On the Performance of Redundant Residue Number System Codes Assisted STBC Design

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***Abstract*—In this paper, we propose a novel application of Redundant Residue Number System (RRNS) codes to Space-Time Block Codes (STBCs) design. Based on the so-called "Direct-Mapping" scheme, the link between residues and complex signal constellations is optimized. We derive upper bounds on the codeword error probability of RRNS-STBC and characterize its achievable diversity gain assuming maximum likelihood decoding (MLD). The knowledge of apriori probabilities of residue generation is utilized to implement a probability based Distance-Aware Direct Mapping scheme for M-ary modulation which further improves the error performance of the RRNS-STBC coding scheme.**

***Index Terms*—Redundant residue number system, MIMO, Space-time block code, Distance-Aware Direct Mapping.**

I. Introduction

Redundant residue number system (RRNS) is a non-weighted, carry-free number system that is well known for its robust self-checking, error detection and error correction properties [1]-[3]. Prior efforts in application of RRNS as a channel coding scheme have been limited due to the complexity of decoding. But with recent advance in computing, RRNS can be used as a channel code to improve error detection and correction. The applications of RRNS as a channel coding scheme and its similarities and advantages over other non-binary codes like Reed-Solomon (RS) codes has been discussed in detail in [1]-[6]. The benefits of incorporating RRNS in MIMO (multiple-input multiple-output) have not been investigated. The main contribution of this paper is understanding and quantifying the role of RRNS in MIMO systems.

In this paper, non-systematic RRNS codes [3],[7] along with a "direct mapping" scheme are formulated as space-time block codes (STBCs) to support high performance and high-rate transmission. In contrast to orthogonal space-time block codes (OSTBCs) [8], RRNS-STBC does not require the channel to remain constant during its coding/decoding period. In the proposed transmission scheme, information bits are first mapped to integers and based upon a predetermined moduli set, the corresponding residues are calculated. Then, the obtained residues are mapped to complex constellation symbols using the "direct mapping" scheme. While mapping bits to residues, not all residues occur with equal probability. Since the direct mapping scheme maps residues directly to constellation points, the unequal apriori probabilities of residues can be effectively exploited. We propose an unique prior probability based distance-aware direct mapping (DA) scheme. In this scheme residues are mapped to complex symbol constellations in a distance aware manner based on their apriori probabilities. Finally, the mapped symbols are constructed as space-time block codes and transmitted over multiple antennas. At the receiver end, inverse operations including the use of Chinese Remainder Theorem (CRT) to convert residues back to integers are implemented to recover the original information. We derive upper bounds on the codeword error probability of RRNS-STBC assuming ML detection and M-*ary* QAM constellation. The achievable coding gain and diversity gain are clearly revealed from this analysis. Using simulations, we compare the performance of RRNS-STBC and OSTBC i.e. Alamouti's scheme for a 2×2 MIMO system and show the improvement given by RRNS-STBC. With distance aware mapping, a further improvement in SNR (signal to noise ratio) of about 2dB for spatially multiplexed case and about 1dB for full diversity transmission case is obtained. It is important to note that this gain comes at the cost of increased receiver complexity as compared to the Alamouti's scheme.

This paper is organized as follows. Section II describes the system model, with emphasis on RNS arithmetic, the RRNS coding scheme and the direct mapping scheme. Then, we evaluate the performance of the RRNS-STBC coding scheme in section III and finally, in section IV, we compare the performance of RRNS-STBC with OSTBC using simulation results.

II. System Model

The block diagram of RRNS-STBC coded MIMO system is given in Fig.1. The system is assumed to have M_T transmit antennas and M_R receive antennas. Binary inputs are first converted to integer residues following the RRNS arithmetic (explained in subsection A). The generated residues are directly mapped to M-*ary* complex constellation points forming

Fig. 1. The block diagram of RRNS-STBC coded MIMO systems

the so-called "direct-mapping" scheme, where each residue corresponds to one complex symbol. In the STBC mapping block in Fig.1, the mapped complex symbols are grouped into $M_T \times T$ code blocks, where T is the length of each block. The receiver decodes the transmitted symbols using ML decoding and consequently performs demapping and inverse RNS transform using the Chinese Remainder Theorem to obtain the binary output. The next section discusses RNS and RRNS arithmetic and the coding scheme.

