On the Performance and Optimization for MEC Networks Using Uplink NOMA

Yinghui Ye¹, Guangyue Lu², Rose Qingyang Hu³, Liqin Shi¹

¹School of Telecommunications Engineering, Xidian University, China.

² Shaanxi Key Laboratory of Information Communication Network and Security,

Xi'an University of Posts and Telecommunications.³ Department of ECE, Utah State University, U.S.A.

Abstract-In this paper, we investigate a non-orthogonal multiple access (NOMA) based mobile edge computing (MEC) network, in which two users may partially offload their respective tasks to a single MEC server through uplink NOMA. We propose a new offloading scheme that can operate in three different modes, namely the partial computation offloading, the complete local computation, and the complete offloading. We further derive a closed-form expression of the successful computation probability for the proposed scheme. As part of the proposed offloading scheme, we formulate a problem to maximize the successful computation probability by jointly optimizing the time for offloading, the power allocation of the two users and the offloading ratios which decide how many tasks should be offloaded to the MEC server. We obtain the optimal solutions in the closed forms. Simulation results show that our proposed scheme can achieve the highest successful computation probability than the existing schemes.

I. INTRODUCTION

Mobile edge computing (MEC) has been deemed a promising technique to enhance computation service so that future wireless communications are able to realize computationintensive and delay-sensitive services, e.g., virtual reality and autonomous driving [1]–[3]. The basic idea of MEC is to let mobile users perform computation offloading, i.e., mobile users can offload partial or complete tasks to the nearby access points with more powerful computation capabilities. There are two operation modes for MEC: binary computation offloading and partial computation offloading [1]. For the former operation mode, the computation tasks are either fully locally computed or completely offloaded to the MEC server. For the latter one, the computation tasks can be divided into two parts, where one part is locally executed and the other part is offloaded to the MEC server.

On the other hand, non-orthogonal multiple access (NOMA) has been viewed as a key technology for the future wireless communication networks [4], [5]. It allows multiple users to operate in the same frequency band simultaneously with different power levels to improve the spectral efficiency and connectivity. By employing NOMA technology instead of orthogonal



Fig. 1: System Model.

multiple access (OMA) to offload tasks, the offloading latency can be reduced and the performance of MEC can be improved. Therefore, the aforementioned two communication techniques, MEC and NOMA, can be combined to obtain gains in terms of latency [6], energy consumption [7], [8].

The combination of NOMA and MEC was studied in [7], where the authors minimized the weighted sum of the energy consumption at all mobile users subject to their computation latency constraints for both the partial computation offloading and the binary computation offloading modes. A similar problem was studied in [8] by considering the user clustering for uplink NOMA. In [9], the authors provided a guideline to choose the best mode among OMA, pure NOMA, and hybrid NOMA based MEC networks in terms of the energy consumed by complete offloading. Different from the previous works, where the main focus was on the energy computation minimization by optimizing the network parameters based on instantaneous channel state information (CSI), the authors in [10] studied the impact of NOMA's parameters, e.g., user channel conditions and transmit powers, on the complete offloading by deriving the expression of the successful computation probability. To the best of our knowledge, there is no open work to study the successful computation probability of NOMA based MEC networks for a hybrid operation mode, which takes the partial computation offloading, the full local computation and the complete offloading into consideration, based on the statistic CSI.

In this paper, we consider a NOMA based MEC network, where two users may offload their computation tasks to a single MEC server through uplink NOMA. The proposed offloading scheme can operate in one of following three modes, namely partial computation offloading, complete local computation, and complete offloading. As part of the proposed offloading scheme, we firstly derive a closed-form expression for the successful computation probability based on the statistic CSI. Then a problem is formulated to maximize the successful computation by optimizing the time for offloading,

Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org.

This work was supported by the scholarship from China Scholarship Council, the Natural Science Foundation of China (61801382), the Science and Technology Innovation Team of Shaanxi Province for Broadband Wireless and Application (2017KCT-30-02), the US National Science Foundations grants under the grants NeTS-1423348 and the EARS-1547312. The corresponding author is Liqin Shi (liqinshi@hotmail.com).

the power allocation of the two users to perform uplink NOMA and the offloading ratios which decide how many tasks should be offloaded to the MEC server. Although this problem is non-convex, we obtain the optimal solutions in closed forms. Simulations are provided to support our work.

II. SYSTEM MODEL AND WORKING FOLW

We consider a NOMA based MEC network consisting of one MEC server (i.e., gateway or base state) and two users A and B, as shown in Fig. 1. Each user has tasks with M_k (k = A, B) bits to be executed and the users may not be able to execute their tasks locally within the latency budget due to the limited local computational capabilities. We assume that the task-input bits are bit-wise independent and can be arbitrarily divided into different groups [1]. More specifically, for the user k, $\beta_k M_k$ bits are offloaded to the MEC server and the remaining ones, $(1 - \beta_k) M_k$, are locally executed, where $0 \le \beta_k \le 1$. Therefore, the MEC server may schedule users to offload partial tasks through uplink NOMA so that all the tasks can be computed within the delay budget. We propose a new offloading scheme¹ that operates in a hybrid operation mode, which can support all three modes, namely partial computation offloading, full local computation and complete offloading. Moreover, all the channels are assumed to be quasi-static Rayleigh fading, where the channel coefficients are constant for each block but vary independently between different blocks. Also, the maximum supportable latency of the NOMA based MEC network is assumed as T seconds and T is considered to be less than the coherence interval.

