# Reduced Complexity Super-Trellis Decoding for Convolutionally Encoded Transmission Over ISI-Channels

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Abstract—In this paper we propose a matched encoding (ME) scheme for convolutionally encoded transmission over intersymbol interference (usually called ISI) channels. A novel trellis description enables to perform equalization and decoding jointly, *i.e.*, enables efficient super-trellis decoding. By means of this matched non-linear trellis description we can significantly reduce the number of states needed for the receiver-side Viterbi algorithm to perform maximum-likelihood sequence estimation. Further complexity reduction is achieved using the concept of reduced-state sequence estimation.

*Index Terms*—ISI-channel; convolutionally encoded transmission; super-trellis decoding; matched decoding; reduced state sequence estimation; trellis-coded modulation;

## I. INTRODUCTION

Convolutional coded pulse-amplitude modulation (PAM) is an attractive digital communication scheme for transmission over intersymbol interference (ISI) channels, when low latency is desired. Low latency, required *e.g.*, for real-time bidirectional communication, is obtained by the use of convolutional codes (instead of block codes, *cf.* [1]) and dispense with interleaving (as opposed to conventional bit-interleaved coded modulation [2]).

For this setup, the optimum receiver performs equalization of the ISI-channel and decoding of the convolutional code jointly in a single super-trellis [3]. This technique, however, is commonly regarded prohibitively complex due to the large overall number of states of a super-trellis. Hence, equalization and decoding are usually performed subsequently in two separate processing steps, each based on its own trellis description. As long as no interleaving can be applied, iterative (Turbo-) decoding/equalization does not work satisfactorily.

In this paper, we merge the convolutional encoder and the ISI-channel into a single non-linear trellis encoder with binary delay elements only. It is shown, that the total number of states of this equivalent non-linear trellis description is significantly smaller than the number of states in the usual super-trellis. Consequently, this non-linear trellis description enables very efficient implementation of optimum super-trellis decoding (STD) based on maximum-likelihood sequence estimation (MLSE) using the Viterbi algorithm (VA) [4] or other trellis-based decoding algorithms.

Combining this approach with reduced-state sequence estimation (RSSE) [5]–[7] enables to further reduce the computational complexity and thus offers a flexible trade-off between performance (in terms of required signal-to-noise power ratio to guarantee a target bit error rate) and receiver complexity.

The only requirement necessary for our approach is that the rate- $\frac{K}{n}$  convolutional code is matched to the *M*-ary modulation via  $M = 2^n$ , *i.e.*, trellis-coded modulation (TCM). For sake of simplicity, we here consider real-valued ASK only.

Note that recently, we have adopted a similar approach to receiver design of continuous phase modulation (CPM) in combination with non-coherent differential detection [8].

This paper is organized as follows: After the definition of the system model in Sec. II, we derive the equivalent nonlinear trellis description in Sec. III. In Sec. IV the reduced computational complexity is discussed and Sec. V employs reduced-state sequence estimation (RSSE) for our approach. The effectiveness of the proposed approach is validated by means of numerical simulations in Sec. VI. The paper concludes with a summary.

#### II. SYSTEM MODEL

We first introduce convolutionally encoded PAM transmission over ISI-channels (cf. example of Fig. 1). The discrete-



Fig. 1: Concatenation of an ISI-channel with a rate- $\frac{1}{2}$  convolutional encoder  $[g_{1,\text{oct}}; g_{2,\text{oct}}] = [5_{\text{oct}}; 7_{\text{oct}}]$  and an ISI-channel (L = 2).

time transmitter is composed of a rate- $\frac{K}{n}$  binary convolutional encoder with generator polynomials  $[g_i]$ ,  $1 \le i \le n$ , with K input symbols and n parallel output symbols at each time instant, a mapper and M-ary PAM transmission. The transmit signal traverses through a memory-L discretetime ISI-channel with L + 1 channel coefficients h[k] with

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k denoting the time index. In the convolutional encoder  $\boxplus$  symbolizes the addition operation over the Galois field  $\mathbb{F}_2$ , *i.e.*, calculations are performed mod 2, whereas  $\oplus$  and  $\otimes$  indicate the addition and multiplication operation over the real numbers, respectively.

#### III. MATCHED ENCODING APPROACH

In the conventional approach one would process the receiver input signal first by a MLSE or a symbol-by-symbol trellisbased equalizer for the FIR filter h[k] and forward soft- or hard-output symbols of this trellis equalization to the decoder for the channel code, *i.e.*, solve the equalization and decoding tasks in two separate processing steps. An optimum receiver however would perform MLSE in the super-trellis, decoding the binary channel encoder and the ISI-channel impulse response h[k] of length L jointly. In a straight-forward approach the super-trellis would have  $Z_{enc} \cdot M^L$  states, where  $Z_{enc}$  is the number of states of the convolutional encoder.

