Outage Analysis and Relay Allocation for Multi-stream OFDMA Decode-and-Forward Rayleigh Fading Networks

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***Abstract*—In this paper, we study a clustered two-hop decode-and-forward (DF) network consisting of a set of source-destination pairs, and a cluster of relays. We consider the case where channels are Rayleigh frequency selective, orthogonal frequency division multiple access (OFDMA) is employed, and there is no line of sight (LOS) between source and destination clusters. Approximating the capacity of a single source-relay-destination link by a Gaussian random variable (RV), the global outage probability of this network is characterized allowing for correlated OFDM subcarrier gains and arbitrary number of bits on each subcarrier. The obtained global probability of outage is used as an objective function to formulate an optimization problem to allocate relays to source-destination pairs. The outage probability minimization problem through relay allocation then is converted to a standard assignment problem, for which a low complexity algorithm based on Hungarian method is proposed. The numerical results show the precision and effectiveness of our analysis and proposed relay allocation technique.**

***Index Terms*—Decode-and-forward, OFDMA, outage probability.**

I. Introduction

Cooperative relaying is recognized as an outstanding candidate for replacing bulky multi-antenna systems, by exploiting geographically separated nodes as relays. Introducing relays to wireless networks opens up many interesting research problems. Relay allocation to source-destination streams is among these topics, which can be tackled to satisfy different kinds of objectives such as minimizing global outage probability of the network. Outage behavior of fundamental relaying protocols, namely decode-and-forward (DF) and amplify-and-forward (AF), was first addressed in [1] in high signal-to-noise ratio (SNR) regimes. Then in [2], [3], outage probability of DF and AF protocols are obtained in closed form. It should be noted that authors of these works assumed non frequency selective Rayleigh fading channels, which suits narrow band communications. However, the desire for higher data rates has forced the wireless industry into accommodating wide frequency bands, where frequency selectivity is inevitable. Frequency selective fading can be suppressed to a great extend by employing orthogonal frequency division multiplexing (OFDM) for single streams, and its multiple access variant, orthogonal frequency-division multiple access (OFDMA) for multiple simultaneous streams. In OFDM, the user is free to transmit on all the available subcarriers, while in OFDMA, multi-access channel is achieved by allowing the users to occupy only a subset of available subcarriers. Analyzing the outage event in OFDM and OFDMA relay systems imposes another challenge upon researchers.

In the context of single stream OFDM cooperative communication with single or multiple relays, most researchers assume that equal number of bits is allocated to all the OFDM subcarriers. Following this assumption, the event of outage turns out if and only if at least one OFDM subcarrier is in outage [4], [5], [6]. However, equal bit allocation assumption does not encompass the general case where higher number of bits with the same level of power can be allocated to an OFDM subcarrier of a good channel gain, to compensate for OFDM subcarriers in deep fade. Moreover, it is common to model OFDM subcarrier gains as statistically independent random variables (RV) to simplify the outage analysis [4], [5], [6], [7]. In [8], we have presented an outage analysis for a general case of arbitrary number of bits on OFDM subcarriers with correlated gains in a DF two-hop relaying system in which OFDM modulation is used.

The problem of analyzing the outage behavior of OFDM cooperative networks becomes more challenging when simultaneous streams are allowed to coexist. In this case, assigning relays to individual source-destination pairs becomes crucial, as well. Regarding the multi-stream scenarios, an immense amount of research exists on up-link and down-link of cellular systems, where multiple nodes are communicating with a single base station. For instance in [9], cellular multi-user cooperative networks under OFDM modulation are addressed, where a circular cell is divided into equal area sectors, and one relay serves all the nodes in a single sector. The outage probability of this system is also analyzed and used as the performance metric. Compared to cellular systems, non-cellular systems which are the focus of this paper, e.g., ad hoc and mesh topologies, where simultaneous streams can exist among multiple sources and destinations are less explored.

In this paper, we study a clustered OFDMA two-hop DF network consisting of a set of source-destination pairs, and a cluster of relays. We consider the case of no line of sight (LOS) between source cluster and the destination cluster, and it is assumed that individual source-relay and relay-destination channels suffer from Rayleigh frequency selective fading. The contributions of this paper are summarized in the following: The global outage probability of such networks is

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approximated assuming correlated OFDM subcarrier gains on individual source-relay and relay-destination channels, along with the freedom of allocating arbitrary number of bits to each OFDM subcarrier. It is also shown that the outage probability on a single source-relay-destination link does not depend on the choice of OFDMA subcarrier subset, as long as there is equal number of subcarriers in each subset, and a given subset is formed from contiguous subcarriers. The problem of relay allocation to minimizing the obtained global outage probability is formulated, and it is shown it can be converted to a standard assignment problem. Then, a novel low complexity centralized relay allocation scheme based on Hungarian method is proposed to minimize the global outage probability. The proposed relay allocation scheme is based on knowing only some statistical characteristics of the channel, as opposed to high complexity methods based on having access to full channel state information.

The rest of this paper is organized as follows. In the following section, the system model is defined. In section III, The outage probability of a single source-destination stream through a relay is obtained. Then, in section IV, we characterize the global probability of outage and propose our relay allocation scheme. The performance of our proposed method is examined through numerical simulations in section V, and we conclude the paper by section VI.

