Control of a Nonlinear Teleoperation System by State Convergence

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I. Introduction

THE teleoperator device allows the human operator to perform mechanical actions that are usually performed by human hands and arms. Since the introduction of the first modern master/slave manipulator in the late 1940s, teleoperation systems have been used for a number of different tasks, for example, the handling of toxic and harmful material in remote environments, such as submarines or space, and perform tasks that require extreme precision, manipulators will continue to play a role in this type of applications in the future [1].

Stability is the most important aspect to build a teleoperation system with a high level of telepresence. If a system exhibits unstable or closely unstable behavior, the illusion of the operator being virtually present at the remote will be destroyed, making the task difficult or impossible to complete. For applications of teleoperation in which the remote side is physically remote, time delays are the major cause of stability issues.

The first work dealing with the problem of delay was published in 1965 [2], where the system was operated in open-loop, so it did not present the issue of instability [3].

In 1966 and later it was determined that a time equal to or less than 50 ms delay can destabilize bilateral controllers [3]. The problem is due to the generation of energy in the communication channel which makes this component of the system not passive [3].

A way of solving this problem is the addition of damping to the master and slave in order to absorb the energy generated in the system.

However, this technique does not guarantee stability and results in poor performance [4], [5]. As an alternative, it is possible to modify bilateral control in such a way such that the communication channel acts as a line without loss of transmission [3].

Several control schemes are proposed in literature to deal with specific problems in the field of robotics teleoperation [6]. Control schemes have been proposed which use different non-linear control techniques, such as passivity, sliding mode control, adaptive control or robust [7] [8], [9], [10] which result in stable master-slave systems when the communication channel presents small delays and the environment is considered soft. However, in the design of the control algorithms is considered a linear dynamic for the teleoperator and the effect of the delay are analyzed using linear approaches [5], [6].

A first step towards the unification of the analysis of stability for teleoperators with time delay was presented in [11]. They proposed a general Lyapunov – like function as a candidate, and analyzed the stability of different control schemes, ranging from constant time delay to variable, with or without transformation of dispersion, and with or without position tracking.

The work presented here is a continuation of the method of design and control presented in [12], which is based on the development of the teleoperation system as a linear system of order n in state space, the control signal allows the remote manipulator follow to the local manipulator through the state convergence even if it has a delay in the communication channel.

The goal of this study is to advanced the previous algorithm by modeling the behavior of the local and remote manipulator, and the channel communication by applying nonlinear state space equations, and proposed a control strategy to show the stability of a nonlinear bilateral teleoperation system for both local and remote manipulators with constant time delay.

II. NONLINEAR SYSTEM OF STATE CONVERGENCE

The local and remote manipulator robot are modeled using theLagrange - Euler formulation as a couple of serial links of n degrees of freedom with rotational joints.

$$
\begin{aligned}
M_l(q_l)\ddot{q}_l + C_l(q_l,\dot{q}_l)\dot{q}_l + g_l(q_l) &= \tau_{lc} + F_h \\
M_r(q_r)\ddot{q}_r + C_r(q_r,\dot{q}_r)\dot{q}_r + g_r(q_r) &= \tau_{rc} - F_e
\end{aligned} \tag{1}
$$

Where $\ddot{q}_i, \dot{q}_i, q_i \in R^n$ correspond to the acceleration, speed and position vectors of the joint $i = \{l, r\}$

$l \rightarrow$ local manipulator robot
$r \rightarrow$ remote manipulator robot

$M_i(q_i) \in R^{n \times n}$	Inertia matrix
$C_i(q_i,\dot{q}_i) \in R^{n \times n}$	Coriolis and centrifugal forces matrices
$g_i(q_i) \in R^n$	Gravitational forces vector
$\tau_{ic} \in R^n$	Torque vector
$F_h \in R^n$	Operator interaction force vector
$F_e \in R^n$	Environment interaction force vector

In the block diagram of the teleoperator system, Fig. 1, the dynamics of the local and remote manipulator is given by (1). It is presumed that the interaction of the human operator with the local handle is a constant force in the following way [13]:

$$
F_h = F_{op} \text{ Constant vector } \in R^n \tag{2}
$$

The interaction of the environment with the remote manipulator is considered passive.

$$
F_e = K_e q_r + B_e \dot{q}_r \tag{3}
$$

K_e, B_e are definite positive matrix $\in R^{n \times n}$

We proposed the control law (4) [14], as show in Fig. 1 this control law compensates for gravitational forces, so that the torques τ_{ic} are given by:

$$
\tau_{lc} = \tau_l + g_l(q_l), \quad \tau_{rc} = \tau_r + g_r(q_r) \tag{4}
$$

Where $\tau_i \in R^n$ correspond to the signal control torque.