In order to reduce the computational complexity of STD, we introduce a matched trellis description for convolutionally encoded PAM transmission over ISI-channels (so-called matched encoding). This non-linear trellis encoder can be used to build the matched decoding (MD) trellis for the joint equalization and decoding process.

If the number of output symbols from the encoder is related to the size of the modulation alphabet M so that  $n = \log_2(M)$  holds, the following trellis description can achieve exactly the same performance at reduced complexity, *i.e.*, with fewer states in the trellis. To see this, note that in each encoding step, n-K output symbols of the encoder are redundant and depend on K input symbols. *E.g.*, in Fig. 1 one of the two channel encoder output symbols contains no further information.

The restriction that the size of the modulation alphabet has to match the number of output symbols of the convolutional encoder only allows to combine a  $\frac{K}{2}$ -rate encoder with a 4-ary modulation, a  $\frac{K}{3}$ -rate encoder with a 8-ary modulation, and so on. However, we showed in [9] that when puncturing is performed to increasing the rate of the convolutional encoder the matched decoding approach can still be applied using a time-variant non-linear trellis description and a slightly modified VA.

We here show, how to merge the binary channel encoder with the M-ary channel impulse response to form a single time-invariant binary non-linear trellis encoder. To this end, we transform the transmission scheme step-by-step.

First, we describe the mapping process analytically.<sup>1</sup> For clarity, we restrict ourselves to M = 4, but note that the concept easily extends to arbitrary  $M = 2^n$ .

In this example M = 4, *i.e.*, n = 2, with natural mapping (here equals a set partitioning mapping), the upper branch corresponds to the most significant bit (MSB) whereas the lower branch describes the least significant bit (LSB). Having the MSB and LSB at time instant k we now have to perform the mapping to the symbols of the modulation alphabet. For the 4-ary natural labeling we multiply the MSB by 2 and add the LSB, *i.e.*, c[k] = 2MSB[k] + LSB[k]. The conversion from unipolar binary symbols c[k] into bipolar symbols b[k] within an alphabet of size M can be done with  $b[k] = (c[k] \cdot 2) - (M-1)$ . The resulting block diagram, for natural labeling is depicted in Fig. 2.

Different mappings are easily incorporated, *e.g.*, a Gray labeling can be achieved by c[k] = (1 - MSB[k])(2MSB[k] + LSB[k]) + (MSB[k])(2MSB[k] + (1 - LSB[k])). Furthermore, a 4-ary quadrature amplitude modulation (QAM) can be represented using the MSB as real part and the LSB as imaginary part (or vice versa), *e.g.*, b[k] = (2MSB - 1) + j(2LSB - 1), with  $j = \sqrt{-1}$ .



Fig. 2: Equivalent description of the convolutional encoding and the ISI-channel (exemplarily M = 4; natural labeling).

For the second step, recall that the mod-operation can be represented using the floor-function. In terms of Gaussian notation we can thus write

$$x \mod n = x - n \cdot \left\lfloor \frac{x}{n} \right\rfloor$$
 (1)

where  $\lfloor . \rfloor$  denotes the floor-function. In addition, we see that the main branch (after the summation of MSB and LSB) has a multiplication and summation which can be moved to the output of the convolution. With  $C = -\sum_{k=0}^{L} h[k](M-1)$  and the Gauss representation of the modulo operation we can sketch the transmission system as depicted in Fig. 3. All calculations can therefore be performed in the real numbers.



Fig. 3: Replacement of the mod 2 addition with the non-linear representation using the floor function.

Finally, note that now the convolution can be moved into the MSB branch and LSB branch, respectively, which enables to

<sup>&</sup>lt;sup>1</sup>To simplify notation, we do no longer strictly distinguish elements and operations from the Galois field and the real numbers.

use binary delay elements instead of M-ary ones. The mapping can be moved to the end of the branches.

This representation now has n independent binary branches, *i.e.*, an MSB and an LSB branch in the case of n = 2, which all depend on the same K input values (here K = 1). Due to the memory elements of the ISI-channel being binary and depending on either the MSB or the LSB we can combine them with the memory elements of the convolutional encoder and distinguish them using the generator polynomials  $g_1$  and  $g_2$ . This results in a single non-linear filter combining the calculations in each branch.



Fig. 4: The matched encoder (ME) as a non-linear encoder representation of coded PAM transmission over an ISI-channel.

