Performance Bound for Generalized *M*-QAM Constellations in Time-discrete Multipath Rayleigh Fading Channels with Channel Estimation Errors

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Abstract— **In this paper we derive a lower bound on the bit error rate (BER) for the individual bit streams that can be mapped simultaneously in an hierarchical way onto generalized multilevel quadrature amplitude modulations (M-QAM). The expressions are valid for time-discrete multipath Rayleigh fading environments with imperfect channel estimation and can be regarded as an extension of the well-known matched filter bound (MFB).**

Keywords- matched filter bound, hierarchical constellations, quadrature amplitude modulation, Rayleigh fading, channel estimation.

I. INTRODUCTION

In the design of wireless communication networks, when the objective is the broadcast of information to several users, *M*-HQAM constellations can be used instead of typical M-QAM, as in DVB-T [1]. *M*-HQAM constellations constitute a very simple method to provide unequal bit error protection and improve the efficiency of a network in broadcast transmissions. Several data streams can be transmitted simultaneously with different error protection levels and, depending on the propagation conditions, a given user can attempt to demodulate only the more protected bits or also the less protected bits carrying additional information. This approach is possible whenever the information is scalable like the cases of coded voice or video signals, as studied in[2][3]. Hierarchical 16-QAM and 64-QAM constellations are being considered for Universal Mobile Telecommunications System Terrestrial Radio Access Long Term Evolution (UTRA LTE) [4].

Due to the difficulty in dealing with multipath channels, instead of trying to obtain exact performance expressions for multipath channels it can be satisfactory to just have some analytical expressions representing an ideal performance that may not be possible to achieve in practice but can work as a very important benchmark for the evaluation of a specific receiver. The MFB is found assuming perfect channel knowledge and the transmission of only on pulse, i.e., no intersymbol interference (ISI) occurs, and can be regarded as a lower limit on the BER for a particular communication channel. Recently there was as a renewed interest in the computation of the MFB since the performance in time-dispersive channels with ISI can be close to the MFB, provided that suitable equalization schemes are employed combined with diversity techniques. An example is the IB-DFE (Iterative Block Decision Feedback Equalizer) [5][6], which is a promising iterative frequency-domain equalizer for cyclic-prefix-assisted block transmission schemes. MFB expressions for BPSK and QPSK transmission over two beam Rayleigh fading channels were obtained in [7] . These expressions were then extended to continuous-time delay profile channels in [8] and also to time-discrete multipath channels in [9] and [10] . Diversity reception was also taken into consideration in [8] and [10]. Using a similar approach to [10], MFB expressions were derived for the symbol error probability of QAM in [11]. If one of the assumptions of the MFB is removed, namely the perfect channel knowledge, we can obtain a tighter lower bound. In fact, imperfect channel estimation can severely degrade the performance of a transmission and thus it is important to have ways of evaluating its impact.

In this paper we take into account the presence of channel estimation errors and derive bounds for the BERs of the different types of bits (in terms of error protection level) in generalized hierarchical QAM constellations. The presented expressions are valid in time-discrete multipath Rayleigh fading channels where the different paths can be correlated. The paper is organized as follows. Section II gives a small definition of *M*-HQAM constellations and describes the model of the communication system considered. Section III derives performance bound expressions for square constellations and extends them to rectangular constellations. Section IV presents and analyzes some numerical results using the bounds and finally conclusions are given in Section V.

II. SYSTEM AND CHANNEL MODEL

A. Generalized Hierarchical QAM Signal Constellations

With a proper mapping of the different information bit streams any *M*-QAM constellation can become a hierarchical one (even an uniform QAM constellation [12]) with $1/2 \cdot \log_2 M$ different classes of bits ($\log_2 M$ if the constellation is rectangular). By using non uniformly spaced signal points it is possible to modify the different error protection levels of the streams. M-HQAM constellations can be constructed as nested constellation and can be characterized by the ratios of the distances between the inner component constellations

$$k_i = \frac{D_i}{D_{i+1}}, \quad i = 1, \ldots, \log_2 M/2 - 1 \tag{1}$$

For example, Figure 1 shows a non uniform 16-HQAM constellation which is defined with $k_1=D_1/D_2$ ($k_1=0.5$ corresponds to a uniform 16-QAM).

