Accurate Energy-Efficient Power Control for Uplink NOMA Systems under Delay Constraint

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Abstract—Machine-type communications (MTC) devices in 5G will use the Non-orthogonal multiple access (NOMA) technology for massive connections. These devices switch between the transmission mode and the sleep mode for battery saving; and their applications may have diverse quality of service (QoS) requirements. In this paper, we develop a new uplink energyefficient power control scheme for multiple MTC devices with the above mode transition capability and different QoS requirements. By using the effective bandwidth and the effective capacity models, the system's energy efficiency can be formulated as the ratio of the sum effective capacity to the sum energy consumption. Two new analytical models are used in system's energy efficiency maximization problem: 1) two-mode circuitry model and 2) accurate delay-outage approximation model. Simulation shows our proposed scheme is capable of providing exact delay QoS guarantees for NOMA systems.

Index Terms—MTC, NOMA, power control, delay constraint, QoS requirements

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) is a powerdomain multiplexing technology that allows users to transmit signals on the same time-frequency resources [1]. This technology supports scenarios with massive connectivity, therefore, it is a 5G candidate technology for uplink massive machinetype communications (MTC) [2]. MTC devices are usually power limited but have applications with diverse quality of service (QoS) requirements, e.g., public monitoring, real-time localization, industrial automation. Therefore, energy efficient transmission is a crucial requirement in most cases. Therefore, it is important to design an energy-efficient power control scheme in uplink NOMA systems with delay QoS constraint.

Extensive research has been done on power control for NOMA systems by taking link-layer QoS requirements into consideration. Yang et al. [3] and Cai et al. [4] proposed a power allocation scheme for NOMA systems under minimum rate constraints. When modeling a wireless communication system as a queueing system, the delay QoS requirements can be approximated by effective capacity model [5]. Yu et.al [6] used effective model to analyze the performance of a twouser downlink NOMA network. Furthermore, Choi [7], Liu et.al [8] and Chen et al. [9] used the effective capacity model and proposed a cross-layer power control policy to guarantee a certain delay QoS requirement in NOMA systems. However, in these work, the transmitter is always in transmission mode, which will over estimate transmit power consumption in their power control schemes. In 2016, Sinaie et al. [10] proposed a cross-layer power consumption model by considering a twomode circuitry (a circuitry works in the transmission mode or sleep mode). Xu et al. [11], [12] followed Sinaie's work and proposed a new energy efficiency analytical model by using a two-mode circuitry. The model is simple but only apply to the point-to-point wireless communication systems, which is not suitable for NOMA systems.

The aim of this paper is to design an energy-efficient power control scheme with delay QoS constraint in uplink NOMA systems. By continuing Xu's work, we develop Xu's energy efficiency analytical model in uplink NOMA systems and formulate an optimization problem to maximize the energy efficiency of NOMA systems under delay QoS requirements and peak power constraints. We further use the Dinkelbach method and develop a new power control scheme that can solve the above optimization problem.

The rest of this paper is organized as follows. Section II describes the uplink NOMA system model as well as the effective bandwidth and the effective capacity model. In section III, we analyze the effective capacity in NOMA systems. In section IV, we will formulate our power control problem under target delay-outage constraints. In section V, the problem will be solved by Dinkelbach algorithm. Performance of the proposed algorithm is evaluated in Section VI by simulations. Section VII summarizes our work.

II. SYSTEM MODEL

A. NOMA System Mode

As shown in Fig. 1, we consider an uplink NOMA system, in which a NOMA base station serves K MTC users or user equipments (UEs) (in this paper, we use the terms MTC users and UE interchangeably). A UE works either in a transmission mode if there is data to transmit or in a sleep mode otherwise. Transitions between two modes can be modeled as a state machine, which is illustrated in Fig. 2. All the UEs utilize the same time and frequency resources but different power domain



upper layers Fig. 2: Two-mode UE

to transmit data. The received signal at the base station is given by

$$y = \sum_{k=1}^{K} \sqrt{P_k^{tx}} h_k s_k + n, \tag{1}$$

where $h_k (k \in 1, \dots, K)$ is the Rayleigh fading coefficient with unit variance between the base station and user k, P_k^{tx} denotes the transmit power for user k, s_k denotes the transmit signal for user k, and n is the additive white Gaussian noise (AWGN) with mean μ and variance σ^2 .