The proposed offloading scheme is introduced as follows. In the first phase t_0 , the MEC server determines the parameters of the proposed offloading scheme such as the offloading ratios and the time for offloading. Then users A and B offload their tasks to the MEC server via uplink NOMA at the second phase t_1 . After successful offloading, the offloaded and the local tasks are computed at the MEC server and the users respectively during the third phase t_2 . Finally, the MEC server feeds back the computed results to A and B within t_3 . Following [6]–[8], [10], t_0 and t_3 , are assumed very small and thus are neglected. Accordingly, our proposed offloading scheme consists of two main phases: offloading phase t_1 and tasks executing phase t_2 , i.e., $t_1 + t_2 \leq T$. 1) Offloading Phase: During the offloading phase, A and

B transmit their respective tasks with $\beta_A M_A$ and $\beta_B M_B$ bits to the MEC server simultaneously. The received signal at the MEC server is given as

$$y_{\rm MEC}^{\rm up} = \sqrt{\frac{P_{\rm A}}{1 + d_{\rm A}^{\alpha}}} h_{\rm A} x_{\rm A} + \sqrt{\frac{P_{\rm B}}{1 + d_{\rm B}^{\alpha}}} h_{\rm B} x_{\rm B} + w, \qquad (1)$$

where P_k is the transmit power of user k; $\frac{1}{1+d_k^{\alpha}}$ denotes the large-scale fading with the distance d_k from the MEC server

to the user k and the path loss exponent α ; $h_k \sim \mathbb{CN}(0,1)$ models the small-scale Rayleigh fading between the MEC server and the user k; x_k is the transmit signal at the user k with $\mathbb{E}\left|\left|x_{k}\right|^{2}\right| = 1$; w is the received additive Gaussian white noise at the MEC server with variance σ^2 .

In order to improve the performance of the uplink NOMA, we introduce the power allocation coefficient at the users, denoted by λ , to realize the power control at the users. Let $P_{\rm A} = \lambda P$ and $P_{\rm B} = (1 - \lambda)P$ [12], where P is the total power of the two users². According to the principle of uplink NOMA, the MEC server firstly decodes x_A and then subtracts this component from the received signal to decode $x_{\rm B}$. The achievable capacity from the user A to the MEC server is expressed as

$$\tau_{x_{\mathrm{A}}} = t_1 B_c \log_2 \left(1 + \gamma_{\mathrm{SINR}}^{x_{\mathrm{A}}} \right). \tag{2}$$

 B_c is the bandwidth and $\gamma^{x_{\rm A}}_{\rm SINR}$ is the received signal to interference and noise ratio (SINR) at the MEC server to decode x_A , given by

$$\gamma_{\rm SINR}^{x_{\rm A}} = \frac{(1+d_{\rm B}^{\alpha})\lambda\rho|h_{\rm A}|^2}{(1-\lambda)\left(1+d_{\rm A}^{\alpha}\right)\rho|h_{\rm B}|^2 + (1+d_{\rm A}^{\alpha})\left(1+d_{\rm B}^{\alpha}\right)},\qquad(3)$$

where $\rho = \frac{P}{\sigma^2}$ is the input signal to noise ratio (SNR). If x_A is successfully decoded, i.e., $\tau_{x_A} \ge \beta_A M_A$, x_A will be subtracted by applying successive interference cancellation (SIC) and the achievable capacity from the user B to the MEC server can be written as

$$\tau_{x_{\rm B}} = t_1 B_c \log_2 \left(1 + \gamma_{\rm SNR}^{x_{\rm B}} \right),\tag{4}$$

where $\gamma_{\text{SNR}}^{x_{\text{B}}} = \frac{(1-\lambda)\rho|h_{\text{B}}|^2}{1+d_{\text{B}}^{\alpha}}$ is the SNR to decode x_{B} . If x_{A} and x_{B} are decoded successfully at the MEC server,

the total tasks to be executed in the MEC server, denoted by $\tau^{\rm total}$, is given as

$$\tau^{\text{total}} = \sum_{k=A,B} \beta_k M_k.$$
 (5)

2) Task Executing Phase: During the task executing phase, the local tasks, $(1 - \beta_k)M_k$, and the offloaded tasks, τ^{total} , are carried out at the user k and the MEC server, respectively. Therefore, the required time for task execution at the user kand the MEC server can be expressed respectively as:

$$t_{2}^{k} = \frac{(1 - \beta_{k}) M_{k}C}{f_{\text{user}}}, \ t_{2}^{\text{MEC}} = \frac{\tau^{\text{total}}C}{f_{\text{MEC}}},$$
 (6)

where C denotes the number of CPU cycles required for computing one input bit; f_{user} and f_{MEC} are the CPU frequencies at the users and the MEC server, respectively. Without loss of generality, we assume that $f_{\text{MEC}} = N f_{\text{user}}$ with N > 1 to characterize the difference of their computational capabilities.

III. SUCCESSFUL COMPUTATION PROBABILITY MAXIMIZATION

We introduce a successful computation probability to evaluate the performance of the considered NOMA based MEC system. On this basis, we firstly answer how many tasks should

¹The existing works [2], [7], [8] assume that the users are able to offload and execute tasks simultaneously. Actually, the users are usually with limited computational capabilities, inferring that they may be with a single core CPU. In this case, the assumption above may not hold since a single core CPU can not execute more than one task (thread) simultaneously [11]. Accordingly, in our proposed scheme, we assume that the users are installed with a single core CPU and switch the operation modes between tasks offloading and tasks executing. This is one of main differences compared with the existing schemes.

²In many practical scenarios, one crucial criteria is the total transmission power. When multiple users share the same bandwidth in a cell, the constraint of the total transmission power is important to manage inter-cell interference. Besides, as pointed out in [13], the total power constraint is beneficial to measure the inter-group interference. These facts motivate us to consider this model, where the total power of NOMA users is no more than a threshold, in a uplink NOMA system.

