Incentive Mechanism Design for Unbiased Federated Learning with Randomized Client Participation

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Abstract-Incentive mechanism is crucial for federated learning (FL) when rational clients do not have the same interests in the global model as the server. However, due to system heterogeneity and limited budget, it is generally impractical for the server to incentivize all clients to participate in all training rounds (known as full participation). The existing FL incentive mechanisms are typically designed by stimulating a fixed subset of clients based on their data quantity or system resources. Hence, FL is performed only using this subset of clients throughout the entire training process, leading to a biased model because of data heterogeneity. This paper proposes a game-theoretic incentive mechanism for FL with randomized client participation, where the server adopts a customized pricing strategy that motivates different clients to join with different participation levels (probabilities) for obtaining an unbiased and high-performance model. Each client responds to the server's monetary incentive by choosing its best participation level, to maximize its profit based on not only the incurred local cost but also its intrinsic value for the global model. To effectively evaluate clients' contribution to the model performance, we derive a new convergence bound which analytically predicts how clients' arbitrary participation levels and their heterogeneous data affect the model performance. By solving a non-convex optimization problem, our analysis reveals that the intrinsic value leads to the interesting possibility of bidirectional payment between the server and clients. Experimental results using real datasets on a hardware prototype demonstrate the superiority of our mechanism in achieving higher model performance for the server as well as higher profits for the clients.

I. INTRODUCTION

Federated learning (FL) has recently emerged as an attractive distributed machine learning paradigm, which enables many clients to collaboratively train a machine learning model

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under the coordination of a central server, while keeping the training data private [1]. In FL, each client exploits its local dataset to compute a local model update, and the server periodically aggregates these local model updates to obtain a global model [2]. Because clients and the server in FL usually belong to different entities, clients may not have as much interest in obtaining a high-performance model (i.e., higher accuracy and lower loss) as the server. For instance, a company could adopt the FL framework to train a commercial model by letting its customers train the model with their local data and on their own devices, while the customers (clients) may not be interested in the model. Hence, without sufficient compensation, clients may not be willing to participate due to the associated local cost (e.g., resource consumption for computation and communication), which makes the incentive mechanism design crucial in FL systems [3]-[5].

However, designing an efficient and effective incentive mechanism for FL is challenging due to two unique FL features [6]: 1) clients in FL systems are usually massively distributed with different local resources and independent availability (known as *system heterogeneity*); 2) the training data are unbalanced and non-i.i.d. across the clients (known as *statistical/data heterogeneity*).

Due to system heterogeneity, it is generally impractical to design an incentive mechanism for FL that requires all clients to participate in *all training rounds* (known as full client participation). This is because FL usually involves a large number of clients and multiple training rounds, and incentivizing full client participation requires a considerable monetary compensation, which may be beyond the server's budget. Moreover, clients may be only intermittently available due to their usage patterns, which prevents them from participating in every training round.

In the existing studies (e.g., [7]–[14]), incentive mechanism with partial client participation mainly stimulate a *deterministic* (fixed) subset of "valuable" clients based on their data quantity, computation or communication resources, where FL is performed only using this subset of clients throughout the entire training process. However, these mechanisms may result

in a severely *biased model* because of statistical heterogeneity (e.g., the data at incentivized clients may not be representative of all clients' data), which *fails to converge to the optimal model that would be obtained if all the clients participate in training.* Therefore, we are motivated to study the first key challenging question:

Question 1: *How to design a practical incentive mechanism for FL with partial client participation, to ensure the convergence to a globally optimal unbiased model?*

In tackling the bias issue due to partial client participation, recent works have proposed various client sampling and model aggregation algorithms (e.g., [15]–[22]), where the clients are presumed to be always active (available) upon the server's request. This assumption may not always hold in practice, as clients are independent decision-makers with different local interests. This motivates us to explore the role of *clients' independent participation levels* (probabilities) in incentive mechanism design, where the server offers monetary rewards to stimulate all clients to join with desired participation levels.

From the server's perspective, intuitively, it can achieve a higher model performance if all clients participate at higher levels, whereas the resulting payment may be beyond the limited budget. In addition, due to statistical heterogeneity, clients with higher participation levels may have low data quality (e.g., small datasize or skewed data distribution) and thus have little contribution to the model performance. Therefore, it is critical to design an efficient payment strategy where clients who contribute more towards the global training objective receive more rewards. However, without actually training the model, it is generally impossible to determine how clients' participation level and their non-i.i.d. data affect the model performance. This leads to our second key question:

Question 2: How to incentivize the clients' active participation in FL by measuring the contribution of each client's participation level and local (non i.i.d.) data on the model performance, so as to design an efficient payment strategy?

From the clients' perspective, instead of purely providing training data and services to gain profit, clients may have *intrinsic value (motivation)* to participate in FL, e.g., to obtain a more powerful global model for their own use when they have few or skewed local data samples. Depending on a client's intrinsic value, the server may adopt a flexible pricing mechanism and achieve the proper trade-off between encouraging clients' participation and reducing the overall cost. *To our best knowledge, the role of intrinsic value has not been studied in the FL incentive mechanism.*

In light of the above discussion, this paper proposes a novel incentive mechanism for FL with *randomized partial client participation*, where the server motivates clients to join FL with different participation levels (probabilities) as shown in Fig. 1. Specifically, we model the interaction between the server and clients as a sequential two-stage *Stackelberg game*, where the server (leader) decides a *customized pricing* scheme for each client's participation level to maximize the global



Fig. 1. Incentive mechanism for federated learning with randomized client participation, where the server adopts a customized payment strategy to motivate clients to join with different participation levels q.

model performance. Each client (follower) then responds to the server's monetary incentive by independently choosing its best participating level to maximize its own profit, based on the incurred local cost as well as the intrinsic value for the global model.

We first develop an adaptive model aggregation scheme to guarantee the model unbiasedness for arbitrary independent client participation levels. With the unbiased design, we obtain a new FL convergence bound, which analytically establishes the relationship between the global model performance and clients' arbitrary participation levels with unbalanced and noni.i.d. data without actually training the model.

Then, based on the convergence bound, we obtain the Stackelberg Equilibrium (SE) via solving a non-convex optimization problem, which reveals interesting design principles. Counterintuitively, we show that in order to achieve an unbiased global model, the server will set higher prices for clients who have larger local costs. For the clients, those with higher intrinsic values may receive lower prices (compensations) from the server and choose lower participation levels. When the clients' intrinsic values are higher than a certain threshold, the clients may need to pay for the server for participation, which leads to a *bi-directional payment*.