The resulting, non-linear trellis encoder, as depicted in Fig. 4, can be used to generate the hypothesis and the state transitions of a finite state machine (FSM). The receiver is depicted in Fig. 5 and uses the hypothesis to calculate the metrics  $\lambda_i[k]$ , *e.g.*, Euclidean distances, for the noisy received signal and performs optimum MLSE via the VA. At this point a suboptimum state reduction can be applied, as described in Sec. V.



Fig. 5: The full-state matched decoder (MD) using the nonlinear encoder representation for metric calculation and the VA for decoding.

#### IV. COMPLEXITY COMPARISON

The main advantage of matched encoding is the reduction of the convolution by the ISI-channel from an M-ary input sequence into  $\log_2(M)$  binary parallel convolutions in each branch. As the number of convolutions affects the calculation of metrics at the receiver but does not influence the number of resulting MLSE states, we will now examine the complexity of separated equalization and decoding, the super-trellis decoding (STD), and matched decoding (MD). Clearly, as a measure for the computational complexity the total number of states required for receiver-side processing can be adopted. For our comparison we need to distinguish the number of states that result from the convolutional encoder and the ISI-channel from the receiver complexity. The latter can either be a result of separated equalization and decoding, super-trellis decoding, or matched decoding.

## A. Separated Equalization and Decoding

For separated equalization and decoding the receiver complexity is defined as the sum of states in the equalization and the decoding, *i.e.*,

$$Z_{\text{separate}} = Z_{\text{equ}} + Z_{\text{enc}}.$$
 (2)

In our simulations we distinguish between hard- and softoutput trellis-based equalization using DFSE, or the BCJR algorithm, respectively, and decoding is performed using the VA in the full-state trellis.

## B. Super-Trellis Decoding

In a super-trellis we consider encoder states and channel states jointly resulting in a total number of states in the supertrellis of

$$Z_{\text{STD}} = Z_{\text{enc}} \cdot Z_{\text{equ}} = 2^{\nu} \cdot M^L = 2^{\nu} \cdot 2^{(n \cdot L)}.$$
 (3)

Apparently, already for moderate  $\nu$ , n and/or L, super-trellis decoding becomes intractable.

#### C. Matched Decoding

There are two differences compared to STD when considering the proposed matched encoding/decoding approach. First, the convolution with the channel impulse response is done with binary delay elements in contrast to M-ary elements. Second, as the MSB and LSB depend on each other (as of the channel encoder) not all state transitions are allowed anymore. As can be seen from Fig. 4 the total number of delay elements does not increase although we use binary delay elements, only. Thus, we still have  $2^{\nu}$  possible states for the binary channel encoder (which is fully integrated into the non-linear encoder) but only  $2^{L}$  possible states for the convolution resulting in a total number of states of

$$Z_{\rm MD} = 2^{\nu} \cdot 2^L. \tag{4}$$

Recall that for n = 2 there are two convolutions in parallel for the computation of the hypothesis. Finally, employing RSSE (*cf.* Sec. V), the complexity depends on the partitioning as will be described below, *i.e.*,  $Z_{\text{MD-RSSE}} = Z_{\text{R}} = 2^{r}$  with arbitrary integer r > 1.

#### D. Comparison

The main advantage of MD compared to STD is the reduction of states without loss in performance. The resulting trellis still describes the super-trellis but with fewer states. The gain of this state reduction therefore calculates to

$$G_{\rm MD} = \frac{Z_{\rm STD}}{Z_{\rm MD}} = \frac{2^{(n \cdot L)}}{2^L} = 2^{L(n-1)}.$$
 (5)

Table I summarizes several examples for different encoders and channel lengths for the special case of n = 2 (M = 4). Obviously the gain increases with the length of the ISIchannel.

TABLE I: Number of states for PAM transmission with M = 4, n = 2 and for the super-trellis representation and MD, respectively.

Encoder	L	$Z_{ m STD}$	$Z_{\rm MD}$	$G_{\rm MD}$
16 states $\nu = 4$ <i>e.g.</i> , [23 <sub>oct</sub> ; 04 <sub>oct</sub> ]	0	16	16	1
	1	64	32	2
	2	256	64	4
	3	1024	128	8
	4	4096	256	16
	5	16384	512	32
64 states $\nu = 6$ <i>e.g.</i> , [103 <sub>oct</sub> ; 024 <sub>oct</sub> ]	1	64	64	1
	0	256	128	2
	2	1024	256	4
	3	4096	512	8
	4	16384	1024	16
	5	65536	2048	32

## V. REDUCED-STATE SEQUENCE ESTIMATION

We have shown that the super-trellis of convolutionally encoded transmission over ISI-channels can be represented using significantly fewer states by parallelizing the M-ary convolution. At this point we can use reduced-state sequence estimation (RSSE) [6] to further reduce the number of states at the cost of small loss in Euclidean distance.

