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On Routing Algorithms With End-To-End Delay Guarantees†

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On Routing Algorithms With End-to-End Delay Guarantees ¹

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Abstract

We consider the transmission of a message of size r from a source to a destination with guarantees on the end-to-end delay over a computer network with n nodes and m links. There are three sources of delays: (a) propagation delays along the links, (b) delays due to bandwidth availability on the links, and (c) queuing delays at the intermediate nodes. First, we consider that delays on various links and nodes are given as functions of the message size. If the delay in (b) is a non-increasing function of the bandwidth, we propose $O(m^2 + mn \log n)$ time algorithm to compute a path with the minimum end-to-end delay for any given message size r . We then consider that the queuing delay in (c) is a random variable correlated with the message size according to an *unknown* distribution. At each node, the measurements of queuing delays and message sizes are available. We propose two algorithms to compute paths whose delays are close to optimal delays with a high probability, irrespective of the distribution of the delays, and based entirely on the measurements of sufficient size.

Keywords and Phrases: End-to-end delay, quality of service, routing, probably approximately correct, path-table.

1 Introduction

End-to-end delay guarantees are very important in computer networks that are expected to support a variety of services requiring transmission of messages of varying sizes. In the present networks, especially the Internet, such guarantees cannot be provided, and a message can indeed be arbitrarily delayed under certain load conditions. In the future, however, newer applications such as medical image transfer and real-time instrumentation control over networks, require that messages be sent with certain guarantees on the end-to-end delay. Since the requirements can be quite varied, different “dedicated” network architectures might be required for various applications. The exact nature of the guarantees that can be provided over such networks depends on the delay sources and their precise contribution to the end-to-end delay.

In this paper, we propose polynomial-time source-based algorithms for computing a path with end-to-end delay guarantees under fairly general delay profiles, which are specified either as (deterministic) functions or as measurements of random delay variables. In the first case, we present an exact algorithm to compute a path with the minimum end-to-end delay for any given message

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size. In the second case, we consider that the transmission is subject to random queuing delays with *unknown* distributions. Based on the measurements of queuing delays, we propose two algorithms that yield paths which approximate the optimal paths (computable only under a complete knowledge of delay distributions) with a high probability, irrespective of delay distributions. The closeness of approximation depends on the sample size and the size of the network.

We consider a computer network represented by a graph $G = (V, E)$ with n nodes and m edges or links. A message of size r must be transmitted from a source node s to a destination node d with performance guarantees on the end-to-end delay. In a *deterministic formulation*, a message transmitted on the network incurs three types of delays:

- (a) *Link Propagation Delay*: For each link $e = (v_1, v_2)$, there is a *link-delay* $d(e) \geq 0$ such that a message of unit length sent via e from node v_1 at time t will arrive at node v_2 at time $t + d(e)$. The link-delay essentially includes the propagation time of the link.
- (b) *Bandwidth Constrained Delay*: Each link $e \in E$ has a *bandwidth* $b(e) \geq 0$. Once initiated, a message of r units can be sent along link e in $g(r, b(e)) + d(e)$ time, where $g(r, b)$ is non-decreasing in r and non-increasing in b . For a simple bandwidth constraint, we have $g(r, b(e)) = r/b(e)$ [5] so that the delay along link e is $r/b(e) + d(e)$.
- (c) *Deterministic Queuing Delay*: At any node v , there is a deterministic queuing delay $q_v(r)$ which is a function of the message size r .

Consider a *simple path* P , from source $s = v_0$ to destination $d = v_k$, given by $(v_0, v_1), (v_1, v_2), \dots, (v_{k-1}, v_k)$, where $(v_j, v_{j+1}) \in E$, for $j = 0, 1, \dots, (k-1)$, and v_0, v_1, \dots, v_k are distinct. Subsequently, a simple path is referred to simply as a path. Let $e_j = (v_j, v_{j+1})$. The *bandwidth* of this path, denoted by $b(P)$, is given by $\min_{j=0}^{k-1} b(e_j)$, and the *delay due to bandwidth* is given by $g(r, b(P))$. Then, the *end-to-end delay* of path P in transmitting a message of size r is given by

$$t(r, P) = g(r, b(P)) + \sum_{j=0}^{k-1} d(e_j) + \sum_{j=0}^{k-1} q_{v_j}(r). \quad (1.1)$$

Let us denote $d(P) = \sum_{j=0}^{k-1} d(e_j)$ and $q(r, P) = \sum_{j=0}^{k-1} q_{v_j}(r)$ such that $t(r, P) = g(r, b(P)) + d(P) + q(r, P)$. We show that a path with the minimum end-to-end delay for any given message size r can be computed with time complexity $O(m^2 + mn \log n)$. A special case of this problem, when $q(r, P) = 0$ and $g(r, b(P)) = r/b(P)$, for all P , has been extensively studied under the appellation of the *quickest path problem* [5, 14, 9].