Finally, we evaluate the result of our proposed game with both real and synthetic datasets on a hardware prototype. Experimental results demonstrate the superiority of our proposed mechanism in achieving higher global model performance for the server as well as higher profits for the clients. For example, for MNIST dataset, our proposed pricing spends 69% less time than the baseline uniform pricing for reaching the same target accuracy under the same budget.

In summary, our key contributions are as follows:

- Unbiased Incentive Mechanism for FL with Randomized Client Participation: To the best of our knowledge, we are the first to study FL incentive mechanism with practically randomized client participation that guarantees the unbiasedness of the obtained global model towards full client participation.
- Convergence Bound for Evaluating Clients' Contribution:

We obtain a new FL convergence upper bound for clients' arbitrary participation levels with unbalanced and noni.i.d. data, which allows us to effectively measure each client's contribution to the global model performance.

- *Intrinsic Value Design:* We prove that the intrinsic value can lead to a flexible bi-directional payment between the server and clients. As far as we know, this is the first work that models the impact of clients' intrinsic value in the FL incentive mechanism.
- Insightful Equilibrium Properties:

We obtain insightful design principles that characterize the impact of the server's and clients' system parameters. In particular, our analysis reveals a counter-intuitive result that the sever may set higher prices for clients who have larger local costs in order to guarantee the model unbiasedness.

• *Experimental Evaluation with Hardware Prototype*: We provide hardware experimental results that demonstrate the superiority of our proposed mechanism with a 69% time savings compared to baseline uniform pricing for MNIST dataset under the same budget.

II. RELATED WORKS

Incentive mechanism design for FL has received significant attention over recent years (for a comprehensive review please refer to [23]). In the literature, existing works mainly focus on designing incentives mechanism with full client participation (e.g., [24]–[27]).

Considering the server's limited budget and clients' heterogeneous local resources, recent FL incentive mechanisms (e.g., [7]–[9]) have considered more practical partial client participation scenarios. However, these works mainly stimulate a *deterministic* subset of "valuable" clients and train the global model only with these clients' data. Although such mechanisms may speed up the training process at a certain level and have merits in eliciting clients' private information, they may result in a biased global model if the selected clients have skewed data. We propose to incentivize all clients to join FL with independent participation probabilities, which enables the server to obtain all clients' data contributions to guarantee the model's unbiasedness.

In addition, most existing incentive mechanisms evaluate clients' contribution based on their data quantity (e.g., [7]–[9], [12], [13]) or computation/communication resources (e.g., [9]–[12], [14], [24], [25], [27]), which do not capture the critical heterogeneous data distribution across clients. Our mechanism measures clients' contribution via a theoretical convergence result for general statistical heterogeneity with unbalanced and non-i.i.d. data.

The organization of the rest of the paper is as follows. Section III introduces federated learning and game model. Section IV presents our new error-convergence bound with randomized client participation level. Section V analyzes the equilibrium of the proposed game and solution insights. Section VI provides hardware-based cross-device experimental results. We conclude this paper in Section VII.

III. FEDERATED LEARNING AND GAME MODEL

We first introduce the basics of FL with partial client participation and the proposed unbiased and independent client participation scheme in Section III-A. Then, we present the incentive mechanism design for client participation level (probability), and describe the decision problems for the server and clients, respectively in Section III-B. Finally, Section III-C presents the strategic interactions between the server and clients with the proposed Stackelberg game as well as the challenges in solving the game.

A. FL with Randomized Partial Client Participation

We consider a typical FL scenario, where a central server wants to learn a model based on the data from a set $\mathcal{N} = \{1, \ldots, N\}$ clients. Each client $n \in \mathcal{N}$ has d_n data samples, $(\boldsymbol{x}_{n,1}, \ldots, \boldsymbol{x}_{n,d_n})$, which is distributed in a non-i.i.d. fashion. Define $f(\boldsymbol{w}; \boldsymbol{x}_{n,i})$ as the loss function, indicating how the machine learning model parameter \boldsymbol{w} performs on the data sample $\boldsymbol{x}_{n,i}$. Thus, the local loss function of client n is

$$F_n(\boldsymbol{w}) := \frac{1}{d_n} \sum_{i=1}^{d_n} f(\boldsymbol{w}; \boldsymbol{x}_{n,i}).$$
(1)

We further denote $a_n = d_n / \sum_{n=1}^N d_n$ as the weight of the *n*-th device, where $\sum_{n=1}^N a_n = 1$. By denoting F(w) as the global loss function, the goal of FL is to solve the following optimization problem [1]:

$$\min_{\boldsymbol{w}} F(\boldsymbol{w}) := \sum_{n=1}^{N} a_n F_n(\boldsymbol{w}).$$
 (2)

Due to limited system bandwidth and clients' diverse availability, the most popular and de facto optimization algorithm to solve (2) is FedAvg [2]. In FedAvg, the server randomly samples a fraction of K clients (known as *partial client participation*) in each round, and each selected client performs multiple (e.g., E) steps of local stochastic gradient descent (SGD) iterations on (1). Then, the server aggregates their resulting local model updates periodically for a given deadline of R rounds or until the global loss (2) converges.

However, considering that clients in FL are independent decision-makers, each client n can decide its own participation level (probability) q_n , instead of using the sampling probability decided by the server. Nevertheless, without a careful algorithm design, the obtained global model can be severely biased due to statistical heterogeneity.

In the following, before we present our incentive mechanism design, we first propose an unbiased model aggregation scheme under an arbitrary independent client participation level $q = \{q_1, \ldots, q_N\}$, such that the obtained model based on our mechanism is unbiased towards full client participation. We define the weighted aggregated model with full client participation for any round r as $\overline{w}^{r+1} := \sum_{n=1}^{N} a_n w_n^{r+1}$. With this, we have the following result.