Replacing (4) in (1) yields:

$$
\begin{aligned}
M_l(q_l)\ddot{q}_l + C_l(q_l,\dot{q}_l)\dot{q}_l &= \tau_l + F_{op} \\
M_r(q_r)\ddot{q}_r + C_r(q_r,\dot{q}_r)\dot{q}_r &= \tau_r - F_e
\end{aligned} \tag{5}
$$

Consider the local and remote manipulator (1) connected via a communication channel with a constant delay, T, as show in Fig.1.

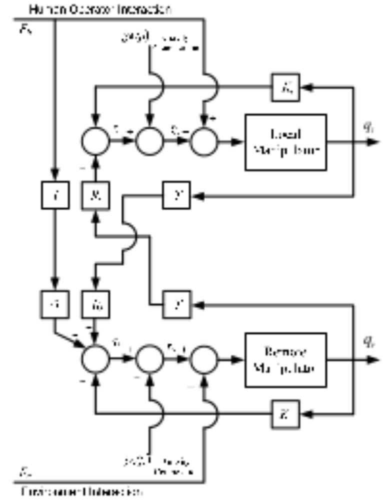

Fig. 1. Block diagram of nonlinear control of teleoperation system considering delay.

Consider that control algorithm, the coupling torque for the local and remote manipulator is given by:

$$
\begin{aligned}
\tau_l &= K_{l1}q_l + K_{l2}\dot{q}_l + R_{l1}q_r(t-T) + R_{l2}\dot{q}_r(t-T) \\
\tau_r &= K_{r1}q_r + K_{r2}\dot{q}_r + R_{r1}q_l(t-T) + R_{r2}\dot{q}_l(t-T) + G_2 F_{op}(t-T)
\end{aligned} \tag{6}
$$

Where:

$$
K_l = [K_{l1} \quad K_{l2}], R_l = [R_{l1} \quad R_{l2}], K_r = [K_{r1} \quad K_{r2}], R_r = [R_{r1} \quad R_{r2}]
$$

Where: K_{l1}, K_{l2}, R_{l1}, R_{l2}, K_{r1}, K_{r2}, R_{r1} and R_{r2} are order nxn matrices constant diagonal positive definite. G_2 is a constant.

The equilibrium points of the position of local and remote manipulator are defined as $\bar{q}_l \in R^n$ y $\bar{q}_r \in R^n$ then by using (2), (3), (5) and (6).

$$
\begin{aligned}
0 &= K_{l1}\bar{q}_l + R_{l1}\bar{q}_r + \bar{F}_{op} \\
0 &= K_{r1}\bar{q}_r + R_{r1}\bar{q}_l + G\bar{F}_{op}(t-T) - K_e\bar{q}_r
\end{aligned} \tag{7}
$$

Defining new position variables:

$$
\tilde{q}_l(t) = q_l(t) - \bar{q}_l \rightarrow q_l(t) = \tilde{q}_l + \bar{q}_l \tag{8}
$$

$$
\tilde{q}_r(t) = q_r(t) - \bar{q}_r \rightarrow q_r(t) = \tilde{q}_r + \bar{q}_r \tag{9}
$$