The successful computation probability, denoted by \mathcal{P}_s , is defined as the probability that all the tasks M_k are successfully executed within a given time T, given by

$$\mathcal{P}_{s} = \Pr\left(\tau_{x_{\mathrm{B}}} \ge \beta_{\mathrm{B}}M, \tau_{x_{\mathrm{A}}} \ge \beta_{\mathrm{A}}M, \max\left\{t_{2}^{\mathrm{A}}, t_{2}^{\mathrm{B}}, t_{2}^{\mathrm{MEC}}\right\} \le t_{2}\right).$$
(7)

Proposition 1. For any given parameters, i.e., t_1 , t_2 , β_A , β_B and λ , the closed-form expression of the successful computation probability can be written as

$$\mathcal{P}_{s} = \begin{cases} \exp\left(-\frac{(1+d_{\mathrm{B}}^{\alpha})\gamma_{2}}{(1-\lambda)\rho} - \frac{\gamma_{1}(1+d_{\mathrm{A}}^{\alpha})(1+\gamma_{2})}{\lambda\rho}\right) \times \\ \frac{(1+d_{\mathrm{B}}^{\alpha})\lambda}{(1+d_{\mathrm{B}}^{\alpha})\lambda + \gamma_{1}(1-\lambda)\left(1+d_{\mathrm{A}}^{\alpha}\right)}, \text{ if } \max\left\{t_{2}^{\mathrm{A}}, t_{2}^{\mathrm{B}}, t_{2}^{\mathrm{MEC}}\right\} \leq t_{2}, \\ 0, & \text{otherwise} \end{cases}$$

$$(8)$$

where $\gamma_1 = 2\frac{\beta_A M}{t_1 B_c} - 1$ and $\gamma_2 = 2\frac{\beta_B M}{B_c t_1} - 1$. *Proof.* Please see Appendix A.

Remark 1. Proposition 1 serves the following purposes. Firstly, we provide a closed-form expression to characterize the successful transmission probability of uplink NOMA based MEC networks. Besides, the closed-form expression in Proposition 1 offers a possibility to obtain the optimal parameters in terms of the maximum successful transmission probability, which are particularly helpful for designing our considered NOMA based MEC network. It is worth noting that, different from the existing works [2], [6]–[8] with a focus on the design of NOMA based MEC networks based on the instantaneous CSI, our designed network is based on the statistic CSI and removes the need to know the accurate instantaneous CSI, alleviating the burdens of signallings.

Corollary 1. If $\frac{MC}{f_{\text{user}}} \leq T$ holds, we have $\mathcal{P}_s = 1$. In this case, the users execute all the tasks locally within the maximum supportable latency T and the desirable working mode is the complete local computation in terms of successful computation probability. Therefore, the optimal network parameters are as follows: $\beta_{\text{A}}^* = \beta_{\text{B}}^* = 0$, $t_1^* = 0$, $t_2^* = \frac{M_k C}{f_{\text{user}}}$, and λ^* can be taken at any value.

In order to provide more insights in designing our considered network, we formulate an problem to maximize the successful computation probability by jointly optimizing the parameters of the proposed offloading parameters, i.e., t_1 , t_2 , β_A , β_B and λ , for the case³ with $\frac{MC}{f_{user}} > T$ in the following.

$$\mathbf{P_0}: \max_{\substack{t_1, t_2, \beta_A, \beta_B, \lambda \\ t_1 + t_2 \leq T , \\ \frac{MC}{f_{\text{user}}} > T \ 0 < \lambda < 1, \\ 0 < t_1, t_2, \beta_A, \beta_B \leq 1, \\ \max\left\{t_2^A, t_2^B, t_2^{\text{MEC}}\right\} \leq t_2.$$
(9)

It is not hard to find that the problem P_0 is a non-convex problem due to the non-convex objective function. In general,

there is no standard algorithm to solve non-convex optimization problems efficiently. We provide the following two propositions to obtain the optimal solutions as follows.

Proposition 2. The equality $t_1 + t_2 = T$ holds for the successful computation probability maximization.

Proof. Please see Appendix B.

Proposition 3. The maximum successful computation probability can be always achieved by satisfying the following equality, i.e., $t_2^{\text{A}} = t_2^{\text{B}} = t_2^{\text{MEC}} = t_2$.

Proof. Please see Appendix C.

Remark 2. The intuition behind Proposition 3 is that the two users and the MEC server complete the tasks executing within the same time in order to make the best use of the tasks executing time t_2 , reducing the length of the tasks needed to be offloaded. This is beneficial to enhance the successful computation probability, as shown in Lemma 1 (Please find Lemma 1 in the fourth paragraph of Appendix C).

Based on propositions 2 and 3, we can derive the optimal offloading time t_1 , the task executing time t_2 , β_A and β_B , as summarized in Theorem 1.

Theorem 1. To maximize the successful computation probability, we have

$$\begin{cases} t_1^* = T - \frac{2MC}{f_{\rm Mer}(N+2)}, \\ t_2^* = \frac{2MC}{f_{\rm user}(N+2)}, \\ \beta_{\rm A}^* = \beta_{\rm B}^* = \frac{N}{2+N}, \end{cases}$$
(10)

where * denotes the optimal solution corresponding to the optimization variables.

Proof. When $t_2^{A*} = t_2^{B*} = t_2^{MEC*} = t_2^*$ is satisfied, we have $t_2^* = \frac{2MC}{f_{user}(N+2)}$ and $\beta_A^* = \beta_B^* = 1 - \frac{f_{user}t_2^*}{MC}$. Therefore, we have $\beta_A^* = \beta_B^* = \frac{N}{2+N}$. Combing Theorem 1 and Proposition 2, t_1^* can also be determined and the proof is complete.

Remark 3. Theorem 1 reveals the following facts. Firstly, the optimal parameters, i.e., t_1^* , t_2^* , β_A^* , β_B^* , are independent of the locations of the users and the power allocation coefficient λ . This means that the derived results in Theorem 1 can be directly used for randomly deployed users, and that λ^* can be obtained by solving $\frac{\partial \mathcal{P}_s}{\partial \lambda} = 0$ if λ^* exists. Secondly, N determines how many tasks needed to be offloaded for the two users. In particular, the users are willing to offload more tasks to improve the successful computation probability as Nincreases. When $N \to \infty$, we have $\beta_{\rm A}^* = \beta_{\rm B}^* = 1$ and the complete offloading is desirable, while the partial offloading is better than complete offloading when N is finite. This means that the complete offloading is not an optimal working mode if the users have strong computational capability. Thirdly, for a given T, we have derived an optimal time allocation scheme to achieve the tradeoff between the offloading time t_1 and the tasks executing time t_2 in terms of successful computation probability. Lastly, for a given M, the latency can be reduced without decreasing the successful computation probability when we increase f_{user} and f_{MEC} while N remains unchanged. The reason is as follows. When f_{user} increases, $\beta_{\rm A}^*$, $\beta_{\rm B}^*$ and the length of tasks to be offloaded remain unchanged, while the tasks executing time t_2 decreases. In this case, the successful computation probability remains unchange if the offloading time t_1 remains unchanged. Therefore, t_1+t_2 decreases and the latency is reduced.