Before proceeding further, some notations used in this paper are clarified. We use $|\cdot|$ to mean magnitude of a complex argument, and $\delta(\cdot)$ and $U(\cdot)$ to represent the unit impulse and step functions. The Cartesian product of two sets is represented by $(\cdot) \times (\cdot)$, and defined as the set of all ordered pairs whose first element is a member of the left argument, and the second element is a member of the right argument. Moreover, $\bar{\cdot}$ and $\mathrm{E}[\cdot]$ indicate the expected value, $\mathrm{var}(\cdot)$ symbolizes the variance, $\mathrm{cov}(\cdot,\cdot)$ indicates the covariance of the two arguments, while the probability distribution function (PDF) of X is denoted by $f_X(\cdot)$, and $\mathcal{N}(0,\sigma^2)$ and $\mathcal{CN}(0,\sigma^2)$ denote the normal, and complex normal PDFs with zero mean and variance σ^2. Vectors and matrices are represented by bold letters, the i^{th} element of vector $\mathbf{V}$ is denoted by $\mathbf{V}(i)$, $\mathrm{diag}(\cdot)$ is a matrix with elements on the main diagonal taken from the elements of the input vector and zeros elsewhere, and $\mathrm{trace}(\cdot)$ is a scalar obtained by adding up the elements on the main diagonal of the input matrix.

II. System Model

In a Rayleigh frequency selective wireless environment, N_{s} sources (s_i, $i \in \{1,\ldots,N_{\mathrm{s}}\}$) wish to communicate to their intended destinations (d_i, $i \in \{1,\ldots,N_{\mathrm{s}}\}$), establishing N_{s} source-destination pairs, using the help of N_{r} relays (r_j, $j \in \{1,\ldots,N_{\mathrm{r}}\}$) in between, as shown in Fig. 1. Each relay can support only one s_i-d_i pair, i.e. at least N_{s} relays are needed to support all s_i-d_i pairs. It is assumed that there is no LOS between sources and destinations, and all the nodes are half-duplex, capable of only transmitting or receiving data at a specific time and frequency band. Therefore, a two time slot OFDMA scheme is employed. The frequency band consisting of N_{sc} flat faded OFDM subcarriers is divided into M non-overlapping subsets of subcarriers $\{\phi_1,\ldots,\phi_M\}$, each formed

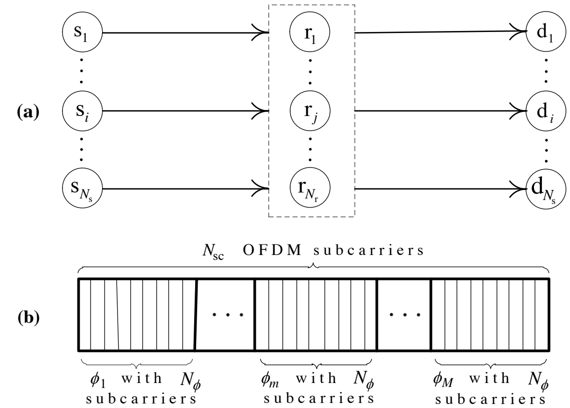

Fig. 1. System model (a) system configuration (b) OFDMA setup

from N_ϕ contiguous OFDM subcarriers (i.e. $M \cdot N_\phi = N_{\mathrm{sc}}$)[1]. These OFDMA subcarrier subsets are allocated to s_i-d_i pairs, one to each pair, in order to realize more than one s_i-d_i pair at a time. In the first time slot, the sources transmit on their allocated OFDMA subset, and relays listen. Assuming that one relay is allocated to each source, in the second time slot, the relays retransmit the information on the allocated OFDMA subset to the destinations under DF protocol. We also assume perfect time and frequency synchronization along with a cyclic prefix that is long enough to overcome the channel delay spread.

There are $2 \cdot N_{\mathrm{s}} \cdot N_{\mathrm{r}}$ physical channels in this system, $N_{\mathrm{s}} \cdot N_{\mathrm{r}}$ channels between sources and relays, and $N_{\mathrm{s}} \cdot N_{\mathrm{r}}$ channels between relays and destinations, where all these channels are statistically independent, due to the random nature of multi-paths fading in wireless channels. Indicating by $L_{\mathrm{s}_i\mathrm{r}_j}$ and $L_{\mathrm{r}_j\mathrm{d}_i}$ the number of channel taps on s_i-r_j and r_j-d_i channels, respectively, the channel impulse responses are

$$\begin{aligned} h_{\mathrm{s}_i\mathrm{r}_j}(\tau) &= \sum_{l=0}^{L_{\mathrm{s}_i\mathrm{r}_j}-1} \mathbf{h}_{\mathrm{s}_i\mathrm{r}_j}(l)\delta(\tau - l \cdot T_c), \\ h_{\mathrm{r}_j\mathrm{d}_i}(\tau) &= \sum_{l=0}^{L_{\mathrm{r}_j\mathrm{d}_i}-1} \mathbf{h}_{\mathrm{r}_j\mathrm{d}_i}(l)\delta(\tau - l \cdot T_c), \end{aligned} \tag{1}$$

where T_c indicates the coherence time, and the channel tap coefficients are taken from the following $L_{\mathrm{s}_i\mathrm{r}_j}$ and $L_{\mathrm{r}_j\mathrm{d}_i}$ element random vectors

$$\begin{aligned} \mathbf{h}_{\mathrm{s}_i\mathrm{r}_j} &= [\mathbf{h}_{\mathrm{s}_i\mathrm{r}_j}(1)\ \ \mathbf{h}_{\mathrm{s}_i\mathrm{r}_j}(2)\ \ \ldots\ \ \mathbf{h}_{\mathrm{s}_i\mathrm{r}_j}(L_{\mathrm{s}_i\mathrm{r}_j})]^T, \\ \mathbf{h}_{\mathrm{r}_j\mathrm{d}_i} &= [\mathbf{h}_{\mathrm{r}_j\mathrm{d}_i}(1)\ \ \mathbf{h}_{\mathrm{r}_j\mathrm{d}_i}(2)\ \ \ldots\ \ \mathbf{h}_{\mathrm{r}_j\mathrm{d}_i}(L_{\mathrm{r}_j\mathrm{d}_i})]^T. \end{aligned} \tag{2}$$