Replacing (6), (8) and (9) in (5), the dynamics of a bilateral teleoperation system in closed-loop is given by:

$$
\begin{aligned}
M_l\ddot{\tilde{q}}_l + C_l\dot{\tilde{q}}_l &= K_{l1}\tilde{q}_l + R_{l1}\tilde{q}_r(t-T) + K_{l2}\dot{\tilde{q}}_l + + R_{l2}\dot{\tilde{q}}_r(t-T) \\
M_r\ddot{\tilde{q}}_r + C_r\dot{\tilde{q}}_r &= K_{r1}\tilde{q}_r + R_{r1}\tilde{q}_l(t-T) + K_{r2}\dot{\tilde{q}}_r + R_{r2}\dot{\tilde{q}}_l(t-T) - K_e\tilde{q}_r - B_e\dot{\tilde{q}}_r
\end{aligned} \tag{10}
$$

Theorem 2.1

For the bilateral teleoperation system given by (10), making the following considerations

$$
\begin{aligned}
&K_{l1} = -K, \quad K_{l2} = -3K_1, \quad K_{r1} = -K, \quad R_{l2} = 2K_1 \\
&R_{l1} = K, \quad K_{r2} = -3K_1, \quad R_{r1} = K, \quad R_{r2} = 2K_1
\end{aligned}
\tag{11}
$$

Where: K_1 and K are positive definite constant diagonal matrices.

If the following is satisfied:

$$
K_1 - \frac{\alpha_1}{2}K - \frac{T^2}{2\alpha_2}K > 0, \quad K_1 - \frac{\alpha_2}{2}K - \frac{T^2}{2\alpha_1}K > 0 \tag{12}
$$

Where α_1, α_2 and T are scalar constants, then

$$
\lim_{t\to\infty} \tilde{q}_l = \lim_{t\to\infty} \tilde{q}_r = \lim_{t\to\infty} \dot{\tilde{q}}_l = \lim_{t\to\infty} \dot{\tilde{q}}_r = 0
$$

Proof

For the stability analysis considering the constant delay, we used a Lyapunov-Krasovskii functional [15], [16], [17].

Defining V, a positive definite function, as:

$$
\begin{aligned}
V\left(\dot{\tilde{q}}_l, \dot{\tilde{q}}_r, \tilde{q}_l, \tilde{q}_r\right) = {}& \frac{1}{2}\dot{\tilde{q}}_l^T M_l \dot{\tilde{q}}_l + \frac{1}{2}\dot{\tilde{q}}_r^T M_r \dot{\tilde{q}}_r + \frac{1}{2}(\tilde{q}_l - \tilde{q}_r)^T K (\tilde{q}_l - \tilde{q}_r) + \frac{1}{2}\tilde{q}_r^T K_e \tilde{q}_r \\
&+ \int_{t-T}^{t} \dot{\tilde{q}}^T(\xi) K_1 \dot{\tilde{q}}_l(\xi) d\xi + \int_{t-T}^{t} \dot{\tilde{q}}_r^T(\xi) K_1 \dot{\tilde{q}}_r(\xi) d\xi
\end{aligned}
\tag{13}
$$

T: Constant delay

K, K_e y K_1 positive definite constant diagonal matrices

The time derivative of (13) along the system trajectories described by (10) is given by,

$$
\begin{aligned}
\dot{V} = {}& \dot{\tilde{q}}_l^T M_l \ddot{\tilde{q}}_l + \frac{1}{2}\dot{\tilde{q}}_l^T \dot{M}_l \dot{\tilde{q}}_l + \dot{\tilde{q}}_r^T M_r \ddot{\tilde{q}}_r + \frac{1}{2}\dot{\tilde{q}}_r^T \dot{M}_r \dot{\tilde{q}}_r + \left(\dot{\tilde{q}}_l - \dot{\tilde{q}}_r\right)^T K (\tilde{q}_l - \tilde{q}_r) + \dot{\tilde{q}}_r^T K_e \tilde{q}_r \\
&+ \dot{\tilde{q}}_l^T K_1 \dot{\tilde{q}}_l + \dot{\tilde{q}}_r^T K_1 \dot{\tilde{q}}_r - \dot{\tilde{q}}_l^T(t-T) K_1 \dot{\tilde{q}}_l(t-T) - \dot{\tilde{q}}_r^T(t-T) K_1 \dot{\tilde{q}}_r(t-T)
\end{aligned}
\tag{14}
$$