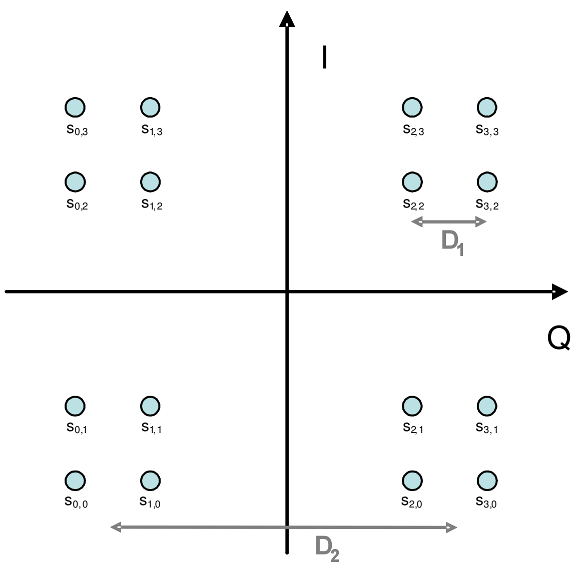

Figure 1. Hierarchical 16-QAM constellation.

In our analysis we assume that each information stream is split in two, so that half of the stream goes for the in-phase branch and the other for the quadrature branch of the modulator. The resulting bit sequence in each branch is Gray encoded and mapped onto the respective $\sqrt{M}$-PAM constellation symbols. The symbols from the in-phase and quadrature branches are then grouped together forming complex M-QAM symbols. The Gray coding for each $\sqrt{M}$-PAM constellation is performed following the procedure described in [13]. First the constellation symbols are represented in horizontal axis and are labeled from left to right with integers starting from 0 to $\sqrt{M}$-1. These labels are then converted to their binary representation so that each symbol s_j can be represented by a $\log_2 M/2$-digit binary sequence: $b_1^j, b_2^j, \ldots, b_{\log_2 M/2}^j$. The corresponding Gray code is then computed using

$$g_1^j = b_1^j$$
$$g_m^j = b_m^j \oplus b_{m-1}^j, \qquad m = 2, 3, \ldots, \frac{\log_2 M}{2} \tag{2}$$

where $\oplus$ represents modulo-2 addition.

B. System model

Let us consider the transmission over a multipath Rayleigh fading channel composed of L discrete taps. The corresponding transmit/receive block diagram is shown in Figure 3.

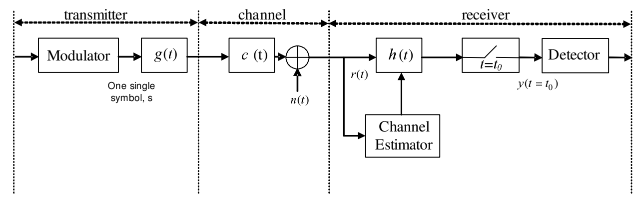

Figure 2. Baseband Transmit/Receive scheme.

The channel response at time t to an impulse applied at instant t-τ can be modelled as

$$c(\tau, t) = \sum_{i=1}^{L} \alpha_i(t)\, \delta(\tau - \tau_i), \tag{3}$$

where $\alpha_i(t)$ is a zero-mean complex Gaussian random process, $\delta(t)$ is the Dirac function and τ_i is the delay (assumed constant) of tap i. Regarding the cross-correlation between different taps, no restriction is imposed, i.e., the taps can be correlated. For the derivation of the bound we assume the transmission of only one pulse $s \cdot g(t)$, where s is an HQAM symbol and $g(t)$ is the impulse response of the transmit filter. If the channel is slowly time-varying it can be considered approximately constant inside this interval and the sampled signal at time t_0 after the receiver filter can be written as

$$y(t = t_0) = s \cdot \sum_{i=1}^{L} \sum_{i'=1}^{L} \alpha_i \hat{\alpha}_{i'}^* R(\tau_i - \tau_{i'}) + \sum_{i=1}^{L} \hat{\alpha}_i^* w_i, \tag{4}$$

where w_i represents Additive White Gaussian Noise (AWGN) samples with power spectral density N_0, $\hat{\alpha}_i$ is the channel estimate for tap i and $R(\tau)$ is the autocorrelation function of the transmit filter. It should be noted that assuming a linear channel estimator, the channel estimates $\hat{\alpha}_i$ will be complex Gaussian random variables.

