Secure *k*-Nearest Neighbor Query over Encrypted Data in Outsourced Environments

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Abstract

For the past decade, query processing on relational data has been studied extensively, and many theoretical and practical solutions to query processing have been proposed under various scenarios. With the recent popularity of cloud computing, users now have the opportunity to outsource their data as well as the data management tasks to the cloud. However, due to the rise of various privacy issues, sensitive data (e.g., medical records) need to be encrypted before outsourcing to the cloud. In addition, query processing tasks should be handled by the cloud; otherwise, there would be no point to outsource the data at the first place. To process queries over encrypted data without the cloud ever decrypting the data is a very challenging task. In this paper, we focus on solving the k-nearest neighbor (kNN) query problem over encrypted database outsourced to a cloud: a user issues an encrypted query record to the cloud, and the cloud returns the k closest records to the user. We first present a basic scheme and demonstrate that such a naive solution is not secure. To provide better security, we propose a secure kNN protocol that protects the confidentiality of the data, user's input query, and data access patterns. Also, we empirically analyze the efficiency of our protocols through various experiments. These results indicate that our secure protocol is very efficient on the user end, and this lightweight scheme allows a user to use any mobile device to perform the kNN query.

Keywords: Security, k-NN Query, Encryption, Cloud Computing

1 Introduction

As an emerging computing paradigm, cloud computing attracts many organizations to consider utilizing the benefits of a cloud in terms of cost-efficiency, flexibility, and offload of administrative overhead. In cloud computing model [13, 16], a data owner outsources his/her database T and the DBMS functionalities to the cloud that has the infrastructure to host outsourced databases and provides access mechanisms for querying and managing the hosted database. On one hand, by outsourcing, the data owner gets the benefit of reducing the data management costs and improves the quality of service. On the other hand, hosting and query processing of data out of the data owner control raises security challenges such as preserving data confidentiality and query privacy.

One straightforward way to protect the confidentiality of the outsourced data from the cloud as well as from the unauthorized users is to encrypt data by the data owner before outsourcing [1, 15, 19]. By this way, the data owner can protect the privacy of his/her own data. In addition, to preserve query privacy, authorized users require encrypting their queries before sending them to the cloud for evaluation. Furthermore, during query processing, the cloud can also derive useful and sensitive information about the actual data items by observing the data access patterns even if the data and query are encrypted [4, 27]. Therefore, following from the above discussions, secure query processing needs to guarantee (1) confidentiality of the encrypted data (2) confidentiality of a user's query record and (3) hiding data access patterns.

Using encryption as a way to achieve data confidentiality may cause another issue during the query processing step in the cloud. In general, it is very difficult to process encrypted data without ever having to decrypt it. The question here is how the cloud can execute the queries over encrypted data while the data stored at the cloud are encrypted at all times. In the literature, various techniques related to query processing over encrypted data have been proposed, including range queries [2, 11, 12, 24] and other aggregate queries [10, 17]. However, these techniques are either not applicable or inefficient to solve advanced queries such as the *k*-nearest neighbor (*k*NN) query.

In this paper, we address the problem of secure processing of k-nearest neighbor query over encrypted data (SkNN) in the cloud. Given a user's input query Q, the objective of the SkNN problem is to securely identify the k-nearest data tuples to Q using the encrypted database of T in the cloud, without allowing the cloud to learn anything regarding the actual contents of the database T and the query record Q. More specifically, when encrypted data are outsourced to the cloud, we observe that an effective SkNN protocol needs to satisfy the following properties:

Table 1: Sample Heart Disease Dataset T

record-id	age	sex	cp	trestbps	chol	fbs	slope	ca	thal	num
t_1	63	1	1	145	233	1	3	0	6	0
t_2	56	1	3	130	256	1	2	1	6	2
t_3	57	0	3	140	241	0	2	0	7	1
t_4	59	1	4	144	200	1	2	2	6	3
t_5	55	0	4	128	205	0	2	1	7	3
t_6	77	1	4	125	304	0	1	3	3	4

- ullet Preserve the confidentiality of T and Q at all times
- Hiding data access patterns from the cloud
- ullet Accurately compute the k-nearest neighbors of query Q
- Incur low computation overhead on the end-user

In the past few years, researchers have proposed various methods [13,28,31] to address the SkNN problem. However, we emphasize that the existing SkNN methods proposed in [13,28] violate at least one of the above mentioned desirable properties of a SkNN protocol. On one hand, the methods in [13,28] are insecure because they are vulnerable to chosen and known plaintext attacks. On the other hand, recent method in [31] returns non-accurate kNN result to the end-user. More precisely, in [31], the cloud retrieves the relevant encrypted partition instead of finding the encrypted exact k-nearest neighbors. Furthermore, in [13,31], the end-user involves in heavy computations during the query processing step. By doing so, the method in [31] utilizes cloud as just a storage medium, i.e., no significant work is done on the cloud side. More details about the existing SkNN methods are provided in Section 2.

Along this direction, with the goal of providing better security, this paper proposes a novel SkNN protocol that satisfies the above properties altogether.

1.1 Problem Definition

Suppose the data owner Alice owns a database T of n records, denoted by t_1, \ldots, t_n , and m attributes. Let $t_{i,j}$ denote the j^{th} attribute value of record t_i . In our problem setting, we assume that Alice initially encrypts her database attribute-wise, that is, she computes $E_{pk}(t_{i,j})$, for $1 \le i \le n$ and $1 \le j \le m$, where E_{pk} denotes the encryption function of a public-key cryptosystem that is semantically secure [18]. Let the encrypted database be denoted by $E_{pk}(T)$. We assume that Alice outsources $E_{pk}(T)$ as well as the future querying processing services to the cloud.

Consider an authorized user Bob who wants to ask the cloud for k-neighbor records that are closest to his input query $Q = \langle q_1, \ldots, q_m \rangle$ based on $E_{pk}(T)$. During this process, Bob's query Q and contents of database T should not be revealed to the cloud. In addition, the access patterns to the data should be protected from the cloud. We refer to such a process as Secure kNN (SkNN) query over encrypted data in the cloud. Without loss of generality, let $\langle t'_1, \ldots, t'_k \rangle$ denote the k-nearest records to Q. Then, we formally define the SkNN protocol as follows:

$$SkNN(E_{nk}(T),Q) \rightarrow \langle t'_1,\ldots,t'_k \rangle$$

We emphasize that, at the end of the SkNN protocol, the output $\langle t'_1, \dots, t'_k \rangle$ should be revealed only to Bob. We now present a real-life application of the SkNN protocol.

Example 1. Consider a physician who would like to know the risk factor of heart disease in a specific patient. Let T denote the sample heart disease dataset with attributes record-id, age, sex, cp, trestbps, chol, fbs, slope, ca, thal, and num as shown in Table 1. The description and range for each of these attributes are shown in Table 2. The heart disease dataset given in Table 1 is obtained from the UCI machine learning repository [14].

Initially, the data owner (hospital) encrypts T attribute-wise, outsources the encrypted database $E_{pk}(T)$ to the cloud for easy management. In addition, the data owner delegates the future query processing services to the cloud.