The successive interference cancelation (SIC) technique is used at base station to eliminate multiuser interference. The SIC decoding order is based on the channel gain information. The highest channel gain user's signal is decoded at first since it is strongest at the base station. Then removing the strongest signal from the received signal to decode the second highest channel gain user's signal. That is, to decode the k^{th} user's signal first decodes the i^{th} (i < k) user's signal and removes this signal from received signal, in the order $i = 1, 2, \dots, k -$ 1; the signals from the i^{th} (i > k) users are treated as noise.

Let $|h_k|^2$ denote the channel gain of the k^{th} user, where $|\cdot|$ is the absolute value of a complex number. $|h_k|^2$ has an in-



Fig. 3: Buffer Model

dependent and identically distributed exponential distribution with unit mean. For simplicity, we assume that and the channel gains are sorted in a descending order, i.e., $|h_1|^2 \ge |h_2|^2 \ge$ $\dots \ge |h_K|^2$. Therefore, the signal-to-interference-plus-noise ratio (SINR) experienced when decoding k^{th} user's signal is

$$\gamma_k = \frac{S_k}{I_k + \sigma^2} = \frac{P_k^{tx} |h_k|^2}{\sum_{i=k+1}^K |h_i|^2 P_k^{tx} + \sigma^2},$$
 (2)

where S_k is the received k^{th} user's signal and $I_k = \sum_{i=k+1}^{K} |h_i|^2 P_i$ represents other users interference. Specially, the K^{th} user's SINR is

$$\gamma_K = \frac{P_K^{tx} |h_K|^2}{\sigma^2},\tag{3}$$

We assume that the instantaneous channel gain information is perfectly known at each UE. Based on the Shannon theory, the achievable rate of the k^{th} user C_k is given by

$$C_k = B \log_2 \left(1 + \gamma_k \right), \tag{4}$$

where B is the total bandwidth utilized by all the users.

B. The preliminaries of effective bandwidth and the effective capacity model

All the NOMA UEs have its own buffer with infinite buffer size. The buffer model is shown at Fig. 3. The k^{th} UE arrival data from the data source at slot n is $A_k[n]$ ($k = \{1, 2, \dots, K\}, n = \{1, 2, 3, \dots\}$). Furthermore, we follow the work [11] and assume that

- 1) the k^{th} UE source arrival data confirms to a Bernoulli process with a data arrival probability p_k $(p_k \in (0, 1])$
- 2) and the arrival $A_k[n]$ is exponentially distributed with a mean data length L.

Based on the above assumptions, the k^{th} UE arrivals $A_k[1], A_k[2], \cdots$ are identically distributed (IID) random variables (RVs) identical to a RV A_k ; the probability density function (PDF) of the arrival A_k is

$$f_{A_k}(a) = \begin{cases} p_k \frac{1}{L} \exp(-\frac{1}{L}a), & a > 0\\ 1 - p_k, & a = 0 \end{cases}$$
(5)

the average data arrival rate for UE k is

$$\mu_k = \frac{p_k L}{T_s}.$$
(6)

where T_s denotes the duration of a slot.

Due to the fact that the service rate in any time-varying wireless channels fluctuates, we assume that the k^{th} user services $S_k[1], S_k[2], \cdots$ are IID RVs identical to a RV S_k

$$S_k = T_s C_k = T_s B \log_2(1 + \gamma_k). \tag{7}$$

In order to guarantee a QoS requirement, the effective bandwidth is defined as the minimum constant service rate and the effective capacity is defined as maximum constant arrival rate specified by exponent u [5]. Based on the effective bandwidth model and the effective capacity model, the k^{th} UE's data arrival rate and service rate can be characterized by their own effective bandwidth $\alpha_k^{(b)}(u_k)$ and effective capacity $\alpha_k^{(c)}(u_k)$:

$$\alpha_k^{(b)}(u_k) = \frac{\log E(e^{u_k A_k})}{u_k T_s} = \frac{1}{u_k T_s} log \left(\frac{p_k}{1 - u_k L} + 1 - p_k\right).$$
(8)
$$\alpha_k^{(c)}(u_k) = -\frac{\log E(e^{-u_k S_k})}{u_k T_s} = -\frac{\log E(e^{-u_k T_s B \log_2(1 + \gamma_k)})}{u_k T_s}.$$
(9)