III. PERFORMANCE BOUND DERIVATION

A. Square QAM Constellations

Our target is to obtain the bound for the BER of each different bit type i_m (m=1,…,$\log_2(M)/2$) in a constellation. Assuming a square constellation and due to the mapping considered, the I and Q branches are symmetric and thus we can develop our study using the decision variable for only one of the branches. In the following we will consider only the decision variable for the I branch. Figure 3 shows the decision regions for each bit along the I axis in the case of a 16-QAM constellation.

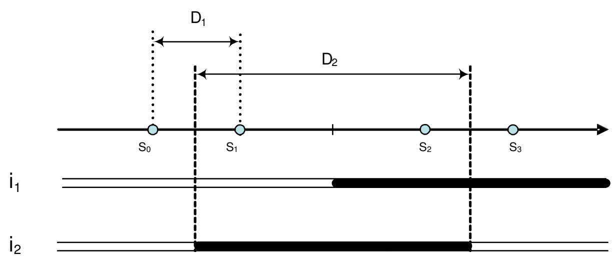

Figure 3. Decision regions for a 16-QAM constellation projected into the I branch. The dark areas correspond to a '1' and the white areas to '0'.

The reliability of the channel estimates can depend on the symbol position in the transmitted frame, which means the error probability will also depend on this position. Therefore, the BER must be averaged over all the possible locations t in a frame of size N. From [12], and taking into account that with the matched filter the decision borders should be weighted with $\sum_{i=1}^{L}\sum_{i'=1}^{L}\hat{\alpha}_i\hat{\alpha}_{i'}^{*}R\left(\tau_i-\tau_{i'}\right)$, we can write the BER expression for bit i_m in a generalized M-QAM constellation as (15) where

$$\mathbf{B}_m(l)=\frac{\mathbf{D_s}\left(\left(2l-1\right)2^{1/2\cdot\log_2 M-m}\right)+\mathbf{D_s}\left(\left(2l-1\right)2^{1/2\cdot\log_2 M-m}+1\right)}{2} \quad (5)$$

and

$$\mathbf{D}_\mathbf{s}\left(j\right)=\sum_{i=1}^{1/2\cdot\log_2 M}\left(2b_i^j-1\right)D_{1/2\cdot\log_2 M-i+1}\,. \quad (6)$$

To compute the BER bounds it is necessary to obtain an expression for the probability inside (15). In the following derivation we will drop the index t (symbol position in the frame for simplicity of notation, though we should keep in mind that the channel estimates can depend on it). To avoid that the decision borders depend explicitly on the channel estimate, the probability expression can be rewritten as

$$\text{Prob}\left\{\text{Re}\left[y\left(t=t_0\right)\right]<\mathbf{B}_m\left(l\right)\sum_{i=1}^{L}\sum_{i'=1}^{L}\hat{\alpha}_i\hat{\alpha}_{i'}^{*}R\left(\tau_i-\tau_{i'}\right)\middle|s=s_{j,f},t\right\}=$$
$$\text{Prob}\left\{y_{re}<0\middle|s=s_{j,f},t\right\} \quad (7)$$

where

$$y_{re}=\text{Re}\left[y\left(t=t_0\right)\right]-\mathbf{B}_m\left(l\right)\sum_{i=1}^{L}\sum_{i'=1}^{L}\hat{\alpha}_i\hat{\alpha}_{i'}^{*}R\left(\tau_i-\tau_{i'}\right). \quad (8)$$