Lemma 1. (Unbiased FL with Independent Client Participation Level) For clients under an arbitrary participation level *q*, we aggregate the participants' local updates as

$$\boldsymbol{w}^{r+1} \leftarrow \boldsymbol{w}^r + \sum_{n \in \mathcal{S}(\boldsymbol{q})^r} \frac{a_n}{q_n} \left(\boldsymbol{w}_n^{r+1} - \boldsymbol{w}^r \right),$$
 (3)

where $S(q)^r$ is the participating client set in round r. Then, we have

$$\mathbb{E}_{\mathcal{S}(\boldsymbol{q})^r}[\boldsymbol{w}^{r+1}] = \overline{\boldsymbol{w}}^{r+1}.$$
(4)

Proof. We give the proof in Appendix A. \Box

Remark: The interpretation of our aggregation scheme is similar to that of *importance sampling*. More specifically, we inversely re-weight the participant's updated model gradient in the aggregation step (e.g., $\frac{1}{q_n}$ for client n), such that the aggregated model is unbiased towards the true update with full client participation. We note that simply inversely weighting the model updates from the sampled clients does not yield an unbiased global model, i.e. $\mathbb{E}_{\mathcal{K}(q)^{(r)}} [\sum_{i \in \mathcal{K}(q)^{(r)}} \frac{p_i}{Kq_i} \mathbf{w}_i^{(r+1)}] \neq \overline{\mathbf{w}}^{(r+1)}$, as the equality holds only when clients are sampled uniformly at random, i.e., $q_i = 1/N$. Particularly, when $q_n = 1$ for all n, $\mathcal{S}(q)^r$ is the full set with all N clients, and \boldsymbol{w}^{r+1} in (3) recovers $\overline{\boldsymbol{w}}^{r+1}$. Nevertheless, unlike most active sampling schemes (e.g., [15]–[17], [22]) where clients' sampling probabilities q_n^{sam} are dependent with $\sum_{n=1}^N q_n^{\text{sam}} = 1$, the clients' participation levels q_n in our model is independent from each other with the sum $\sum_{n=1}^N q_n$ varying between 0 to N.

B. Mechanism Design for Randomized Client Participation

As clients are independent decision-makers, we will explore the impact of *clients' independent participation levels* (*probabilities*) $q = \{q_1, \ldots, q_N\}$ in the incentive mechanism design. Specifically, under a limited payment budget, the server designs a customized pricing scheme for each client's participation level to maximize the model performance. In the following, we present the decision problems for the server and the clients.

1) Server's Decision Problem: The goal of the server is to minimize the training loss defined in (2) for a certain number of training rounds. To achieve this, the server imposes a set of prices $P = \{P_1, \ldots, P_N\}$ to incentivize each client's independent participation level (probability) $q = \{q_1, \ldots, q_N\}$ under a payment budget B, where P_i represents the price per unit of client *i*'s participating level. Hence, the payment for each client n is P_nq_n .

Let us denote $w^{R}(q)$ as the obtained model after R rounds when clients participate with level q under pricing strategy P. We can formulate the server's problem as the following P1:

P1: min
$$U_s(\boldsymbol{P}, \boldsymbol{q}) := \mathbb{E}\left[F\left(\boldsymbol{w}^R(\boldsymbol{q})\right)\right],$$
 (5a)

s.t.
$$\sum_{n=1}^{N} P_n q_n \le B.$$
 (5b)

The expectation of the objective in (5a) comes from the randomness in client's participation level q and local SGD.

Since the total budget is limited, it is important for the server to design the optimal pricing strategy P to maximize its utility.

Remark: Unlike existing incentive mechanisms in FL where the server always pays for the clients (e.g., $P_n \ge 0$, for all n) [7]–[14], we allow the payment to be bi-directional. This is because, instead of purely providing training data and computing services, if some client n has a high appreciation for the global model as we will show later, it may be willing to pay for the server (i.e., $P_n < 0$).

2) **Clients' Decision Problem**: Each client's goal is to choose its participation level q_n to maximize its utility function U_n , based on its incurred *local cost* and its *intrinsic value* for the global model, which we will explain next.

Local Cost Model. The cost of client n involves resource consumption for model computation and communication as well as the lost opportunity for joining other activities for monetary reward. Intuitively, the higher participation level the higher cost will be, so we model the cost function C_n as

$$C_n = c_n q_n^{\tau}, \ \tau > 1, \ 0 \le q_n \le q_{n,\max}.$$
 (6)

The exponent $\tau > 1$ captures a broad class of convex cost functions, indicating an increasing rate as q_n increases. We let $\tau = 2$ for analytical tractability in the rest of the paper, which is also a standard assumption in economic models when the decision variable is constrained [28], e.g., $q_n \leq q_{n,\text{max}}$. Nevertheless, we claim that our theoretical results in this paper also hold for an arbitrary $\tau > 1$. Parameter $c_n > 0$ is the local cost parameter.

Intrinsic Value Model. In addition to incurring the resource cost, clients may have intrinsic motivation to participate in FL, e.g., obtaining the powerful global model. In order to effectively model the intrinsic value, we denote $F(w_n^*)$ as the loss when client n applies its locally optimal model w_n^* into the global loss function (2), where $w_n^* = \arg \min_w F_n(w)$ is solved using client's local training data on its local loss function $F_n(w)$ as defined in (1). Then, we model the intrinsic value V_n for client n as

$$V_n := v_n \left(F(\boldsymbol{w}_n^*) - \mathbb{E} \left[F\left(\boldsymbol{w}^R(\boldsymbol{q}) \right) \right] \right).$$
(7)

The value of $F(w_n^*) - \mathbb{E} \left[F(\boldsymbol{w}^R(\boldsymbol{q})) \right]$ represents the improvement of model performance due to the participation in FL. Parameter $v_n \ge 0$ is the preference level for the improvement, since clients may have different preferences for the same model improvement. Intuitively, given that $F(\boldsymbol{w}_n^*)$ is a constant value and independent of \boldsymbol{q} , a lower $\mathbb{E} \left[F(\boldsymbol{w}^R(\boldsymbol{q})) \right]$ yields a higher intrinsic value V_n , indicating that client n has an internal drive to minimize the server's utility in (5a).

Based on the above, we formulate each client n's decision problem as follows:

$$\mathbf{P2}: \max_{q_n} U_n(q_n, P_n) := P_n q_n - C_n + V_n \qquad (8a)$$

s.t.
$$0 \le q_n \le q_{n,\max}$$
. (8b)

Remark: As we discussed in the previous subsection, due to the existence of the intrinsic value, clients may have incentives to participate in FL even without monetary reward, i.e.,



Fig. 2. Stackelberg game between the server and Clients.

 $P_n = 0$. In certain cases, when some client n has a very high intrinsic value v_n , it is possible for client n to pay for the server for participation, i.e., $P_n < 0$.

C. Stackelberg Game Formulation

As shown in Fig. 2, we model the sequential decisionmaking between the server and clients as a two-stage Stackelberg game [29], where the server acts as the Stackelberg leader and decides the pricing variables $\mathbf{P} = \{P_1, \ldots, P_N\}$ to minimize its utility defined in (5a) in Stage I. Then, given the server's pricing strategy \mathbf{P} , each client acts as a Stackelberg follower and chooses its reactive participation level q_n to maximize its utility defined in (8a) in Stage II. In the following, we refer to the proposed Stackelberg game as Client Participation Level Game (CPL Game).¹

1) Solution Concept of the Proposed CPL Game: The common solution concept of the CPL Game is Stackelberg equilibrium (SE), which we define as follows.