³As the optimal parameters of the proposed scheme have been obtained in Corollary 1 when $\frac{MC}{f_{\text{user}}} \leq T$, here we only focus on the case with $\frac{MC}{f_{\text{user}}} > T$.



Fig. 2: \mathcal{P}_s versus the length of tasks. Fig. 3: \mathcal{P}_s versus total power of the users.

Fig. 4: The way to reduce latency.

Let us turn our attention to study how the MEC server allocates the power for the two fixed users to maximize the successful computation probability.

Theorem 2. There is a unique optimal power allocation coefficient λ^* for \mathbf{P}_0 and its closed-form expression is written as

$$\lambda^{*} = \begin{cases} x_{N} + \sqrt[3]{\frac{-y_{N} + \sqrt{y_{N}^{2} - g^{2}}}{2m_{1}}} + \sqrt[3]{\frac{-y_{N} - \sqrt{y_{N}^{2} - g^{2}}}{2m_{1}}}, & \text{if } y_{N}^{2} > g^{2} \\ \{0 < \lambda < 1 \mid \lambda = x_{N} + \delta \text{ or } x_{N} - 2\delta\}, & \text{if } y_{N}^{2} = g^{2} \\ \{0 < \lambda < 1 \mid \lambda = x_{N} + 2\delta \cos\left(\phi - \frac{2\pi i}{3}\right), & i = 0, 1, 2\}, \\ & \text{otherwise} \end{cases}$$
(11)

where $a_1 = \frac{\gamma_1(1+d_A^{\alpha})}{1+d_B^{\alpha}}$, $a_2 = \frac{(1+d_B^{\alpha})\gamma_2}{\rho}$, $a_3 = \frac{\gamma_1(1+d_A^{\alpha})(1+\gamma_2)}{\rho}$, $m_1 = a_1 + a_3 - a_2 + a_1a_2 - a_1a_3$, $m_2 = 3a_1a_3 - 2a_1 - a_1a_2 - 2a_3$, $m_3 = a_1 - 3a_1a_3 + a_3$, $\theta^2 = 3\delta^2$, $g = 2m_1\delta^3$, $\delta^2 = \frac{m_2^2 - 3m_1m_3}{9m_1^2}$, $x_N = -\frac{m_2}{3m_1}$, $y_N = \frac{2m_2^3}{27m_1^2} - \frac{m_2m_3}{3m_1} + a_1a_3$, $\phi = \frac{1}{3}\arccos\left(-\frac{y_N}{g}\right)$ and the sign of δ is the same of the sign of $\sqrt[3]{\frac{y_N}{2m_1}}$.

Proof. Please see Appendix D.

Based on the above analysis, the optimal network parameters can be determined and summarized as

$$\begin{pmatrix} \beta_{\rm A}^*, \beta_{\rm B}^*, t_1^*, t_2^{b^*}, \lambda^* \end{pmatrix} = \begin{cases} \begin{pmatrix} 0, 0, 0, \frac{MC}{f_{\rm user}}, \text{any value} \end{pmatrix}, & \text{if } \frac{MC}{f_{\rm user}} \leq T, \\ \text{shown in Theorem1 and Theorem2, if } \frac{MC}{f_{\rm user}} > T. \end{cases}$$
(12)

Substituting (12) into (8), the maximum successful computation probability is given as

$$\mathcal{P}_{s}^{*} = \exp\left(-\frac{\left(1+d_{\mathrm{B}}^{\alpha}\right)\gamma^{*}}{\left(1-\lambda\right)\rho} - \frac{\gamma^{*}\left(1+d_{\mathrm{A}}^{\alpha}\right)\left(1+\gamma_{2}\right)}{\lambda\rho}\right) \times \frac{\left(1+d_{\mathrm{B}}^{\alpha}\right)\lambda^{*}}{\left(1+d_{\mathrm{B}}^{\alpha}\right)\lambda^{*} + \gamma^{*}\left(1-\lambda^{*}\right)\left(1+d_{\mathrm{A}}^{\alpha}\right)}, \tag{13}$$

where $\gamma^* = \begin{cases} 2^{\left(T(2+N) - \frac{2MC}{f_{\text{user}}}\right)B_c} - 1, \text{ if } \frac{MC}{f_{\text{user}}} > T, \\ 0, & \text{otherwise.} \end{cases}$

Corollary 2. If $\rho \to \infty$, the maximum successful computation probability approaches one. This means our considered NOMA based MEC network with optimal parameters is able to meet any required successful computation probability by adjusting the transmit power.

Proof. If $\rho \to \infty$, a_2 and a_3 approach zero and (D.3) can be approximated as $a_1\lambda^3 - 2a_1\lambda^2 + a_1\lambda \approx 0$. In this case, $\lambda^* \to 1$ holds. Combing (13), we have $\lim_{\rho \to \infty} \mathcal{P}_s^* = 1$.

IV. SIMULATION RESULTS

In this section, simulation results are provided to verify our derived results and investigate the successful computation probability the proposed offloading scheme can achieve. Unless otherwise specified, the following parameters are used throughout the simulation: N = 5, T = 10ms, $f_{user} =$ 0.5GHz, C = 1000cycle/bit, M = 10kbits, $\sigma^2 = 10^{-9}$ W, $d_A = 5$ m, $d_B = 25$ m, $\alpha = 4$, $t_1 = t_1^*$, $t_2 = t_2^*$, $\lambda = \lambda^*$ and $\beta_A = \beta_B = \beta^*$.