Table 2: Attribute Description of Heart Disease Dataset T

age	age in years					
sex	1=male, 0=female					
cp	chest pain type: 1=typical angina, 2=atypical angina,					
	3=non-anginal pain, 4=asymptomatic					
trestbps	resting blood pressure (mm Hg)					
chol	serum cholesterol in mg/dl					
fbs	fasting blood sugar > 120 mg/dl (1=true; 0=false)					
slope	slope of the peak exercise ST segment					
	(1=upsloping, 2=flat, 3=downsloping)					
ca	number of major vessels (0-3) colored by flourosopy					
thal	3=normal, 6=fixed defect, 7=reversible defect					
num	diagnosis of heart disease from 0 (no presence) to 4					

Now, we consider a doctor working at the hospital, say Bob, who would like to know the risk factor of heart disease in a specific patient based on T. Let the patient medical information be $Q = \langle 58, 1, 4, 133, 196, 1, 2, 1, 6 \rangle$. In the SkNN protocol, Bob first need to encrypt Q (to preserve the privacy of his query) and send it to the cloud. Then the cloud searches on the encrypted database $E_{pk}(T)$ to figure out the k-nearest neighbors to the user's request. For simplicity, let us assume k = 2. Under this case, the 2 nearest neighbors to Q are t_4 and t_5 (by using Euclidean distance as the similarity metric). After this, the cloud sends t_4 and t_5 (in encrypted form) to Bob. Here, the cloud should identify the nearest neighbors of Q in an oblivious manner without knowing any sensitive information, i.e., all the computations have to be carried over encrypted records. Finally, Bob receives t_4 and t_5 that will help him to make medical decisions.

1.2 Our Contribution

In this paper, we propose a novel SkNN protocol to facilitate the k-nearest neighbor search over encrypted data in the cloud that preserves both the data privacy and query privacy. In our protocol, once the encrypted data are outsourced to the cloud, Alice does not participate in any computations. Therefore, no information is revealed to Alice. In particular, the proposed protocol meets the following requirements:

- Data confidentiality Contents of T or any intermediate results should not be revealed to the cloud.
- Query privacy Bob's input query Q should not be revealed to the cloud.
- Correctness The output $\langle t'_1, \dots, t'_k \rangle$ should be revealed only to Bob. In addition, no information other than t'_1, \dots, t'_k should be revealed to Bob.
- Low computation overhead on Bob After sending his encrypted query record to the cloud, Bob involves only in a little computation compared with the existing works [13,28,31]. More details are given in Section 4.2.
- **Hidden data access patterns** Access patterns to the data, such as the records corresponding to the k-nearest neighbors of Q, should not be revealed to Alice and the cloud (to prevent any inference attacks).

We emphasize that the intermediate results seen by the cloud in our protocol are either newly generated randomized encryptions or random numbers. Thus, which data records correspond to the k-nearest neighbors of Q are not known to the cloud. In addition, after sending his encrypted query record to the cloud, Bob does not involve in any computations (less workload at Bob's local machine). Hence, data access patterns are further protected from Bob. More details are given in Section 4.2.

The rest of the paper is organized as follows. We discuss the existing related work and some background concepts in Section 2. A set of security primitives that are utilized in the proposed protocols and their possible implementations are provided in Section 3. The proposed protocols are explained in detail in Section 4. Section 5 discusses the performance of the proposed protocols based on various experiments. We conclude the paper along with future work in Section 6.

2 Related Work and Background

In this section, we first present an overview of the existing secure k-nearest neighbor techniques. Then, we present the security definition adopted in this paper and the Paillier cryptosystem along with its additive homomorphic properties as a background.

2.1 Existing SkNN Techniques

Retrieving the k-nearest neighbors to a given query Q is one of the most fundamental problem in many application domains such as similarity search, pattern recognition, and data mining. In the literature, many techniques have been proposed to address the SkNN problem, which can be classified into two categories based on whether the data are encrypted or not: centralized and distributed.

2.1.1 Centralized Methods

In the centralized methods, we assume that the data owner outsources his/her database and DBMS functionalities (e.g., kNN query) to an untrusted external service provider which manages the data on behalf of the data owner where only trusted users are allowed to query the hosted data at the service provider. By outsourcing data to an untrusted server, many security issues arise, such as data privacy (protecting the confidentiality of the data from the server as well as from query issuer). To achieve data privacy, data owner is required to use data anonymization models (e.g., k-anonymity) or cryptographic (e.g., encryption and data perturbation) techniques over his/her data before outsourcing them to the server.

Encryption is a traditional technique used to protect the confidentiality of sensitive data such as medical records. Due to data encryption, the process of query evaluation over encrypted data becomes challenging. Along this direction, various techniques have been proposed for processing range [2, 11, 12, 24] and aggregation queries [10, 17] over encrypted data. However, in this paper, we restrict our discussion to secure evaluation of kNN query.

In the past few years, researchers have proposed different methods [13,28,31] to address the SkNN problem. Wong et al. [28] proposed a new encryption scheme called asymmetric scalar-product-preserving encryption (ASPE) that preserves scalar product between the query vector Q and any tuple vector t_i from database T for distance comparison which is sufficient to find kNN. In [28], data and query are encrypted using slightly different encryption schemes before outsourcing to the server. As an alternative, Hu et al. [13] proposed a method based on Privacy Homomorphism (PH) encryption scheme. More specifically, they used a provably secure privacy homomorphism encryption scheme from [5] that supports modular addition, subtraction and multiplication over encrypted data. They addressed the SkNN problem under the following setting: the client has the ciphertexts of all data points in database T and the encryption function of T whereas the server has the decryption function of T and some auxiliary information regarding each data point. However, both methods in [13, 28] are not secure because they are vulnerable to chosen-plaintext attacks. We refer the reader to [31] for more details on these security issues.

Recently, Yao et al. [31] designed a new SkNN method based on partition-based secure Voronoi diagram (SVD). Instead of asking the cloud to retrieve the exact kNN, they required, from the cloud, to retrieve a relevant encrypted partition $E_{pk}(G)$ for $E_{pk}(T)$ such that G is guaranteed to contain the k-nearest neighbors of Q. However, in our work, we are able to solve the SkNN problem accurately by letting the cloud to retrieve the exact k-nearest neighbors of Q (in encrypted form). In addition, most of the computations during the query processing step in [13, 31] are performed locally by the end-user (i.e., query issuer) which conflicts the very purpose of outsourcing the DBMS functionalities to the cloud. Since our proposed protocol solves the problem of finding k-nearest neighbors of $E_{pk}(Q)$

over encrypted data, it can also be used in other relevant data mining tasks such as secure clustering, classification, and outlier detection.

2.1.2 Data Distribution Methods

In the data distributed methods, data are assumed to be partitioned either vertically or horizontally and distributed among a set of independent, non-colluding parties. In the literature, the data distributed methods rely on secure multiparty computation (SMC) techniques that enable multiple parties to securely evaluate a function using their respective private inputs without disclosing the input of one party to the others. Many efforts have been made to address the problem of kNN query in a distributed environment. Shaneck et al. [23] proposed privacy-preserving algorithm to perform k-nearest neighbor search. The protocol in [23] is based on secure multiparty computation for privately computing kNN points in a horizontally partitioned dataset. Qi et al. [20] proposed a single-step kNN search protocol that is provably secure with linear computation and communication complexities. Vaidya et al. [25] studied privacy-preserving top-k queries in which the data are vertically partitioned. Ghinita et al. [6] proposed a private information retrieval (PIR) framework for answering kNN queries in location-based services. However, their solution protects only the query privacy, i.e., it does not address data confidentiality and access pattern issues.

We emphasize that the above data distribution methods are not applicable to perform kNN queries over encrypted data for two reasons: (1). In our work, we deal with encrypted form of database and query which is not the case in the above methods (2). The database in our case is encrypted and stored on the cloud whereas in the above methods it is partitioned (in plaintext format) among different parties.

2.2 Security Definition

In this paper, privacy/security is closely related to the amount of information disclosed during the execution of a protocol. There are many ways to define information disclosure. To maximize privacy or minimize information disclosure, we adopt the security definitions in the literature of secure multiparty computation (SMC) first introduced by Yao's Millionaires' problem for which a provably secure solution was developed [29,30]. In this paper, we assume that parties are semi-honest; that is, a semi-honest party (also referred to as honest-but-curious) follows the rules of the protocol using its correct input, but is free to later use what it sees during execution of the protocol to compromise security. We refer the reader to [7,8] for detailed security definitions and models. Briefly, the following definition captures the above discussion regarding a secure protocol under the semi-honest model.

