**Formal Ontology Generation by Deep Machine Learning**

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***Abstract*** — An ontology is a taxonomic hierarchy of lexical terms and their syntactic and semantic relations for representing a framework of structured knowledge. Ontology used to be problem-specific and manually built due to its extreme complexity. Based on the latest advances in cognitive knowledge learning and formal semantic analyses, an Algorithm of Formal Ontology Generation (AFOG) is developed. The methodology of AFOG enables autonomous generation of quantitative ontologies in knowledge engineering and semantic comprehension via deep machine learning. A set of experiments demonstrates applications of AFOG in cognitive computing, semantic computing, machine learning and computational intelligence.

***Keywords*** **—** *Ontology, formal models, autonomic generation, concept algebra, machine learning, knowledge representation, cognitive robot, denotational semantics, cognitive computing, AI, computational intelligence*

# I. INTRODUCTION

Recent basic studies in formal semantic theories and mathematical rules in concept and semantic algebra have led to novel technologies in cognitive machine learning from human knowledge in a rigorous and quantitative approach. Ontology was a branch of metaphysics dealing with the nature of being in philosophy [1, 5, 9, 10, 12, 14, 19, 25, 28, 35, 42, 43, 44]. Studies on ontology are recently extended into the emerging fields of knowledge science [28] and cognitive linguistics [24, 36, 37]. In the contemporary disciplines, ontology is a taxonomic hierarchy of knowledge that uses lexical terms and their syntactic and semantic relations to represent a framework of structured knowledge. Applied ontology used to be manually built for dealing with small-scale knowledge, which were often tedious, complicated, subjective, redundant, nonquantitative, non-rigorous, inconsistent, and incomplete in practice.

Typical ontological structures developed in knowledge engineering are represented by WordNet, Dublin Core, and GOLD. WordNet is a lexical knowledgebase [8, 9, 14]. Dublin Core is an ontology for documents publishing standardization in ISO 15836 [6]. GOLD is a general ontology for linguistic description [4]. Traditional ontological technologies may only represent a set of static knowledge and are highly application-specific and subjective. Towards a general-purpose linguistic knowledge base, WordNet provides an unweighted or subjective lexical network. Each node in the network stands for a specific ‘*sense*’ and is expressed by a lexical structure called ‘*synset*’ that consists of multiple synonyms. In addition to WordNet, ConceptNet [7] is introduced as commonsense knowledge base developed at MIT. ConceptNet extends WordNet’s ‘*sense*’ from individual words to complex concepts [2, 3, 10]. However, ConceptNet merely describes various relations among informal and subjective concepts in a planar structure without distinguishing the fundamental difference between the essential intensions and extensions of concepts.

In order to rigorously improve the structure and methodology of ontology theories, a mathematical model of *formal concept* [26] is created that denotes any concept in human knowledge as a triple of sets of attributes, objects, and relations. Based on the mathematical model of concepts, a formal methodology for manipulating knowledge is enabled known as *concept algebra* [16, 26, 38], which provides a rigorous methodology for formal knowledge manipulations by a set of algebraic operators on abstract concepts. Concept algebra enables the quantification of concepts by semantic weights [26] and the measurement of knowledge by the basic unit of *binary relation* (*bir*) [28] towards knowledge science.

This paper presents a fundamental theory of formal ontology for knowledge representation in cognitive computing and cognitive knowledge learning. In the remainder of this paper, the mathematical model of ontology is created in Section II. A rigorous methodology for formal ontology generation is developed in Section III embodied by the algorithm of formal ontology generation. A set of experimental results are reported in Section IV based on case studies and quantitative semantic analyses.

II. MATHEMATICAL MODELS OF FORMAL ONTOLOGY

In cognitive computing and cognitive linguistics, ontology is both a method for modeling a domain of knowledge and a set of collective concepts for representing a structured knowledge in knowledge engineering [1, 4, 26, 28, 32, 39]. The mathematical model of ontology is underpinned by formal semantics theories [24] and denotational mathematics [18, 21, 23, 27] such as concept algebra [26], semantic algebra [24], inference algebra [22], big data algebra [29], fuzzy logical algebra [30], and probability algebra [31].