It is assumed that the channel tap coefficients on a given physical channel are statistically independent, zero mean, and complex normally distributed RVs. Therefore, channel tap coefficient vectors $\mathbf{h}_{\mathrm{s}_i\mathrm{r}_j}$ and $\mathbf{h}_{\mathrm{r}_j\mathrm{d}_i}$ are modeled as complex

[1] In this paper, all indices i refer to sources, and to be more precise, source-destination pairs, all indices j refer to relays, and indices n, n_1, and n_2 refer to the OFDM subcarriers. Where not indicated, indices i and j can take any value in $\{1,\ldots,N_{\mathrm{s}}\}$ and $\{1,\ldots,N_{\mathrm{r}}\}$, respectively, while n, n_1, and n_2 can take values in $\{1,\ldots,N_{\mathrm{sc}}\}$.

normal vectors with covariance matrices $\boldsymbol{\Gamma}_{\mathrm{s}_i\mathrm{r}_j}$ and $\boldsymbol{\Gamma}_{\mathrm{r}_j\mathrm{d}_i}$, as follows

$$\mathbf{h}_{\mathrm{s}_i\mathrm{r}_j} \sim \mathcal{CN}(0, \boldsymbol{\Gamma}_{\mathrm{s}_i\mathrm{r}_j}) \quad , \quad \mathbf{h}_{\mathrm{r}_j\mathrm{d}_i} \sim \mathcal{CN}(0, \boldsymbol{\Gamma}_{\mathrm{r}_j\mathrm{d}_i}), \tag{3}$$

where

$$\begin{aligned}\boldsymbol{\Gamma}_{\mathrm{s}_i\mathrm{r}_j} &= \mathrm{diag}\left[\mathrm{var}\left(\mathbf{h}_{\mathrm{s}_i\mathrm{r}_j}(1)\right) \ \ldots \ \mathrm{var}\left(\mathbf{h}_{\mathrm{s}_i\mathrm{r}_j}(L_{\mathrm{s}_i\mathrm{r}_j})\right)\right],\\ \boldsymbol{\Gamma}_{\mathrm{r}_j\mathrm{d}_i} &= \mathrm{diag}\left[\mathrm{var}\left(\mathbf{h}_{\mathrm{r}_j\mathrm{d}_i}(1)\right) \ \ldots \ \mathrm{var}\left(\mathbf{h}_{\mathrm{r}_j\mathrm{d}_i}(L_{\mathrm{r}_j\mathrm{d}_i})\right)\right].\end{aligned} \tag{4}$$

Having defined the channels in time domain, the Rayleigh OFDM subcarrier gains can be modeled as N_{sc} element vectors, which are obtained by performing N_{sc}-point Discrete Fourier Transform ($\mathrm{N_{sc}}$- DFT) on channel tap coefficient vectors,

$$\begin{aligned}\mathbf{H}_{\mathrm{s}_i\mathrm{r}_j} &= [\mathbf{H}_{\mathrm{s}_i\mathrm{r}_j}(1), \ldots, \mathbf{H}_{\mathrm{s}_i\mathrm{r}_j}(N_{\mathrm{sc}})] = N_{\mathrm{sc}}\text{- DFT}\{\mathbf{h}_{\mathrm{s}_i\mathrm{r}_j}\},\\ \mathbf{H}_{\mathrm{r}_j\mathrm{d}_i} &= [\mathbf{H}_{\mathrm{r}_j\mathrm{d}_i}(1), \ldots, \mathbf{H}_{\mathrm{r}_j\mathrm{d}_i}(N_{\mathrm{sc}})] = N_{\mathrm{sc}}\text{- DFT}\{\mathbf{h}_{\mathrm{r}_j\mathrm{d}_i}\}.\end{aligned} \tag{5}$$

The channel power gain vectors on s_i-r_j and r_j-d_i channels are represented by $\mathbf{X}_{ij}$ and $\mathbf{Y}_{ji}$, respectively. These two N_{sc} element vectors are formally defined as

$$\begin{aligned}\mathbf{X}_{ij} &= [|\mathbf{H}_{\mathrm{s}_i\mathrm{r}_j}(1)|^2, \ldots, |\mathbf{H}_{\mathrm{s}_i\mathrm{r}_j}(N_{\mathrm{sc}})|^2],\\ \mathbf{Y}_{ji} &= [|\mathbf{H}_{\mathrm{r}_j\mathrm{d}_i}(1)|^2, \ldots, |\mathbf{H}_{\mathrm{r}_j\mathrm{d}_i}(N_{\mathrm{sc}})|^2].\end{aligned} \tag{6}$$