A. RNS AND RRNS ARITHMETIC

RNS is defined by the choice of v number of positive integers m_i ($i = 1, 2, \cdots, v$), referred to as moduli [9]. If all the moduli are pairwise relative primes to each other, any integer N which falls in the range of [0, M_r) can be uniquely and unambiguously denoted by the residue sequence $\{r_1, r_2, \cdots, r_v\}$, where $M_r = \prod_{i=1}^{v} m_i$ and $r_i = N \text{mod}\{m_i\}$ for $i = 1, 2, \cdots, v$. To recover the integer information N, Chinese Remainder Theorem (CRT) [9] is generally used.

In order to incorporate error control, RNS has to be designed with certain number of redundant moduli. An RRNS (redundant RNS) code is obtained by appending $(u - v)$ number of moduli $\{m_{v+1}, m_{v+2}, \cdots, m_u\}$ to the RNS in order to form an RRNS code of u positive, pairwise relative prime moduli, where $\min\{m_{v+1}, m_{v+2}, \cdots, m_u\} \geq \max\{m_1, m_2, \cdots, m_v\}$ must hold. Now, the integer N is represented by a residue sequence $\{r_1, r_2, \cdots, r_u\}$ based on u number of moduli, forming a so-called RRNS(u,v) code.

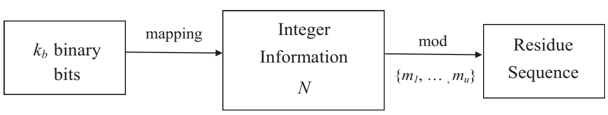

Fig. 2. Encoding procedure

Fig.2 shows the block diagram of the encoding procedure i.e. binary to RRNS conversion. In the encoder, k_b bits are mapped to one integer N, where $k_b = \lfloor \log_2 M_r \rfloor$ and $\lfloor x \rfloor$ denotes the largest integer smaller than x. Then, corresponding residues are obtained by taking the modulus of N based upon the chosen set of moduli.

B. DIRECT MAPPING SCHEME

In the M-*ary* direct mapping block shown in Fig.1, calculated residues are directly mapped to constellation points and then grouped into $M_T \times T$ code blocks, where T is the length of each block. We notice that the available residue digits are in the range of $[0, \max\{m_1, \cdots, m_u\} - 1]$. Therefore, we propose to map residue digits directly to M-*ary* (PSK/QAM) complex constellation points forming the so-called "direct-mapping" scheme. Here, $M \geq \max\{m_1, \cdots, m_u\} - 1$ must hold. Consequently, complex symbols corresponding to each of the residue digits are transmitted via multiple antennas.

TABLE I
DIRECT RESIDUES TO COMPLEX SYMBOLS MAPPING USING MODULUS 4

Residues	Bit Assignment	QPSK Symbols
0	00	1
1	01	j
2	10	$-j$
3	11	-1

One illustrative example is given in Table I to characterize the "direct mapping" scheme in the case of $\max\{m_1, \cdots, m_u\} - 1 = 3$ and gray coded mapping is considered. The residues are mapped directly to the constellation points such that the $log_2 M$-bit binary value of the residue is equal to binary gray coded bit assignment of the constellation point.

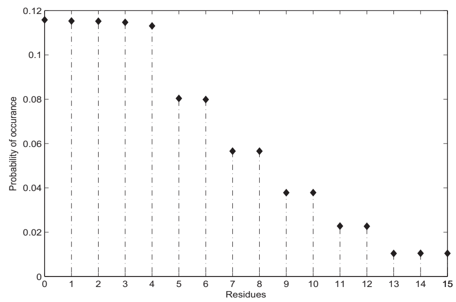

Fig. 3. Probability mass function for $m_o = \{5, 7, 9\}$ and $m_r = \{11, 13, 16\}$.

In case of RRNS(u,v) code, when a random bit sequence is given as input, the probability of occurrence of each of the u residues corresponding to each decimal number of k_b bits is different for every set of u moduli. These probabilities of occurrence can be calculated apriori for a predetermined set of v primary moduli and $u - v$ redundant moduli. For example, consider a moduli set where $m_o = \{5, 7, 9\}$ and $m_r = \{11, 13, 16\}$. The pmf (probability mass function) of the residues is shown in Fig.3. It can be seen from Fig 3. that the residues can be ranked according to probability in the order $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 9, 11, 12, 14, 15, 13\}$. Here we see that $\max\{r\} = \max\{m_1, \cdots, m_u\} - 1 = 15$. In this case we can use a 16-QAM constellation for mapping the residues.