Replacing (10), applying the property of the robots dynamics [13], by simplifying and grouping terms, yields:

$$
\begin{aligned}
\dot{V} = {}& \dot{\tilde{q}}_l^T (K_{l1} + K)\tilde{q}_l + \left[\dot{\tilde{q}}_l^T R_{l1} \tilde{q}_r(t-T) - \dot{\tilde{q}}_l^T K \tilde{q}_r\right] \\
&+ \left[\dot{\tilde{q}}_l^T K_{l2} \dot{\tilde{q}}_l + \dot{\tilde{q}}_l^T R_{l2} \dot{\tilde{q}}_r(t-T) - \dot{\tilde{q}}_r^T(t-T) K_1 \dot{\tilde{q}}_r(t-T)\right] + \dot{\tilde{q}}_l^T K_1 \dot{\tilde{q}}_l \\
&+ \dot{\tilde{q}}_r^T (K_{r1} + K)\tilde{q}_r + \left[\dot{\tilde{q}}_r^T R_{r1} \tilde{q}_l(t-T) - \dot{\tilde{q}}_r^T K \tilde{q}_l\right] \\
&+ \left[\dot{\tilde{q}}_r^T K_{r2} \dot{\tilde{q}}_r + \dot{\tilde{q}}_r^T R_{r2} \dot{\tilde{q}}_l(t-T) - \dot{\tilde{q}}_l^T(t-T) K_1 \dot{\tilde{q}}_l(t-T)\right] + \dot{\tilde{q}}_r^T K_1 \dot{\tilde{q}}_r \\
&- \dot{\tilde{q}}_r^T B_e \dot{\tilde{q}}_r
\end{aligned}
\tag{15}
$$

Replacing (11) in (15) one obtain

$$
\begin{aligned}
\dot{V} = {}& \dot{\tilde{q}}_l^T K\left[\tilde{q}_r(t-T) - \tilde{q}_r(t)\right] - \left[\dot{\tilde{q}}_r(t-T) - \dot{\tilde{q}}_l\right]^T K_1 \left[\dot{\tilde{q}}_r(t-T) - \dot{\tilde{q}}_l\right] + \dot{\tilde{q}}_r^T K\left[\tilde{q}_l(t-T) - \tilde{q}_l(t)\right] \\
&- \left[\dot{\tilde{q}}_l(t-T) - \dot{\tilde{q}}_r\right]^T K_1 \left[\dot{\tilde{q}}_l(t-T) - \dot{\tilde{q}}_r\right] - \dot{\tilde{q}}_r^T B_e \dot{\tilde{q}}_r - \dot{\tilde{q}}_l^T K_1 \dot{\tilde{q}}_l - \dot{\tilde{q}}_r^T K_1 \dot{\tilde{q}}_r
\end{aligned}
\tag{16}
$$

Taking

$$
\dot{e}_{\tilde{q}_l} = \dot{\tilde{q}}_r(t-T) - \dot{\tilde{q}}_l, \quad \dot{e}_{\tilde{q}_r} = \dot{\tilde{q}}_l(t-T) - \dot{\tilde{q}}_r \tag{17}
$$

We obtain:

$$
\begin{aligned}
\dot{V} = {}& \dot{\tilde{q}}_l K\left[\tilde{q}_r(t-T) - \tilde{q}_r(t)\right] - \dot{e}_{\tilde{q}_l}^T K_1 \dot{e}_{\tilde{q}_l} + \dot{\tilde{q}}_r^T K\left[\tilde{q}_l(t-T) - \tilde{q}_l(t)\right] \\
&- \dot{e}_{\tilde{q}_r}^T K_1 \dot{e}_{\tilde{q}_r} - \dot{\tilde{q}}_r^T B_e \dot{\tilde{q}}_r - \dot{\tilde{q}}_l^T K_1 \dot{\tilde{q}}_l - \dot{\tilde{q}}_r^T K_1 \dot{\tilde{q}}_r
\end{aligned}
\tag{18}
$$

Using the following relations

$$
\tilde{q}_l(t-T) - \tilde{q}_l(t) = -\int_0^T \dot{\tilde{q}}_l(t-\sigma) d\sigma, \quad \tilde{q}_r(t-T) - \tilde{q}_r(t) = -\int_0^T \dot{\tilde{q}}_r(t-\sigma) d\sigma
$$