Here, we can deduce that the elements of $\mathbf{X}_{ij}$ are exponentially distributed, since the channel gains in frequency are Rayleigh distributed. Moreover, elements of $\mathbf{X}_{ij}$ for a specific i and j are identical RVs, thus all associated with exponential parameter $\lambda_{\mathrm{s}_i\mathrm{r}_j}$. The same arguments hold for $\mathbf{Y}_{ji}$, its elements, and $\lambda_{\mathrm{r}_j\mathrm{d}_i}$. Hence, the elements of $\mathbf{X}_{ij}$ and $\mathbf{Y}_{ji}$ are taken from the following distributions

$$\begin{aligned}f_{\mathbf{X}_{ij}(n)}(x) &= \lambda_{\mathrm{s}_i\mathrm{r}_j} e^{-(\lambda_{\mathrm{s}_i\mathrm{r}_j})x} \cdot U(x), \ \forall n \in \{1, \ldots, N_{\mathrm{sc}}\},\\ f_{\mathbf{Y}_{ij}(n)}(y) &= \lambda_{\mathrm{r}_j\mathrm{d}_i} e^{-(\lambda_{\mathrm{r}_j\mathrm{d}_i})y} \cdot U(y), \ \forall n \in \{1, \ldots, N_{\mathrm{sc}}\},\end{aligned} \tag{7}$$

where the exponential distribution parameters are obtained to be

$$\lambda_{\mathrm{s}_i\mathrm{r}_j} = \frac{1}{\mathrm{trace}[\boldsymbol{\Gamma}_{\mathrm{s}_i\mathrm{r}_j}]} \quad , \quad \lambda_{\mathrm{r}_j\mathrm{d}_i} = \frac{1}{\mathrm{trace}[\boldsymbol{\Gamma}_{\mathrm{r}_j\mathrm{d}_i}]}. \tag{8}$$

Furthermore, denoting by $\Theta_{X_{\mathrm{s}_i\mathrm{r}_j}}(n_1, n_2)$ and $\Theta_{Y_{\mathrm{r}_j\mathrm{d}_i}}(n_1, n_2)$ the $\mathrm{cov}\left(\mathbf{X}_{ij}(n_1), \mathbf{X}_{ij}(n_2)\right)$ and $\mathrm{cov}\left(\mathbf{Y}_{ji}(n_1), \mathbf{Y}_{ji}(n_2)\right)$, respectively, they easily can be obtained to be [10]

$$\begin{aligned}\Theta_{X_{\mathrm{s}_i\mathrm{r}_j}}(n_1, n_2) &= \Big| \sum_{l=0}^{L_{\mathrm{s}_i\mathrm{r}_j}-1} \mathrm{var}\left(\mathbf{h}_{\mathrm{s}_i\mathrm{r}_j}(l)\right) e^{-\hat{j}2\pi l \frac{n_1-n_2}{N_{\mathrm{sc}}}} \Big|^2,\\ \Theta_{Y_{\mathrm{r}_j\mathrm{d}_i}}(n_1, n_2) &= \Big| \sum_{l=0}^{L_{\mathrm{r}_j\mathrm{d}_i}-1} \mathrm{var}\left(\mathbf{h}_{\mathrm{r}_j\mathrm{d}_i}(l)\right) e^{-\hat{j}2\pi l \frac{n_1-n_2}{N_{\mathrm{sc}}}} \Big|^2,\end{aligned} \tag{9}$$

where $\hat{j}$ is the imaginary unit $\sqrt{-1}$. It should be noted that $\mathrm{cov}\left(\mathbf{X}_{ij}(n_1), \mathbf{X}_{\acute{i}\acute{j}}(n_2)\right) = 0$ if $i \neq \acute{i}$ or $j \neq \acute{j}$, because in this case, the arguments of $\mathrm{cov}(.,.)$ are obtained from two independent events. Following the same reasoning, it is trivial that $\mathrm{cov}\left(\mathbf{X}_{ij}(n_1), \mathbf{Y}_{\acute{i}\acute{j}}(n_2)\right) = 0$ for all $(i,j) \in \{1, \ldots, N_{\mathrm{s}}\} \times \{1, \ldots, N_{\mathrm{r}}\}$ and $(\acute{i}, \acute{j}) \in \{1, \ldots, N_{\mathrm{s}}\} \times \{1, \ldots, N_{\mathrm{r}}\}$.

Now, let us assume that s_i with power budget of P_{s_i} wants to transmit its information to d_i using the help of r_j with power budget of P_{r_j}. Further, let us assume that this communication is taking place through m^{th} OFDMA subcarrier set, i.e., s_i and r_j are allowed to use OFDM subcarriers $n \in \phi_m$. Also, s_i and r_j equally distribute their power budget among their N_ϕ available OFDM subcarriers, allocating $\frac{P_{\mathrm{s}_i}}{N_\phi}$ and $\frac{P_{\mathrm{r}_j}}{N_\phi}$ to each subcarrier in ϕ_m, respectively. Having defined these parameters, the capacity of s_i-r_j-d_i link on the n^{th} OFDM subcarrier, where $n \in \phi_m$, is [1]

$$\begin{aligned}C_{ij}(n) =& \frac{1}{2} \log\left(1 + \min\left\{\mathbf{X}_{ij}(n)\frac{P_{\mathrm{s}_i}}{N_\phi \sigma_{\mathrm{r}}^2}, \mathbf{Y}_{ji}(n)\frac{P_{\mathrm{r}_j}}{N_\phi \sigma_{\mathrm{d}}^2}\right\}\right),\\ & n \in \phi_m,\end{aligned} \tag{10}$$

where σ_r^2 and σ_d^2 are noise variances at relays and destinations, respectively. Finally, the overall capacity of s_i-r_j-d_i link through ϕ_m is

$$C_{ij}^m = \frac{1}{N_\phi} \sum_{n=(m-1)N_\phi+1}^{mN_\phi} C_{ij}(n). \tag{11}$$

In this paper, the eventual goal is to allocate relays to s_i-d_i pairs such that the global outage probability of the system is minimized. Hence, obtaining the global outage probability as the main objective function is one of the crucial steps. On the way to achieving this purpose, evaluating the outage probability of a single s_i-r_j-d_i link through ϕ_m is of higher priority. We denote this probability by $P_{\text{out}}^m(i,j)$, and analyze it in the next section. Then, in section IV, we formulate the overall outage probability minimization problem through relay allocation and propose our solution.