We exploit the apriori probability of occurrence of the residues by proposing a unique probability based distance-aware (DA) direct mapping scheme. A set-partitioning approach is used to decompose constellation points into sets with maximum separation for the highest ranked residues according to their probability of occurrence.

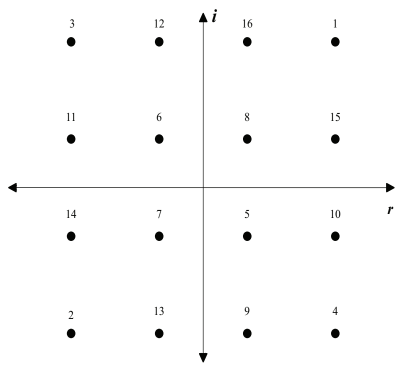

Fig. 4. Probability Based Distance-Aware Mapping Scheme for 16-QAM

Fig.4 illustrates the mapping scheme for 16-QAM. The set of four most probable residues are mapped to the four corners of the constellation. The next set of four are mapped diagonally across from them such that the least probable in this set is nearest to the most probable in the first set thus maintaining maximum distance possible among the most probable residues. The next two sets are mapped to points such that the convention is followed and the least probable surround the most probable residues from the first set. The scheme is general for any M-*ary* QAM constellation. The numbers correspond to the probability of occurrence *i.e.* the rank of the particular residue. Number 1 denotes the most probable (0, in this example) and 16 the least probable (13, in this example) residue/symbol. For an 8-PSK constellation, the first set of four residues are mapped to points on the four axes and the next four are again mapped to the remaining constellation points such that the two least probable ones surround the most probable residue. For QPSK and BPSK the same mapping scheme is applicable and the extension is trivial.

III. Performance Analysis of RRNS-STBC Design

In this section, we analyze the performance of RRNS based STBC coded MIMO system. The RRNS-STBC is a concatenated channel coding scheme [10], with M-*ary* modulation and STBC together acting as the inner code and RRNS being the outer code. We denote the transmitted $M_T \times T$ codeword as $\mathbf{X}$. The received $M_R \times T$ block code $\mathbf{Y}$ is then given by

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}, \tag{1}$$

where $\mathbf{H}$ is the $M_R \times M_T$ channel matrix with each entry distributed according to $\mathcal{CN}(0, 1)$, $\mathbf{N}$ is a $M_R \times T$ complex Gaussian noise matrix with each entry distributed according to $\mathcal{CN}(0, \frac{1}{2}N_0)$.

At the receiver, assuming MLD, the output codeword $\hat{\mathbf{X}}$ corresponds to

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in \mathcal{R}^{M_T \times T}} \|\mathbf{Y} - \mathbf{H}\mathbf{X}\|^2, \tag{2}$$

where the minimization is taken over all possible input STBCs $\mathcal{R}^{M_T \times T}$. For the case of DA mapping, the M symbols are not equiprobable. So, the output codeword $\hat{\mathbf{X}}$ corresponds to

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in \mathcal{R}^{M_T \times T}} \|\mathbf{Y} - \mathbf{H}\mathbf{X}\|^2 - 2\sigma^2 \ln P(\mathbf{X}), \tag{3}$$

where $\sigma^2 = \frac{1}{2}N_0$ is the variance of the channel noise and $P(\mathbf{X})$ is the apriori probability of generation of codeword $\mathbf{X}$.

Assuming M-QAM constellation, the probability that the transmitted codeword $\mathbf{X}_i$ is mistaken for another codeword $\mathbf{X}_j$ (also known as the pairwise error probability (PEP)) is

$$P(\mathbf{X}_i, \mathbf{X}_j | \mathbf{H}) = Q\left(\sqrt{\frac{\rho \|\mathbf{H}\mathbf{E}_{i,j}\|_F^2}{2M_T}}\right), \tag{4}$$

where $\mathbf{E}_{i,j} = \mathbf{X}_i - \mathbf{X}_j$ is the $M_T \times T$ codeword difference matrix, $Q(\cdot)$ is the Q-function and $\rho = \frac{E_b k_b}{u N_0}$ is the average SNR at the receive antenna in a SISO fading link, which accounts for the change in SNR due to redundancy in the RRNS code.