Variable y_{re} can be expressed in a convenient quadratic form as

$$y_{re}=\sum_{i=1}^{L}\sum_{i'=1}^{L}\left(\alpha_i\hat{\alpha}_{i'}^{*}\frac{R\left(\tau_i-\tau_{i'}\right)}{2}s+\alpha_i^{*}\hat{\alpha}_{i'}\frac{R^{*}\left(\tau_i-\tau_{i'}\right)}{2}s^{*}-\right.$$
$$\left.-\mathbf{B}_m\left(l\right)\hat{\alpha}_i\hat{\alpha}_{i'}^{*}R\left(\tau_i-\tau_{i'}\right)\right)+\sum_{i=1}^{L}\left(\frac{\hat{\alpha}_i^{*}w_i}{2}+\frac{\hat{\alpha}_i w_i^{*}}{2}\right)=\mathbf{z}^H\mathbf{\Sigma}\mathbf{z}\,. \quad (9)$$

The last equality in (9) corresponds to the respective matrix representation ($\mathbf{A}^{\mathrm{H}}$ denotes conjugate transpose of matrix $\mathbf{A}$). In this matrix format $\mathbf{z}$, is a $3L\times 1$ column vector defined as

$$\mathbf{z}=\begin{bmatrix}\alpha_1 & \hat{\alpha}_1 & \cdots & \alpha_L & \hat{\alpha}_L & \cdots & w_1 & w_L\end{bmatrix}^T \quad (10)$$

and $\mathbf{\Sigma}$ is a $3L\times 3L$ matrix constructed according to

$$\mathbf{\Sigma}=\begin{bmatrix}
0 & R_{11}^{*} & 0 & R_{12}^{*} & 0 & \cdots & 0 & R_{1L}^{*} & 0 & 0 & \cdots & 0\\
R_{11} & \tilde{R}_{11} & R_{21} & \tilde{R}_{21} & R_{31} & \cdots & R_{L1} & \tilde{R}_{L1} & 1/2 & 0 & \cdots & 0\\
0 & R_{21}^{*} & 0 & R_{22}^{*} & 0 & \cdots & 0 & R_{2L}^{*} & 0 & 0 & \cdots & 0\\
R_{12} & \tilde{R}_{12} & R_{22} & \tilde{R}_{22} & R_{32} & \cdots & R_{L2} & \tilde{R}_{L1} & 0 & 1/2 & \cdots & 0\\
0 & R_{31}^{*} & 0 & R_{32}^{*} & 0 & \cdots & 0 & R_{3L}^{*} & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots\\
0 & R_{L1}^{*} & 0 & R_{L2}^{*} & 0 & \cdots & 0 & R_{LL}^{*} & 0 & 0 & \cdots & 0\\
R_{1L} & \tilde{R}_{1L} & R_{2L} & \tilde{R}_{2L} & R_{3L} & \cdots & R_{LL} & \tilde{R}_{LL} & 0 & 0 & \cdots & 1/2\\
0 & 1/2 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0\\
0 & 0 & 0 & 1/2 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots\\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1/2 & 0 & 0 & \cdots & 0
\end{bmatrix}, \quad (11)$$

with

$$R_{ii'}=\frac{s\cdot R\left(\tau_i-\tau_{i'}\right)}{2} \quad (12)$$

and

$$\tilde{R}_{ii'}=-\mathbf{B}_m\left(l\right)\cdot R\left(\tau_i-\tau_{i'}\right). \quad (13)$$

Note that $R\left(\tau_i-\tau_{i'}\right)=R^{*}\left(\tau_{i'}-\tau_i\right)$ and hence $\mathbf{\Sigma}$ is Hermitian.