**Definition 1.** An *ontology* is a taxonomic hierarchy of knowledge, which uses lexical terms and their syntactic and semantic relations to represent a framework of structured knowledge.

An ontology on a specific domain of knowledge can be quantitatively determined by the relations of formal concepts according to concept algebra. The semantic relations [11, 20, 24, 35, 36] among formal concepts can be classified in the categories of synonym, partial synonym, hypernym/hyponym, and holonym/meronym as illustrated in Figure 1.



Fig. 1. The general semantic pattern of ontology

In formal ontology, knowledge science and cognitive linguistics, the universe of discourse of knowledge can be formally embodied by a semantic space *Θ* as a hierarchical concept network [28, 40]. In the semantic space of knowledge, a formal ontology is a denotational mathematical structure of semantical relations between formal concepts according concept algebra.

**Definition 2.** The *mathematical model of an ontology*, Ω|SM, in Θ|SM is a hyperstructure of semantic relations of knowledge between a set of formal concepts *C* embodied by a pair of square matrixes known as those of *the semantic weights W* and *semantic levels* *L*, i.e.:

 (2)

where each concept *C* is formally specified in the semantic space of ontology Θ|SM.

The properties of ontology and semantical relations of formal concepts in a Cognitive Knowledge Base (CKB) are hierarchical, relational, quantitative, weighted and nonnegative. A set of calibrated formal concepts in a sample semantic space *Θ*1|SM is given in Table 1, which is generated by a machine knowledge learning system [38, 39, 41, 45] powered by concept algebra.

# III. THE METHODOLOGY AND ALGORITHM FOR FORMAL ONTOLOGY GENERATION

On the basis of the mathematical models of formal ontology as elaborated in preceding section, a methodology for formal ontology generation is developed. It enables an autonomous algorithm of formal ontology generation implemented by deep machine knowledge learning technologies.

# 3.1 Determining Weights of Concept Relations in

# an Ontology

The semantics of a formal concept is determined by its intension characterized by a set of attributes according to concept algebra [26]. Therefore, the equivalency between a pair of formal concepts is determined by the similarity of their intensional attributes.

**Theorem 1.** The *equivalency* of a pair of formal concepts, *Ci* ~ *Cj*, is determined by the intensions of the concepts *Ci*.*Ai* and *Ci*.*Aj* and constrained in a normalized unit interval = [0, 1], i.e.:

** (3)

***Proof.*** According to concept algebra, the semantic relation between a pair of formal concepts can be verified in three forms known as equivalent (=), independent (), or related () due to partial equivalency, i.e.:

 (4)

**Definition 3.** The*matrix of semantic weights*, *W*, is a set of weighted semantic relation among a set of *n* formal concepts, which are quantitatively determined by Theorem 1, i.e.:

 (5)

## Case Study 1. A set of 20 formal concepts, *G*1 = {*pen, pencil, stationery, computer, printer, bank, bank(river), bank (storage), animal, mammal, feline, cat, lion, tiger, leopard, knowledge, information, cognition, intelligence, science*}, as generated by machine learning in Table 1 are elicited from CKB [25, 41, 45]. The 20 x 20 matrix of semantic weights *W*1 is determined according to Theorem 1 as shown in Eq. 6. It is obvious in *W*1 that the reflexive equivalency between identical concepts *wii* is always 1.0 on the diagonal cells. A nil similarity in the semantic matrix indicates an irrelevant semantic relation between two independent concepts. For instance, *C*1(*pen*) and *C*9(*animal*) are independent, i.e., *w19* = 0, because their sets of attributes have no intersection according to Table 1. A threshold may be introduced to eliminate low similarity pairs of formal concepts in the matrix in order to reduce the complexity of semantic analyses.

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Fig. 2 The contour plot of weighted semantic relations among

20 formal concepts in *W*1

The experimental data obtained in *W*1 can be illustrated by a contour plot [13] as shown in Figure 2. A contour vector (*vc1*) is adopted to select certain semantic weights to be plotted, e.g., *vc*1= [0.05, 0.1, 0.2, 0.3, …, 1.0]. Figure 2 demonstrates that the top similarities of formal concepts are those of the reflexive concepts along the diagonal where all Ci~Cj ≡ 1.0. It is noteworthy that the equivalency plot is symmetric because the reflexivity of concept equivalence, i.e., Ci~Cj ≡ Cj~Ci. There are reginal coherencies with respect to the semantic relations of sample concepts in each of the three clusters such as those of realistic entities, animals and abstract artifacts.