Applying Chernoff bound and averaging over all channel realizations, (4) can be upper bounded as [8]

$$P(\mathbf{X}_i, \mathbf{X}_j) \leq \prod_{k=1}^{r(\mathbf{G}_{i,j})} \left(\frac{1}{1 + \rho \lambda_k(\mathbf{G}_{i,j})/4M_T}\right)^{M_R}, \tag{5}$$

where λ_k is the k-th non-zero eigenvalue of $\mathbf{G}_{i,j} = \mathbf{E}_{i,j}\mathbf{E}_{i,j}^H$, and $r(\mathbf{G}_{i,j})$ is the rank of $\mathbf{G}_{i,j}$. Hence, the symbol/residue error probability can be upper bounded as [11]

$$P_s \leq \sum_{i=1}^{N_c} P(\mathbf{X}_i) \sum_{j=1, j \neq i}^{N_c} P(\mathbf{X}_i, \mathbf{X}_j), \tag{6}$$

where N_c is the total number of codewords and $P(\mathbf{X}_i)$ is the probability that the codeword $\mathbf{X}_i$ is transmitted.

RRNS (u, v) code is a non-linear, non-binary code which is similar to RS Codes provided $\min\{m_{v+1}, m_{v+2}, \cdots, m_u\} \geq \max\{m_1, m_2, \cdots, m_v\}$ holds [3]. If the moduli are relatively close to each other, the weight distribution of RRNS codes can be approximated using the weight distribution of RS codes [12]. For "error-correction-only" [13], the codeword error probability of un-interleaved RRNS (u, v)-STBC with hard-decision decoding can be further expressed and upper bounded as [14]

$$\begin{aligned} P_c(\mathbf{X}_i, \mathbf{X}_j) &\leq \sum_{k=t+1}^{u} \binom{u}{k} (P_s)^k (1 - P_s)^{u-k} \\ &\leq (N_c - 1) 2^u P_s^{\chi/2}, \end{aligned} \tag{7}$$

where $t = \lfloor \frac{1}{2}(d_{min} - 1) \rfloor$ is the number of errors that can be corrected by the RRNS (u, v) code and $d_{min} = \chi = u - v + 1$ is the maximum possible minimum-distance of the RRNS (u, v) code. By substituting (6) into (7), we have

$$\begin{aligned}
& P_c(\mathbf{X}_i, \mathbf{X}_j) \leq (N_c - 1) 2^u P_s^{\chi/2} \\
\leq \quad & (N_c - 1) 2^u \\
\times \quad & \left[\frac{1}{N_c} \sum_{j=1}^{N_c} \sum_{i=1, i \neq j}^{N_c} \prod_{k=1}^{r(\mathbf{G}_{i,j})} \left(\frac{1}{1 + \rho \lambda_k(\mathbf{G}_{i,j})/4M_T} \right)^{M_R} \right]^{\chi/2} \\
\leq \quad & (N_c - 1) N_c^{-\chi/2} 2^u \\
\times \quad & \left[\prod_{k=1}^{r(\mathbf{G}_{a,b})} \sum_{j=1}^{N_c} \sum_{i=1, i \neq j}^{N_c} \left(\frac{1}{1 + \rho \lambda_k(\mathbf{G}_{a,b})/4M_T} \right)^{M_R} \right]^{\chi/2} \\
\leq \quad & (N_c - 1) N_c^{-\chi/2} 2^u [N_c(N_c - 1)]^{r(\mathbf{G}_{a,b})\chi/2} \\
\times \quad & \prod_{k=1}^{r(\mathbf{G}_{a,b})} \left[\left(\frac{1}{1 + \rho \lambda_k(\mathbf{G}_{a,b})/4M_T} \right)^{M_R \chi/2} \right],
\end{aligned} \tag{8}$$

where,

$$\lambda_k(\mathbf{G}_{a,b}) = \min_{1 \leq i,j \leq N_c, i \neq j} \lambda_k(\mathbf{G}_{i,j}).$$