For obtaining the bound it is necessary to know the probability density function (PDF) of y_{re}. To find this PDF we can apply a characteristic function based method but first we need to write y_{re} as a sum of uncorrelated random variables with known PDFs. Although the components of $\mathbf{z}$ are assumed Gaussian, they can be correlated: we can have correlation between different taps, between channel coefficients and channel estimates and also between noise and channel estimates (depending on the channel estimation method employed). Denoting $\mathbf{\Psi}$ as the covariance matrix of $\mathbf{z}$ ($\mathbf{\Psi}=\text{Cov}\left[\mathbf{z}\right]$), this matrix is Hermitian and positive-semidefinite and, consequently, it is possible to find at least one matrix $\mathbf{Q}$ so that $\mathbf{\Psi}$ can be decomposed into $\mathbf{\Psi}=\mathbf{Q}\mathbf{Q}^H$. In particular, if we apply the Cholesky decomposition, $\mathbf{Q}$ will be a lower triangular matrix.

$$P_b\left(i_m\right)=\frac{1}{N}\frac{2}{\sqrt{M}}\sum_{t=1}^{N}\sum_{j=0}^{\sqrt{M}/2-1}\left[1-g_m^j+\left(-1\right)^{g_m^j}\sum_{l=1}^{2^{(m-1)}}\left[\left(-1\right)^{l+1}\frac{2}{\sqrt{M}}\times\sum_{f=0}^{\sqrt{M}/2-1}\text{Prob}\left\{\text{Re}\left[y\left(t=t_0\right)\right]<\mathbf{B}_m\left(l\right)\sum_{i=1}^{L}\sum_{i'=1}^{L}\hat{\alpha}_i\hat{\alpha}_{i'}^{*}R\left(\tau_i-\tau_{i'}\right)\middle|s=s_{j,f},t\right\}\right]\right], \quad (14)$$

Using matrix $\mathbf{Q}$ we can define a new vector $\mathbf{z}' = \mathbf{Q}^{-1}\mathbf{z}$ and it can easily be shown that its covariance matrix corresponds to the identity matrix. Therefore the components of $\mathbf{z}'$ will be uncorrelated unit-variance complex Gaussian variables. Introducing this transformation into (9) leads to

$$y_{re} = \mathbf{z}'^H \mathbf{Q}^H \boldsymbol{\Sigma} \mathbf{Q} \mathbf{z}' = \mathbf{z}'^H \boldsymbol{\Sigma}' \mathbf{z}', \qquad (15)$$

with

$$\boldsymbol{\Sigma}' = \mathbf{Q}^H \boldsymbol{\Sigma} \mathbf{Q}. \qquad (16)$$

Matrix $\boldsymbol{\Sigma}'$ is still Hermitian and, as a result, can be decomposed as

$$\boldsymbol{\Sigma}' = \boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Phi}^H. \qquad (17)$$

In (17), $\boldsymbol{\Lambda}$ is a diagonal matrix whose elements are the eigenvalues λ_i (i=1,..,3L) of $\boldsymbol{\Sigma}'$ and $\boldsymbol{\Phi}$ is a matrix whose columns are the eigenvectors of $\boldsymbol{\Sigma}'$, which are orthogonal ($\boldsymbol{\Phi}\boldsymbol{\Phi}^H = \mathbf{I}$). We can then rewrite as

$$y_{re} = \mathbf{z}''^H \boldsymbol{\Lambda} \mathbf{z}'' = \sum_{i=1}^{3L} \lambda_i \left|z_i''\right|^2, \qquad (18)$$

where we have defined another vector, $\mathbf{z}'' = \boldsymbol{\Phi}^H \mathbf{z}'$. It can be easily shown that its components are still uncorrelated, unit-variance complex Gaussian variables. Since z_i'' are Gaussian, the individual variables $x_i = \lambda_i \left|z_i''\right|^2$ follow exponential distributions with PDFs given by

$$p(x_i) = \begin{cases} \dfrac{1}{\lambda_i}\exp\left(-\dfrac{x_i}{\lambda_i}\right)u(x_i) & \lambda_i > 0 \\ \dfrac{1}{|\lambda_i|}\exp\left(\dfrac{x_i}{|\lambda_i|}\right)u(-x_i) & \lambda_i < 0 \end{cases}, \qquad (19)$$

where $u(x)$ represents the unit step function (i.e., $u(x)$=0 for x<0 and 1 for x>0). According to (18), y_{re} can be expressed as a sum of independent random variables with exponential distributions and the respective characteristic function is

$$E\left\{e^{-j\upsilon \cdot y_{re}}\right\} = \prod_{i=1}^{3L} \frac{1}{1 + j\lambda_i \upsilon}. \qquad (20)$$