Fig. 2 shows the successful computation probability versus the length of tasks, where three schemes are considered: (i) the proposed scheme, (ii) the full local computation scheme, (iii) the complete offloading scheme. The power allocation λ is set as 0.3. It can be observed that the successful computation probability \mathcal{P}_s decreases with the increase of the length M of tasks. This is because the offloading time t_1 also decreases with the increase of M, resulting in the decrease of \mathcal{P}_s . It can also be seen that the proposed scheme achieves the highest successful computation probability than the complete local computation scheme and the complete offloading scheme. This is because the proposed scheme can decide how many tasks should be offloaded by considering the length of tasks, the difference of their computational capabilities, etc. For example, when M is small, the tasks tend to be locally executed, while when M is large enough, the tasks tend to be completely offloaded.

Fig. 3 shows the successful computation probability versus the total power P under five schemes: (1) the proposed scheme, (2) the fixed offloading via NOMA scheme, (3) the complete offloading via NOMA scheme, (4) the complete offloading via OMA scheme, (5) the full local computation scheme. For the fixed offloading via NOMA scheme, the offloading ratio is fixed as 0.65, 0.85 and 0.95, respectively. One observation is that the successful computation probability increases with the increase of P. This is due to the fact that a larger P brings a larger ρ , resulting in a larger \mathcal{P}_s . Among these schemes, the proposed scheme achieves the highest successful computation probability.

Fig. 4 illustrates the effectiveness of the proposed scheme to reduce latency in *Remark 3* and shows the successful computation probability \mathcal{P}_s (the left vertical axis) and the latency (the right vertical axis) trend with the increase of f_{user} . The offloading time is set as $t_1 = 10 - \frac{2MC}{5 \times 10^8 (N+2)}$ ms. It

$$\mathcal{P}_{s} = \Pr\left(\max\left\{t_{2}^{A}, t_{2}^{B}, t_{2}^{\text{MEC}}\right\} \le t_{2}\right) \Pr\left(\frac{\left(1 + d_{B}^{\alpha}\right)\gamma_{2}}{\left(1 - \lambda\right)\rho} \le |h_{B}|^{2} \le \frac{\left(1 + d_{B}^{\alpha}\right)\lambda\rho|h_{A}|^{2} - \gamma_{1}\left(1 + d_{A}^{\alpha}\right)\left(1 + d_{B}^{\alpha}\right)}{\gamma_{1}\left(1 - \lambda\right)\left(1 + d_{A}^{\alpha}\right)\rho}\right| \max\left\{t_{2}^{A}, t_{2}^{B}, t_{2}^{\text{MEC}}\right\} \le t_{2}\right)$$

$$= \int_{\frac{\gamma_{1}\left(1 + d_{A}^{\alpha}\right)\left(1 + \gamma_{2}\right)}{\lambda\rho}}^{\infty} \exp\left(-\frac{\left(1 + d_{B}^{\alpha}\right)\gamma_{2}}{\left(1 - \lambda\right)\rho} - x\right) - \exp\left(-\frac{\left(1 + d_{B}^{\alpha}\right)\lambda\rho x - \gamma_{1}\left(1 + d_{A}^{\alpha}\right)\left(1 + d_{B}^{\alpha}\right)}{\gamma_{1}\left(1 - \lambda\right)\left(1 + d_{A}^{\alpha}\right)\rho} - x\right)dx$$

$$= \exp\left(-\frac{\left(1 + d_{B}^{\alpha}\right)\gamma_{2}}{\left(1 - \lambda\right)\rho} - \frac{\gamma_{1}\left(1 + d_{A}^{\alpha}\right)\left(1 + \gamma_{2}\right)}{\lambda\rho}\right) - \exp\left(\frac{1 + d_{B}^{\alpha}}{\left(1 - \lambda\right)\rho}\right)\int_{\frac{\gamma_{1}\left(1 + d_{A}^{\alpha}\right)\left(1 + \gamma_{2}\right)}{\lambda\rho}}^{\infty} \exp\left(-\left(\frac{\left(1 + d_{B}^{\alpha}\right)\lambda}{\gamma_{1}\left(1 - \lambda\right)\left(1 + d_{A}^{\alpha}\right)} + 1\right)x\right)dx$$

$$= \frac{\left(1 + d_{B}^{\alpha}\right)\lambda}{\left(1 + d_{B}^{\alpha}\right)\lambda + \gamma_{1}\left(1 - \lambda\right)\left(1 + d_{A}^{\alpha}\right)}\exp\left(-\frac{\left(1 + d_{B}^{\alpha}\right)\gamma_{2}}{\left(1 - \lambda\right)\rho} - \frac{\gamma_{1}\left(1 + d_{A}^{\alpha}\right)\left(1 + \gamma_{2}\right)}{\lambda\rho}\right).$$

$$(A.2)$$

can be observed that with the increase of f_{user} , \mathcal{P}_s remains unchanged while the latency decreases. This is because both the offloading ratios, β_A^* and β_B^* , as well as the offloading time t_1 keep unchanged while the tasks executing time t_2 decreases with the increase of f_{user} . It can be also seen that for the same set of f_{user} , with the increase of N, \mathcal{P}_s increases with the cost of higher latency. This is because with a larger N, the users tend to offload more tasks to achieve a higher \mathcal{P}_s , leading to the increase of the offloading time t_1 .

V. CONCLUSIONS

We have proposed a new offloading scheme for a NOMA based MEC network, which can operate in the partial computation offloading, the full local computation or the complete offloading. We have derived the successful computation probability for the proposed offloading scheme. We also formulated an optimization problem to maximize the successful computation probability by jointly optimizing the parameters of the proposed offloading scheme and obtained the optimal solutions in closed forms. Simulation results were presented to show that our proposed scheme outperforms the existing schemes in terms of successful computation probability.