In high SNR regime, (8) can be further simplified as

$$\begin{aligned}
P_c(\mathbf{X}_i, \mathbf{X}_j) \quad \leq \quad & \frac{2^u N_c^{[r(\mathbf{G}_{a,b})\chi/2 - \chi/2]} (N_c - 1)^{[r(\mathbf{G}_{a,b})\chi/2 + 1]}}{\left[\prod_{k=1}^{r(\mathbf{G}_{a,b})} \lambda_k(\mathbf{G}_{a,b}) \right]^{M_R \chi/2}} \\
\times \quad & \left(\frac{\rho}{4M_T} \right)^{-r(\mathbf{G}_{a,b}) M_R \chi/2}.
\end{aligned} \tag{9}$$

When we apply the DA mapping scheme, the union bound in (6) changes due unequal prior probabilities of the codewords and the decision boundaries for ML symbol detection are scaled according to the prior probabilities. $\hat{\mathbf{X}}$ becomes more accurate and $P_c(\mathbf{X}_i, \mathbf{X}_j)$ decreases resulting in better error performance than normal gray coded QAM mapping with ML decoding assuming equiprobable symbols.

From (9), we observe that for RRNS(u, v)-STBC, the coding gain mainly depends on the term $\left[\prod_{k=1}^{r(\mathbf{G}_{i,j})} \lambda_k(\mathbf{G}_{a,b}) \right]^{M_R \chi/2}$, while $r(\mathbf{G}_{a,b}) M_R \chi/2$ determines the achievable diversity gain. In contrast to OSTBC, $\mathbf{G}_{a,b}$ in RRNS(u, v)-STBC can not be guaranteed to be full rank. However, the additional term $\chi/2$ introduces a new degree of freedom for designing STBC and can provide higher diversity gain than OSTBC. It is worth noting here that the achievable diversity gain of RRNS(u, v)-STBC is obtained assuming MLD. However, low complexity sub-optimal decoders suggested, e.g., in [3],[12], can be implemented without significant loss of diversity/coding gain. The upper bound on BER of RRNS(u, v)-STBC can be calculated by using the fact that

$$P_{ber} \approx \frac{1}{T} \sum_{j=1}^{N_c} \sum_{i=1, i \neq j}^{N_c} \frac{e_{i,j}}{k_b} P_c(\mathbf{X}_i, \mathbf{X}_j), \tag{10}$$

where $e_{i,j}$ is the number of error bits due to the error event of $\mathbf{X}_i \rightarrow \mathbf{X}_j$.

IV. Simulation Results

A 2×2 MIMO system is considered. Alamouti scheme [15] is simulated as an example of OSTBC. In order to develop a fair comparison scenario between Alamouti scheme and RRNS (u, v)-STBC, we use global codes efficiency [16] as a standardizing metric. The global codes efficiency is defined as the product of the spatial and temporal coding rates i.e., $\eta_G = R_c R_s$, where R_c is the coding rate of the error correcting code (RRNS in this case) and R_s is the STBC coding rate. STBC coding rate [16] is defined as the number of useful modulation symbols over the number of space time coded symbols used in the STBC. First of all, we set the same η_G for both RRNS(u, v)-STBC and OSTBC. Secondly, modulation scheme used in RRNS-STBC is set to be the same as in OSTBC. Bit error rate (BER) performance is evaluated.

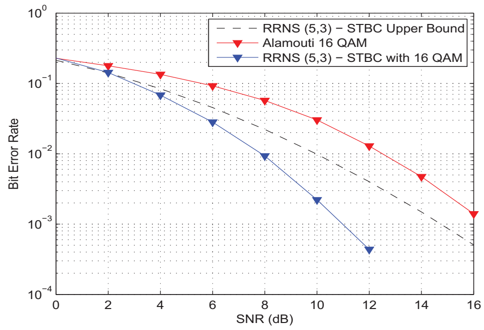

Fig. 5. BER performance of RRNS (5,3)- STBC with moduli set of $m_o = \{7, 9, 11\}$, $m_r = \{13, 16\}$.