Definition 1. Let a_i be the input of party P_i , $\prod_i(\pi)$ be P_i 's execution image of the protocol π and b_i be the result computed from π for P_i . π is secure if $\prod_i(\pi)$ can be simulated from $\langle a_i, b_i \rangle$ and distribution of the simulated image is computationally indistinguishable from $\prod_i(\pi)$.

2.3 Paillier Cryptosystem

The Paillier cryptosystem is an additive homomorphic and probabilistic asymmetric encryption scheme [18]. Let E_{pk} be the encryption function with public key pk given by (N,g), where N is a product of two large primes and g is in $\mathbb{Z}_{N^2}^*$. Also, let D_{sk} be the decryption function with secret key sk. Given $a,b \in \mathbb{Z}_N$, the Paillier encryption scheme exhibits the following properties:

- a. Homomorphic Addition $E_{pk}(a+b) \leftarrow E_{pk}(a) * E_{pk}(b) \mod N^2$;
- b. Homomorphic Multiplication $E_{pk}(a*b) \leftarrow E_{pk}(a)^b \mod N^2$;
- c. **Semantic Security** The encryption scheme is semantically secure [8, 9]. Briefly, given a set of ciphertexts, an adversary cannot deduce any additional information about the plaintext.

In this paper, we assume that a data owner encrypted his or her data using Paillier cryptosystem before outsourcing them to a cloud. Some common notations that are used extensively in this paper are shown in Table 3.

Table 3: Common Notations

Alice	The data owner of database T					
$E_{pk}(T)$	Attribute-wise encryption of T					
Bob	An authorized user who can access $E_{pk}(T)$ in the cloud					
n	Number of data records in T					
m	Number of attributes in T					
t_i	i^{th} record in T					
Q	Bob's query record					
t_i'	i^{th} nearest record to ${\cal Q}$ based on ${\cal T}$					
l	Domain size (in bits) of the squared Euclidean distance					
	based on T					
$\langle z_1, z_l \rangle$	The most and least significant bits of integer z					
[z]	Vector of encryptions of the individual bits of z					

3 Basic Security Primitives

In this section, we present a set of generic protocols that will be used as sub-routines while constructing our proposed SkNN protocol in Section 4.2. All of the below protocols are considered under two-party semi-honest setting. In particular, we assume the existence of two semi-honest parties P_1 and P_2 such that the Paillier's secret key sk is known only to P_2 whereas pk is treated as public.

- Secure Multiplication (SM) Protocol:
 - This protocol considers P_1 with input $(E_{pk}(a), E_{pk}(b))$ and outputs $E_{pk}(a*b)$ to P_1 , where a and b are not known to P_1 and P_2 . During this process, no information regarding a and b is revealed to P_1 and P_2 . The output $E_{pk}(a*b)$ is known only to P_1 .
- Secure Squared Euclidean Distance (SSED) Protocol:
 - P_1 with input $(E_{pk}(X), E_{pk}(Y))$ and P_2 securely compute the encryption of squared Euclidean distance between vectors X and Y. Here X and Y are m dimensional vectors where $E_{pk}(X) = \langle E_{pk}(x_1), \dots, E_{pk}(x_m) \rangle$ and $E_{pk}(Y) = \langle E_{pk}(y_1), \dots, E_{pk}(y_m) \rangle$. At the end, the output $E_{pk}(|X-Y|^2)$ is known only to P_1 .
- Secure Bit-Decomposition (SBD) Protocol:
 - P_1 with input $E_{pk}(z)$ and P_2 securely compute the encryptions of the individual bits of z, where $0 \le z < 2^l$. The output $[z] = \langle E_{pk}(z_1), \dots, E_{pk}(z_l) \rangle$ is known only to P_1 . Here z_1 and z_l denote the most and least significant bits of integer z respectively.
- Secure Minimum (SMIN) Protocol:
 - P_1 with input ([u], [v]) and P_2 with sk securely compute the encryptions of the individual bits of minimum number between u and v. That is, the output is $[\min(u, v)]$ which will be known only to P_1 . During this protocol, no information regarding u and v is revealed to P_1 and P_2 .
- Secure Minimum out of n Numbers (SMIN $_n$) Protocol: In this protocol, P_1 has n encrypted vectors $([d_1], \ldots, [d_n])$ and P_2 has sk. Here $[d_i] = \langle E_{pk}(d_{i,1}), \ldots, E_{pk}(d_{i,l}) \rangle$ such that $d_{i,1}$ and $d_{i,l}$ are the most and least significant bits of integer d_i respectively, for $1 \le i \le n$. P_1 and P_2 jointly compute the output $[\min(d_1, \ldots, d_n)]$. At the end of this protocol, $[\min(d_1, \ldots, d_n)]$ is known only to P_1 . During the SMIN $_n$ protocol, no information regarding any of d_i 's is revealed to P_1 and P_2 .
- Secure Bit-OR (SBOR) Protocol: P_1 with input $(E_{pk}(o_1), E_{pk}(o_2))$ and P_2 securely compute $E_{pk}(o_1 \vee o_2)$, where o_1 and o_2 are two bits. The

Algorithm 1 SM $(E_{pk}(a), E_{pk}(b)) \rightarrow E_{pk}(a * b)$

Require: P_1 has $E_{pk}(a)$ and $E_{pk}(b)$; P_2 has sk

- 1: P_1 :
 - (a). Pick two random numbers $r_a, r_b \in \mathbb{Z}_N$
 - (b). $a' \leftarrow E_{pk}(a) * E_{pk}(r_a)$
 - (c). $b' \leftarrow E_{pk}(b) * E_{pk}(r_b)$; send a', b' to P_2
- 2: P_2 :
 - (a). Receive a' and b' from P_1
 - (b). $h_a \leftarrow D_{sk}(a')$; $h_b \leftarrow D_{sk}(b')$
 - (c). $h \leftarrow h_a * h_b \mod N$
 - (d). $h' \leftarrow E_{pk}(h)$; send h' to P_1
- 3: P_1 :
 - (a). Receive h' from P_2
 - (b). $s \leftarrow h' * E_{pk}(a)^{N-r_b}$
 - (c). $s' \leftarrow s * E_{pk}(b)^{N-r_a}$
 - (d). $E_{pk}(a*b) \leftarrow s' * E_{pk}(r_a*r_b)^{N-1}$

output $E_{pk}(o_1 \vee o_2)$ is known only to P_1 .

We now discuss each of these protocols in detail. Also, we either propose new solution or refer to the most efficient known implementation to each one of them.

Secure Multiplication (SM). Consider a party P_1 with private input $(E_{pk}(a), E_{pk}(b))$ and a party P_2 with the secret key sk. The goal of the secure multiplication (SM) protocol is to return the encryption of a*b, i.e., $E_{pk}(a*b)$ as output to P_1 . During this protocol, no information regarding a and b is revealed to P_1 and P_2 . The basic idea of the SM protocol is based on the following property which holds for any given $a, b \in \mathbb{Z}_N$:

$$a * b = (a + r_a) * (b + r_b) - a * r_b - b * r_a - r_a * r_b$$
(1)

where all the arithmetic operations are performed under \mathbb{Z}_N . The overall steps in SM are shown in Algorithm 1. Briefly, P_1 initially randomizes a and b by computing $a' = E_{pk}(a) * E_{pk}(r_a)$ and $b' = E_{pk}(b) * E_{pk}(r_b)$, and sends them to P_2 . Here r_a and r_b are random numbers in \mathbb{Z}_N known only to P_1 . Upon receiving, P_2 decrypts and multiplies them to get $h = (a + r_a) * (b + r_b) \mod N$. Then, P_2 encrypts h and sends it to P_1 . After this, P_1 removes extra random factors from $h' = E_{pk}((a + r_a) * (b + r_b))$ based on Equation 1 to get $E_{pk}(a * b)$. Note that, for any given $x \in \mathbb{Z}_N$, "N - x" is equivalent to "-x" under \mathbb{Z}_N . Hereafter, we use the notation $r \in_R \mathbb{Z}_N$ to denote r as a random number in \mathbb{Z}_N .