In RSSE, Z MLSE states, each with  $M = 2^K$  possible transitions to adjacent states, are combined into  $Z_R = \frac{Z}{2^J}$ ;  $J \in \mathbb{N}$ hyperstates [3], [5] each having  $2^J$  substates and  $2^K \cdot 2^J$ state transitions as depicted in Fig. 6 with K = 1 and J = 1. A certain assignment of states to hyper states is called *partitioning* [5].

Instead of selecting a survivor from  $2^{K}$  arriving transitions at each of the Z MLSE states we now select a single survivor from a set of  $2^{K} \cdot 2^{J}$  transitions at  $Z_{R}$  hyper states. The number of metrics that have to be calculated remains  $2^{K} \cdot Z$ .

The main difference is, that we decide for a surviving path prematurely resulting in a truncation of error events. A loss in Euclidean distance appears if an error event with minimum Euclidean distance gets truncated. Therefore the performance of RSSE strongly depends on the partitioning of the states into hyperstates.

One approach to find the optimum partitioning is to determine the mutual state distances and iteratively maximize the intra hyperstate distance [5]. Unfortunately the exhaustive search for the state distances is impractical for a larger number of states.

Fortunately the channel impulse response h[k] is fully integrated into the non-linear trellis description. W.l.o.g we assume that the channel impulse response h[k] is minimum phase, which can be achieved by the application of a proper all-pass filter. For a minimum phase channel impulse response the MLSE equalization with a reduced number of states is well-known as delayed decision-feedback sequence estimation



Fig. 6: Illustration of hyper states and sub states in a trellis.

(DFSE) [6], [7], [10], [11]. On an ISI-channel with delay length L, DFSE generates the trellis on the first  $q_h < L$  coefficients only. Some post cursors of the discrete-time impulse response are cancelled using a decision feedback in each state using the state register of the VA. This can be interpreted as a particular solution of RSSE using a methodical partitioning.

As the minimum phase ISI-channel is fully integrated into the non-linear trellis we can apply the methodical DFSE partitioning to use RSSE for MD for convolutionally encoded PAM transmission over ISI-channels.

#### VI. NUMERICAL RESULTS

The effectiveness of the approach of MD is now verified by means of numerical simulations. We restrict ourselves to rate- $\frac{1}{2}$  encoding schemes and a 4-ary modulation alphabet. As convolutional encoder we apply the generator polynomials given in Table I, which, in combination with natural labeling, result in a trellis coded modulation scheme (TCM) for *M*ary ASK.<sup>2</sup> The ISI-channel is described by (6) using  $L \in$  $\{2; 5\}$  with  $Z_{cha} = 16$  states and 1024 states, respectively. For simplicity an exemplary minimum phase ISI-channel is generated by

$$h[k] = \frac{1}{\alpha} \cdot \frac{L-k+1}{L+1}; \qquad 0 \le k \le L$$
 (6)

$$\alpha^{2} = \sum_{k=0}^{L} \left( \frac{L-k+1}{L+1} \right)^{2}$$
(7)

and normalized to unit energy. Please note that due to the normalization the equivalent energy per bit  $E_{b}$  is identical at transmitter output and receiver input.

Our MD approach of RSSE operating on the equivalent nonlinear trellis description is compared to separate equalization and decoding employing DFSE/BCJR [13] for equalization and the full-state VA for decoding. Here, the full-state BCJR equalization is used to compare our approach with softdecision equalization and decoding, whereas DFSE employs a hard-decision reduced-state equalization [6], [7], [10], [11]

<sup>&</sup>lt;sup>2</sup>Note that [12] reveals an equivalence between TCM encoders. There, it is shown that a [5; 7] encoder with gray labeling is identical to a [5; 2] encoder with natural labeling.



Fig. 7: Bit error performance for convolutionally encoded 4-ASK transmission (natural labeling) over the ISI-AWGN-channel. Left: Generator polynomials  $[23_{oct}; 04_{oct}]$  ( $Z_{enc} = 16$ ), L = 2 ( $Z_{cha} = 16$ ). Right: Generator polynomials  $[103_{oct}; 024_{oct}]$  ( $Z_{enc} = 64$ ), L = 5 ( $Z_{cha} = 1024$ ).

based on a VA. Both equalization techniques require a fullstate VA for decoding the convolutional code.