APPENDIX A

PROOF OF THE PROPOSITION 1

According to the Law of total probability, we derive the successful computation probability from the following two cases, i.e., Case I: $\max\{t_2^A, t_2^B, t_2^{\text{MEC}}\} > t_2$ and Case II: $\max\{t_2^A, t_2^B, t_2^{\text{MEC}}\} \le t_2$. Since there are no variables, i.e., $|h_A|^2$ and $|h_B|^2$, involved in the following two inequalities: $\max\{t_2^A, t_2^B, t_2^{\text{MEC}}\} > t_2$ and $\max\{t_2^A, t_2^B, t_2^{\text{MEC}}\} \le t_2$, we have

Case I:
$$\Pr\left(\max\left\{t_{2}^{A}, t_{2}^{B}, t_{2}^{MEC}\right\} > t_{2}\right) = 1,$$

Case II: $\Pr\left(\max\left\{t_{2}^{A}, t_{2}^{B}, t_{2}^{MEC}\right\} \le t_{2}\right) = 1.$ (A.1)

Applying the Law of total probability and considering (A.1), it is not hard to find that the successful computation probability of Case I equals zero. By the same argument, the successful computation probability of Case II can be rewritten as (A.2), as shown at the top of the next page.

APPENDIX B

PROOF OF THE PROPOSITION 2

Proposition 2 is proven by using contradiction. Let $(t_1^*, t_2^*, \beta_A^*, \beta_B^*, \lambda^*)$ denote the optimal solution for $\mathbf{P_0}$ and the optimal solution satisfies $t_1^* + t_2^* < T$. In addition to the optimal

solution, we construct another feasible solution, denoted by $(t_1^+, t_2^+, \beta_A^+, \beta_B^+, \lambda^+)$, for \mathbf{P}_0 , where we have $t_2^* = t_2^+$, $\beta_A^* = \beta_A^+, \beta_B^* = \beta_B^+, \lambda^* = \lambda^+, t_1^+ > t_1^*$ and $t_1^+ + t_2^+ = T$. Also, the corresponding successful computation probabilities with $(t_1^*, t_2^*, \beta_A^*, \beta_B^*, \lambda^*)$ and $(t_1^+, t_2^+, \beta_A^+, \beta_B^+, \lambda^+)$ are denoted as \mathcal{P}_s^* and \mathcal{P}_s^+ , respectively. Obviously, the solution with $(t_1^+, t_2^+, \beta_A^+, \beta_B^+, \lambda^+)$ satisfies all the constraints of \mathbf{P}_0 . Next, we discuss the relationship between \mathcal{P}_s and t_1 when

Next, we discuss the relationship between \mathcal{P}_s and t_1 when other parameters are fixed. The first-order derivative of \mathcal{P}_s with respect to t_1 is calculated as

$$\frac{\partial \mathcal{P}_s}{\partial t_1} = \frac{\partial \mathcal{P}_s}{\partial \gamma_1} \times \frac{\partial \gamma_{1_s}}{\partial t_1} = \frac{\partial \mathcal{P}_s}{\partial \gamma_1} \times \left(-\frac{\beta_A M}{t_1^2 B_c} 2^{\frac{\beta_A M}{t_1 B_c}} \right). \tag{B.1}$$

Thus, the monotonicity of \mathcal{P}_s with respect to t_1 depends on $\frac{\partial \mathcal{P}_s}{\partial \gamma_1}$, given by

$$\frac{\partial \mathcal{P}_s}{\partial \gamma_1} = -\left(\frac{k_1}{\left(1 + k_1 \gamma_1\right)^2} + \frac{k_3}{1 + \gamma_1 k_1}\right) \exp\left(-k_2 - k_3 \gamma_1\right), \quad (B.2)$$

where $k_1 = \frac{(1-\lambda)(1+d_A^{\alpha})}{(1+d_B^{\alpha})\lambda}$, $k_2 = \frac{(1+d_B^{\alpha})\gamma_2}{(1-\lambda)\rho}$, and $k_3 = \frac{(1+d_A^{\alpha})(1+\gamma_2)}{\lambda\rho}$. Obviously, $\frac{\partial \mathcal{P}_s}{\partial t_1} > 0$ and \mathcal{P}_s increases with t_1 . When $t_1^+ > t_1^*$, we have $\mathcal{P}_s^+ > \mathcal{P}_s^*$, indicating that $(t_1^*, t_2^*, \beta_A^{\alpha}, \beta_B^*, \lambda^*)$ is not the optimal solution.

APPENDIX C PROOF OF THE PROPOSITION 3

We prove Proposition 3 from two steps as follows. In *Step 1*, we demonstrate that the maximum successful probability could be achieved when max $\{t_2^A, t_2^B, t_2^{\text{MEC}}\} = t_2$ by contradiction. In *Step 2*, we show that $t_2^A = t_2^B = t_2^{\text{MEC}}$ holds for successful computation probability maximization. For convenience, suppose that $(t_1^*, t_2^*, \beta_A^*, \beta_B^*, \lambda^*)$ achieves the maximum successful computation probability \mathcal{P}_s^* and that $(t_1^+, t_2^+, \beta_A^+, \beta_B^+, \lambda^+)$ is the other feasible solution for \mathbf{P}_0 . The corresponding successful computation probability with the constructed solution is denoted as \mathcal{P}_s^+ . We also assume that $(t_1^*, t_2^*, \beta_A^*, \beta_B^*, \lambda^*) \neq (t_1^+, t_2^+, \beta_A^+, \beta_B^+, \lambda^+)$. Thus, we have $\mathcal{P}_s^* > \mathcal{P}_s^+$. Proof of *Step 1*. Suppose that max $\{t_2^A, t_2^B, t_2^{\text{MEC}}\} < t_2^*$

Proof of Step 1. Suppose that $\max \{t_2^A, t_2^B, t_2^{MEC}\} < t_2^*$ holds. It can be drawn from Proposition 2 that $t_1^* + t_2^* = T$. Also, it is assumed that $\lambda^* = \lambda^+$, $t_1^+ + t_2^+ = T$, $t_2^+ = \max \{t_2^A, t_2^B, t_2^{MEC}\}$, $\beta_A^+ = \beta_A^*$ and $\beta_B^+ = \beta_B^*$ are satisfied. Obviously, the constructed solution satisfies all the constraints of \mathbf{P}_0 . Besides, since $\beta_A^+ = \beta_A^*$ and $\beta_B^+ = \beta_B^*$ are satisfied, we have $t_2^+ < t_2^*$ and $t_1^+ > t_1^*$. As pointed out in the Proof of the Proposition 2, \mathcal{P}_s increases with t_1 . Hence, we have $\mathcal{P}_s^+ > \mathcal{P}_s^*$. This contradicts the original assumption that $\mathcal{P}_s^* > \mathcal{P}_s^*$. In summary, $\max \{t_2^A, t_2^B, t_2^{MEC}\} = t_2$ holds from successful computation probability maximization.