The algorithm for the above deterministic formulation is applicable to special networks wherein delay functions are accurately known. In more general networks, however, good estimates may be available only for link delays and bandwidths, but the queuing delays are only known approximately at best. Typically, the queuing delays depend on the messages from other sources competing to be sent on the same link. In practice, however, empirical measurements of the queuing delays can be collected at each node by logging the delays and sizes of all messages routed through it. To address the problems of this type, we consider a *probabilistic formulation* of the problem with the queuing delays characterized as follows:

(c') A message of size R arrives at the source s according to an *unknown* distribution P_R ². At any node v , Q_v and R_v are the random variables³ denoting the queuing delay and message size distributed according to *unknown* distributions P_{Q_v} and P_{R_v} , respectively. Note that Q_v is not a simple function of R_v in that for any given value of R_v there could be more than one value for Q_v . The measurements $(Q_{v;1}, R_{v;1}), (Q_{v;2}, R_{v;2}), \dots, (Q_{v;l}, R_{v;l})$ that are independently and identically distributed (iid) according to the joint distribution P_{Q_v, R_v} , are known at each node $v \in V$. For any given message size $R_v = r$, let $\bar{q}_v(r)$ denote the expectation of Q_v , i. e. $\bar{q}_v(r) = E[Q_v | R_v = r]$. For the second algorithm, we assume that $\bar{q}_v(r)$ is a non-decreasing function of r which informally means that longer messages are likely to be more delayed than shorter ones⁴. Note that this condition does not imply that the values of $Q_{v;i}$ plotted against $R_{v;i}$ are non-decreasing.

We consider two routing problems under the probabilistic formulation for which the algorithm developed under the conditions (a)-(c) can be utilized. The *end-to-end delay* of path P in transmitting a message of size R is

$$T(R, P) = g(R, b(P)) + d(P) + \sum_{j=0}^{k-1} Q_{v_j} \quad (1.2)$$

which is a random variable of the joint distributions of queuing delays at various nodes and the message size. The *expected delay* of path P averaged over all message sizes is given by

$$\bar{T}(P) = \int g(R, b(P)) dP_R + d(P) + \sum_{j=0}^{k-1} \int Q_{v_j} dP_{Q_{v_j}, R_{v_j}}.$$

Let P^* denote a path with the minimum expected end-to-end delay, i.e. $\bar{T}(P^*) = \min_{P \in \mathcal{P}} \bar{T}(P)$, where \mathcal{P} is set of all paths between s and d . We present an algorithm to compute a path \hat{P} based entirely on iid measurements, such that given a sample of size

$$l = \frac{32(n+1)^2 \tau^2}{\epsilon^2} [4 \ln(4(n+1)\tau/\epsilon) + \ln(2^n 18(n+1)/\delta)],$$

we have

$$P[|\bar{T}(\hat{P}) - \bar{T}(P^*)| \geq \epsilon] \leq \delta, \quad (1.3)$$

where $\sup_{r,b} g(r, b) \leq \tau_1$, $\sup_v Q_v \leq \tau_2$, and $\tau = \max(\tau_1, \tau_2)$. This performance guarantee means that with a high probability the expected delay of the computed path is within ϵ of the optimal expected delay irrespective of the underlying distributions.

We consider the end-to-end delay of a path P for a *given message size* R given by

$$T_R(P) = g(R, b(P)) + d(P) + \sum_{j=0}^{k-1} Q_{v_j}|_R.$$

²We denote the random variables by capital letters (e. g. R) and their deterministic counterparts by the corresponding lower case letter (e. g. r).

³For the existence of Q_v the sizes of the external messages arriving at all source nodes (not just s) that are routed via v be probabilistic (in particular, the existence of P_R at s alone is not sufficient). Furthermore, it is assumed that for any given R the routing path is deterministically chosen for all sources in the network.