Definition 1. The Stackelberg equilibrium (SE) of the CPL Game is a set of decisions $\{P^{SE}, q^{SE}\}$ satisfying

$$q_n^{SE}(\boldsymbol{P}) = \arg \max_{q_n(\boldsymbol{P})} U_n\left(q_n(\boldsymbol{P})\right), \forall n \in \mathcal{N},$$
(9a)

$$\boldsymbol{P}^{SE} = \arg\min_{\boldsymbol{P}} U_s\left(\boldsymbol{P}, \boldsymbol{q}^{SE}(\boldsymbol{P})\right). \tag{9b}$$

At a SE, neither the server or the clients has incentive to deviate for better choice. A powerful technique to obtain SE is backward induction [30], where we first solve for clients' decision-making q(P) given the server's pricing scheme P in Stage II, and then move back to Stage I to determine the server's pricing strategy P.

2) Challenges in Solving the CPL Game: Solving the CPL Game, however, is challenging due to the lack of an analytical expression of $\mathbb{E}[F(\boldsymbol{w}^R(\boldsymbol{q}))]$ to characterize the impact of \boldsymbol{q} . Hence, it is difficult for the server to evaluate the clients' contributions and make an efficient pricing decision. Moreover, in general, without actually training the model, it is impossible to find out how \boldsymbol{q} affects the final model $\boldsymbol{w}^R(\boldsymbol{q})$ and the corresponding loss $\mathbb{E}[F(\boldsymbol{w}^R(\boldsymbol{q}))]$.

We will show how to address the above challenge in Section IV, and then we will find the SE solution for the CPL Game in Section V.

IV. CONVERGENCE ANALYSIS FOR RANDOMIZED CLIENT PARTICIPATION LEVEL

In this section, we address the challenge mentioned in Section III-C2 by deriving a new tractable error-convergence bound. The convergence bound establishes the analytical relationship between q and $\mathbb{E}[F(w^R(q))]$, which allows us to approximate $\mathbb{E}[F(w^R(q))]$ in U_s and U_n and analytically solve the PCL Game.

A. Key Assumptions

Assumption 1. For each client $n \in \mathcal{N}$, F_n is L-smooth and μ -strongly convex.

Assumption 2. For each client $n \in N$, the stochastic gradient of F_n is unbiased with its variance bounded by σ_n^2 .

Assumption 3. For each client $n \in N$, the expected squared norm of its stochastic gradient is bounded by G_n^2 .

Assumptions 1 and 2 are commonly made in many existing studies of convex FL problems, such as ℓ_2 -norm regularized linear regression, logistic regression (e.g., [16]–[18], [21], [31], [32]). Nevertheless, in Assumption 3, we make an assumption for each client n with the bound G_n instead of the bound G for all the clients as in [16]–[18], [21], [31], [32]. This is because if clients' data are i.i,d., then G_n would be the same across the clients since each client locally performs SGD from the same data distribution. However, when clients have non-i.i.d. data distribution, the values of G_n would be different, which not only characterizes the data heterogeneity in our convergence result but also yields a more accurate pricing design (as we will show in Section V). In practice, we can estimate G_n by letting the participated clients send back their actual local stochastic gradient norms computed along the trajectory of the model updates.

B. Bounded Model Variance

We first present the introduced variance of the aggregated model w^{r+1} in (3) due to randomized client participation.

Lemma 2. The variance between the aggregated model w^{r+1} in (3) and the global model with full participation \overline{w}^{r+1} is bounded as

$$\mathbb{E}_{\mathcal{S}(\boldsymbol{q})^{r}} \left\| \boldsymbol{w}^{r+1} - \overline{\boldsymbol{w}}^{r+1} \right\|^{2} \leq 4 \sum_{n=1}^{N} \frac{(1-q_{n}) a_{n}^{2} G_{n}^{2}}{q_{n}} \left(\eta^{r} E \right)^{2}.$$
(10)

Proof Sketch. The proof follows a similar argument of Lemma 5 in [16], except that our client participation levels q_n are independent among each other, which does not satisfy the assumption of $\sum_{n=1}^{N} q_n = 1$ made in [16].

Remark: When $q_n = 1$ for all *n*, the variance in (10) is equal to zero, since the aggregated model w^{r+1} in the left hand side of (10) is the same as \overline{w}^{r+1} because $q_n = 1$ implies full participation.

C. Main Convergence Result

Theorem 1. (Convergence Upper Bound under an Arbitrary q) Consider any given client participation level $q = \{q_1, \ldots, q_N\}$ and the unbiased aggregation in Lemma 1, if we

¹Our mechanism assumes complete information, focusing on evaluating clients' contribution and designing an effective payment scheme. For the more realistic incomplete information scenario, we can adopt *Bayesian* method to model and analyze the performance similarly with a higher complexity.

choose the decaying learning rate $\eta_r = \frac{2}{\max\{8L, \mu E\} + \mu r}$, the optimality gap after R rounds satisfies

$$\mathbb{E}[F(\boldsymbol{w}^{R}(\boldsymbol{q}))] - F^{*} \leq \frac{1}{R} \left(\alpha \sum_{n=1}^{N} \frac{(1-q_{n}) a_{n}^{2} G_{n}^{2}}{q_{n}} + \beta \right), \quad (11)$$

where $\alpha = \frac{8LE}{\mu^2}$, $\beta = \frac{2L}{\mu^2 E} A_0 + \frac{12L^2}{\mu^2 E} \Gamma + \frac{4L^2}{\mu E} \| \boldsymbol{w}_0 - \boldsymbol{w}^* \|^2$, $A_0 = \sum_{n=1}^N a_n^2 \sigma_n^2 + 8 \sum_{n=1}^N a_n G_n^2 (E-1)^2$, and $\Gamma = F^* - \sum_{n=1}^N a_n F_n^*$.

Proof. We give the proof in Appendix B.