where u_k is k^{th} UE's QoS exponent. If the assumptions of the Gartner-Ellis theorem hold, there is a unique QoS exponent $u_k^* > 0$ that satisfies

$$\alpha_k^{(b)}(u_k^*) = \alpha_k^{(c)}(u_k^*), \tag{10}$$

then the complementary cumulative distribution function (CCDF) of backlog size can be approximated by [5]:

$$P(Q_k > B) \approx p_k^b e^{-u_k^* B},\tag{11}$$

where p_k^b is the nonempty buffer probability for k^{th} UE. The parameter u_k^* plays a critically important role for statistical QoS guarantees, which indicates the exponential decay rate of the QoS violation probability. A smaller u_k^* corresponds to a slower decay rate, which implies that the k^{th} user can only provide a looser QoS guarantee, while a larger u_k^* leads to a faster decay rate, which means that a more stringent QoS requirement can be supported. When the arrival A_k is exponentially distributed with a mean data length L, the value of the nonempty buffer probability can be calculated from Xu' s work [11] as

$$p_k^b = 1 - u_k^* L. (12)$$

Denote the k^{th} UE circuits power consumption and transmission probability by P_k^c and p_k^{tx} respectively. The k^{th} UE total power consumption P_k is

$$P_k = P_k^c + p_k^{tx} P_k^{tx}, aga{13}$$

The probability of a UE being in transmission mode is equivalent to the probability that traffic arrives from the upper layer or the buffer storage is non-empty. Denote the event of traffic arrival by A and the event of buffer is non-empty by B. Since two events are mutually independent, therefore, p_k^{tx} can be expressed as

$$p_k^{tx} = P(A) + P(B) - P(AB) = p_k + p_k^b - p_k^b p_k.$$
 (14)

III. EFFECTIVE CAPACITY ANALYSIS IN NOMA

A. Effective capacity analysis for two-user NOMA

We first consider a two-user NOMA system, where two users utilize the same time-frequency resources. This scenario has been extensively studied in many papers [7], [13], which is called multiuser superposition transmission or paired NOMA. In a two-user NOMA system, the corresponding distribution of SINR for the first user is given by:

Case 1: If second user is in sleep mode, the PDF of the SINR is

$$f_1(x) = \frac{\sigma^2}{P_1^{tx}} e^{-\frac{\sigma^2}{P_1^{tx}}x}.$$
(15)

Case 2: If second user is in transmission mode, the PDF of the SINR is [14]

$$f_2(x) = \left(\frac{\sigma^2}{P_1^{tx} + P_2^{tx}} + \frac{P_2^{tx}P_1^{tx}}{\left(P_1^{tx} + P_2^{tx}x\right)^2}\right)e^{-\frac{\sigma^2 x}{P_1^{tx}}}.$$
 (16)

Therefore, the effective capacity for the first user is given by

$$\begin{aligned} \alpha_1^{(c)}(u_1) &= -\frac{1}{u_1 T_s} \log \left[E(e^{-u_1 S_1}) \right] \\ &= -\frac{1}{u_1 T_s} \log \left[p_2^{tx} \int_0^{+\infty} e^{-u_1 T_s B \log_2(1+x)} f_2(x) dx \right. \\ &+ (1-p_2^{tx}) \int_0^{+\infty} e^{-u_1 T_s B \log_2(1+x)} f_1(x) dx \right]. \end{aligned}$$
(17)

Since the second user's signal is only interfered by the noise, therefore, the effective capacity for the second user is

$$\begin{aligned} \alpha_2^{(c)}(u_2) &= -\frac{1}{u_2 T_s} \log \left[E(e^{-u_2 S_2}) \right] \\ &= -\frac{1}{u_2 T_s} \log \left[e^{-u_2 T_s B \log_2(1+x)} \frac{\sigma^2}{P_2^{tx}} e^{-\frac{\sigma^2}{P_2^{tx}} x} dx \right]. \end{aligned}$$
(18)