Replacing in (18)

$$
\begin{aligned}
\int_0^{t_f} \dot{V} ds = {}& -\int_0^{t_f} \dot{e}_{\tilde{q}_l}^T K_1 \dot{e}_{\tilde{q}_l} ds - \int_0^{t_f} \dot{e}_{\tilde{q}_r}^T K_1 \dot{e}_{\tilde{q}_r} ds - \int_0^{t_f} \dot{\tilde{q}}_l^T K_1 \dot{\tilde{q}}_l ds - \int_0^{t_f} \dot{\tilde{q}}_r^T K_1 \dot{\tilde{q}}_r ds \\
&- \int_0^{t_f} \dot{\tilde{q}}_l^T K \int_0^T \dot{\tilde{q}}_r(t-\sigma) d\sigma ds - \int_0^{t_f} \dot{\tilde{q}}_r^T K \int_0^T \dot{\tilde{q}}_l(t-\sigma) d\sigma ds - \int_0^{t_f} \dot{\tilde{q}}_r^T B_e \dot{\tilde{q}}_r ds
\end{aligned}
\tag{19}
$$

For any vector signals x, y, any variable time delay T, α_1, $\alpha_2 > 0$ we have that:

$$
\begin{aligned}
-2\int_0^{t_f} \dot{x}^T K \int_0^T \dot{y}(t-\sigma) d\sigma dt &= \alpha_1 \int_0^{t_f} \dot{x}^T K \dot{x} dt + \frac{T^2}{\alpha_1}\int_0^{t_f} \dot{y}^T K \dot{y} dt \\
&= \alpha_1 \lambda_{\min}(K) \|\dot{x}\|_2^2 + \frac{T^2}{\alpha_1}\lambda_{\min}(K) \|\dot{y}\|_2^2
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
-2\int_0^{t_f} \dot{y}^T K \int_0^T \dot{x}(t-\sigma) d\sigma dt &= \alpha_2 \int_0^{t_f} \dot{y}^T K \dot{y} dt + \frac{T^2}{\alpha_2}\int_0^{t_f} \dot{x}^T K \dot{x} dt \\
&= \alpha_2 \lambda_{\min}(K) \|\dot{y}\|_2^2 + \frac{T^2}{\alpha_2}\lambda_{\min}(K) \|\dot{x}\|_2^2
\end{aligned}
\tag{21}
$$

Where $\lambda_{\min}(A)$ specifies the smallest eigenvalue of A, and the notation $\|\cdot\|_2$ specifies L_2 norm of a signal in the interval $[0, t_f]$.

Replacing (20) and (21) in (19)

$$
\begin{aligned}
\int_0^{t_f} \dot{V} ds = {}& -\lambda_{\min}(K_1) \left\|\dot{e}_{\tilde{q}_l}\right\|_2^2 - \lambda_{\min}(K_1) \left\|\dot{e}_{\tilde{q}_r}\right\|_2^2 - \lambda_{\min}(B_e) \left\|\dot{\tilde{q}}_r\right\|_2^2 \\
&- \lambda_{\min}\left(K_1 - \frac{\alpha_1}{2}K - \frac{T^2}{2\alpha_2}K\right) \left\|\dot{\tilde{q}}_l\right\|_2^2 \\
&- \lambda_{\min}\left(K_1 - \frac{\alpha_2}{2}K - \frac{T^2}{2\alpha_1}K\right) \left\|\dot{\tilde{q}}_r\right\|_2^2
\end{aligned}
\tag{22}
$$

From (22), $\int_0^{t_f} \dot{V} ds \le 0$ if the relations in (12) are satisfied, considering $\lim t_f = \infty$, $\int_0^{\infty} \dot{V} ds \le 0$, we conclude that the signals $\left\{\dot{\tilde{q}}_l, \dot{\tilde{q}}_r, \tilde{q}_r - \tilde{q}_l, \tilde{q}_r\right\} \in L_\infty$ and $\left\{\dot{\tilde{q}}_l, \dot{\tilde{q}}_r, \dot{e}_{\tilde{q}_l}, \dot{e}_{\tilde{q}_r}\right\} \in L_2$.