We summarize the key insights of Theorem 1 as follows:

- The bound in (11) is valid for arbitrary independent client participating level *q* and variant number of participating clients in each round (i.e., ∑_{n=1}^N q_n can vary between 0 to N), which is the key difference from existing convergence results with active client sampling [15]–[19], [21], [22].
- The bound in (11) characterizes how randomized partial client participation (i.e., q_n < 1) worsens the convergence rate compared to full client participation. It also indicates that in order to obtain an unbiased global model, all clients need to participate with non-zero probability for model convergence, i.e., q_n > 0, for all n. This is because when q_n → 0, it will take infinite number of rounds R for convergence. Our bound also explains why only incentivizing and training part of the clients in existing mechanisms (e.g., [7]–[14]) may fail to converge to the optimal global model.
- The convergence bound in (11) establishes the relationship between the expected loss function $\mathbb{E}[F(\boldsymbol{w}^{R}(\boldsymbol{q}))]$, clients' participation level \boldsymbol{q} , and their heterogeneous data a_nG_n . In other words, how clients' unbalanced data (a_n) and non-i.i.d. data distribution (G_n) affect the model training. This not only provides analytical utility expressions for the server U_s and clients U_n , but also enables an efficient pricing strategy as we will show in Section V.

The use of G_i in our convergence analysis (instead of more accurate instantaneous gradient norms) is mainly due to the challenging "chicken and egg" problem. Specifically, before the model has been fully trained, it is generally impossible to know exactly how different FL configurations (e.g., clients' different participation levels in this paper) affect the FL performance. Therefore, to optimize the FL performance, we need to have a lightweight surrogate that can (approximately) predict what will happen if we choose a specific configuration. A common surrogate used for this purpose is the convergence upper bound, which has become a common practice in the communications and networking community (e.g., [22], [33]-[37]). Moreover, our experiments in Section VI demonstrate the superiority of our designed pricing scheme based on the convergence bound, in terms of achieving higher global model performance and higher client profits compared to baseline schemes.

V. STACKELBERG EQUILIBRIUM ANALYSIS

In this section, we use the obtained convergence bound in Theorem 1 to approximate $\mathbb{E}[F(\boldsymbol{w}^{R}(\boldsymbol{q}))]$ in the server's utility U_s and clients' utility U_n , and solve the CPL Game via backward induction. We first solve clients' decision-making \boldsymbol{q} given the server's pricing scheme \boldsymbol{P} in Stage II, and then move back to Stage I to determine \boldsymbol{P} . Finally, we characterize the key insights of the optimal solution.

A. Client's Decision at Stage II

We approximate $\mathbb{E}[F(\boldsymbol{w}^{R}(\boldsymbol{q}))]$ with the convergence bound in (11), and rewrite client *n*'s problem given the server's pricing strategy \boldsymbol{P} as the following Problem **P2**':

$$\mathbf{P2}': \max_{q_n} U_n(q, P_n) = P_n q_n - c_n q_n^2 + v_n \left[F(w_n^*) - F^* - \frac{1}{R} \left(\alpha \sum_{n=1}^N \frac{(1-q_n) a_n^2 G_n^2}{q_n} + \beta \right) \right],$$
(12a)
s.t. $0 \le q_n \le q_{n,\max}.$ (12b)

We observe that the objective function (12a) is concave in q_n . Along with the linear constraints in (12b), we conclude that Problem **P2'** is *concave*. Therefore, the optimal solution of Problem **P2'** is unique, which is the best choice of q_n to maximize its own utility.

Based on the first order condition, the optimal choice of $q_n^*(P_n)$ for client *n* satisfies

$$P_n + v_n \frac{\alpha}{R} \frac{a_n^2 G_n^2}{q_n^{*2}} - 2c_n q_n^* = 0.$$
(13)

Although the closed form solution of $q_n^*(P_n)$ is complicated because (13) is a cubic equation, we can show that $q_n^*(P_n)$ is a monotonically increasing convex function in P_n .

Based on the client's optimal solution in (13), we move to Stage I of the CPL game.

B. Server's Decision at Stage I

In Stage I, the server chooses its pricing strategy based on all the clients' best responses. In other words, the server substitutes $q_n^*(P_n)$ into its utility function in (5) for obtaining the optimal price vector P under the budget constraint B.

However, since the analytical expression of $q_n(P_n)$ is complicated, it is difficult to obtain the optimal P in the server's decision problem. As $q_n^*(P_n)$ is unique, we can write its inverse function based on (13), i.e., $P_n(q_n) = 2c_nq_n - a_n^2G_n^2/q_n^2$. Then, we substitute this expression of $P_n(q_n)$ into the server's utility function and solve Stage I problem in (5). Therefore, with the obtained convergence bound and the constraints of q_n , we rewrite problem in (5) as the following Problem **P1**':

P1': min
$$F^* + \frac{1}{R} \left(\alpha \sum_{n=1}^{N} \frac{(1-q_n)a_n^2 G_n^2}{q_n} + \beta \right)$$
 (14a)

s.t.
$$\sum_{n=1}^{N} \left(2c_n q_n - \frac{\alpha}{R} \frac{v_n a_n^2 G_n^2}{q_n^2} \right) q_n \le B,$$
 (14b)

$$0 \le q_n \le q_{n,\max}.\tag{14c}$$

Although the objective function in (14a) is convex in q_n , the budget constraint in (14b) is not convex in q_n . Thus, Problem **P1'** is *non-convex*. To efficiently solve Problem **P1'**, we define a new control variable

$$M := \sum_{n=1}^{N} c_n q_n^2,$$
 (15)

where $0 \le M \le \sum_{n=1}^{N} c_n$. Then, we rewrite Problem **P1**' as the following Problem **P1**'':²

$$\mathbf{P1}'': \quad \min_{\boldsymbol{q},M} g(\boldsymbol{q},M) := \frac{\alpha}{R} \sum_{i=1}^{N} \frac{(1-q_n)a_n^2 G_n^2}{q_n}$$

s.t. $2M - \sum_{n=1}^{N} \frac{\alpha}{R} \frac{v_n a_n^2 G_n^2}{q_n} \le B,$
 $\sum_{n=1}^{N} c_n q_n^2 = M, \quad 0 \le q_n \le q_{n,\max}.$ (16)

For any fixed feasible value of M, Problem $\mathbf{P1}''$ is convex because the objective function and the constraints are convex. Hence, we can approximately solve Problem $\mathbf{P1}''$ in two steps. First, for any fixed M, we solve for the optimal $q^*(M)$ in Problem $\mathbf{P1}''$ via a convex optimization tool, e.g., CVX [38]. This allows us to write the objective function of Problem $\mathbf{P1}''$ as $g(q^*(M), M)$. Then we will further solve the problem by using a linear search method with a fixed step-size ϵ_0 over the interval $\left[0, \sum_{n=1}^{N} c_n q_{n,\max}^2\right]$, which leads to $q^*(M^*(\epsilon_0)) = \arg\min_{M(\epsilon_0)} q^*(M(\epsilon_0))$.