Example 2. Suppose a = 59 and b = 58. For simplicity, let $r_a = 1$ and $r_b = 3$. Initially, P_1 computes $a' = E_{pk}(60) = E_{pk}(a) * E_{pk}(r_a)$, $b' = E_{pk}(61) = E_{pk}(b) * E_{pk}(r_b)$ and sends them to P_2 . Then, P_2 decrypts and multiplies them to get h = 3660. After this, P_2 encrypts h to get $h' = E_{pk}(3660)$ and sends it to P_1 . Upon receiving h', P_1 computes $s = E_{pk}(3483) = E_{pk}(3660 - a * r_b)$, and $s' = E_{pk}(3425) = E_{pk}(3483 - b * r_a)$. Finally, P_1 computes $E_{pk}(a * b) = E_{pk}(3422) = E_{pk}(3425 - r_a * r_b)$.

Secure Squared Euclidean Distance (SSED). In the SSED protocol, P_1 holds two encrypted vectors $(E_{pk}(X), E_{pk}(Y))$ and P_2 holds the secret key sk. Here X and Y are two m-dimensional vectors where $E_{pk}(X) = \langle E_{pk}(x_1), \dots, E_{pk}(x_m) \rangle$

Algorithm 2 SSED $(E_{pk}(X), E_{pk}(Y)) \rightarrow E_{pk}(|X - Y|^2)$ Require: P_1 has $E_{pk}(X)$ and $E_{pk}(Y)$; P_2 has sk

1: P_1 , for $1 \le i \le m$ do:

(a).
$$E_{pk}(x_i - y_i) \leftarrow E_{pk}(x_i) * E_{pk}(y_i)^{N-1}$$

- 2: P_1 and P_2 , for 1 < i < m do:
 - (a). Compute $E_{pk}((x_i y_i)^2)$ using the SM protocol
- 3: P_1 :

(a).
$$E_{pk}(|X-Y|^2) \leftarrow \prod_{i=1}^m E_{pk}((x_i-y_i)^2)$$

and $E_{pk}(Y) = \langle E_{pk}(y_1), \dots, E_{pk}(y_m) \rangle$. The goal of the SSED protocol is to securely compute $E_{pk}(|X-Y|^2)$, where |X-Y| denotes the Euclidean distance between vectors X and Y. During this protocol, no information regarding X and Y is revealed to P_1 and P_2 . The basic idea of SSED follows from following equation:

$$|X - Y|^2 = \sum_{i=1}^{m} (x_i - y_i)^2$$
 (2)

The main steps involved in SSED are shown in Algorithm 2. Briefly, for $1 \le i \le m$, P_1 initially computes $E_{pk}(x_i - y_i)$ by using the homomorphic properties. Then P_1 and P_2 jointly compute $E_{pk}((x_i-y_i)^2)$ using the SM protocol, for $1 \leq i \leq m$. Note that the outputs of the SM protocol are known only to P_1 . After this, by applying homomorphic properties on $E_{pk}((x_i - y_i)^2)$, P_1 computes $E_{pk}(|X - Y|^2)$ locally based on Equation 2.

Example 3. Refer to Table 1, let us assume that P_1 holds the encrypted data records of t_1 and t_2 as X and Yrespectively. That is, $E_{pk}(X) = \langle E_{pk}(63), E_{pk}(1), E_{pk}(1), E_{pk}(145), E_{pk}(233), E_{pk}(1), E_{pk}(3), E_{pk}(0), E_{pk}(6), E_{pk}(6),$ $E_{pk}(0)$ and $E_{pk}(Y) = \langle E_{pk}(56), E_{pk}(1), E_{pk}(3), E_{pk}(130), E_{pk}(256), E_{pk}(1), E_{pk}(2), E_{pk}(1), E_{pk}(6), E_{pk}(2) \rangle$. During the SSED protocol, P_1 initially computes $E_{pk}(x_1 - y_1) = E_{pk}(7), \dots, E_{pk}(x_{10} - y_{10}) = E_{pk}(-2)$. Then, P_1 and P_2 jointly compute $E_{pk}((x_1 - y_1)^2) = E_{pk}(49) = SM(E_{pk}(7), E_{pk}(7)), \dots, E_{pk}((x_{10} - y_{10})^2) = SM(E_{pk}(-2), E_{pk}(-2)) = E_{pk}(4)$. P_1 locally computes $E_{pk}(|X - Y|^2) = E_{pk}(\sum_{i=1}^{10} (x_i - y_i)^2) = E_{pk}(813)$.

Secure Bit-Decomposition (SBD). We assume that P_1 has $E_{pk}(z)$ and P_2 has sk, where z is not known to both parties and $0 \le z < 2^l$. The goal of the secure bit-decomposition (SBD) protocol is to compute the encryptions of the individual bits of binary representation of z [21,22]. That is, the output is $[z] = \langle E_{pk}(z_1), \dots, E_{pk}(z_l) \rangle$, where z_1 and z_l denote the most and least significant bits of z respectively. At the end, the output [z] is known only to P_1 .

Since the goal of this paper is not to investigate existing SBD protocols, we simply use the most efficient SBD protocol that was recently proposed in [21].

Example 4. Let us suppose that
$$z=55$$
 and $l=6$. Then the SBD protocol with private input $E_{pk}(55)$ gives $[55]=\langle E_{pk}(1), E_{pk}(1), E_{pk}(1), E_{pk}(1), E_{pk}(1), E_{pk}(1) \rangle$ as the output to P_1 .

Secure Minimum (SMIN). In this protocol, P_1 with input ([u], [v]) and P_2 with sk securely compute the encryptions of the individual bits of $\min(u, v)$, i.e., the output is $[\min(u, v)]$. Here $[u] = \langle E_{pk}(u_1), \dots, E_{pk}(u_l) \rangle$ and $[v] = \langle E_{pk}(u_1), \dots, E_{pk}(u_l) \rangle$ $\langle E_{pk}(v_1), \dots, E_{pk}(v_l) \rangle$, where u_1 (resp., v_1) and u_l (resp., v_l) are the most and least significant bits of u (resp., v). At the end, the output $[\min(u, v)]$ is known only to P_1 .

We assume that $0 \le u, v < 2^l$ and propose a novel SMIN protocol. The basic idea of the proposed SMIN protocol is for P_1 to randomly choose the functionality F (by flipping a coin), where F is either u > v or v > u, and to obliviously execute F with P_2 . Since F is randomly chosen and known only to P_1 , the output of the functionality F is oblivious to P_2 . Based on the output and chosen F, P_1 computes $[\min(u, v)]$ locally using homomorphic properties.