From close-loop dynamic (10), thus $\ddot{\tilde{q}}_l, \ddot{\tilde{q}}_r \in L_\infty$, are uniformly continuous [18], using Barbalat's Lemma [19], we conclude that:

$$
\lim_{t\to\infty} \dot{\tilde{q}}_l = \lim_{t\to\infty} \dot{\tilde{q}}_r = \lim_{t\to\infty} \dot{e}_{\tilde{q}_l} = \lim_{t\to\infty} \dot{e}_{\tilde{q}_r} = 0 .
$$

Probing that $\dddot{\tilde{q}}_l, \dddot{\tilde{q}}_r \in L_\infty$, hence the signals are uniformly continuous. The signal continuity imply that the integral exist and is bounded.

As show above $\lim_{t\to\infty} \dot{\tilde{q}}_l = \lim_{t\to\infty} \dot{\tilde{q}}_r = 0$. Using Barbalat's Lemma [19] $\lim_{t\to\infty} \ddot{\tilde{q}}_l = \lim_{t\to\infty} \dot{\tilde{q}}_r = 0$.

As a result for the dynamics of the system (10) with $K_{l1} = -K, \quad K_{r1} = -K, \quad R_{l1} = K, \quad R_{r1} = K$ we have that

$$\lim_{t\to\infty} \left\| \tilde{q}_r(t-T) - \tilde{q}_l \right\| = 0, \quad \lim_{t\to\infty} \left\| \tilde{q}_l(t-T) - \tilde{q}_r \right\| = K^{-1} K_e \lim_{t\to\infty} \tilde{q}_r \quad (23)$$

Using the fact that

$$\tilde{q}_l(t-T) = \tilde{q}_l - \int_{t-T}^{t} \dot{\tilde{q}}_l dt, \quad \tilde{q}_r(t-T) = \tilde{q}_r - \int_{t-T}^{t} \dot{\tilde{q}}_r dt$$

And the $\lim_{t\to\infty} \dot{\tilde{q}}_l = \lim_{t\to\infty} \dot{\tilde{q}}_r = 0$, yields

$$\lim_{t\to\infty} \left\| \tilde{q}_r - \tilde{q}_l \right\| = 0, \quad \lim_{t\to\infty} \left\| \tilde{q}_l - \tilde{q}_r \right\| = K^{-1} K_e \lim_{t\to\infty} \tilde{q}_r \quad (24)$$

The above equations imply that $\lim_{t\to\infty} \tilde{q}_l = \lim_{t\to\infty} \tilde{q}_r = 0$.

Therefore the origin of the system $\dot{\tilde{q}}_l, \dot{\tilde{q}}_r, \tilde{q}_l, \tilde{q}_r$ is asymptotically stable and $\lim_{t\to\infty} q_l(t) = \overline{q}_l, \lim_{t\to\infty} q_r(t) = \overline{q}_r$.

This guarantees the stability of the teleoperation system.

A. *Reflection Static Force*

Consider the system described by (5) and the control law (6) for the range of control (12), we have the following:

$$0 = \overline{F}_{op} + K_{l1}\overline{q}_l + R_{l1}\overline{q}_r$$

Where $K_{l1} = -K, \quad R_{l1} = K$, $K_{r1} = -K, \quad R_{r1} = K$

$$\overline{F}_{op} = K(\overline{q}_l - \overline{q}_r)$$

$$0 = -F_e + K_{r1}\overline{q}_r + R_{r1}\overline{q}_l + G_2\overline{F}_{op} \quad (25)$$

$$0 = -F_e + K(\overline{q}_l - \overline{q}_r) + G_2\overline{F}_{op}$$

$$\overline{F}_{op} = \frac{F_e}{(1+G_2)} \quad (26)$$

B. *Local-Remote Manipulator Position Coordination*

If $F_{op} = F_e = 0$, (25) and (26) can be written as $\overline{q}_l - \overline{q}_r = 0$.