Once we obtain q^* from Problem P1", we immediately have the optimal price P^* via (13) as follows,

$$P_n^* = 2c_n q_n^* - v_n \frac{\alpha}{R} \frac{a_n^2 G_n^2}{q_n^{*2}}.$$
 (17)

Finally, we conclude that the obtained solution pair $\{q^*, P^*\}$ based on backward induction are the SE $\{P^{SE}, q^{SE}\}$ for the proposed CPL game [29]. Notably, based on the relationship between q_n^{SE} and P_n^{SE} in (17), we highlight that clients with low participation level q_n^{SE} will receive a low price P_n^{SE} , which leads to a low payment $P_n^{SE}q_n^{SE}$ from the server. In particular, if these clients with low participation levels also have high intrinsic value v_n , they may need to pay for the server, i.e., $P_n^{SE} < 0$, which we show in the next section.

C. Properties of SE in the CPL Game

This subsection presents some interesting properties of the obtained SE $\{P^{SE}, q^{SE}\}$. Before that, we first give the following lemma which leads to our property analysis.

Lemma 3. At the SE of the CPL Game, the server's budget constraint is tight.

Proof Sketch. The idea of this proof is to use contradiction, which we omit due to page limitation. \Box

1) Impact of the server's budget B: We first show the impact of the server's budget B on SE $\{P^{SE}, q^{SE}\}$.

Proposition 1. Both P^{SE} and q^{SE} increase in budget B.

Proof. Given
$$B = \sum_{n=1}^{N} \left(2c_n q_n^{*2} - \frac{\alpha}{R} \frac{v_n a_n^2 G_n^2}{q_n^*} \right)$$
 from Lemma 3,

we have $\partial B/\partial q_n^* = 4c_n q_n^* + \frac{\alpha}{R} \frac{v_n a_n^* G_n^*}{q_n^*} > 0$. Thus, q_n^* increases in *B*. Next, based on the monotonically increasing relationship between q_n^* and P_n^* in (17), we conclude that P_n^* also increases in *B*.

Proposition 1 shows that when the budget B is higher, the server could increase its price P_n to incentivize higher client participation level q_n to decrease the global loss for improving the model performance. Similarly, clients also have incentive to increase their participation level for more profit.

2) Impact of clients' parameters: This subsection shows how client's parameters, including data quality, local cost, and intrinsic value affect the values of $\{P^{SE}, q^{SE}\}$ at SE. For simplicity, we consider those clients whose equilibrium choices are in the (strict) interior of domains, i.e., $0 < q_n^{SE} < q_{n,max}$. **Theorem 2.** (Impact of clients' parameters on q^{SE}) For any clients i and j whose equilibrium q_i^{SE} and q_j^{SE} are in the interior of the domains, we must have $\frac{c_i q_i^{*3}}{a_i^2 G_i^2} + v_i = \frac{c_j q_j^{*3}}{a_j^2 G_j^2} + v_j$.

Proof. We give the proof in Appendix C.

We summarize the key insights of Theorem 2 as follows:

- (*Heterogeneous Data quality*) Clients with larger a_nG_n (e.g., large data size and gradient norm upper-bound) have higher participation level q_n^{SE} , given the same parameters c_n and v_n among clients.
- (Local cost) Clients with large local cost parameter c_n have lower participate level q_n^{SE} , given the same parameters a_nG_n and v_n among clients.
- (Intrinsic value) It is counter-intuitive that a client with a larger intrinsic value parameter v_n has a lower participation rate q_n^{SE} , given the same a_nG_n and c_n among clients. This is because although a higher q_n benefits its intrinsic value if its v_n is large, the server will set a lower price P_n (as we show in the next theorem) which yields a smaller payment P_nq_n . Considering that the larger q_n also incurs larger cost $c_nq_n^2$, q_n should not be large.

Theorem 3. (Impact of clients' parameters on \mathbf{P}^{SE}) For any client *n* whose equilibrium q_n^{SE} is in the interior domain, we must have

$$P_n^* = \left(\frac{2\alpha c_n^2 a_n^2 G_n^2}{R}\right)^{\frac{1}{3}} \left[\left(\frac{1}{\lambda^*} - v_n\right)^{\frac{1}{3}} - 2\left(\frac{v_n^{\frac{3}{2}}}{\frac{1}{\lambda^*} - v_n}\right)^{\frac{4}{3}} \right].$$
(18)

In addition, there exists a threshold v^t , such that $P_n^{SE} > 0$, if $v_n \leq v^t$, and $P_n^{SE} < 0$ otherwise.

Proof. We give the proof in Appendix D.

²We omit constants F^* and β in the objective of **P1'** for simplicity.



Fig. 3. Cross-device FL prototype with a laptop serving as the central server and 40 Raspberry Pis serving as clients.

Before we present the insights of Theorem 3, we show how parameters c_n and $a_n G_n$ affect P_n^{SE} with the following result.

Corollary 1. With the same threshold v^t in Theorem 3, and for any clients *i* and *j* whose equilibrium q_i^{SE} and q_j^{SE} are in the interior of the domains and satisfy $c_i a_i G_i > c_j a_j G_j$,

- 1) if $v_i < v_j < v^t$, then $P_i^{SE} > P_j^{SE} > 0$; 2) if $v_i > v_j > v^t$, then $P_i^{SE} < P_j^{SE} < 0$.

We summarize the key insights of Theorem 3 and Corollary 1 as follows:

- (Intrinsic value) Our result provides a quantitative criterion, which indicates the payment direction of P_n^{SE} between the server and the clients.
- (Data quality) We have shown that, no matter which payment direction, the price P_n^{SE} is higher for clients who have large values of $a_n G_n$, given the same parameters c_n and v_n among clients. This result demonstrates the suboptimality of existing mechanisms with uniform pricing or data quantity-based pricing.
- (Local cost) It is counter-intuitive that clients with large c_n will have higher price P_n^{SE} when parameters $a_n G_n$ and v_n are the same as other clients. However, this result makes sense because a client with a large local cost c_n tends to join with low participation level q_n , which can cause a negative impact on the server's utility. Hence, to prevent this, the server will set a higher price.