⁴This is not true for certain queuing disciplines such as the first-in and first-out.

The *expected delay* of path P for a *given message size* R is given by

$$\bar{T}_R(P) = g(R, b(P)) + d(P) + \sum_{j=0}^{k-1} \int Q_{v_j} dP_{Q_{v_j}|R}.$$

Let P_R^* be the path with minimum end-to-end delay for message size R , i.e. $\bar{T}_R(P_R^*) = \min_{P \in \mathcal{P}} \bar{T}_R(P)$.

By using regression estimation, we present an algorithm to compute \hat{P}_R such that

$$P[|T_R(\hat{P}_R) - T_R(P_R^*)| \geq \epsilon] \leq \delta, \quad (1.4)$$

for a sufficiently large sample. Informally, this condition guarantees that the delay of \hat{P}_R is close to that of P_R^* with a high probability. Performance guarantees of the type in Eq (1.3) and (1.4) are similar in spirit to those in Probably Approximately Correct (PAC) learning [15, 16]. Such results are about the best one can expect in scenarios where very little information is available about the underlying probability distributions.

The present problem is a special case of the well-known optimal routing problem [3], and can also be posed as a special case of the classical minimum cost flow problem [1] studied in transportation and operations research. The main contributions of this paper are polynomial-time algorithms, and PAC guarantees based only on measurements, i.e. without the knowledge of distributions.

A number of *Quality of Service* (QoS) routing algorithms that provide bounds on end-to-end delay and/or transmission rates have been studied based on single paths [18, 17] and multiple paths [10, 12]. Optimization of network-level parameters is discussed in [10], and their QoS parameters are not deterministic nor distribution-free. In spirit, our problem is a special case of the one studied in [7], where the transmission task is specified by several parameters, but our algorithms are polynomial-time computable and source-based. More general variations of the problem are discussed in [2, 8], but their guarantees are “soft” and are not measurement-based.

The organization of this paper is as follows. An algorithm for computing the minimum end-to-end delay path under the conditions (a)-(c) is presented in Section 2. The problem of random queuing delays under the conditions (a), (b), and (c') is addressed in Section 3.

2 Minimum End-To-End Delay Routing

We now describe an algorithm to compute a path with the minimum end-to-end delay according to Eq. (1.1) for transmitting a message of given size r from s to d . We define an *augmented delay* of an edge $e = (v_1, v_2)$ as $d_A(e) = d(e) + q_{v_1}(r)$. Let b_1, b_2, \dots, b_c denote the distinct values of the bandwidths $b(e)$, $e \in E$. Let $G(a) = (V, E(a))$ denote the subnetwork where $e \in E(a)$ if and only if $b(e) \geq a$. Let a $s - d$ *shortest path* in $G(a)$ denote the shortest delay path based *only* on the augmented delay of the edges (i.e. with minimum $d_A(\cdot)$ value). Our algorithm Min-Path is based on the algorithm of [14] which was originally proposed for the special case when $q_v(r) = 0$, and $g(r, b(P)) = r/b(P)$.

Step 1 of this algorithm is executed by c invocations of Dijkstra's shortest path algorithm with a total complexity $O(cm + cn \log n)$ using Fibonacci heaps [6]. The cost of step 2 is $O(c)$. Thus the complexity of this algorithm is $O(m^2 + mn \log n)$. We establish the correctness of this algorithm in the following theorem, which is an extension of the results from [14].

Theorem 2.1 Let P^t be a path with the minimum end-to-end delay in G for message size r , i. e. $t(P^t) = \min_{P \in \mathcal{P}} t(P)$. Then the following statements are true:

- (i) P^t is a $s-d$ shortest path in $G(b(P^t))$ according to the augmented delay $d_A(\cdot)$.
- (ii) Let P_j be a $s-d$ shortest path in $G(b_j)$, $j = 1, 2, \dots, c$, and

$$g(r, b(P_1)) + d(P_1) + q(r, P_1) = \min_{j=1}^c \{g(r, b(P_j)) + d(P_j) + q(r, P_j)\}. \quad (2.1)$$

Then P_1 is a minimum end-to-end delay path in G for the message size r .