**Case Study 2.** Let *G*2 be a group of 23 arbitrary concepts, i.e., *G*2 = {*office, drawing, text, pen, writing, nib, paper, tool, printing, expression, material, stationery, pencil, printer, device, instrument, data, word, ink, system, emotion, showing, computer*}. The semantic weight matrix *W*2 is obtained according to Theorem 1 based on the CKB as illustrated in Figure 3 where the contour vector *vc2*= [0.05, 0.1, 0.15, 0.2, 0.3, 0.5].



Fig. 3 The contour plot of weighted semantic relations

among 23 formal concepts in *W*2

The machine learning results on concept equivalency provide a rigorous and quantitative semantic analysis of arbitrary set of concepts represented by individual words in natural language based on the machine acquired CKB. The topology of the semantic space demonstrates the characteristics of semantic weight coherency and symmetry in formal ontology. It reveals the cluster property of formal concepts in a hierarchical ontology denoted by the circle areas in the relational semantic space *Θ*|SM.

# 3.2 The Algorithm for Autonomous Ontology Generation by Machine Learning

The methodology for formal ontology generation is implemented by an unsupervised machine learning algorithm driven by cognitive robots supported by CKB [13, 25, 41, 45]. The algorithm autonomously generates a formal and quantitative ontology on an arbitrary set of target concepts via cognitive machine learning.

 (6)



Fig. 4 The Algorithm of Formal Ontology Generation (AFOG) in RTPA

**Algorithm 1.** The *algorithm of formal ontology generation* (AFOG) autonomously builds a formal and quantitative ontology Ω|SM by machine learning as formally described in Figure 4 adopting the formal notations of Real-Time Process Algebra (RTPA) [15, 17]. The input of the AFOG algorithm is a set of formal concepts C|Ξ learnt by machine, which may be either related or unrelated. The outputs of the AFOG algorithm are a relational hierarchical of formal ontology Ω|SM where the semantical relations of all concepts in the input are quantitatively analyzed. The automatically generated ontology is visually plotted as a hierarchical and weighted semantic network.

AFOG|PM first initializes the expected ontologyparticularly its sets of semantic weights and expected semantic levels based on Definition 3. It then determines the semantic equivalency  between each pair of given concepts according to Theorem 1. As a result, each formal concept is semantically classified and allocated in a proper position in the hierarchical semantic space of the formal ontologyquantitatively generated by machine learning. On the basis of , specific semantical relations and relative semantic levels are determined among potential syntactic relations between a pair of formal concepts such as those of synonyms , partial synonyms (), hypernyms (), hyponyms (), and irrelevant concepts (). The AFOG algorithm is supported by the CKB embodied by Θ|SM, which is established by quantitative machine learning and semantic analyses [40, 44].

The AFOG algorithm is underpinned by a set of concept, semantic and hierarchical rules according to concept algebra [24], semantic algebra [26] and the theory of formal ontology [33]. AFOG is supported by the hierarchical rules of algebraic semantic manipulations [33] and a visualization tool [13].

# IV. EXPERIMENTAL RESULTS OF

# FORMAL ONTOLOGY GENERATION BY

# MACHINE LEARNING

Machine learning for semantic relations among arbitrary concepts in the semantic hierarchy Θ|SM is carried out by the cognitive algorithm of AFOG implemented in MATLAB. The AFOG algorithm is fully autonomous and unsupervised for machine learning in order to build a relational hierarchy of formal concepts supported by a CKB acquired by machines via cognitive knowledge learning [13, 41, 45]. The formal ontology generator, AFOG|PM, visually represents an ontology as a hierarchical semantic network where the node represents a term (or the identifier of a formal concept *C*|SM.*id*|S) and the edge denotes a weighted semantic associations between a pair of nodes. Applying the AFOG algorithm, any ontology can be autonomously generated and quantitatively visualized.