VI. EXPERIMENTAL EVALUATION

In this section, we empirically evaluate the performance of our proposed mechanism with different datasets on our hardware-based cross-device FL prototype, as illustrated in Fig. 3. Our prototype consists of N = 40 Raspberry Pis serving as clients and a laptop computer acting as the central server. All devices are interconnected via an enterprise-grade Wi-Fi router. We develop a TCP-based socket interface for the communication between the server and clients.

In the following, we first present the evaluation setup and then show the experimental results.

TABLE I SYSTEM PARAMETERS FOR DIFFERENT SETUPS

Setup\Parameter	budget B	local cost \overline{c}	intrinsic value \overline{v}
Setup 1	200	50	4,000
Setup 2	40	20	30,000
Setup 3	500	80	10,000

A. Experimental Setup

1) Datasets and Implementations: Following similar setups as in [16], [22], we evaluate our results on three datasets, with detailed implementations as follows:

- Setup 1: The first experiment uses the Synthetic dataset, which generates 60-dimensional random vectors as input data with a non-i.i.d. Synthetic (1,1) setting. We generate 22,377 data samples and distribute them among the devices in an unbalanced power-law distribution.
- Setup 2: The second experiment uses MNIST dataset, where we randomly subsample 14, 463 data samples from MNIST and distribute them among the devices in an unbalanced (following the power-law distribution) and non-i.i.d. (i.e., each device has 1-6 classes) fashion.
- Setup 3: The third experiment uses EMNIST dataset, where we randomly subsample 35, 155 lower case character samples from the EMNIST dataset and distribute among the devices in an unbalanced (i.e., numbers of data samples at each device follows a power-law distribution) and non-i.i.d. fashion (i.e., each device has a randomly chosen number of classes, ranging from 1 to 10).

2) Model and Parameters: For all experiments, we adopt the convex multinomial logistic regression model, with $w_0 = 0$ and SGD batch size 24. We use an initial learning rate of $\eta_0 = 0.1$, a decay rate of 0.996, and a local iteration number E = 100. We estimate the task-related parameters α and data quality-related parameter G_n for each setup following a similar approach as [22]. We let $q_{n,max} = 1$ for all n and training round R = 1000 for all three setups. Table I shows the parameter settings for budget B, mean local cost parameter \overline{c} , and mean intrinsic value parameter \overline{v} for each setup, with \overline{c} and \overline{v} following exponential distribution among clients. We note that the parameters and results are general, and the specific parameters for a scenario can often be obtained through measurement in practice.

B. Experimental Results

We first compare the performance of our proposed optimal pricing with other benchmark pricing schemes, and then we evaluate the impact of different system parameters on the model performance and Equilibrium solution. For all three setups, we average each experiment over 20 independent runs.

1) Comparison with Different Pricing Schemes: We compare our proposed optimal pricing P^* with two benchmark pricing schemes: uniform pricing P^{u} where the server sets the same price for all devices, and weighted pricing P^{W} where



Fig. 4. Model performance of loss and accuracy for Setups 1-3 for different pricing schemes.

clients' prices are proportional to their datasize. Then, we run experiments for all three Setups using the corresponding optimal clients' participation levels q^* , q^u , and q^w , based on the above three pricing schemes. Fig. 4 illustrates the global model performance of global loss and test accuracy for different pricing schemes for Setups 1–3 with evaluating time 1,000, 2,000, and 3,000 seconds, respectively.

Loss. As shown in Figs. 4(a), 4(c), and 4(e), our proposed optimal pricing scheme *achieves lower global loss with smaller variance* throughout the evaluating process compared to the other benchmarks under the same budget. Specifically, for EMNIST dataset, Fig. 4(c) shows that our scheme reaches a target loss of 0.33 using around 33.1% less time than weighted pricing and around 52.9% less time than the uniform pricing. We summarize the superior performances of loss in Table II.

Accuracy. Figs. 4(b), 4(d), and 4(f) show that our proposed optimal pricing scheme *achieves higher test accuracy with smaller variance* compared to the other benchmarks under the same budget. In particular, for MNIST dataset, Fig. 4(d) shows that our proposed pricing scheme spends around 20.0% less time than the weighted pricing and around 46.5% less time than the uniform pricing for reaching a target accuracy of 69%. We summarize the superior performances of accuracy

TABLE II Running Time for reaching the target loss (shown in Fig. 4) for different pricing schemes

Setup \ Pricing Schemes	Proposed	Weighted	Uniform
Setup 1	711 s	791 s	903 s
Setup 2	926 s	1,384 s	1,969 s
Setup 3	1,448 s	$1,758 \ s$	2,735 s

TABLE III Running time for reaching the target accuracy (shown in Fig. 4) for different pricing schemes

Setup \ Pricing Schemes	Proposed	Weighted	Uniform
Setup 1	668 s	759 s	871 s
Setup 2	411 s	1,096 s	$1,767 \ s$
Setup 3	1,412 s	1,766 s	2,645 s

TABLE IV Total clients' utility gain of our proposed pricing scheme over the other two benchmark pricing schemes

Setup \ Gain	$\sum_{n=1}^N U_n^* - \sum_{n=1}^N U_n^{u}$	$\sum_{n=1}^{N} U_n^* - \sum_{n=1}^{N} U_n^*$	
Setup 1	10,839	5,175	
Setup 2	77,975	75,909	
Setup 3	86,200	22,413	

in Table III.

Clients' Utility. Table IV shows the gain of clients' total utility of our proposed pricing over the other two benchmarks. We see that our proposed optimal pricing scheme *yields higher total clients' utility* compared to the other two benchmark pricing schemes.

The above observations validate that our designed mechanism with optimal pricing can *incentivize high-quality clients to participate with higher participation levels* under the same budget, which benefits both the server and clients' utilities.

2) *Impact of System Parameters:* Fig. 5 - Fig. 7 show the impact of system parameters on the global model performance of our proposed mechanism with evaluating time 600 seconds. Due to page limit, we evaluate each parameter with one setup.

Impact of \overline{v} : Fig. 5 shows that as clients' mean intrinsic value \overline{v} increases in Setup 1, the obtained model achieves a lower loss and a higher accuracy, respectively. This observation is because when clients have more interest in the global model, they have internal motivation to participate in higher levels. In addition, as predicted by our theory, we show in Table V that as \overline{v} increases, the number of clients whose payment is negative also increases, e.g., from 0 to 5.

Impact of \overline{c} : Fig. 6 shows that as clients' mean local cost \overline{c} decreases in Setup 2, the obtained model achieves a lower loss, a higher accuracy, and a smaller variance. This is because large cost prevents clients from participating in higher levels.