This implies that the equilibrium points of the local and remote manipulator are identical. Then, the position coordination error $\tilde{q}(t) = q_l(t) - q_r(t)$

Tends to zero like $\lim_{t\to\infty} \tilde{q}(t) = \lim_{t\to\infty} (q_l(t) - q_r(t)) = 0$

Then, there is positions coordination between the local and remote manipulator.

III. Simulation

Control law (4) and (6) along with the dynamics of the teleoperation system (1) have been simulated using Matlab™ and Simulink®. For a local manipulator will use a PHANTOM Omni® haptic device from Sensable Technologies. For a remote manipulator we employed a planar serial arm with three degrees of freedom, actuated by DC motors [20]:

$$M_l(q_l)\ddot{q}_l + C_l(q_l, \dot{q}_l)\dot{q}_l + g_l(q_l) + f_l(\dot{q}_l) = \tau_{lc} + F_{op}$$
$$M_r(q_r)\ddot{q}_r + C_r(q_r, \dot{q}_r)\dot{q}_r + g_r(q_r) + f_r(\dot{q}_r) = \tau_e - F_e$$

$f(\dot{q}) \in R^n$ It is a static model of joints friction, defined by [14]:

$$f_l(\dot{q}_l) = f_r(\dot{q}_r) = f(\dot{q}) = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.05 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

All the simulations have been realized using a communication channel time delay of the $T = 0.5$ s.

The inertia matrix M_r, the coriolis and centrifugal forces matrix C_r, the force of gravity matrix g_r of remote manipulator are defined by:

$$M_r = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}, \quad C_r = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}, \quad g_r = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}$$

Where:

$$M_{11} = 0.045879 + 0.03176\cos(q_2)$$
$$M_{12} = M_{21} = 0.012801 + 0.01588\cos(q_2)$$
$$M_{13} = M_{31} = 0.0014037$$
$$M_{22} = 0.012801$$
$$M_{23} = M_{32} = 0.0014037$$
$$M_{33} = 0.0014037$$
$$C_{11} = C_{13} = 0$$
$$C_{12} = -0.01588\sin(q_2)(\dot{q}_2 + 2\dot{q}_1)$$
$$C_{21} = 0.01588\sin(q_2)\dot{q}_1$$
$$C_{22} = C_{23} = 0, \quad C_{31} = C_{32} = C_{33} = 0$$
$$g_1 = -0.739\sin(q_1)\cos(q_2) - 0.739\cos(q_1)\sin(q_2) - 1.6409\sin(q_1)$$
$$g_2 = -0.739\cos(q_1)\sin(q_2) - 0.739\sin(q_1)\cos(q_2)$$
$$g_3 = 0$$

Considering the gains K y K_1 in (13) as:

$$K_1 = \begin{bmatrix} 40 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \quad K = \begin{bmatrix} 79 & 0 & 0 \\ 0 & 59 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

The controller parameters K_{l1}, K_{l2}, K_{r1}, K_{r2}, R_{l1}, R_{l2}, R_{r1} and R_{r2} they are determined by (12), in addition $G = 1$.

In order to assess the stability of the contact, in simulations, we considered a soft environment modeled by means of a spring -damper system, with the spring and damper gains as:

$$K_e = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 25 \end{bmatrix} N/m, \quad B_e = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} N \cdot s/m$$

Fig. 2 show the force (torque) applied by the human operator to the local manipulator.

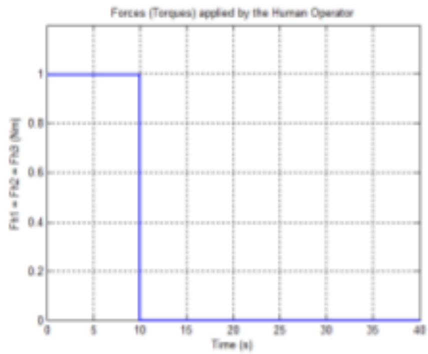

Fig. 2. Force (torque) applied by the human operator.

A. Case A: Without Environment Interaction

Fig. 3 and Fig. 4 show the joints positions of the local and remote manipulator. From simulations can be show that is guaranteed stability for the considered time-delay.