B. Effective capacity analysis for K-user NOMA

The k^{th} user effective capacity in a NOMA systems depends on the distribution of γ_k , which is difficult to derive the close-form expression when the number of user is larger than three. Gu et al. [14] derive a simple method to reduce the computational complexity to calculate the effective capacity of k^{th} user in a full-interference scenario. But in NOMA systems, due to the SIC mechanism, only the first user's signal interfered by other K - 1 users' signal. The k^{th} user's signal only interfered with i^{th} (i > k) user's signal. Based on the above facts and integrated with Gu's work, $\alpha_k^{(c)}(u_k)$ can be calculated as

$$\alpha_{k}^{(c)}(u_{k}) = -\frac{1}{u_{k}T_{s}} \log \left\{ 1 - \int_{0}^{1} e^{-s} \prod_{i \in N, i > k} p_{i}^{tx} \frac{\sigma^{2}}{\sigma^{2} + sP_{i}} dt \right\}$$
(19)

where N denotes the set of transmission mode users and

$$s = \frac{\sigma^2 \left(2^{-\frac{1}{u_k B T_s} \ln t} - 1\right)}{P_i^{tx}}.$$
 (20)

IV. PROBLEM FORMULATION OF THE POWER CONTROL

In this work, we use bits per Joule to measure the system's energy efficiency, which is the ratio of the system's sum effective capacity to the total power consumption:

$$\eta = \frac{\sum_{k=1}^{K} \alpha_k^{(c)}(u_k)}{\sum_{k=1}^{K} P_k}.$$
(21)

Now let us consider the delay-QoS requirement. The total delay D_k experienced by user k consists of queueing delay D_k^q and transmission delay that equals T_s :

$$D_k = D_k^q + T_s. (22)$$

When the maximum delay bound D_{max} and a tolerance ε are specified by a typical MTC application, the system is in delay-outage if it cannot guarantee the following inequality:

$$P\left(D_k > D_{max}\right) \le \varepsilon. \tag{23}$$

where $P(\cdot)$ denotes the probability of a random event.

Let $\mathbf{P}^{tx} = [P_1^{tx}, P_2^{tx}, \cdots, P_K^{tx}]$ and $\mathbf{u}=[u_1, u_2, \cdots, u_K]$ denote the vector of transmit power and QoS exponent, respectively. The optimal power control problem can be formulated as **P1**:

$$\mathbf{P1}: \max \ \eta(\mathbf{P^{tx}}, \mathbf{u}), \tag{24}$$

s.t.
$$P(D_k > D_{max}) \le \varepsilon.$$
 (25)

$$P_k^{tx} \le P_{max} . \tag{26}$$

where (25) is the delay-outage constraint and (26) is peak power constraint for each UE.

V. POWER CONTROL STRATEGY

In this section, we will solve **P1** to obtain the optimal power control strategy for maximizing uplink NOMA energy efficiency.

According to [12], the CCDF of queueing delay D_k^q can be approximated by

$$P(D_k^q > t) = \left(\frac{1 - u_k^* L}{1 - u_k^* L + p_k u_k^* L}\right)^{\frac{t}{T_s} + 1}.$$
 (27)

By substituting t into (27) with $D_{max} - T_s$

$$P(D_k > D_{max}) = P(D_k^q > D_{max} - T_s)$$

= $\left(\frac{1 - u_k^* L}{1 - u_k^* L + p_k u_k^* L}\right)^{\frac{D_{max}}{T_s}}$. (28)

The constraint (25) can be re-written as

$$\left(\frac{1-u_k L}{1-u_k L+p_k u_k L}\right)^{\frac{D_max}{T_s}} \leq \varepsilon$$

$$\Leftrightarrow u_k \geq \frac{\beta-1}{(p_k+\beta-1)L}.$$
(29)

where \Leftrightarrow is the equivalent sign and $\beta = \varepsilon^{-\frac{T_s}{D_{max}}}$. Result (29) indicates that constraint (25) give a lower bound of QoS

exponent u_k . A large value of u_k indicates a stringent delay QoS requirement and thus requires more power consumption. Based on the above observation, we first have the following result:

Result 1: When the average data arrival rate is μ_k , the energy efficiency of the uplink NOMA system is a decreasing function of QoS exponent u_k .

For a proof of Result 1, see Appendix A. Based on Result 1, the optimal QoS exponent u_k is the boundary value

$$u_k = \frac{\beta - 1}{(p_k + \beta - 1)L}.$$
(30)

By substituting u_k^* in (30) into (24) in **P1**, the problem **P1** becomes **P2**:

$$\mathbf{P2}: \max \eta(\mathbf{P^{tx}}, \mathbf{u^*}), \tag{31}$$

$$P_k^{tx} \le P_{max}.\tag{32}$$

where $\mathbf{u}^* = [u_1^*, u_2^*, \cdots, u_K^*]$ is the optimal QoS exponent vector. The problem **P2** is to find such a $\mathbf{P^{tx}}$ that satisfy the constraint (32). We have the following result to formally characterize this problem:

Result 2: In an uplink NOMA system, for given QoS exponent the vector of QoS exponents of different UEs, the sum of effective capacity is concave of the transmission power P_k^{tx} .