Algorithm 3 SMIN([u], [v]) \rightarrow $[\min(u, v)]$

Require: P_1 has [u] and [v], where $0 \le u, v < 2^l$; P_2 has sk

- 1: P_1 :
 - (a). Randomly choose the functionality F
 - (b). **for** i = 1 to l **do**:
 - $E_{pk}(u_i * v_i) \leftarrow SM(E_{pk}(u_i), E_{pk}(v_i))$
 - if F: u > v then:

$$- W_i \leftarrow E_{pk}(u_i) * E_{pk}(u_i * v_i)^{N-1}$$
$$- \Gamma_i \leftarrow E_{pk}(v_i - u_i) * E_{pk}(\hat{r}_i); \hat{r}_i \in_R \mathbb{Z}_N$$

else

$$- W_i \leftarrow E_{pk}(v_i) * E_{pk}(u_i * v_i)^{N-1}$$
$$- \Gamma_i \leftarrow E_{pk}(u_i - v_i) * E_{pk}(\hat{r}_i); \hat{r}_i \in_R \mathbb{Z}_N$$

- $G_i \leftarrow E_{pk}(u_i \oplus v_i)$
- $H_i \leftarrow H_{i-1}^{r_i} * G_i$; $r_i \in_R \mathbb{Z}_N$ and $H_0 = E_{pk}(0)$
- $\Phi_i \leftarrow E_{pk}(-1) * H_i$
- $L_i \leftarrow W_i * \Phi_i^{r_i'}; r_i' \in_R \mathbb{Z}_N$
- (c). $\Gamma' \leftarrow \pi_1(\Gamma)$
- (d). $L' \leftarrow \pi_2(L)$; send Γ' and L' to C
- 2: P_2 :
 - (a). Receive Γ' and L' from P_1
 - (b). $M_i \leftarrow D_{sk}(L_i')$, for $1 \le i \le l$
 - (c). if $\exists j$ such that $M_j = 1$ then $\alpha \leftarrow 1$ else $\alpha \leftarrow 0$
 - (d). $M_i' \leftarrow {\Gamma_i'}^{\alpha}$, for $1 \le i \le l$
 - (e). Send M' and $E_{pk}(\alpha)$ to P_1
- 3: P_1 :
 - (a). Receive M' and $E_{pk}(\alpha)$ from P_2
 - (b). $\widetilde{M} \leftarrow \pi_1^{-1}(M')$
 - (c). **for** i = 1 to l **do**:
 - $\lambda_i \leftarrow \widetilde{M}_i * E_{pk}(\alpha)^{N-\hat{r}_i}$
 - $\bullet \ \, \textbf{if} \, \, F: u > v \, \, \textbf{then} \, \, E_{pk}(\min(u,v)_i) \leftarrow E_{pk}(u_i) * \lambda_i \\ \quad \textbf{else} \, \, E_{pk}(\min(u,v)_i) \leftarrow E_{pk}(v_i) * \lambda_i \\$

The overall steps involved in the SMIN protocol are shown in Algorithm 3. To start with, P_1 initially chooses the functionality F as either u > v or v > u randomly. Then, using the SM protocol, P_1 computes $E_{pk}(u_i * v_i)$ with the help of P_2 , for $1 \le i \le l$. Now, depending on F, P_1 proceeds as follows, for $1 \le i \le l$:

• If F: u > v, compute

$$W_{i} = E_{pk}(u_{i}) * E_{pk}(u_{i} * v_{i})^{N-1}$$

$$= E_{pk}(u_{i} * (1 - v_{i}))$$

$$\Gamma_{i} = E_{pk}(v_{i} - u_{i}) * E_{pk}(\hat{r}_{i})$$

$$= E_{pk}(v_{i} - u_{i} + \hat{r}_{i})$$

• If F: v > u, compute:

$$W_{i} = E_{pk}(v_{i}) * E_{pk}(u_{i} * v_{i})^{N-1}$$

$$= E_{pk}(v_{i} * (1 - u_{i}))$$

$$\Gamma_{i} = E_{pk}(u_{i} - v_{i}) * E_{pk}(\hat{r}_{i})$$

$$= E_{pk}(u_{i} - v_{i} + \hat{r}_{i})$$

where \hat{r}_i is a random number in \mathbb{Z}_N

- Observe that if F: u > v, then $W_i = E_{pk}(1)$ only if $u_i > v_i$, and $W_i = E_{pk}(0)$ otherwise. Similarly, when F: v > u, we have $W_i = E_{pk}(1)$ only if $v_i > u_i$, and $W_i = E_{pk}(0)$ otherwise. Also, depending of F, Γ_i stores the encryption of randomized difference between u_i and v_i which will be used in later computations.
- Compute the encrypted bit-wise XOR between the bits u_i and v_i as $G_i = E_{pk}(u_i \oplus v_i)$ using the below formulation:

$$G_i = E_{pk}(u_i) * E_{pk}(v_i) * E_{pk}(u_i * v_i)^{N-2}$$

In general, for any two given bits o_1 and o_2 , we have $o_1 \oplus o_2 = o_1 + o_2 - 2(o_1 * o_2)$

- Compute an encrypted vector H by preserving the first occurrence of $E_{pk}(1)$ (if there exists one) in G by initializing $H_0 = E_{pk}(0)$. The rest of the entries of H are computed as $H_i = H_{i-1}^{r_i} * G_i$. We emphasize that at most one of the entry in H is $E_{pk}(1)$ and the remaining entries are encryptions of either 0 or a random number. Also, if there exists an index j such that $H_j = E_{pk}(1)$, then index j is the first position (starting from the most significant bit) at which the corresponding bits of u and v differ.
- Then, P_1 computes $\Phi_i = E_{pk}(-1) * H_i$. Note that "-1" is equivalent to "N-1" under \mathbb{Z}_N . From the above discussions, it is clear that $\Phi_i = E_{pk}(0)$ at most once since H_i is equal to $E_{pk}(1)$ at most once. Also, if $\Phi_j = E_{pk}(0)$, then index j is the position at which the bits of u and v differ first.
- Compute an encrypted vector L by combining W and Φ . Note that W_i stores the result of $u_i > v_i$ or $v_i > u_i$ which depends on F known only to P_1 . Precisely, P_1 computes $L_i = W_i * \Phi_i^{r_i}$, where r_i' is a random number in \mathbb{Z}_N . The observation here is if \exists an index j such that $\Phi_j = E_{pk}(0)$, denoting the first flip in the bits of u and v, then W_j stores the corresponding desired information, i.e., whether $u_j > v_j$ or $v_j > u_j$ in encrypted form.

After this, P_1 permutes the encrypted vectors Γ and L using two random permutation functions π_1 and π_2 . Specifically, P_1 computes $\Gamma' = \pi_1(\Gamma)$ and $L' = \pi_2(L)$, and sends them to P_2 . Upon receiving, P_2 decrypts L' component-wise to get $M_i = D_{sk}(L_i')$, for $1 \le i \le l$, and checks for index j (decide the output of F). That is, if $M_j = 1$, then the output of F is 1, and 0 otherwise. Let the output be α . Note that since F is not known to P_2 , the output α is oblivious to P_2 . In addition, P_2 computes a new encrypted vector M' where $M_i' = \Gamma_i'^{\alpha}$, for $1 \le i \le l$, sends M' and $E_{pk}(\alpha)$ to P_1 . After receiving M' and $E_{pk}(\alpha)$, P_1 computes the inverse permutation of M' as $\widetilde{M} = \pi_1^{-1}(M')$. Then, P_1 performs the following homomorphic operations to compute the encryption of i^{th} bit of $\min(u,v)$, i.e., $E_{pk}(\min(u,v)_i)$, for $1 \le i \le l$:

Table 4: P_1 chooses F as $v > u$ where $u = 55$ and $v = 58$												
[u]	[v]	W_i	Γ_i	G_i	H_i	Φ_i	L_i	Γ_i '	L_i'	M_i	λ_i	\min_i
1	1	0	r	0	0	-1	r	1+r	r	r	0	1
1	1	0	r	0	0	-1	r	r	r	r	0	1
0	1	1	-1 + r	1	1	0	1	1+r	r	r	-1	0
1	0	0	1+r	1	r	r	r	-1 + r	r	r	1	1
1	1	0	r	0	r	r	r	r	1	1	0	1
	_	_										

• Remove the randomness from \widetilde{M}_i by computing

$$\lambda_i = \widetilde{M}_i * E_{pk}(\alpha)^{N - \hat{r}_i}$$

• If F: u > v, compute the i^{th} encrypted bit of $\min(u, v)$ as $E_{pk}(\min(u, v)_i) = E_{pk}(u_i) * \lambda_i = E_{pk}(u_i + \alpha * (v_i - u_i))$. Otherwise, compute $E_{pk}(\min(u, v)_i) = E_{pk}(v_i) * \lambda_i = E_{pk}(v_i + \alpha * (u_i - v_i))$.

