2 and IRA-3 respectively. For IRA-2 a comparison with the approximation derived in [15] is also provided (dashed line). The irregular user degree distributions are $\Lambda_1 = 0.263x^2 + 0.344x^3 + 0.393x^5$ and $\Lambda_2 = 0.51x^2 + 0.49x^4$.

received with equal power, and a signal-to-noise ratio (SNR) of $P/N = 6$ dB. As per [15], the receiver operates on a sliding window of duration $3T_f$ and is moved by $0.1T_f$ forward when no further packets can be successfully recovered in the current decoding window. We plot the PLR for two regular user degree distributions, $\Lambda = x^2$ and $\Lambda = x^3$, named IRA-2 and IRA-3 in the presented results, respectively, and for two irregular user degree distributions, namely $\Lambda_1 = 0.263x^2 + 0.344x^3 + 0.393x^5$ and $\Lambda_2 = 0.51x^2 + 0.49x^4$.

In Fig. 3, we depict the simulation results for the aforementioned user degree distributions. The VF is of duration $T_f = 200 T_p$ and the rate is $R = 1.5$ [b/sym]. Consequently, $n_p = T_f/T_p = 200$ and according to (2), for the selected SNR and rate, $\varphi \cong 0.44$ and thus $n_v = \lfloor T_f/T_v \rfloor = 225$.

The solid marked lines correspond to Monte Carlo simulations while dash-dotted lines identify the PLR approximation. For reference, the approximation of [15] corresponding to (9) is also depicted for IRA-2 (dashed line). Remarkably, the derived PLR provides an accurate prediction of the simulated PLR for all considered user degree distributions in the error floor region. For IRA-2, the derived approximation closes the gap to the simulated curve of the approximation in [15]. It is also noteworthy that the approximation of [15] cannot address any irregular distribution. Finally, we underline that despite the computation of the number of disjoint vulnerable periods n_v disregards collisions among more than two replicas, the approximation remains particularly tight also in the presence of user degrees larger than two.

In order to investigate the robustness of the error floor approximation, we delve into two slightly modified scenarios. The first one reduces the replicas protection against noise and interference by selecting a larger rate of $R = 2$ [b/sym] for

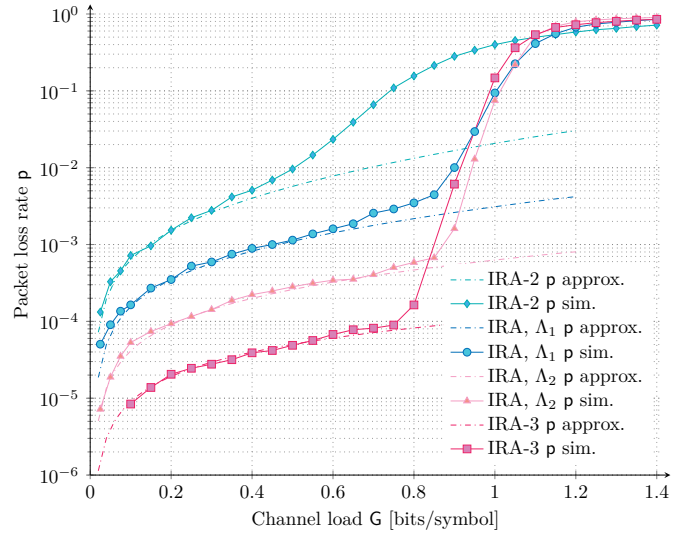


Fig. 4. PLR performance simulated (marked solid lines) compared to the derived approximation (dash-dotted lines), for $P/N = 6$ dB, $R = 2$ [b/sym] and $T_f = 200 T_p$. Regular user degree distributions with $\Lambda = x^2$ and $\Lambda = x^3$ are denoted with IRA-2 and IRA-3 respectively. The irregular user degree distributions are $\Lambda_1 = 0.263x^2 + 0.344x^3 + 0.393x^5$ and $\Lambda_2 = 0.51x^2 + 0.49x^4$.

the same SNR of $P/N = 6$ dB and is shown in Fig. 4. A higher rate results in packets with shorter duration and abbreviates the VF duration, but also reduces the interference that can be counteracted. As a result, $\varphi \cong 0.78$ and thus $n_v = \lfloor T_f/T_v \rfloor = 127$. The same color and marker code of Fig. 3 is adopted. We can notice that also in this case a very tight match between the derived approximation and the numerical results for all the user degree distributions is obtained. Also the sharp drop in the PLR performance at very low channel load is precisely predicted and confirms once more the great potential of the analytical approximation.