For a proof of Result 2, see Appendix B. Based on result 2, the numerator of η in (21) is concave. Since the denominator of η is an affine function, the fractional function η is qusi-concave [15]. A fractional quasi-concave problem can be solved by Dinkelbach's algorithm as a sequence of parameterized concave problems [16]. Let q_i^* be the optimal value of original problem, q_i^* can be expressed as

$$q_i^* = \max_{\mathbf{P}^{\mathsf{tx}}} \left\{ \frac{\sum\limits_{k=1}^{K} \alpha^{(c)}(u_k^*)}{\sum\limits_{k=1}^{K} P_k} \right\}.$$
 (33)

Problem **P2** can be transformed to the following parametric concave problem:

$$F(q_i) = \max_{\mathbf{Ptx}} \left\{ \sum_{k=1}^{K} \alpha^{(c)}(u_k^*) - q_i \sum_{k=1}^{K} P_k \right\}.$$
 (34)

The maximal value of q_i^* in (34) is a root of the equation $F(q_i^*) = 0$. The value of q_i^* can be found by solving the parameterized problem in (34) according to the Dinkelbach method. For a given q_i^* in (34), we solve the problem as **P3**:

$$\mathbf{P3} : \max_{\mathbf{Ptx}} \left\{ \sum_{k=1}^{K} \alpha^{(c)}(u_k^*) - q_i^* \sum_{k=1}^{K} P_k \right\},$$
(35)

$$P_k^{tx} \le P_{max}.\tag{36}$$

The problem P3 is a concave optimization problem and constraint (36) satisfies Slater's condition [15], one that can



Fig. 4: Simulation and approximation results of delay violation probability for different UEs.

be efficiently solved by the Lagrange dual method. The Lagrangian function of problem **P3** can be written as

$$L(P_k^{tx}, \lambda_k) = \sum_{k=1}^{K} \alpha^{(c)}(u_k^*) - q_i^* \sum_{k=1}^{K} P_k + \sum_{k=1}^{K} \lambda_k (P_k^{tx} - P_{max}),$$
(37)

where λ_k is the nonnegative Lagrange multipliers. The equivalent dual problem can be decomposed into two parts: 1) the maximization problem solves the power control problem and 2) the minimization problem solves corresponding Lagrange multiplier, which is given by

$$\min_{\lambda_k \ge 0} \max_{\substack{P_k^{tx}}} L(P_k^{tx}, \lambda_k)$$
(38)

By using the Lagrange dual decomposition, the maximization problem can be solved by differentiating $L(P_k^{tx}, \lambda_k)$ with P_k^{tx} . We denote the optimal power control by P_k^{tx*} . The Karush–Kuhn–Tucker (KKT) conditions for **P3** are given by

$$P_k^{tx*} \le P_{max},\tag{39}$$

$$\lambda_k^* \ge 0, \tag{40}$$

$$\lambda_k^* (P_k^{tx*} - P_{max}) = 0, \qquad (41)$$

$$\frac{dL}{dP_k^{tx*}} = \frac{B}{\log 2} E\left(\frac{\gamma_k e^{-u_k^* R_k}}{P_k^{tx*}(1+\gamma_k)E(e^{-u_k^* R_k})}\right) - q_i^* p_k + \lambda_k^* = 0$$
(42)

where $(\cdot)^*$ represents the value of corresponding variable at the optimal point. Equation (39) and (40) are feasibility conditions, (41) is the complementary slackness condition, and (42) is the stationary condition. The optimal power P_k^{tx*} can be solved as

$$P_k^{tx*} = \left[\frac{B\gamma_k}{\log 2(1+\gamma_k)(q_i^*p_k - \lambda_k^*)}\right]^+,\tag{43}$$

where $[x]^+$ denotes $max\{x, 0\}$ and λ_k^* is the optimal Lagrangian multiplier, which need to ensure the system to meet the peak power constraint of each UE.