In the SMIN protocol, one main observation (upon which we can also justify the correctness of the final output) is that if F: u > v, then $\min(u, v)_i = (1 - \alpha) * u_i + \alpha * v_i$ always holds, for $1 \le i \le l$. Similarly, if F: v > u, then $\min(u, v)_i = \alpha * u_i + (1 - \alpha) * v_i$ always holds.

Example 5. Consider that u = 55, v = 58, and l = 6. Assume that P_1 holds $[55] = \langle E_{pk}(1), E_{pk}(1), E_{pk}(0), E_{pk}(1), E_{p$

$$i = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\pi_1(i) = 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1$$

$$\pi_2(i) = 2 \quad 1 \quad 5 \quad 6 \quad 3 \quad 4$$

F: v > u. Then, various intermediate results based on the SMIN protocol are as shown in Table 4. Following from Table 4, we observe that:

• At most one of the entry in H is $E_{pk}(1)$ (= H_3) and the remaining entries are encryptions of either 0 or a random number in \mathbb{Z}_N .

- Index j = 3 is the first position at which the corresponding bits of u and v differ.
- $\Phi_3 = E_{vk}(0)$ since H_3 is equal to $E_{vk}(1)$. Also, since $M_5 = 1$, P_2 sets α to 1.

At the end, only P_1 knows $[\min(u, v)] = [u] = [55]$.

Secure Minimum out of n Numbers (SMIN $_n$). Consider P_1 with private input $([d_1],\ldots,[d_n])$ and P_2 with sk, where $0 \le d_i < 2^l$ and $[d_i] = \langle E_{pk}(d_{i,1}),\ldots,E_{pk}(d_{i,l})\rangle$, for $1 \le i \le n$. The goal of the SMIN $_n$ protocol is to compute $[\min(d_1,\ldots,d_n)] = [d_{\min}]$ without revealing any information about d_i 's to P_1 and P_2 . Here we construct a new SMIN $_n$ protocol by utilizing SMIN as the building block. The proposed SMIN $_n$ protocol is an iterative approach and it computes the desired output in an hierarchical fashion. In each iteration, minimum between a pair of values is computed and are feeded as input to the next iteration. Therefore, generating a binary execution tree in a bottom-up fashion. At the end, only P_1 knows the final result $[d_{\min}]$.

```
Algorithm 4 SMIN<sub>n</sub>([d_1], \ldots, [d_n]) \rightarrow [d_{\min}]
```

```
Require: P_1 has ([d_1], \dots, [d_n]); P_2 has sk

1: P_1:

(a). [d'_i] \leftarrow [d_i], for 1 \le i \le n

(b). num \leftarrow n

2: P_1 and P_2, for i = 1 to \lceil \log_2 n \rceil:

(a). for 1 \le j \le \lfloor \frac{num}{2} \rfloor:

• if i = 1 then:

- [d'_{2j-1}] \leftarrow \text{SMIN}([d'_{2j-1}], [d'_{2j}])

- [d'_{2j}] \leftarrow 0

else

- [d'_{2i(j-1)+1}] \leftarrow \text{SMIN}([d'_{2i(j-1)+1}], [d'_{2ij-1}])

- [d'_{2ij-1}] \leftarrow 0

(b). num \leftarrow \lceil \frac{num}{2} \rceil

3: P_1:

(a.) [d_{\min}] \leftarrow [d'_1]
```

The overall steps involved in the proposed SMIN $_n$ protocol are highlighted in Algorithm 4. Initially, P_1 assigns $[d_i]$ to a temporary vector $[d_i']$, for $1 \le i \le n$. Also, he/she creates a global variable num and initialize it to n, where num represents the number of (non-zero) vectors involved in each iteration. Since the SMIN $_n$ protocol executes in a binary tree hierarchy (bottom-up fashion), we have $\lceil \log_2 n \rceil$ iterations, and in each iteration, the number of vectors involved varies. In the first iteration (i.e., i=1), P_1 with private input $([d'_{2j-1}], [d'_{2j}])$ and P_2 with sk involve in the SMIN protocol, for $1 \le j \le \left \lfloor \frac{num}{2} \right \rfloor$. At the end of the first iteration, only P_1 knows $[\min(d'_{2j-1}, d'_{2j})]$ and nothing is revealed to P_2 , for $1 \le j \le \left \lfloor \frac{num}{2} \right \rfloor$. Also, P_1 stores the result $[\min(d'_{2j-1}, d'_{2j})]$ in $[d'_{2j-1}]$, updates $[d'_{2j}]$ to zero and num to $\left \lceil \frac{num}{2} \right \rceil$.

During the i^{th} iteration, only the non-zero vectors are involved, for $2 \le i \le \lceil \log_2 n \rceil$. For example, during second iteration (i.e., i=2), only $[d_1'], [d_3']$, and so on are involved. Note that in each iteration, the output is revealed only to P_1 and num is updated to $\lceil \frac{num}{2} \rceil$. At the end of the SMIN_n protocol, P_1 assigns the final encrypted binary vector of global minimum value, i.e., $[\min(d_1,\ldots,d_n)]$ which is stored in $[d_1']$ to $[d_{\min}]$.

For example, assume that P_1 holds $\langle [d_1], \ldots, [d_6] \rangle$ (i.e., n=6). Then, based on the SMIN_n protocol, the binary execution tree (in a bottom-up fashion) to compute $[\min(d_1, \ldots, d_6)]$ is as shown in Figure 1. Note that, $[d_i']$ is initially set to $[d_i]$, for $1 \le i \le 6$.

Secure Bit-OR (SBOR). Let us assume that P_1 holds $(E_{pk}(o_1), E_{pk}(o_2))$ and P_2 holds sk, where o_1 and o_2 are two bits not known to both parties. The goal of the SBOR protocol is to securely compute $E_{pk}(o_1 \vee o_2)$. At the end of this protocol, only P_1 knows $E_{pk}(o_1 \vee o_2)$. During this process, no information related to o_1 and o_2 is revealed to P_1 and P_2 . Given the secure multiplication (SM) protocol, P_1 can compute $E_{pk}(o_1 \vee o_2)$ as follows:

• P_1 with input $(E_{pk}(o_1), E_{pk}(o_2))$ and P_2 with sk involve in the SM protocol. At the end of this step, the output $E_{pk}(o_1 * o_2)$ is known only to P_1 . Note that, since o_1 and o_2 are bits, $E_{pk}(o_1 * o_2) = E_{pk}(o_1 \wedge o_2)$.

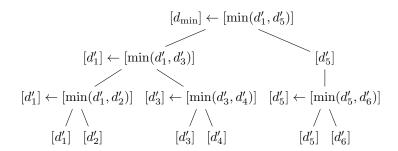


Figure 1: Binary execution tree for n=6 based on the SMIN_n protocol

•
$$E_{pk}(o_1 \vee o_2) = E_{pk}(o_1 + o_2) * E_{pk}(o_1 \wedge o_2)^{N-1}$$
.

We emphasize that, for any given two bits o_1 and o_2 , the property $o_1 \lor o_2 = o_1 + o_2 - o_1 \land o_2$ always holds. Note that, by homomorphic addition property, $E_{pk}(o_1 + o_2) = E_{pk}(o_1) * E_{pk}(o_2)$.

4 The Proposed Protocols

In this section, we first present a basic SkNN protocol and demonstrate why such a simple solution is not secure. Then, we discuss our second approach, a fully secure kNN protocol. Both protocols are constructed using the security primitives discussed in Section 3 as building blocks.

As mentioned earlier, we assume that Alice's database consists of n records, denoted by $T=\langle t_1,\ldots,t_n\rangle$, and m attributes, where $t_{i,j}$ denotes the j^{th} attribute value of record t_i . Initially, Alice encrypts her database attribute-wise, that is, she computes $E_{pk}(t_{i,j})$, for $1 \le i \le n$ and $1 \le j \le m$. Let the encrypted database be denoted by $E_{pk}(T)$. We assume that Alice outsources $E_{pk}(T)$ as well as the future query processing service to the cloud. Without loss of generality, we assume that all attribute values and their Euclidean distances lie in $[0,2^l)$.