Impact of B: Fig. 7 shows that as budget B increases in Setup 3, the obtained model achieves a lower loss, a higher

TABLE V Number of negative payment clients for different $\overline{\upsilon}$

Setup 1 (Synthetic)	$\overline{v} = 0$	$\overline{v} = 4,000$	$\overline{v} = 80,000$
Client number with $P_n < 0$	0	3	5



Fig. 5. Model performance for different \overline{v} for Setup 1.



Fig. 6. Model performance for different \overline{c} for Setup 2.



Fig. 7. Model performance for different B for Setup 3.

accuracy, and a smaller variance. This observation is because more budget allows more clients to participate in higher levels.

VII. CONCLUSION AND FUTURE WORK

In this work, we proposed a randomized client participation mechanism for FL, which guarantees that the obtained model is unbiased and converges to the globally optimal model. We derived a new tractable convergence bound that analytically characterizes the impact of clients' participation levels and their non-i.i.d data on the server's model performance, which yields a more accurate and customized pricing scheme. To characterize clients' internal interests in the global model, we introduced and modeled a new *intrinsic value* in clients' utilities, which allows clients to pay for the server. We showed both theoretical and empirical evidence that the intrinsic value plays a critical role in the server and clients' decision-making. We conducted extensive experiments on a developed hardware prototype to demonstrate the superiority of our mechanism.

Our mechanism serves as an initial step towards the incentive mechanism design to achieve an unbiased and convergence guaranteed model with practical randomized client participation. In the future, we will extend our incentive mechanism for *incomplete information* scenarios using *Bayesian* method. We will further refine our cost model by decoupling the local cost into computation and communication consumption, and design an effective measurement to model clients' intrinsic value.

APPENDIX

A. Proof of Lemma 1

Substituting (3) into (4), we have

$$\mathbb{E}_{\mathcal{S}(q)^{r}}\left[\mathbf{w}^{r+1}\right] = \mathbf{w}^{r} + \mathbb{E}\left[\sum_{n \in \mathcal{S}(q)^{r}} \frac{a_{n}}{q_{n}} \left(\mathbf{w}_{n}^{r+1} - \mathbf{w}^{r}\right)\right]$$
$$= \mathbf{w}^{r} + \sum_{n=1}^{N} q_{n} \frac{a_{n}}{q_{n}} \left(\mathbf{w}_{n}^{r+1} - \mathbf{w}^{r}\right)$$
$$= \mathbf{w}^{r} + \sum_{n=1}^{N} a_{n} \left(\mathbf{w}_{n}^{r+1} - \mathbf{w}^{r}\right)$$
$$= \mathbf{w}^{r} + \overline{\mathbf{w}}^{r+1} - \mathbf{w}^{r} = \overline{\mathbf{w}}^{r+1},$$
(19)

where n in the first equation is a random variable in the randomly sampled set $S(q)^{(r)}$, which concludes the proof.

B. Proof Sketch of Theorem 1

Following a similar proof of convergence under full client participation [16], [32], we first show the convergence result under full client participation is $\mathbb{E}[F(\overline{\boldsymbol{w}}^R))] - F^* \leq \beta/R$, where $\mathbb{E}[F(\overline{\boldsymbol{w}}^R))]$ is the expected global loss after R rounds with full participation, and β is the same as in (11). Then, we use mathematical induction to obtain a non-recursive bound on $\mathbb{E}_{\mathcal{S}(\boldsymbol{q})^r} \| \boldsymbol{w}^R - \boldsymbol{w}^* \|^2$, and show that its difference compared to the bound of full participation $\mathbb{E} \| \overline{\boldsymbol{w}}^R - \boldsymbol{w}^* \|^2$ is the variance introduced in (10). After that, we converted the bound of $\mathbb{E}_{\mathcal{S}(\boldsymbol{q})^r} \| \boldsymbol{w}^R - \boldsymbol{w}^* \|^2$ to $\mathbb{E}[F(\boldsymbol{w}^R(\boldsymbol{q}))] - F^*$ using L-smoothness and strong convexity of $F(\cdot)$, which yields the additional term of $\alpha \sum_{n=1}^{N} \frac{(1-q_n)a_n^2 G_n^2}{q_n}$ in (11) compared to the upper bound with full client participation. \Box

C. Proof of Theorem 2

We write the Lagrangian function $L(q, \lambda, \mu_n, \gamma_n)$ of **P1**' as

$$L = F^* + \frac{1}{R} \left(\alpha \sum_{n=1}^{N} \frac{(1-q_n)a_n^2 G_n^2}{q_n} + \beta \right) + \sum_{n=1}^{N} \gamma_n (q_n - q_{n,\max}) - \sum_{n=1}^{N} \mu_n q_n + \lambda \left(\sum_{n=1}^{N} \left(2c_n q_n^2 - \frac{\alpha}{R} \frac{v_n a_n^2 G_n^2}{q_n} \right) - B \right),$$
(20)

where $\lambda > 0$, $\mu_n \ge 0$ and $\gamma_n \ge 0$ are Lagrangian multipliers (dual variables). Although Problem **P1**' is non-convex, the Karush-Kuhn-Tucker (KKT) conditions are *necessary for optimality*. Thus, for the first order condition, we must have

$$-\frac{\alpha}{R}\frac{a_n^2 G_n^2}{q_n^{*2}} + \lambda^* \left(4c_n q_n^* + \frac{\alpha}{R}\frac{v_n a_n^2 G_n^2}{q_n^{*2}}\right) - \mu_n^* + \gamma_n^* = 0.$$
(21)

Given that $0 < q_n^* < q_{n,\max}$, we have $\mu_n^* = 0$ and $\gamma_n^* = 0$ due to the *Complementary Slackness* in the KKT conditions, which we rewrite (21) as follows

$$\frac{1}{\lambda^*} = \frac{4R}{\alpha} \frac{c_n q_n^{*3}}{a_n^2 G_n^2} + v_n.$$
 (22)

The result is obtained since $1/\lambda^*$ is independent of index *n*.

D. Proof of Theorem 3

We exploit the relationship between q_n^* and P_n^* in (17), and substitute it into the first order condition (22), which leads to (18). By letting $P_n^* = 0$, we have $v_n = 1/(3\lambda^*)$. Therefore, given that P_n^* in (18) decreases in v_n , we must have $P_n^* >$ 0 when $v_n < 1/(3\lambda^*)$, and $P_n^* < 0$ when $v_n > 1/(3\lambda^*)$. Hence, we conclude the proof by letting the threshold $v^t =$ $1/(3\lambda^*)$.

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