The first experiment builds an ontology Ω1|SM for concept group *G*1 as given in Case Study 1, which includes 20 concepts in the clusters of realistic entities, abstract artifacts and animals. The cognitive learning algorithm fetches formal concepts corresponding to the given set of words from the CKB as shown in Table 1. The AFOG algorithm rigorously analyzes semantical relations and determines their hierarchical levels among the target concepts. The autonomously generated formal ontology, Ω1|SM, on *G*1 by the AFOG algorithm is plotted in Figure 5.

The AFOG algorithm autonomously classifies arbitrary informal words into three coherent semantic clusters with rigorous measurements of relational weights, semantic relations and hierarchical allocations according to the corresponding formal concepts generated by machine learning. The ontology of structured knowledge indicates semantic connections among formal concepts in terms of synonyms and partial synonyms at the same semantic level, as well as hypernyms/holonyms and hyponyms/meronyms at a higher/lower level with respect to a target concept. In Figure 5, the blue links across semantic levels denote a hypernym or hyponym between a pair of formal concepts. The red links indicate a set of 1-to-*n* holonym/meronyms relations from the top down where a meronym is an object of the holonym. The green links represent a partial synonym quantified by a corresponding semantic weight.

The second experiment deals with concept group *G*2 as given in Case Study 2 for generating the ontology Ω2|SM. The analysis and learning results autonomously yield Ω2|SM as plotted in Figure 6 by AFOG. As shown in the experiment, an arbitrary set of words has successfully classified into a coherent ontological framework among the semantic clusters of writing, instrument, ink, expression and print.

The third experiment builds an overall semantic ontology in the sample space of 663 frequently used concepts in English. The formal ontology generated by the AFOG algorithm is illustrated in Figure 7. Each formal concept in the formal ontology is a hyperstructure according to concept algebra. Clicking on any node of a formal concept will highlight its direct semantic connections to other formal concepts on roles of intension (attributes), extension (objects), synonyms (SN), partial synonyms (PS), hypernyms (HE)/hyponyms (HO) and/or holonyms (HL)/meronyms (MR).

The formal ontologies generated by AFOG can be flexibly visualized controlling by a threshold (*θ*) of semantical weights where . The level of *θ* determines the inclusion of a certain semantic relation in semantic analysis and ontology generation. The lower the threshold, the more the potential relations being displayed, and the higher the analytic complex. Therefore, a higher *θ* is suitable for generating a precise and neat ontology. However, a lower *θ* is useful for generating a detailed ontology by revealing potential semantic relations implied in the semantic space.



Fig. 5 A machine generated ontology Ω1 on *G*1 by AFOG ()



Fig. 6 A machine generated ontology Ω2 on *G*2 by AFOG ()

As demonstrated in the experiments, the AFOG algorithm powered by deep cognitive knowledge learning methodologies enables autonomous ontology generation in knowledge engineering and semantic comprehension. AFOG enables machines to create their own knowledge bases by formal concept generation and quantitative semantic clustering among arbitrary concepts in the hierarchical semantic space beyond human perception on complex and quantitative semantic relations. AFOG demonstrates a deep machine understanding on semantic relations and their quantitative evaluation beyond human empirical perspectives on knowledge and natural language expressions.

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Fig. 7 The overall semantic hierarchy of formal ontology Ω3 with 663 formal concepts generated by AFOG (*θ*3= 0.7)

# V. CONCLUSION

This work has presented a theory and methodology for formal and quantitative ontology generation. The algorithm of formal ontology generation (AFOG) has been developed based on deep cognitive knowledge learning theories powered by concept algebra. Experimental results produced by the AFOG algorithm have demonstrated an autonomous, accurate, rigorous and quantitative methodology for building formal ontology, which outperforms human subjective counterparts. AFOG has enabled a novel approach to autonomous ontology generation in knowledge engineering and semantic comprehension. AFOG has paved a novel approach to free human from complex manual ontology building in knowledge engineering. The breakthrough in machine knowledge learning and semantic comprehension has led to a wide range of applications in cognitive machine learning, computational linguistics, cognitive knowledge bases and hybrid man-machine intelligence.

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Table 1. Formal Concepts in Ontology Ω1 Generated by Machine Learning



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