Proof: For Part (i), consider any path P_1 in $G(b(P^t))$ which implies $b(P_1) \geq b(P^t)$. This condition in turn implies $q(r, b(P_1)) \leq q(r, b(P^t))$. Since P^t is a minimum end-to-end delay path, we have

$$q(r, b(P^t)) + d(P^t) + q(r, P^t) \leq q(r, b(P_1)) + d(P_1) + q(r, P_1),$$

which implies $d(P^t) + q(r, P^t) \leq d(P_1) + q(r, P_1)$. Thus Part (i) is true.

We now consider Part (ii). For the minimum end-to-end delay path P^t , we have $b(P^t) = b_{j_0} \in \{b_1, b_2, \dots, b_c\}$. By Part (i), P^t is shortest in $G(b_{j_0})$ and P_{j_0} is shortest in $G(b_{j_0})$ by hypothesis. Thus we have $d(P^t) + q(r, P^t) = d(P_{j_0}) + q(r, P_{j_0})$. Furthermore, we have $b(P_{j_0}) \geq b(P^t)$ since all edges of $G(b_{j_0})$ have the bandwidth of at least b_{j_0} . Thus, we have $g(r, b(P_{j_0})) \leq g(r, b(P^t))$, which implies $g(r, b(P_{j_0})) + d(P_{j_0}) + q(r, P_{j_0}) \leq g(r, b(P^t)) + d(P^t) + q(r, P^t)$. By the minimization hypothesis of the theorem we have $g(r, b(P_1)) + d(P_1) + q(r, P_1) \leq g(r, b(P_{j_0})) + d(P_{j_0}) + q(r, P_{j_0})$. In summary, we have $t(r, P_1) \leq t(r, P_{j_0}) \leq t(r, P^t)$. \square

Although the algorithm appears to be applicable only to the cases when a complete knowledge of the delays is available, it could be useful when only estimators of the delays are available.

3 Routing Based on Measurements

The *expected end-to-end delay* of path P is given by

$$\bar{T}(P) = \int g(R, b(P)) dP_R + d(P) + \sum_{j=0}^{k-1} \int Q_{v_j} dP_{Q_{v_j}, R_{v_j}}.$$

Since the distributions P_R and $P_Q = P_{Q_{v_1}, \dots, Q_{v_n}}$ are unknown, the best expected end-to-end delay path, \bar{P} , such that $\bar{T}(\bar{P}) = \min_{P \in \mathcal{P}} \bar{T}(P)$, cannot be computed. In this section, we first present a method to compute \hat{P} that guarantees the condition in Eq (1.3). The *expected end-to-end delay* for a given message size R can be written as follows

$$\bar{T}_R(P) = g(R, b(P)) + d(P) + \sum_{j=0}^{k-1} \bar{q}_{v_j}(R)$$

where $\bar{q}_v(r) = E[Q_v | R = r]$ is the regression of Q_v on R . Under the additional condition that $\bar{q}_v(r)$ is non-decreasing in r , we employ a regression estimator $\hat{q}_v(\cdot)$ to compute a path \hat{P}_R such that $T_R(\hat{P}_R)$ is close to $T_R(P_R^*)$ as in Eq (1.4).

algorithm Min-Path(r)

1. for $j = 1, 2, \dots, c$, compute $s-d$ shortest path P_j in $G(b_j)$;
 2. compute index k which minimizes $\{g(r, b(P_j)) + d(P_j) + q(r, P_j) | j = 1, 2, \dots, c\}$;
 3. return P_k as the path with the minimum end-to-end delay;
-

3.1 Best Empirical Path Method

We consider the *empirical end-to-end delay* given by

$$\hat{T}(P) = \frac{1}{l} \sum_{i=1}^l g(R_i, b(P)) + \sum_{j=0}^{k-1} d(e_j) + \sum_{j=0}^{k-1} \frac{1}{l} \sum_{i=1}^l Q_{v_j; i}.$$

Note that $\hat{T}(\cdot)$ can be computed since it involves only the measured quantities. Then the *best empirical end-to-end delay path* is defined by $\hat{P} = \arg \min_{P \in \mathcal{P}} \hat{T}(P)$, which is computed using the algorithm described in Section 2. Note that \hat{P} is best on the average based on measurements alone.