If there are L' distinct eigenvalues, each with a multiplicity of θ_i, i=1...L', and the negative ones occupy the first L'_{neg} positions we can apply the inverse Fourier transform to (20) and obtain the PDF of y as

. Using the change of variables s=$j\upsilon$, the characteristic function of can be written as

$$p_{y_{re}}(y) = \sum_{i=1}^{L'_{neg}} \sum_{k=1}^{\theta_i} (-1)^{\theta_i - k} \frac{A_{i,k}}{|\lambda_i|^{\theta_i} (\theta_i - k)!(k-1)!} (-y)^{k-1} \exp\left(\frac{y}{|\lambda_i|}\right) u(-y)$$
$$+ \sum_{i=L'_{neg}+1}^{L'} \sum_{k=1}^{\theta_i} \frac{A_{i,k}}{\lambda_i^{\theta_i} (\theta_i - k)!(k-1)!} y^{k-1} \exp\left(-\frac{y}{\lambda_i}\right) u(y). \quad (21)$$

with

$$A_{i,k} = \left[\frac{\partial^{\theta_i - k}}{\partial s^{\theta_i - k}} \left(\prod_{\substack{j=1 \\ j \neq i}}^{L'} \frac{1}{\left(1 + s\lambda_j\right)^{\theta_j}}\right)\right]_{s = -\frac{1}{\lambda_i}}. \qquad (22)$$

To compute (7) it is necessary to integrate this PDF from -∞ to *0* resulting

$$\text{Prob}\left\{y_{re} < 0 \middle| s = s_{j,f}, t\right\} = \int_{-\infty}^{0} p_{y_{re}}(y)\,dy$$

$$= \sum_{i=1}^{L'_{neg}} \sum_{k=1}^{\theta_i} \frac{A_{i,k}}{\lambda_i^{\rho_i - k} (\theta_i - k)!} \qquad (23)$$

The performance bounds can then be obtained by introducing (23) into (14).

B. Extension to Rectangular QAM Constellations

Even though in the previous section we only referred to square QAM constellations, the extension to rectangular ones is fairly simple. In this case, the BER of the bits in the I branch of the constellation is computed in exactly the same way. As for the bits mapped to the Q branch, we need to modify the general BER expression (14) so that the probability required becomes $\text{Prob}\left\{y_{im} < 0 \middle| s = s_{j,f}, t\right\}$, i.e., with y_{re} replaced by y_{im}, where y_{im} can be expressed as

$$y_{im} = \sum_{i=1}^{L} \sum_{i'=1}^{L} \left(\alpha_i \hat{\alpha}_{i'}^* \frac{R(\tau_i - \tau_{i'})}{2j} s - \alpha_i^* \hat{\alpha}_{i'} \frac{R^*(\tau_i - \tau_{i'})}{2j} s^* - \right.$$
$$\left. -\mathbf{B}_m(l)\hat{\alpha}_i \hat{\alpha}_{i'}^* R(\tau_i - \tau_{i'}) \right) + \sum_{i=1}^{L} \left(\frac{\hat{\alpha}_i^* w_i}{2j} - \frac{\hat{\alpha}_i w_i^*}{2j}\right) = \mathbf{z}^H \boldsymbol{\Sigma} \mathbf{z}. \quad (24)$$

The only consequence is that matrix $\boldsymbol{\Sigma}$ needs to be modified accordingly, whereas the remainder of the derivation remains the same.