III. Outage probability of a single source-relay-destination link

In our recent work [8], we have presented an analysis to obtain the outage probability of an OFDM DF relay system, consisting of one source, one relay, and one destination, in which LOS between source and destination is also present. Modifying the system model in [8] by removing the LOS and allowing the active nodes to use only a subset of available OFDM subcarriers, $P_{\text{out}}^m(i,j)$ can easily be characterized. In order to make this paper self-contained, the outline of deriving $P_{\text{out}}^m(i,j)$ is briefly described here. For further details, we refer the reader to [8] and references within.

The event of outage on s_i-r_j-d_i link occurs if the link's capacity from (11) drops below a certain threshold γ_i

$$P_{\text{out}}^m(i,j) = \Pr[C_{ij}^m < \gamma_i]. \tag{12}$$

In order to find $\Pr[C_{ij}^m < \gamma_i]$, the PDF of C_{ij}^m is required. According to (11) and (10), the link capacity C_{ij}^m is a summation over identical RVs $C_{ij}^m(n)$, where $n \in \phi_m$. If these RVs were statistically independent, the link capacity would be expected to follow a Gaussian PDF according to the central limit theorem. But, although the RVs $C_{ij}^m(n)$ are correlated through (9), still the link's capacity approximately follows the normal distribution. On the other hand, to fully characterize a normal RV, calculating the mean and variance is adequate.

As for the mean of C_{ij}^m, we introduce the following set of RVs

$$\mathbf{V}_{ij}(n) = \min\Big\{\mathbf{X}_{ij}(n)\frac{P_{\mathrm{s}_i}}{N_\phi \sigma_r^2}, \mathbf{Y}_{ji}(n)\frac{P_{\mathrm{r}_j}}{N_\phi \sigma_d^2}\Big\}, \forall n \in \phi_m. \tag{13}$$

Given the PDFs of $\mathbf{X}_{ij}(n)$ and $\mathbf{Y}_{ji}(n)$ in (7), the PDF of the above RVs is calculated to be

$$f_{\mathbf{V}_{ij}(n)}(v) = (\alpha_{\mathrm{s}_i\mathrm{r}_j} + \alpha_{\mathrm{s}_i\mathrm{r}_j})e^{-(\alpha_{\mathrm{s}_i\mathrm{r}_j}+\alpha_{\mathrm{r}_j\mathrm{s}_i})v} \cdot U(v), \tag{14}$$

where

$$\alpha_{\mathrm{s}_i\mathrm{r}_j} = \frac{\lambda_{\mathrm{s}_i\mathrm{r}_j} \cdot N_\phi \sigma_{\mathrm{r}}^2}{P_{\mathrm{s}_i}} \quad , \quad \alpha_{\mathrm{r}_j\mathrm{d}_i} = \frac{\lambda_{\mathrm{r}_j\mathrm{d}_i} \cdot N_\phi \sigma_{\mathrm{d}}^2}{P_{\mathrm{r}_j}}. \tag{15}$$

By substituting (13) into (10) we have

$$\begin{aligned}\mathrm{E}[C_{ij}^m] &= \frac{1}{N_\phi}\sum_{n=(m-1)N_\phi+1}^{mN_\phi} \mathrm{E}[C_{ij}(n)] = \mathrm{E}[C_{ij}(n)] \\ &= \int_0^\infty \frac{1}{2}\log(1+v) f_{\mathbf{V}_{ij}(n)}(v)\mathrm{d}v, \end{aligned}\tag{16}$$

Finally, by calculating the above integral the mean of C_{ij}^m is

$$\mathrm{E}[C_{ij}^m] = \frac{-\log_2(e)}{2} e^{(\alpha_{\mathrm{s}_i\mathrm{r}_j}+\alpha_{\mathrm{r}_j\mathrm{s}_i})}\mathrm{Ei}\big(-(\alpha_{\mathrm{s}_i\mathrm{r}_j}+\alpha_{\mathrm{r}_j\mathrm{s}_i})\big). \tag{17}$$

in which $\mathrm{Ei}(\cdot)$ is the exponential integral and defined as $\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{\exp(t)}{t}\mathrm{d}t$ [11]. Note that $\mathrm{E}[C_{ij}^m]$ is independent of m and n, i.e. choice of ϕ_m does not change the mean of a link's capacity.

Considering that the joint PDF of any two OFDM subcarrier gains $|\mathbf{H}_{\mathrm{s}_i\mathrm{r}_j}(n_1)|$ and $|\mathbf{H}_{\mathrm{s}_i\mathrm{r}_j}(n_2)|$, or $|\mathbf{H}_{\mathrm{r}_j\mathrm{d}_i}(n_1)|$ and $|\mathbf{H}_{\mathrm{r}_j\mathrm{d}_i}(n_2)|$, follows a bivariate Rayleigh distribution [12], [13], obtaining a solution in closed form to the variance of C_{ij}^m is not straight-forward. Therefore, we use the delta method [14] to estimate the variance as follows