In our proposed protocols, we assume the existence of two non-colluding semi-honest cloud service providers, denoted by C_1 and C_2 , which together form a federated cloud. We emphasize that such an assumption is not new and has been commonly used in the related problem domains [3, 26]. The intuition behind such an assumption is as follows. Most of the cloud service providers in the market are well-established IT companies, such as Amazon and Google. Therefore, a collusion between them is highly unlikely as it will damage their reputation which in turn effects their revenues.

Under this setting, Alice outsources her encrypted database $E_{pk}(T)$ to C_1 and the secret key sk to C_2 . The goal of the proposed protocols is to retrieve the top k records that are closest to the user query in an efficient and secure manner. Briefly, consider an authorized user Bob who wants to find k records that are closest to his query record $Q = \langle q_1, \ldots, q_m \rangle$ based on $E_{pk}(T)$ in C_1 . Bob initially sends his query Q (in encrypted form) to C_1 . After this, C_1 and C_2 involve in a set of sub-protocols to securely retrieve (in encrypted form) the set of k records corresponding to the k-nearest neighbors of the input query Q. At the end of our protocols, only Bob will receive the k-nearest neighbors to Q as the output.

4.1 Basic Protocol

In the basic secure k-nearest neighbor query protocol, denoted by $SkNN_b$, we relax the desirable properties to produce an efficient protocol (more details are given in the later part of this section).

The main steps involved in the $SkNN_b$ protocol are given in Algorithm 5. Bob initially encrypts his query Q attribute-wise, that is, he computes $E_{pk}(Q) = \langle E_{pk}(q_1), \dots, E_{pk}(q_m) \rangle$ and sends it to C_1 . Upon receiving $E_{pk}(Q)$ from Bob, C_1 with private input $(E_{pk}(Q), E_{pk}(t_i))$ and C_2 with the secret key sk jointly involve in the SSED protocol, where $E_{pk}(t_i) = \langle E_{pk}(t_{i,1}), \dots, E_{pk}(t_{i,m}) \rangle$, for $1 \leq i \leq n$. The output of this step, denoted by $E_{pk}(d_i)$, is the encryption of squared Euclidean distance between Q and t_i , i.e., $d_i = |Q - t_i|^2$. As mentioned earlier, $E_{pk}(d_i)$ is known only to C_1 , for $1 \leq i \leq n$. We emphasize that computation of exact Euclidean distance between encrypted

vectors is hard to achieve as it involves square root. However, in our problem, it is sufficient to compare the squared Euclidean distances as it preserves relative ordering. After this, C_1 sends $\{\langle 1, E_{pk}(d_1) \rangle, \ldots, \langle n, E_{pk}(d_n) \rangle\}$ to C_2 , where entry $\langle i, E_{pk}(d_i) \rangle$ correspond to data record t_i , for $1 \le i \le n$. Upon receiving $\langle 1, E_{pk}(d_1) \rangle, \ldots, \langle n, E_{pk}(d_n) \rangle$, C_2 decrypts the encrypted distance in each entry to get $d_i = D_{sk}(E_{pk}(d_i))$. Then, C_2 generates an index list $\delta = \langle i_1, \ldots, i_k \rangle$ such that $\langle d_{i_1}, \ldots, d_{i_k} \rangle$ are the top k smallest distances among $\langle d_1, \ldots, d_n \rangle$. After this, C_2 sends δ to C_1 . Upon receiving δ , C_1 proceeds as follows:

• Select the encrypted records $E_{pk}(t_{i_1}), \ldots, E_{pk}(t_{i_k})$ as the k-nearest records to Q and randomize them attributewise. More specifically, C_1 computes $E_{pk}(\gamma_{j,h}) = E_{pk}(t_{i_j,h}) * E_{pk}(r_{j,h})$, for $1 \le j \le k$ and $1 \le h \le m$. Here $r_{j,h}$ is a random number in \mathbb{Z}_N and $t_{i_j,h}$ denotes the column h attribute value of data record t_{i_j} . Send $\gamma_{j,h}$ to Bob, for $1 \le j \le k$ and $1 \le h \le m$.

Upon receiving $\gamma_{j,h}$, for $1 \leq j \leq k$ and $1 \leq h \leq m$, C_2 decrypts it to get $\gamma'_{j,h} = D_{sk}(\gamma_{j,h})$ and sends them to Bob. Note that, due to randomization by C_1 , $\gamma'_{j,h}$ is always a random number in \mathbb{Z}_N .

Finally, upon receiving $r_{j,h}$ from C_1 and $\gamma'_{j,h}$ from C_2 , Bob computes the attribute values of j^{th} nearest neighbor to Q as $t'_{j,h} = \gamma'_{j,h} - r_{j,h} \mod N$, for $1 \le j \le k$ and $1 \le h \le m$. Note that N is the RSA modulus or part of the public key pk.

4.2 Fully Secure *k*NN Protocol

The above-mentioned $SkNN_b$ protocol reveals the data access patterns to C_1 and C_2 . That is, for any given Q, C_1 and C_2 know which data records correspond to the k-nearest neighbors of Q. Also, it reveals d_i values to C_2 . However, leakage of such information may not be acceptable in privacy-sensitive applications such as medical data. Along this direction, we propose a fully secure protocol, denoted by $SkNN_m$ (where m stands for maximally secure), to retrieve the k-nearest neighbors of Q. The proposed $SkNN_m$ protocol preserves all the desirable properties of a secure kNN protocol as mentioned in Section 1.

The main steps involved in the proposed $SkNN_m$ protocol are as shown in Algorithm 6. Initially, Bob sends his attribute-wise encrypted query Q, that is, $E_{pk}(Q) = \langle E_{pk}(q_1), \dots, E_{pk}(q_m) \rangle$ to C_1 . Upon receiving, C_1 with private input $(E_{pk}(Q), E_{pk}(t_i))$ and C_2 with the secret key sk jointly involve in the SSED protocol. The output of this step is $E_{pk}(d_i) = E_{pk}(|Q-t_i|^2)$ which will be known only to C_1 , for $1 \le i \le n$. Then, C_1 with input $E_{pk}(d_i)$ and C_2 with sk securely compute the encryptions of the individual bits of d_i using the SBD protocol. Note that the output of this step $[d_i] = \langle E_{pk}(d_{i,1}), \dots, E_{pk}(d_{i,l}) \rangle$ is known only to C_1 , where $d_{i,1}$ and $d_{i,l}$ are the most and least significant bits of d_i respectively. Note that $0 \le d_i < 2^l$, for $1 \le i \le n$.

After this, C_1 and C_2 compute the top k (in encrypted form) records that are closest to Q in an iterative manner. More specifically, they compute $E_{pk}(t'_1)$ in the first iteration, $E_{pk}(t'_2)$ in the second iteration, and so on. Here t'_s denotes the s^{th} nearest neighbor to Q, for $1 \le s \le k$. At the end of k iterations, only C_1 knows $\langle E_{pk}(t'_1), \ldots, E_{pk}(t'_k) \rangle$. To start with, in the first iteration, C_1 and C_2 jointly compute the encryptions of the individual bits of the minimum value among d_1, \ldots, d_n using SMIN $_n$. That is, C_1 with input $\langle [d_1], \ldots, [d_n] \rangle$ and C_2 compute $[d_{\min}]$, where d_{\min} is the minimum value among d_1, \ldots, d_n . The output $[d_{\min}]$ is known only to C_1 . Now, C_1 performs the following operations locally:

ullet Compute the encryption of d_{\min} from its encrypted individual bits as below

$$E_{pk}(d_{\min}) = \prod_{\gamma=0}^{l-1} E_{pk}(d_{\min,\gamma+1})^{2^{l-\gamma-1}}$$
$$= E_{pk}(d_{\min,1} * 2^{l-1} + \dots + d_{\min,l})$$

where $d_{\min,1}$ and $d_{\min,l}$ are the most and least significant bits of d_{\min} respectively.