Fig. 5: Optimal energy efficiency under different target delay bound

As for minimization problem, the multiplier λ_k^* can be updated by the subgradient method [15] as follows:

$$\lambda_k(j+1) = [\lambda_k(j) + \beta_k(j)(P_{max} - P_k^{tx*})]^+, \quad (44)$$

where j is the iteration index, $\beta_k(j)$ is the positive step sizes for the j^{th} iteration (e.g., $\frac{1}{\sqrt{j}}$). When the step sizes are chosen properly, the convergence to the optimal solution is guaranteed.

Parameters	Values
Time duration of a slot, T_s	1ms
Noise spectral density, N_0	-174dBm/Hz
System Bandwidth, B_c	18kHz
Circuit power, P_{tr}	10dBm
Maximum transmit power, P_c	46dBm
Predefined threshold, ε	0.1

TABLE I: System Parameters

VI. RESULT AND DISCUSSION

In this section, we evaluate the performance of the proposed algorithm in Rayleigh fading environment [17]. The channel gain $|h_k|^2$ is generated by exponential distribution with parameter $\chi_k = d_k^\beta$ [18], where d_k is the distance between the k^{th} user and the base station, β denotes the path loss exponent and we set $\beta = 4$. Three different QoS requirements UEs are in a NOMA cell. The distance between base station and three users are 300, 600 and 900 meters, respectively. The other parameters are listed in Table I.

Fig. 4 shows the simulated and approximated of average delay violation probability for different UEs when the all the UEs average date rates are 600 Kbps. The delay QoS requirements for three UEs are set as (10ms, 0.1), (20ms, 0.1) and (30ms, 0.1) respectively. As shown in Fig. 4, the approximation results are very close to the simulation results. This verifies the correctness of delay-outage probability and indicates that our power control scheme is capable of providing precise delay-outage probability guarantees.

Fig. 5 shows a comparison of energy efficiency between the two-mode circuitry and single-mode circuitry. The target delay violation probability ε is 0.1. The figure shows the energy efficiency improvement when the two-mode circuitry is used in NOMA systems. The reason is that the single-mode circuitry overestimates transmit power, thus it is apparently less energy efficient.

VII. CONCLUSION

It is important to design an energy efficient power control scheme in uplink NOMA systems under predefined delayoutage constraints for MTC. Previous work on power control in NOMA systems only consider single-mode UEs, which will over estimate transmit power consumption. In this paper, we consider two-mode UEs in uplink NOMA systems, and propose an energy-efficient power control scheme under predefined delay-outage constraints. Simulation results confirm our power control scheme is capable of providing precise delayoutage probability guarantees in uplink NOMA systems.

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APPENDIX A

Since effective capacity $\alpha_k^{(c)}(u_k)$ is a decreasing function of u_k [5], therefore, the sum of effective capacity $\sum_{k=1}^{K} \alpha_k^{(c)}(u_k)$ is also a decreasing function of u_k . When the QoS exponent u_k increases, the transmission power P_k^{tx} increases accordingly to meet a stringent QoS requirement. Thus the total power consumption P_k is an increasing function of the QoS exponent u_k . Because the numerator is a decreasing function and the denominator is an increasing function, therefore the system's energy efficiency η is a decreasing function of the QoS exponent u_k .

APPENDIX B

Let $h(x) = -\frac{1}{u_k T_s} log E\left(e^{-u_k T_s B log_2(1+c_k x)}\right)(x>0)$, where $c_k = \frac{|h_k|^2}{I_k + \sigma^2}$. Since $log_2(1+c_k x)$ is concave for all x > 0, so $u_k T_s B log_2(1+c_k x)$ is concave in the domain set. This implies that $e^{-u_k T_s B log_2(1+c_k x)}$ is log-convex, and $E\left(e^{-u_k T_s B log_2(1+c_k x)}\right)$ is log-convex as well since logconvexity is preserved under sums [15]. Noting that $\log\left(g\left(\cdot\right)\right)$ is convex for log-convex $g\left(\cdot\right)$. Therefore, $h\left(x\right)$ is a concave function of x for x > 0. Meanwhile, the sum effective capacity can be written as $\sum_{k=1}^{K} h\left(P_k^{tx}\right)$. Since concavity is preserved under sums, thus $\sum_{k=1}^{K} h\left(P_k^{tx}\right)$ is a concave function of transmission power P_k^{tx} , completing the proof.

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