IV. Numerical Results

Using the derived bound expressions we evaluated the BER of several HQAM constellations in different conditions. As an example, in this study we consider the use of a root-square raised cosine (RRC) filter with a roll-off bandwidth factor β=0.22. The results obtained are plotted as a function of E_S/N_0 (E_S denotes the average symbol energy). We assumed a Jakes spectrum [14] for each tap, which means that the respective autocovariance functions can be expressed as

$$R_{\alpha_i}(\tau) = E\left\{\alpha_i(t)\alpha_i(t+\tau)\right\} = \Omega_i^2 J_0(2\pi f_D \tau), \qquad (25)$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, f_D is the Doppler frequency and Ω_i is the root mean square value of the magnitude of tap i. Regarding the channel estimate, we will represent it as [15]

$$\hat{\alpha}_i = a \cdot \alpha_i + \xi_i , \qquad (26)$$

where a is a complex value denoting the bias of the estimate and ξ is the channel estimation error, modeled as a zero-mean complex Gaussian variable, which we will assume the same for all bit positions in the frame. In our simulations we considered Indoor A and Vehicular A environments (both from [16]). The symbols duration T_s was 260 ns.

Figure 4 and Figure 5 present performance results of a 64-HQAM constellation (k_1=k_2=0.4) in Indoor A and Vehicular A environments, respectively. Three groups of curves are shown: one for the most protected bits (MPB), one for the intermediate protected bits (IPB) and the third for the least protected bits (LPB). Curves for perfect channel estimation (coinciding with MFB) as well as for imperfect channel estimation with $E\left[\left|\xi_i\right|^2\right] = 0.1 \cdot \mathrm{N}_0$ and $E\left[\left|\xi_i\right|^2\right] = 0.1 \cdot \mathrm{N}_0 + 0.001 \cdot \Omega_i^2$. The latter case corresponds to channel estimates with irreducible errors and leads to performances with irreducible BER floors as can be seen in the graphs for high E_S/N_0 values. For low E_S/N_0 the prevailing effect is the estimation error due to thermal noise and therefore both imperfect channel estimation curves overlap. It is also visible the higher sensitivity of LPB to channel estimation errors which manifests with higher irreducible BER floors. Comparing the results between the two environments, we note a better performance in Vehicular A environment which has a higher delay spread.

In Figure 6, it is shown the effect of the non-uniformity parameter k_1 on the E_S/N_0 degradation due to imperfect channel estimation for a 16-HQAM transmission with a BER of 10^{-4} in Vehicular A environment. The channel estimation error is modeled as $E\left[\left|\xi_i\right|^2\right] = 0.1 \cdot \mathrm{N}_0 + 0.001 \cdot \Omega_i^2$. Note that in this legend the terminology MPB and LPB might be misleading since for k_1>0.5 these two bit types can swap roles in terms of error protection. Nevertheless, it is clear that the most protected bit has lower sensitivity to channel estimation errors, with degradations below 1dB. As for the least protected bit, a very non-uniform constellation (with k_1 close to 0 or 1) results in a very high sensitivity to channel estimation errors that can make the BER of 10^{-4} unattainable.

Figure 7 evaluates the impact of tap correlation in the performance of a 16-HQAM constellation. The environment considered is Vehicular A and the channel estimation error is modeled as $E\left[\left|\xi_i\right|^2\right] = 0.1 \cdot \mathrm{N}_0 + 0.001 \cdot \Omega_i^2$. Curves for different values of the correlation coefficient between taps, ρ (assumed the same for all the taps), are shown. As a reference the results for flat Rayleigh fading are also presented in the same graph. As expected, increased correlations lead to performance degradation due to a loss of multipath diversity. Nevertheless, this degradation is small until ρ=0.5 and even for higher correlation values, like 0.9, the performance is still relatively far from the flat Rayleigh environment (which does not have any diversity gain).

V. Conclusions

In this paper we have derived a performance bound similar to MFB but which also includes the effect of imperfect channel estimation. The expressions obtained allow the computation of the BER for the individual bit classes of any square/rectangular hierarchical M-QAM constellation in multipath Rayleigh fading environments.