In contrast, the MD-RSSE performes the VA on the reduced set of hyperstates and selects the substates using delayed decisions fed back from the path register. The applied partitioning is created equal to the methodical DFSE partitioning and determines the number of states in the receiver trellis.

## A. Bit Error Performance

The bit error rates for transmission over the ISI-AWGNchannel (one sided power spectral noise density  $N_0$ ) are given in Fig. 7. The number of states for the DFSE/BCJR and for the VA are given in the legend, separately. Here the number of states implemented in the receiver trellis and therefore the receiver complexity is given directly for the MD/RSSE receiver.

Please note that by dispensing the interleaver between channel encoding and modulation for the separated approaches, block errors that are caused by the equalization process reduce the ability to decode due to correlated errors. Obviously, the soft-decision separated approach results in improved bit error rates when compared to the hard-decision separated approach. However, the separated equalization and decoding approach is significantly outperformed by our MD. In the case of a 16state convolutional encoder and a 16-state ISI-channel, the MD performance equals STD at 64 MD states.

The behavior of MD-RSSE is equivalent to that of DFSE. With increasing number of states the performance improves (especially at low number of states) and converges to the performance of STD.

For the 64-state encoder the performance converges slower due to increased encoder complexity (STD not shown due to complexity constraints).

## B. Performance Vs. Complexity Trade-Off

We now compare the receiver complexity for separated equalization and decoding to MD. We compare different channel encodings and ISI-channels defined by their number of states in the transmitter. Channel encoders with 16 or 64 states and an ISI-channel with  $L = \{2, 3, 4, 5\}$  (or  $\{16; 64; 256; 1024\}$  states, respectively) are considered. The channel encoding and the ISI-channel are described by their number of states at the transmitter trellis, *i.e.*, the number of states in the channel encoder and equalizer, respectively, and abbreviated with  $Z_{enc}/Z_{equ}$ .

The target bit error rate is  $10^{-3}$  and the receiver complexity is described by the number of states as described in Sec. IV. As our approach enables the use of RSSE, we can compare the performance for arbitrary receiver complexity for the given target error rate.

In Fig. 8a the results for a convolutional encoder with 16 states and an ISI-channel with another 16 states are depicted. The super-trellis would have 256 states. The best performance for separated equalization and decoding is achieved with the soft-decision approach using the (full-state) BCJR for equalization and the VA for decoding. However, the figure also clearly shows that MD supersedes separated equalization and decoding already for only two states in MD-RSSE.

In Fig. 8b the results for multiple transmitter schemes are depicted. Note, that all separated approaches perform worse with more states compared to the proposed MD approach. Obviously, the 64-state convolutional encoder with  $2^7 = 128$  receiver states can achieve best performance due to increased constraint length of the convolutional encoder. In contrast, increasing the memory of the ISI-channel reduces the minimum Euclidean distance resulting in a degradation of the bit error



(a) Receiver complexity for a  $Z_{\rm enc}=16/Z_{\rm cha}=16$  transmission system.



performance.

It becomes also clear that a  $Z_{enc} = 16/Z_{cha} = 16$  scheme achieves a bit error rate of  $10^{-3}$  with fewer number of receiver states, whereas a convolutional encoder with more states, *i.e.*,  $Z_{enc} = 64/Z_{cha} = 16$ , achieves  $10^{-3}$  with less signal-to-noise power ratio. Hence, the proposed decoding schemes enable a flexible trade-off between complexity and noise-robustness, *i.e.*, power efficiency.

In summary, we conclude that it is favorable to choose a convolutional code with a low number of states in combination with MD-RSSE, when low delay and low complexity are required.

## VII. CONCLUSION

In this paper we have shown that it is possible to reduce the number of states for super-trellis decoding without loss in performance by transforming the M-ary channel convolution into  $\log_2(M)$  parallel binary convolutions. Here, a coded ASK transmission is used, but as several other non-interleaved transmission schemes (*e.g.*, QAM over an ISI-channel) can be represented as a separate channel encoder and a channel impulse response, this approach is attractive for such schemes, as well. We showed that with MD the same performance as super-trellis decoding can be achieved with significantly reduced computational complexity. By using RSSE with DFSElike partitioning we obtain an efficient method for a trade-off between complexity and performance. The only restriction of this approach is the strong relation between the alphabet size of the modulation and the code rate.



(b) Receiver complexity for two different convolutional codes (16 and 64 states, first number in legend) and different ISI-channels ( $Z_{cha}$  states, second number in legend).

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