We show in Fig. 5 the second modified scenario which considers a shorter maximum latency among replicas of the same user, i.e., $T_f = 100 T_p$, while keeping the rate $R = 1.5$ [b/sym] and the $P/N = 6$ dB. Although driven by a different purpose—reducing the latency among replicas has an important impact on both transmitter and receiver design—the performance results mirror what we observed when increasing the rate (cf. Fig. 4). Latency reduction may impact the transmitter architecture by alleviating the battery requirements since the device is required to be active for a shorter period. At the same time, the VF duration has a direct influence on the storage capabilities of the receiver. Enabling the SIC procedure demands storage availability proportional to the VF and thus a reduction eases the memory needs. As a result $n_p = T_f/T_p = 100$ and $\varphi \cong 0.44$, which provides a number of vulnerable periods of $n_v = \lfloor T_f/T_v \rfloor = 112$.

The same color and marker code of Fig. 3 are adopted. Also in this case, the error floor is tightly predicted for all the user degree distributions. Compared to the scenario of Fig. 4, the simulated PLR experiences a larger range of channel load values pertaining to the error floor regime. In particular, for IRA-2 the range is extended from $G = 0.4$ [b/sym]

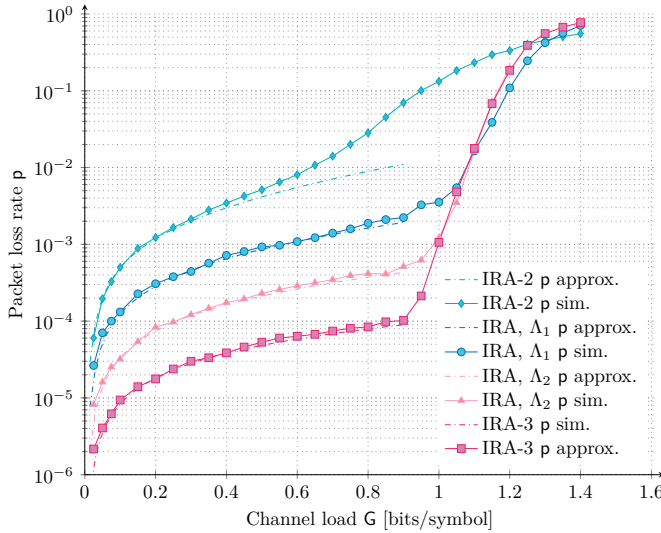


Fig. 5. Simulated PLR performance (marked solid lines) and derived approximation (dash-dotted lines), for $P/N = 6$ dB, $R = 1.5$ [b/sym] and $T_f = 100 T_p$. Regular user degree distributions with $\Lambda = x^2$ and $\Lambda = x^3$ are denoted with IRA-2 and IRA-3 respectively. The irregular user degree distributions are $\Lambda_1 = 0.263x^2 + 0.344x^3 + 0.393x^5$ and $\Lambda_2 = 0.51x^2 + 0.49x^4$.

to $G = 0.6$ [b/sym], while for all other distribution is expanded from $G = 0.75$ [b/sym] to $G = 0.9$ [b/sym]. A shorter VF corresponds to a smaller MAC frame in the slot-synchronous equivalent systems contention resolution diversity slotted ALOHA (CRDSA) [4] and irregular repetition slotted ALOHA (IRSA) [5]. It is known from the literature that smaller MAC frames worsen the PLR performance. Similarly, we can also observe in the case of asynchronous RA that lowering the VF duration has a detrimental impact on the PLR (cf. Fig. 3 and Fig. 5).⁶

V. CONCLUSION

Driven by IoT applications, in this paper we considered an asynchronous RA scheme employing irregular packet replication and SIC at the receiver. We derived an analytical approximation of the PLR of IRA that accurately predicts its performance in the error floor region. The accuracy of the PLR prediction is demonstrated via Monte Carlo simulations for a number of different scenarios. The possibility to analytically evaluate the PLR is particularly appealing, since error floors in the order of 10^{-5} and lower are achievable and numerical evaluations become particularly lengthy. Furthermore, the derived approximation can be used to optimize the degree distribution of IRA for a given scenario of interest in terms of SNR, rate, VF duration, and operative channel load value to achieve a given PLR.

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⁶It shall be noted that ideal interference cancellation has been assumed. When replicas are not perfectly canceled, a residual interference level may be experienced by not yet resolved packets which degrades the performance of the overall scheme.

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