VI. Acknowledgment

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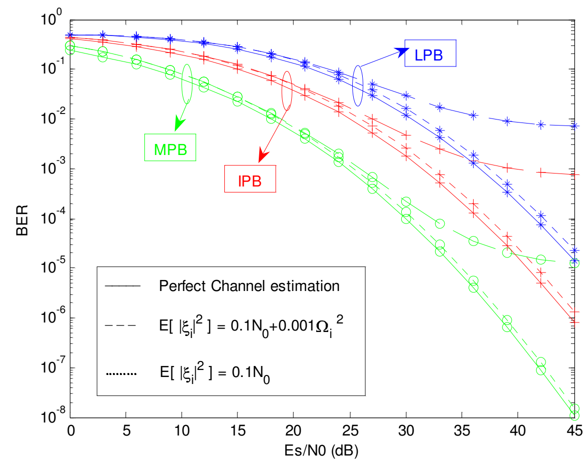

Figure 4. BER bounds for a 64-HQAM transmission with k_1= k_2=0.4 in Indoor A environment.

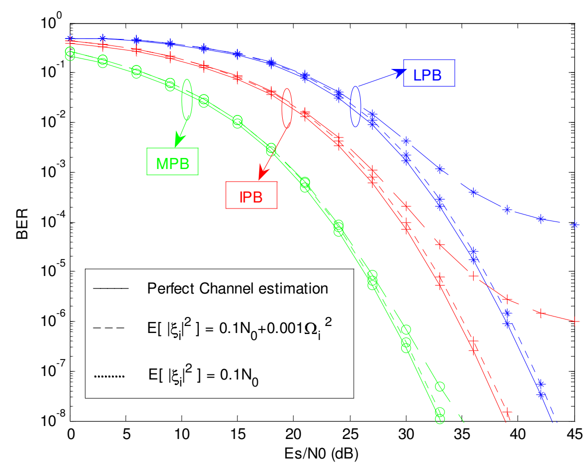

Figure 5. BER bounds for a 64-HQAM transmission with k_1= k_2=0.4 in Vehicular A environment.

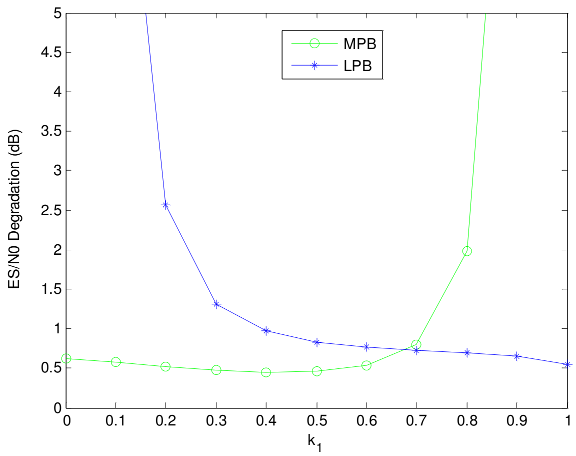

Figure 6. E_S/N_0 degradation due to imperfect channel estimation versus non-uniformity parameter k_1 for a 16-HQAM in Vehicular A environment. (BER=10^{-4})

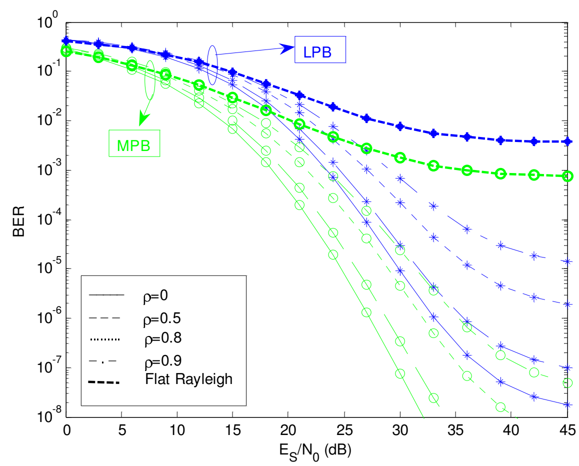

Figure 7. Impact of correlated taps on the BER of a 16-HQAM constellation with k_1=0.4 in Vehicular A environment.

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