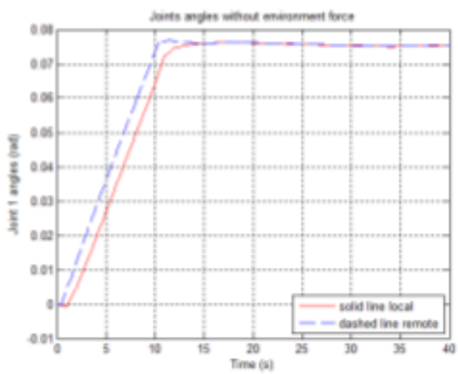

Fig. 3. Joint 1 angles position of local and remote manipulator (rad) Vs. Time (s).

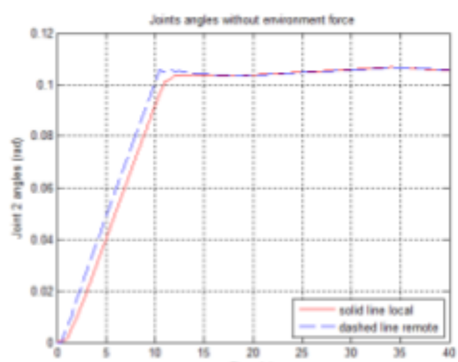

Fig. 4. Joint 2 angles position of local and remote manipulator (rad) Vs. Time (s).

B. Case B: Environment Interaction

Fig. 5, Fig. 6 and Fig. 7 show the joints positions of the local and remote manipulator.

When the remote manipulator does not contact with the environment (0 - 4s and 10-40 s) position coordination of the local and remote manipulator position is achieved.

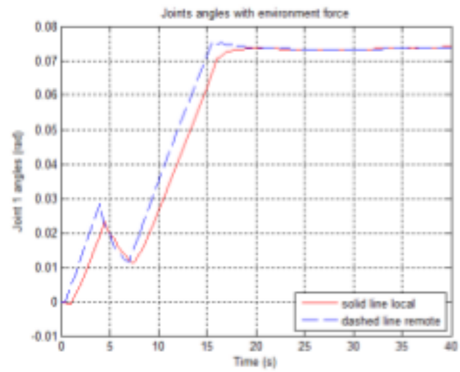

Fig. 5. Joint 1 angles position of local and remote manipulator (rad) Vs. Time (s).

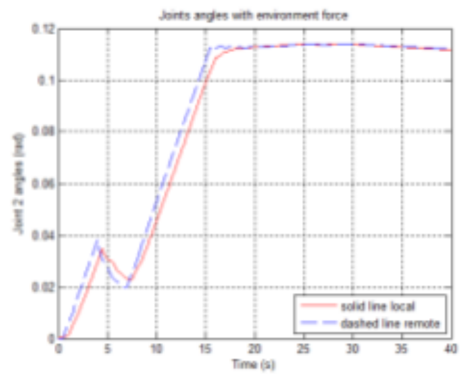

Fig. 6. Joint 2 angles position of local and remote manipulator (rad) Vs. Time (s).

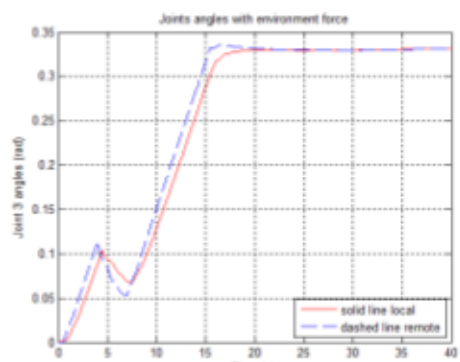

Fig. 7. Joint 3 angles position of local and remote manipulator (rad) Vs. Time (s).

IV. Conclusion

We have shown in this paper that it is possible to control a nonlinear bilateral teleoperator system with the proposed state convergence framework. The method is based on a nonlinear state space formulation and it allows the remote manipulator to follow the local manipulator through state convergence, even when the passivity of the human operator is not guaranteed.

The analysis has shown that, assuming a constant delay, when the local and remote manipulators are coupled using the proposed framework, the result is a stable, nonlinear teleoperation system (both local and remote) were position coordination is achieved.

We also performed simulations which validate the theoretical results of this paper. Experimental results are currently under way and will be reported in the near future.

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