Theorem 3.1 *Under the conditions (a)-(c), given a sample of size*

$$l = \frac{32(n+1)^2 \tau^2}{\epsilon^2} [4 \ln(4(n+1)\tau/\epsilon) + \ln(2^n 18(n+1)/\delta)],$$

the best empirical path \hat{P} can be computed in $O(m^2 + mn \log n)$ time such that

$$P[|\bar{T}(\hat{P}) - \bar{T}(P^*)| \geq \epsilon] \leq \delta,$$

where $\sup_{r,b} g(r,b) \leq \tau_1$, $\sup_v Q_v \leq \tau_2$, and $\tau = \max(\tau_1, \tau_2)$.

Proof: Consider that we have $P \left[\sup_{P \in \mathcal{P}} |\bar{T}(P) - \hat{T}(P)| \geq \epsilon/2 \right] < \delta$

for some $\delta > 0$. Then, with probability at least $1 - \delta$, we have $|\bar{T}(P) - \hat{T}(P)| \leq \epsilon/2$ for all paths simultaneously. By noting that $\hat{T}(\hat{P}) \leq \hat{T}(\bar{P})$, we have

$$\bar{T}(\hat{P}) \leq \hat{T}(\hat{P}) + \epsilon/2 \leq \hat{T}(\bar{P}) + \epsilon/2 \leq \bar{T}(\bar{P}) + \epsilon$$

with probability $1 - \delta$. Thus we have $P[|\bar{T}(\hat{P}) - \bar{T}(P^*)| \geq \epsilon] \leq P \left[\sup_{P \in \mathcal{P}} |\bar{T}(P) - \hat{T}(P)| \geq \epsilon/2 \right]$.

We now bound the right hand side as follows

$$\begin{aligned} P \left[\sup_{P \in \mathcal{P}} |\bar{T}(P) - \hat{T}(P)| \geq \epsilon \right] &\leq P \left[\sup_{P \in \mathcal{P}} |\bar{g}(b(P)) - \hat{g}(b(P))| \geq \frac{\epsilon}{2(n+1)} \right] \\ &\quad + P \left[\sup_{P \in \mathcal{P}} \left| \sum_{j=0}^{k-1} \bar{Q}_{v_j} - \sum_{j=0}^{k-1} \hat{Q}_{v_j} \right| \geq \frac{\epsilon n}{2(n+1)} \right], \end{aligned} \quad (3.1)$$

where we have $\bar{g}(b(P)) = \int g(R, b(P)) dP_R$ and $\hat{g}(b(P)) = \sum_{i=1}^l g(b(P), R_i)$,

$$\bar{Q}_v = \int Q_v dP_{Q_v, R_v} \quad \text{and} \quad \hat{Q}_v = \frac{1}{l} \sum_{i=1}^l Q_{v; i}$$

for $v \in V$.⁵ The first term of Eq. (3.1) is upper bounded as follows

$$\begin{aligned} P \left[\sup_{P \in \mathcal{P}} |\bar{g}(b(P)) - \hat{g}(b(P))| \geq \frac{\epsilon}{2(n+1)} \right] &\leq P \left[\sup_{i=1,2,\dots,m} |\bar{g}(b_i) - \hat{g}(b_i)| \geq \frac{\epsilon n}{2(n+1)} \right] \\ &\leq m 18 l e^{-\frac{\epsilon^2 l}{16(n+1)^2 \tau^2}}, \end{aligned}$$

⁵Here we applied the basic result that $P[A+B > \epsilon_1 + \epsilon_2] \leq P[A > \epsilon_1] + P[B > \epsilon_2]$ since the event $A+B > \epsilon_1 + \epsilon_2$ implies that event that either $A > \epsilon_1$ or $B > \epsilon_2$ (or both); we apply this basic result a number of times in the proofs of this section.

where the last step follows from by Glivenko-Cantelli Lemma ([16], p. 184)). The second term of Eq. (3.1) is upper bounded as follows

$$\begin{aligned}
P \left[\sup_{P \in \mathcal{P}} \left| \sum_{j=0}^{k-1} \bar{Q}_{v_j} - \sum_{j=0}^{k-1} \hat{Q}_{v_j} \right| \geq \frac{\epsilon n}{2(n+1)} \right] &\leq 2^n \sup_{P \in \mathcal{P}} P \left[\left| \sum_{j=0}^{k-1} \bar{Q}_{v_j} - \sum_{j=0}^{k-1} \hat{Q}_{v_j} \right| \geq \frac{\epsilon n}{2(n+1)} \right] \\
&\leq 2^n \sup_{P \in \mathcal{P}} \sup_{v \in P} P \left[|\bar{Q}_v - \hat{Q}_v| \geq \frac{\epsilon}{2(n+1)} \right] \\
&\leq 2^n n 18 l e^{-\frac{\epsilon^2 l}{16(n+1)^2 \tau^2}},
\end{aligned}$$