In Fig.5, BER performance of RRNS $(5, 3)$-STBC using full diversity transmission and Alamouti coding scheme is compared. The moduli set that is used in RRNS $(5, 3)$-STBC is $\{7, 9, 11, 13, 16\}$, where $m_o = \{7, 9, 11\}$ and $m_r = \{13, 16\}$ correspond to the original moduli set and redundant moduli set, respectively. 16-QAM modulation scheme is applied in both RRNS $(5, 3)$-STBC and Alamouti scheme. The global codes efficiency of RRNS $(5, 3)$-STBC ($R_c R_s = 0.5 \times 1 = 0.5$) is the same as Alamouti coding ($R_c R_s = 1 \times 0.5 = 0.5$). The R_c for RRNS-STBC is given by [7]

$$R_c = k_b / \sum_{j=1}^{u} k_{b_j} \tag{11}$$

where $k_b = \lfloor \log_2 M_r \rfloor$ and $k_{b_j} = \lceil \log_2 m_j \rceil$, where $m_j, \{j = 1, 2 \cdots u\}$ are the moduli. It is observed that RRNS $(5, 3)$-STBC outperforms Alamouti coding scheme in terms of BER. This is expected as in this case, $\chi/2 = 1.5$. Therefore, the contribution of the additional term $\chi/2$ provides a similar diversity order relative to Alamouti scheme. Furthermore, RRNS-STBC exploits extra coding gain while Alamouti coding has no coding gain. All these features are clearly revealed in Fig.5. The RRNS-STBC uses the Chinese Remainder Theorem for decoding the codewords. The CRT is computationally intensive as compared to the simple Alamouti scheme. Thus the gain using RRNS-STBC comes at the cost of increased receiver complexity. However in terms of complexity, our implementation is comparable to [16] where RS codes have been used as STBC.

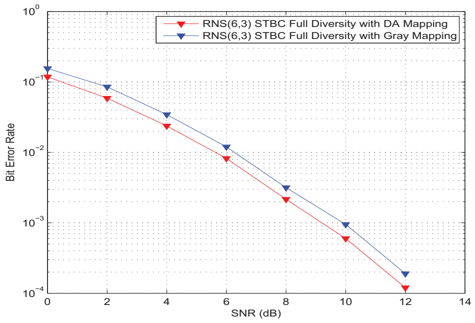

Fig. 6. Comparison of BER performance of Full Diversity RRNS(6,3)-STBC ($T = 3$), with prior probability based Distance-Aware mapping and Gray coded mapping with ML decoding assuming equal priors

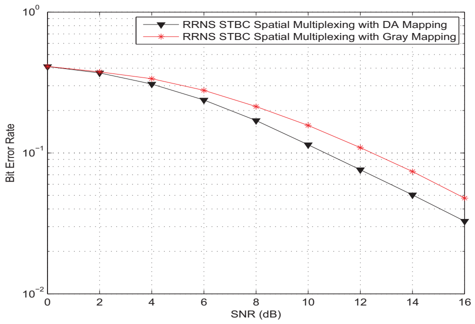

Fig. 7. Comparison of BER performance of Spatially Multiplexed RRNS(6,3)- STBC ($T = 3$) with prior probability based Distance-Aware mapping and Gray coded mapping assuming equal priors

In Fig.6, the comparison between normal gray coded mapping and the new probability based DA mapping for an RRNS (6,3)-STBC with $m_0 = \{5, 7, 9\}$ and $m_r = \{11, 13, 16\}$, as considered in the example in section II.B, is shown for the full diversity transmission case. Here each antenna transmits the same symbol over the same time period. At the receiver end, equal gain combining (EGC) is used along with ML detection. Thus, in this case, $R_s = 1$. It can be seen that the DA mapping scheme gives a consistent improvement of about 1dB in BER performance over the normal gray coded QAM mapping scheme. Fig.7. shows the same comparison in case of a spatial multiplexing scheme where $R_s \geq 1$. It can be seen that a performance improvement of about 2dB is obtained in this case. This gain is achieved due to the DA mapping scheme. The decision boundaries in the signal space are scaled according to the probability of occurrence of a symbol i.e., the most probable symbol has the largest decision region. This leads to a consistent improvement in error performance of the RRNS-STBC when DA mapping is used.

V. Conclusion

In this paper, a novel use of RRNS codes as STBC has been proposed and evaluated. Probability based distance-aware direct mapping scheme has been proposed for RRNS-STBC codes which gives a further improvement in BER performance relative to a naive gray encoded direct mapping scheme, although the decoding of RRNS based ST coding scheme is more complex than OSTBC. Our theoretical and numerical results indicate that significant error rate performance improvement can be offered by RRNS-STBC relative to OSTBC with the complexity being comparable to other concatenated codes such as RS-STBC.

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