$$\begin{aligned}\sigma_{C_{ij}^m}^2 \approx \sum_{n_1\in\phi_m}\sum_{n_2\in\phi_m}\Bigg[&\frac{\partial C_{ij}^m}{\partial \mathbf{X}_{ij}(n_1)}\frac{\partial C_{ij}^m}{\partial \mathbf{X}_{ij}(n_2)}\Theta_{X_{ij}}(n_1,n_2) \\ &+ \frac{\partial C_{ij}^m}{\partial \mathbf{Y}_{ji}(n_1)}\frac{\partial C_{ij}^m}{\partial \mathbf{Y}_{ji}(n_2)}\Theta_{Y_{ji}}(n_1,n_2)\Bigg],\end{aligned}\tag{18}$$

where $\frac{\partial C_{ij}^m}{\partial \mathbf{X}_{ij}(n)}$ and $\frac{\partial C_{ij}^m}{\partial \mathbf{Y}_{ji}(n)}$ denote the partial derivatives of link's capacity C_{ij}^m from (11) and (10) with respect to $\mathbf{X}_{ij}(n)$ and $\mathbf{Y}_{ji}(n)$. The derivatives are evaluated at $\mathrm{E}[\mathbf{X}_{ij}(n)]$ and $\mathrm{E}[\mathbf{Y}_{ji}(n)]$ (more details in [8]).

Inspecting (18), it is observed that $\sigma_{C_{ij}^m}^2$ is also independent of ϕ_m. This is because according to (9), $\Theta_{X_{ij}}(n_1, n_2)$ and $\Theta_{Y_{ji}}(n_1, n_2)$ are functions of OFDM subcarrier spacing $n_1 - n_2$. In other words, as long as there is equal number of OFDM subcarriers in each subset ϕ_m, and these subcarriers are contiguous, choice of ϕ_m does not change the mean and variance of link's capacity. Moreover, since the link's capacity is modeled as a normal RV which is fully represented by its mean and variance, choice of ϕ_m does not affect link's capacity C_{ij}^m, either. Therefore, we omit the index m from C_{ij}^m and $P_{\mathrm{out}}^m(i,j)$, and replace them by C_{ij} and $P_{\mathrm{out}}(i,j)$, respectively.

Having obtained the mean and variance of C_{ij} in (17) and (18), the outage probability on the s_i-r_j-d_i link is

$$P_{\mathrm{out}}(i,j) = \Pr[C_{ij} < \gamma_i] = 1 - Q\left(\frac{\gamma_i - \bar{C}_{ij}}{\sigma_{C_{ij}}}\right). \tag{19}$$

in which $Q(\cdot)$ is the standard Q-function.

IV. Problem Formulation and Proposed Relay Allocation Scheme

In this section, the problem of relay allocation for minimizing the global outage probability is formally stated, and a solution based on Hungarian method is proposed. We say the multi-stream system introduced in section II is in outage if at least one of the N_{s} s_i-d_i links is in outage. In order to obtain an expression for the global outage probability P_{out}, we introduce relay assignment indices $\rho_{ij} \in \{0,1\}$. The relay j is assigned to s_i-d_i pair if and only if $\rho_{ij} = 1$. Moreover, $\rho_{ij} = 1$ implies that $\rho_{\acute{i}\acute{j}} = 0$ if $i \neq \acute{i}$ or $j \neq \acute{j}$, because a relay can help at most one s_i-d_i pair. Following this definition, and considering the fact that the events of outage on single s_i-r_j-d_i links are statistically independent, the probability that none of the s_i-d_i pairs is in outage can be written in the form of

$$1 - P_{\mathrm{out}} = \prod_{i=1}^{N_{\mathrm{s}}}\prod_{j=1}^{N_{\mathrm{r}}}\big(1 - P_{\mathrm{out}}(i,j)\big)^{\rho_{ij}}, \tag{20}$$

in which $P_{\mathrm{out}}(i,j)$ is the outage probability on a single s_i-r_j-d_i link through any of the M OFDMA subcarrier subsets ϕ_m, and this probability has been analyzed in section III. Finally, we are seeking a solution for the following optimization problem

$$\min_{\rho_{ij}} \quad 1 - \prod_{i=1}^{N_{\mathrm{s}}}\prod_{j=1}^{N_{\mathrm{r}}}\big(1 - P_{\mathrm{out}}(i,j)\big)^{\rho_{ij}} \tag{21}$$

$$\text{subject to (s.t.)} \quad \sum_{i=1}^{N_{\mathrm{s}}} \rho_{ij} \leq 1, \tag{22}$$

$$\sum_{j=1}^{N_{\mathrm{r}}} \rho_{ij} \leq 1, \tag{23}$$

$$\rho_{ij} \in \{0,1\}, \forall (i,j) \in \{1,\ldots,N_{\mathrm{s}}\} \times \{1,\ldots,N_{\mathrm{r}}\}. \tag{24}$$

In this optimization problem, the objective function (21) is the global probability of outage, and constraints (22), (23) and (24) guarantee that each relay is at most assigned to one s_i-d_i pair. Further, by removing the constant term in (21) and taking the logarithm of the right hand side, the optimization problem (21)-(24) is transformed into the following optimization problem

$$\max_{\rho_{ij}} \quad \sum_{i=1}^{N_{\mathrm{s}}}\sum_{j=1}^{N_{\mathrm{r}}} \rho_{ij}\log\big(1 - P_{\mathrm{out}}(i,j)\big) \tag{25}$$

$$\text{s.t.} \quad (22), (23) \text{ and } (24), \tag{26}$$

The above optimization problem is a standard 2-dimensional assignment problem [15]. To illustrate the nature of this problem, a gain matrix $\mathbf{A}$ is introduced, where the elements of this matrix are

$$A_{ij} = \log\big(1 - P_{\text{out}}(i,j)\big), \qquad (27)$$
$$\forall (i,j) \in \{1,\ldots,N_{\text{s}}\} \times \{1,\ldots,N_{\text{r}}\}.$$