where the first inequality is due to $|\mathcal{P}| \leq 2^n$, the second inequality is because there are at most n nodes in any P , and the third inequality is due to Glivenko-Cantelli Lemma. In summary, we have

$$P \left[\sup_{P \in \mathcal{P}} |\bar{T}(P) - \hat{T}(P)| \geq \epsilon/2 \right] \leq 18(m + 2^n n) l e^{-\frac{\epsilon^2 l}{16(n+1)^2 \tau^2}} \leq 2^n 18(n+1) l e^{-\frac{\epsilon^2 l}{16(n+1)^2 \tau^2}}.$$

The sample bound follows by noting that the condition $\delta = a l e^{-bl}$ can be ensured by choosing $l \geq \frac{2}{b} \ln \left(\frac{a}{b^2 \delta} \right)$. \square

Notice that the sample size is proportional to $O(n^3)$ for fixed values of parameters ϵ and δ , with no knowledge about the underlying delay distributions. In statistics literature, asymptotic convergence results are more common. Under the condition of Theorem 3.1, we can show that $\bar{T}(\hat{P}) \rightarrow \bar{T}(P^*)$ with probability one by a direct application of Borel-Cantelli Lemma [4].

3.2 Regression Method

The path \hat{P} of previous section does not fully exploit the given value of message size. We now present a method that utilizes the correlation information between message size and queuing delay by utilizing an estimator $\hat{q}_v(\cdot)$ of the regression $\bar{q}_v(\cdot)$. Intuitively, this method “extrapolates” the measurements to the present value of R , and computes the corresponding best path. Consider, for a given message size R ,

$$\hat{T}_R(P) = g(R, b(P)) + d(P) + \sum_{j=0}^{k-1} \hat{q}_{v_j}(R).$$

Let \hat{P}_R minimize $\hat{T}_R(\cdot)$, i.e. $\hat{T}(\hat{P}_R) = \min_{P \in \mathcal{P}} \hat{T}_R(P)$, which can be computed by the algorithm of Section 2. Let \mathcal{Q} denote set of all non-decreasing functions, and we consider a particular estimator $\hat{q}_v(\cdot) \in \mathcal{Q}$ that minimizes the empirical cost function given by $\frac{1}{l} \sum_{q \in \mathcal{Q}} |Q_{v,i} - f(R_{v,i})|$ for $f \in \mathcal{Q}$. We

emphasize that for computing \hat{P}_R it not necessary to have any measurements at the message size R . Thus, a main contribution of the regression method is the computation of paths with performance guarantees at message sizes for which measurements may not be available.

Theorem 3.2 Consider that the regression $\bar{q}_v(r) = E[Q_v | R_v = r]$ is non-decreasing in r . Under the conditions (a), (b) and (c'), given a sample of size

$$l = \frac{32n^2\tau_2^2}{\epsilon^2} [4 \ln(4n\tau_2/\epsilon) + \ln(2^n 72n/\delta)],$$

the path \hat{P}_R can be computed in $O(m^2 + mn \log n)$ time such that $P[|T_R(\hat{P}_R) - T_R(P_R^*)| \geq \epsilon] \leq \delta$, where $\sup_v Q_v \leq \tau_2$.

Proof: We have

$$\begin{aligned} P[|T_R(\hat{P}_R) - T_R(P_R^*)| > \epsilon] &\leq P[|T_R(\hat{P}_R) - \bar{T}_R(\hat{P}_R)| > \epsilon_1] \\ &\quad + P[|\bar{T}_R(\hat{P}_R) - \bar{T}_R(P_R^*)| > \epsilon - \epsilon_1 - \epsilon_2] \\ &\quad + P[|\bar{T}_R(P_R^*) - T_R(P_R^*)| > \epsilon_2]. \end{aligned}$$