Proof of Step 2. We firstly introduce Lemma 1 as follows.

Lemma 1. For any given parameters satisfying all the constraints of \mathbf{P}_0 , \mathcal{P}_s decreases with the increase of β_A (or β_B).

Proof. The first-order derivative of \mathcal{P}_s with respect to β_A is written as

$$\frac{\partial \mathcal{P}_s}{\partial \beta_{\rm A}} = \frac{\partial \mathcal{P}_s}{\partial \gamma_1} \times \frac{\partial \gamma_{1_s}}{\partial \beta_{\rm A}} = \frac{\partial \mathcal{P}_s}{\partial \gamma_1} \times \frac{M}{t_1 B_c} 2^{\frac{\beta_{\rm A}M}{t_1 B_c}}.$$
 (C.1)

Considering (B.2), it is easy to find that \mathcal{P}_s decreases with the increase of β_A . Similarly, we can also prove that \mathcal{P}_s decreases with the increase of β_B .

Now we employ contradiction to verify that the maximum successful computation probability could be achieved when $t_2^{A} = t_2^{B} = t_2^{MEC} = t_2$ holds. Assume that $t_2^{A*} = t_2^{B*} = t_2^{MEC*} = t_2^*$ is not satisfied and $t_2^{A+} = t_2^{B+} = t_2^{MEC+} = t_2^+$ holds. We also assume that $\lambda^* = \lambda^+$ holds. Obviously, the constructed solution satisfies all the constraints of \mathbf{P}_0 . It can be derived from $t_2^{A+} = t_2^{B+} = t_2^{MEC+} = t_2^+$ that $t_2^+ = \frac{2MC}{f_{user}(N+2)}$ and $\beta_A^+ = \beta_B^+ = 1 - \frac{f_{user}t_2^+}{MC}$ are satisfied. Based on the conclusion summarized in *Step 1*, there are four cases for the assumption that $t_2^{A*} = t_2^{B*} = t_2^{MEC*} = t_2^*$ is not satisfied as follows: Case 1 with $t_2^{A*} = t_2^* + t_2$ and $t_2^{MEC*} = t_2^*$, Case 2 with $t_2^{A*} = t_2^*$, $t_2^{B*} < t_2$ and $t_2^{MEC*} < t_2$, and $t_2^{MEC*} < t_2$, $t_2^{B*} = t_2^*$ and $t_2^{MEC*} < t_2^*$.

For Case 1, we have $\beta_A^* = 1 - \frac{f_{\text{user}}t_2^*}{MC}$, $\beta_B \ge 1 - \frac{f_{\text{user}}t_2^*}{MC}$ and $\beta_A^* + \beta_B < \frac{2Nf_{\text{user}}t_2^*}{MC}$. Thus, $2 - \frac{2f_{\text{user}}t_2}{MC} \le \beta_A^* + \beta_B^* < \frac{2Nf_{\text{user}}t_2^*}{MC}$ and $1 - \frac{f_{\text{user}}t_2^*}{MC} < \frac{Nf_{\text{user}}t_2^*}{MC}$ are satisfied. Combing Lemma 1, β_A^* and β_B^* are determined, i.e., $\beta_A^* = \beta_B^* = 1 - \frac{f_{\text{user}}t_2^*}{MC}$. Besides, it can be inferred from $1 - \frac{f_{\text{user}}t_2^*}{MC} < \frac{Nf_{\text{user}}t_2^*}{MC}$ that $t_2^* > \frac{MC}{f_{\text{user}}(N+1)}$ holds. Since $t_2^* > t_2^+$, we have $\beta_A^* < \beta_A^+$ and $\beta_B^* < \beta_B^+$. Based on Proposition 2 and Lemma 1, as well as the fact that \mathcal{P}_s increases with t_1 , the following inequalities in (C.2) are satisfied and thus Case 1 is not optimal.

$$\mathcal{P}_{s}^{+} = \mathcal{P}_{s}\left(t_{1}^{+}, T - t_{1}^{+}, \beta_{A}^{+}, \beta_{B}^{+}, \lambda^{*}\right) > \mathcal{P}_{s}\left(t_{1}^{*}, T - t_{1}^{*}, \beta_{A}^{+}, \beta_{B}^{+}, \lambda^{*}\right) \\ > \mathcal{P}_{s}\left(t_{1}^{*}, T - t_{1}^{*}, \beta_{A}^{+}, \beta_{B}^{+}, \lambda^{*}\right) > \mathcal{P}_{s}\left(t_{1}^{*}, T - t_{1}^{*}, \beta_{A}^{*}, \beta_{B}^{+}, \lambda^{*}\right) \\ > \mathcal{P}_{s}\left(t_{1}^{*}, T - t_{1}^{*}, \beta_{A}^{*}, \beta_{B}^{*}, \lambda^{*}\right) = \mathcal{P}_{s}\left(t_{1}^{*}, t_{2}^{*}, \beta_{A}^{*}, \beta_{B}^{*}, \lambda^{*}\right) = \mathcal{P}_{s}^{*}.$$
(C.2)

By the same argument, the fact that Cases 2-4 are also not satisfied can be proven readily and the proof is omitted due to the space limitation.