These elements are interpreted as the gain achieved by assigning r_j to s_i-d_i pair. Now, the problem of finding ρ_{ij} such that (21) is minimized is reduced to selecting N_{s} elements from $\mathbf{A}$, such that only one element is selected from each row and column, and the summation of these elements is maximized. In other words, if A_{ij} is selected, then $\rho_{ij} = 1$, and $\rho_{ij} = 0$ otherwise. This problem can be solved optimally by adopting the Hungarian method [15] with the complexity of $\mathcal{O}(\max(N_{\text{s}}, N_{\text{r}})^3)$. Forming the gain matrix $\mathbf{A}$ requires $N_{\text{s}} \times N_{\text{r}}$ calculations, giving the complexity of $\mathcal{O}(N_{\text{s}} \times N_{\text{r}})$, thus, the complexity of our proposed solution is dominated by the Hungarian method.

The proposed algorithm of relay allocation for minimizing the global outage probability is briefly described here. Given the statistical properties of the channel impulse responses between sources and relays, and relays and destination, the outage probability of all s_i-r_j-d_i links can be evaluated through the procedure described in section III. It is also observed in section III that the outage probability of a specific link does not depend on the choice of OFDMA subcarrier subset ϕ_m. Therefore, the need for allocating OFDMA subcarrier subsets to s_i-d_i pairs is eliminated, leaving us only with relays to be allocated. In order to allocate relays, the gain matrix in (27) is formed, and then performing the Hungarian method on this matrix readily gives us the relay allocation pattern. Finally, the OFDMA subcarrier subsets ϕ_m can be allocated randomly to the s_i-r_j-d_i links, so that simultaneous communication of the s_i-d_i pairs is not corrupted. This algorithm is evaluated through numerical simulation in the next section.

V. Simulation Results

In this section, the behavior of our system model and the performance of our proposed relay allocation scheme is examined through numerical simulations. In simulations, it is assumed that the number of channel taps on all the channels is equal, i.e., $L_{\text{s}_i\text{r}_j} = L_{\text{r}_j\text{d}_i} = L$, and the channel tap coefficients on a given channel have equal variance, while the variance of channel taps on different channels are not necessarily equal, as the following

$$\text{var}\big(\mathbf{h}_{\text{s}_i\text{r}_j}(1)\big) = \cdots = \text{var}\big(\mathbf{h}_{\text{s}_i\text{r}_j}(L)\big) = \sigma^2_{\text{s}_i\text{r}_j}/L,$$
$$\text{var}\big(\mathbf{h}_{\text{r}_j\text{s}_i}(1)\big) = \cdots = \text{var}\big(\mathbf{h}_{\text{r}_j\text{s}_i}(L)\big) = \sigma^2_{\text{r}_j\text{s}_i}/L. \qquad (28)$$

Furthermore, the total power of channels, $\sigma^2_{\text{s}_i\text{r}_j}$ and $\sigma^2_{\text{r}_j\text{s}_i}$, are taken from a uniformly distributed RV between 0 and 1

$$\sigma^2_{\text{s}_i\text{r}_j} \sim U[0,1] \quad , \quad \sigma^2_{\text{r}_j\text{s}_i} \sim U[0,1]. \qquad (29)$$

Equal power budgets at all the source and relay nodes is also assumed ($P_{\text{s}_i} = P_{\text{r}_j} = P_{\text{s}} = P_{\text{r}}$), and the threshold rate is set to 1 for all s_i-d_i pairs ($\gamma_i = \gamma = 1$ bit/s/Hz), and noise variances at relays and destinations are $\sigma^2_{\text{s}} = \sigma^2_{\text{r}} = 1$. In addition, the users are allowed to use OFDMA subcarrier groups of $N_\phi = 16$, and the SNR is the transmit SNR per OFDM subcarrier.

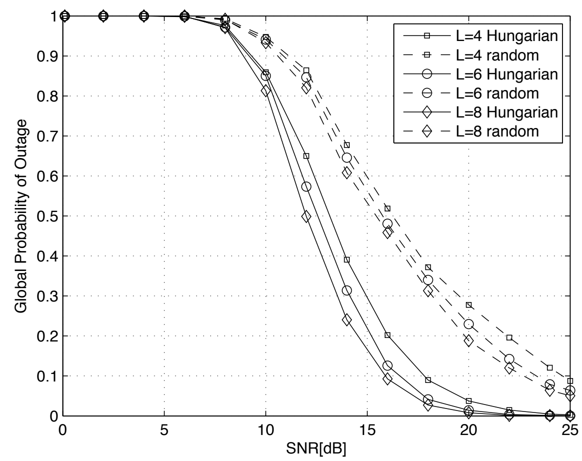

Fig. 2. Simulated global outage probability against SNR when $N_{\text{s}} = N_{\text{r}} = 4$ and $N_\phi = 16$, for $L = 4, 6, 8$.

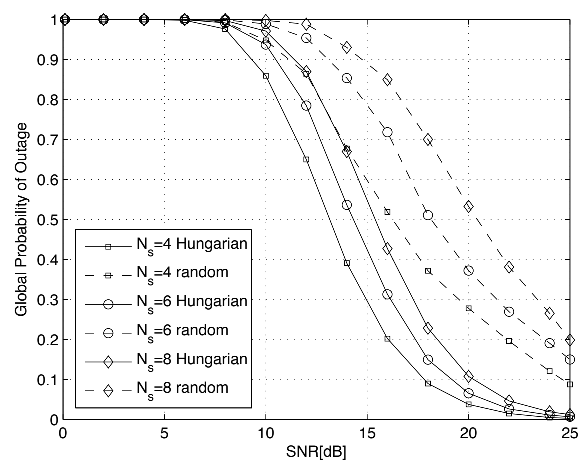

Fig. 3. Simulated global outage probability against SNR when $L = 4$ and $N_\phi = 16$, for $N_{\text{s}} = N_{\text{r}} = 4, 6, 8$.