By Chebyshev's inequality, we have for any $\epsilon_1, \epsilon_2 > 0$,

$$\begin{aligned} P[|T_R(\hat{P}_R) - \bar{T}_R(\hat{P}_R)| > \epsilon_1] &\leq \frac{1}{\epsilon_1} E[|T_R(\hat{P}_R) - \bar{T}_R(\hat{P}_R)|] \\ P[|\bar{T}_R(P_R^*) - T_R(P_R^*)| > \epsilon_2] &\leq \frac{1}{\epsilon_2} E[|\bar{T}_R(P_R^*) - T_R(P_R^*)|] \end{aligned}$$

Since $E|T_R(\hat{P}_R) - \bar{T}_R(\hat{P}_R)| = 0$, and $E|\bar{T}_R(P_R^*) - T_R(P_R^*)| = 0$, we have

$$P[|T_R(\hat{P}_R) - T_R(P_R^*)| > \epsilon] \leq P[|\bar{T}_R(\hat{P}_R) - \bar{T}_R(P_R^*)| > \epsilon].$$

We now upper bound the right hand side. First, we have $\bar{T}_R(\hat{P}_R) - \bar{T}_R(P_R^*) \leq \sum_{j=0}^{k-1} |\bar{q}_{v_j}(r) - \hat{q}_{v_j}(r)|$. By utilizing the result of Vapnik [16] (as in the first part of proof of Theorem 3.1), we have

$$\begin{aligned} P[|\bar{T}_R(\hat{P}_R) - \bar{T}_R(P_R^*)| > \epsilon] &\leq P\left[\sup_{P \in \mathcal{P}} |\bar{T}_R(P) - \hat{T}_R(P)| > \epsilon/2\right] \\ &\leq 2^n \sup_{P \in \mathcal{P}} P\left[|\bar{T}_R(P) - \hat{T}_R(P)| > \epsilon/2\right] \\ &\leq 2^n n \sup_{P \in \mathcal{P}} \sup_{v \in \mathcal{V}} P[|\bar{q}_v(R) - \hat{q}_v(R)| > \epsilon/2n]. \end{aligned}$$

We note that Vapnik's capacity of \mathcal{Q} is 2 which yields the following bound for any $v \in \mathcal{V}$ (see [16] for details)

$$P[|\bar{q}_v(R) - \hat{q}_v(R)| > \epsilon/2n] \leq 18l^2 e^{-\frac{\epsilon^2 l}{16\tau^2 n^2}}.$$

Thus we have $P[|\bar{T}_R(\hat{P}_R) - \bar{T}_R(P_R^*)| > \epsilon] \leq 2^n n 18l^2 e^{-\frac{\epsilon^2 l}{16\tau^2 n^2}}$, which yields the required sample size by noting that the condition $\delta = al^2 e^{-bl}$ can be ensured by choosing $l \geq \frac{2}{b} \ln\left(\frac{4a}{b^2 \delta}\right)$. \square

Using Borel-Cantelli Lemma it can be shown that $|T_R(\hat{P}_R) - T_R(P_R^*)| \rightarrow 0$ with probability one. The main objective of this theorem is to demonstrate that regression estimators can be used to provide performance guarantees based on finite samples. Several other regression estimators can be used in place of \hat{q} . But, to obtain the required sample size, finite sample results are needed for regression estimation; such results are fairly uncommon since most regression results are asymptotic (with some exceptions [13, 11]). For example, using Nadaraya-Watson estimator \hat{q}_v^{NW} , sample sizes are estimated in [13] to ensure the condition $P\left[\sup_r |\bar{q}_v(r) - \hat{q}_v^{NW}(r)| > \epsilon\right] \leq \delta$ under additional smoothness conditions on regressions and densities.

4 Conclusions

We consider the transmission of a message of size r from a source to a destination with guarantees on the end-to-end delay over a computer network with n nodes and m links. Under fairly general

conditions, we proposed polynomial-time algorithms to compute paths whose delays are close to optimal expected delays with a high probability, irrespective of the distribution of delays. These algorithms are obtained by developing efficient algorithms for a general deterministic case, which could be of independent interest. Our main motivation is to establish that QoS guarantees in PAC framework can be provided under fairly general conditions without the knowledge of distributions. Several direct improvements are possible: the sample size estimates can be improved, and more sophisticated regression estimation methods can be employed. The main limitation of the proposed algorithms is the requirement of “stationarity” of the distributions. Extensions of our algorithms to non-stationary cases can be addressed under certain martingale or mixing conditions.

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