APPENDIX D Proof of the Theorem 2

In order to obtain the optimal power allocation coefficient λ^* , we firstly study the first-order derivative of \mathcal{P}_s with respect to λ , given by $\frac{\partial \mathcal{P}_s}{\partial \lambda} = \frac{\exp\left(-\frac{a_2}{1-\lambda} - \frac{a_3}{\lambda}\right)}{\lambda + a_1(1-\lambda)} \left(\frac{a_1}{(1-a_1)\lambda + a_1} + \frac{a_3}{\lambda} - \frac{a_2\lambda}{(1-\lambda)^2}\right)$. Obviously, the sign of $\frac{\partial \mathcal{P}_s}{\partial \lambda}$ corresponds with $\Xi = \left(\frac{a_1}{(1-a_1)\lambda + a_1} + \frac{a_3}{\lambda} - \frac{a_2\lambda}{(1-\lambda)^2}\right)$, and thereby we can obtain λ^* by solving $\Xi = 0$, which can be written as

$$\frac{a_1}{(1-a_1)\lambda + a_1} = \frac{a_2\lambda}{(1-\lambda)^2} - \frac{a_3}{\lambda}.$$
 (D.1)

Constrained as $\lambda \in (0, 1)$, $g(\lambda) = \frac{a_2\lambda}{(1-\lambda)^2} - \frac{a_3}{\lambda}$ increases with λ and the range of $g(\lambda)$ is $(-\infty, +\infty)$; $f(\lambda) = \frac{a_1}{(1-a_1)\lambda+a_1}$ with $a_1 > 1$ increases (or with $a_1 < 1$ decreases) as we increase λ , while $f(\lambda) = 1$ when $a_1 = 1$. In other words, within the feasible region, $f(\lambda)$ with $a_1 > 1$ (or $a_1 < 1$) monotonically increases (or decreases) from 1 to a_1 , and $g(\lambda)$ monotonically decreases from positive infinity to negative infinity. Thus, there exists a unique $\lambda^* \in (0, 1)$ with satisfying (D.1), and the λ^* can be obtained by using simple iterative algorithms. Nevertheless, we still derive the closed-form expression for λ^* as follows. Through mathematical calculations we rewrite (D.1) as

$$m_1\lambda^3 + m_2\lambda^2 + m_3\lambda + a_1a_3 = 0.$$
 (D.2)

According to [14], there are three cases for the solution of (D.2) based on the value of y_N , i.e., Case i. If $y_N^2 - g^2 > 0$, there exists a real root and we have

$$\lambda^* = x_N + \sqrt[3]{\frac{-y_N + \sqrt{y_N^2 - g^2}}{2m_1}} + \sqrt[3]{\frac{-y_N - \sqrt{y_N^2 - g^2}}{2m_1}}.$$
 (D.3)

Case ii. If $y_N^2 - g^2 = 0$, there exist two real roots,

 $\lambda^* = \{ 0 < \lambda_i < 1 \, | \, \lambda_i = x_N + \delta, \, \lambda_i = x_N - 2\delta \, \} \,. \tag{D.4}$

Case iii. If $y_N^2 - g^2 < 0$, there exist three roots,

$$\lambda^* = \left\{ 0 < \lambda_i < 1 \, \middle| \, \lambda_i = x_N + 2\delta \cos\left(\phi - \frac{2\pi i}{3}\right), i = 0, 1, 2 \right\}.$$
 (D.5)

REFERENCES

- Y. Mao *et al.*, "A survey on mobile edge computing: The communication perspective," *IEEE Commun. Surv. Tutor.*, vol. 19, no. 4, pp. 2322–2358, Fourthquarter 2017.
- [2] F. Zhou, Y. Wu, R. Q. Hu, and Y. Qian, "Computation rate maximization in UAV-enabled wireless-powered mobile-edge computing systems," *IEEE J. Sel. Areas Commun.*, vol. 36, no. 9, pp. 1927–1941, Sept 2018.
- [3] H. Sun, F. Zhou, and R. Q. Hu, "Joint offloading and computation energy efficiency maximization in a mobile edge computing system," *IEEE Trans. Veh. Technol.*, vol. 68, no. 3, pp. 3052–3056, March 2019.
- [4] L. Dai, B. Wang, Z. Ding, Z. Wang, S. Chen, and L. Hanzo, "A survey of non-orthogonal multiple access for 5G," *IEEE Commun. Surv. Tutor.*, vol. 20, no. 3, pp. 2294–2323, thirdquarter 2018.
- [5] Z. Zhang, H. Sun, and R. Q. Hu, "Downlink and uplink non-orthogonal multiple access in a dense wireless network," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 12, pp. 2771–2784, Dec 2017.
- [6] Z. Ding, D. W. K. Ng, R. Schober, and H. V. Poor, "Delay minimization for NOMA-MEC offloading," *IEEE Signal Process. Lett.*, vol. 25, no. 12, pp. 1875–1879, Dec 2018.
- [7] F. Wang *et al.*, "Multi-antenna NOMA for computation offloading in multiuser mobile edge computing systems," *IEEE Trans. Commun.*, pp. 1–1, 2018.
- [8] A. Kiani et al., "Edge computing aware NOMA for 5G networks," IEEE Internet Things J., vol. 5, no. 2, pp. 1299–1306, April 2018.
- [9] Z. Ding et al., "Joint power and time allocation for NOMA-MEC offloading," arXiv preprint, arXiv:1807.06306, 2018.
- [10] Z. Ding, P. Fan, and H. V. Poor, "Impact of non-orthogonal multiple access on the offloading of mobile edge computing," *IEEE Trans. Commun.*, vol. 67, no. 1, pp. 375–390, Jan 2019.
- [11] S. Akhter and J. Roberts, "Multi-core programming," 2006.
- [12] Z. Yang *et al.*, "A general power allocation scheme to guarantee quality of service in downlink and uplink noma systems," *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7244–7257, Nov 2016.
- [13] Z. Ding et al., "A general MIMO framework for NOMA downlink and uplink transmission based on signal alignment," *IEEE Trans. Wireless Commun.*, vol. 15, no. 6, pp. 4438–4454, June 2016.
- [14] R. W. Nickalls, "A new approach to solving the cubic: Cardans solution revealed," *The Mathematical Gazette*, vol. 77, no. 480, pp. 354–359, 1993.