In this section, all the demonstrated probabilities, whether analytical or simulated, are probabilities averaged over 1000 samples of $\sigma^2_{\text{s}_i\text{r}_j}$ and $\sigma^2_{\text{r}_j\text{s}_i}$ in (29). In case of analytical results, the global outage probability is obtained using (21) for each sample of $\sigma^2_{\text{s}_i\text{r}_j}$ and $\sigma^2_{\text{r}_j\text{s}_i}$, then the results are averaged. While regarding the simulated results, for each sample of $\sigma^2_{\text{s}_i\text{r}_j}$ and $\sigma^2_{\text{r}_j\text{s}_i}$, 1000000 samples of source-relay and relay-destination channels are generated according to (3) and (4), the number of outage events is counted, and the global outage probability is calculated. Then the outage probabilities associated with samples of $\sigma^2_{\text{s}_i\text{r}_j}$ and $\sigma^2_{\text{r}_j\text{s}_i}$ are averaged.

In Fig 2., the simulated global outage probabilities at different SNRs are shown for random relay allocation and our proposed relay allocation method. In this figure, keeping the number of source-destination pairs and relays at $N_{\text{s}} = N_{\text{r}} = 4$, the number of channel taps L is varied to study the effects of frequency diversity on the global outage probability. As it can be seen, the proposed relay allocation scheme outperforms random relay allocation significantly for all numbers of channel

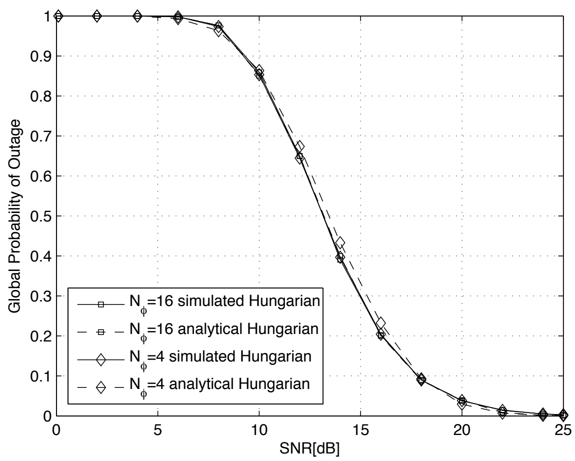

Fig. 4. Simulated and analytical global outage probabilities against SNR when $L = 4$ and $N_\mathrm{s} = N_\mathrm{r} = 4$ for $N_\phi = 4, 16$.

taps, which is due to the improved cooperative diversity in our method. Moreover, increasing the number of channel taps results in better performance in both cases in terms of outage probability, which is due to the increased frequency diversity.

The simulated global outage probabilities at different SNRs for constant $L = 4$ and different numbers of source-destination pairs are demonstrated in Fig 3. Likewise, the results for random relay allocation and our proposed relay allocation method are shown in this figure, where our relay allocation algorithm beats the random allocation scheme with a considerable margin. In addition, increasing the number of users worsen the outage performance. This is because outage event happens even if only one source-destination pair is in outage, which is more likely when more pairs are to be realized. We have observed in our simulations that the analytical outage probabilities agree precisely with simulated results for all the cases of random relay allocation and relay allocation based on Hungarian method, which demonstrates the precision of our outage analysis. However, the analytical results are not depicted in Fig. 2 and Fig. 3 to avoid impairing the readability of the figures. Analytical and simulated results can be compared in Fig. 4.

The effect of low number of subcarriers in OFDMA subsets is investigated in Fig. 4, where we keep both the number of sources and channel taps constant and vary N_ϕ. In this figure, the simulated and analytical outage probabilities with $N_\phi = 4$ and $N_\phi = 16$ are shown where the relays are allocated according to our proposed method. As it is mentioned in [8], approximating the capacity of a single s_i-r_j-d_i link by a Gaussian RV results in tight approximations of outage probability if the number of OFDM subcarriers is not too low. It can be seen in Fig. 4 that the simulated and analytical results match when $N_\phi = 16$, while the analytical results slightly deviate from simulated results when $N_\phi = 4$. However, although the analytical approximated outage probability is not exact for $N_\phi = 4$, we can see that the simulated performance of the system is identical to the case of $N_\phi = 16$. We can deduce here that low number of subcarriers does not provide good approximations for the outage probability, but it does not degrade the effectiveness of our relay allocation method, either.

VI. Conclusion

In this paper, we investigated a clustered two-hop DF network consisting of a set of non-LOS source-destination pairs, and a cluster of relays, where channels are Rayleigh frequency selective, and OFDMA is employed. Given some statistical parameters of channels, the event of outage is analyzed for the general case of correlated OFDM subcarrier gains, and arbitrary number of bits on each subcarrier. It is also shown that the outage probability on a single source-relay-destination link does not depend on the choice of OFDMA subcarrier subset, if all the subsets consist of equal number of contiguous OFDM subcarriers. The obtained global outage probability is then used to formulate a relay allocation optimization problem, for which a low complexity solution based on Hungarian method is proposed. In the end, the numerical results demonstrate the precision of our outage analysis, and effectiveness of the proposed relay allocation method.

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