• Compute the encryption of difference between d_{\min} and each d_i . That is, C_1 computes $\tau_i = E_{pk}(d_{\min}) * E_{pk}(d_i)^{N-1} = E_{pk}(d_{\min} - d_i)$, for $1 \le i \le n$.

Algorithm 5 SkNN_b $(E_{pk}(T),Q) \rightarrow \langle t'_1,\ldots,t'_k \rangle$ Require: C_1 has $E_{pk}(T)$; C_2 has sk; Bob has Q

- 1: Bob:
 - (a). Compute $E_{pk}(q_i)$, for $1 \le j \le m$
 - (b). Send $E_{pk}(Q) = \langle E_{pk}(q_1), \dots, E_{pk}(q_m) \rangle$ to C_1
- 2: C_1 and C_2 :
 - (a). C_1 receives $E_{pk}(Q)$ from Bob
 - (b). **for** i = 1 to n **do**:
 - $E_{pk}(d_i) \leftarrow SSED(E_{pk}(Q), E_{pk}(t_i))$
 - (c). Send $\{\langle 1, E_{pk}(d_1) \rangle, \dots, \langle n, E_{pk}(d_n) \rangle \}$ to C_2
- 3: C_2 :
 - (a). Receive $\{\langle 1, E_{pk}(d_1) \rangle, \dots, \langle n, E_{pk}(d_n) \rangle\}$ from C_1
 - (b). $d_i \leftarrow D_{sk}(E_{pk}(d_i))$, for $1 \le i \le n$
 - (c). Generate $\delta \leftarrow \langle i_1, \dots, i_k \rangle$, such that $\langle d_{i_1}, \dots, d_{i_k} \rangle$ are the top k smallest distances among $\langle d_1, \dots, d_n \rangle$
 - (d). Send δ to C_1
- 4: C_1 :
 - (a). Receive δ from C_2
 - (b). for $1 \le j \le k$ and $1 \le h \le m$ do:
 - $\gamma_{j,h} \leftarrow E_{pk}(t_{i_j,h}) * E_{pk}(r_{j,h})$, where $r_{j,h} \in_R \mathbb{Z}_N$
 - Send $\gamma_{j,h}$ to C_2 and $r_{j,h}$ to Bob
- 5: C_2 :
 - (b). for $1 \le j \le k$ and $1 \le h \le m$ do:
 - Receive $\gamma_{i,h}$ from C_1
 - $\gamma'_{i,h} \leftarrow D_{sk}(\gamma_{j,h})$; send $\gamma'_{i,h}$ to Bob
- 6: Bob:
 - (a). for $1 \le j \le k$ and $1 \le h \le m$ do:
 - Receive $r_{j,h}$ from C_1 and $\gamma'_{j,h}$ from C_2
 - $t'_{j,h} \leftarrow \gamma'_{j,h} r_{j,h} \mod N$

• Randomize τ_i to get $\tau_i' = \tau_i^{r_i} = E_{pk}(r_i * (d_{\min} - d_i))$, where r_i is a random number in \mathbb{Z}_N . Note that τ_i' is an encryption of either 0 or a random number, for $1 \leq i \leq n$. Also, permute τ' using a random permutation function π (known only to C_1) to get $\beta = \pi(\tau')$ and send it to C_2 .

Upon receiving β , C_2 decrypts it component-wise to get $\beta_i' = D_{sk}(\beta_i)$, for $1 \le i \le n$. After this, he/she computes an encrypted vector U of length n such that $U_i = E_{pk}(1)$ if $\beta_i' = 0$, and $E_{pk}(0)$ otherwise. Here we assume that exactly one of the entries in β equals to zero and rest of them are random. This further implies that exactly one of the entries in U is an encryption of 1 and rest of them are encryptions of 0's. However, we emphasize that if β' has more than one 0's, then C_2 can randomly pick one of those indexes and assign $E_{pk}(1)$ to the corresponding index of U and $E_{pk}(0)$ to the rest. Then, C_2 sends U to C_1 . After receiving U, C_1 performs inverse permutation on it to get $V = \pi^{-1}(U)$. Note that exactly one of the entry in V is $E_{pk}(1)$ and the remaining are encryption of 0's. In addition, if $V_i = E_{pk}(1)$, then t_i is the closest record to Q. However, C_1 and C_2 do not know which entry in V corresponds to $E_{pk}(1)$.

Finally, C_1 computes $E_{pk}(t'_1)$, encryption of the closest record to Q, and updates the distance vectors as follows:

- C_1 and C_2 jointly involve in the secure multiplication (SM) protocol to compute $V'_{i,j} = V_i * E_{pk}(t_{i,j})$, for $1 \le i \le n$ and $1 \le j \le m$. The output V' from the SM protocol is known only to C_1 . After this, by using homomorphic properties, C_1 computes the encrypted record $E_{pk}(t'_1) = \langle E_{pk}(t_{1,1}), \ldots, E_{pk}(t_{1,m}) \rangle$ locally, $E_{pk}(t'_{1,j}) = \prod_{i=1}^n V'_{i,j}$, where $1 \le j \le m$. Note that $t'_{1,j}$ denotes the j^{th} attribute value of record t'_1 .
- It is important to note that the first nearest tuple to Q should be obliviously excluded from further computations. However, since C_1 does not know the record corresponding to $E_{pk}(t'_1)$, we need to obliviously eliminate the possibility of choosing this record again in next iterations. For this, C_1 obliviously updates the distance corresponding to $E_{pk}(t'_1)$ to the maximum value, i.e., $2^l 1$. More specifically, C_1 updates the distance vectors with the help of C_2 using the SBOR protocol as below, for $1 \le i \le n$ and $1 \le \gamma \le l$.

$$E_{pk}(d_{i,\gamma}) = SBOR(V_i, E_{pk}(d_{i,\gamma}))$$

Note that when $V_i = E_{pk}(1)$, the corresponding distance vector d_i is set to the maximum value. That is, under this case, $[d_i] = \langle E_{pk}(1), \dots, E_{pk}(1) \rangle$. However, when $V_i = E_{pk}(0)$, the OR operation has no affect on d_i .

The above process is repeated until k iterations, and in each iteration $[d_i]$ corresponding to the current chosen record is set to the maximum value. However, since C_1 does not know which $[d_i]$ is updated, he/she has to re-compute $E_{pk}(d_i)$ in each iteration using their corresponding encrypted binary vectors $[d_i]$, for $1 \le i \le n$. In iteration s, $E_{pk}(t'_s)$ is known only to C_1 .

At the end of the iterative step (i.e., step 3 of Algorithm 6), C_1 has $\langle E_{pk}(t'_1), \ldots, E_{pk}(t'_k) \rangle$ - the list of encrypted records of k-nearest neighbors to the input query Q. The rest of the process is similar to steps 4 to 6 of Algorithm 5. Briefly, C_1 randomizes $E_{pk}(t'_j)$ attribute-wise to get $\gamma_{j,h} = E_{pk}(t'_{j,h}) * E_{pk}(r_{j,h})$ and sends $\gamma_{j,h}$ to C_2 and $r_{j,h}$ to Bob, for $1 \le j \le k$ and $1 \le h \le m$. Here $r_{j,h}$ is a random number in \mathbb{Z}_N . Upon receiving $\gamma_{j,h}$'s, C_2 decrypts them to get the randomized k-nearest records as $\gamma'_{j,h} = D_{sk}(\gamma_{j,h})$ and sends them to Bob, for $1 \le j \le k$ and $1 \le h \le m$. Finally, upon receiving $r_{j,h}$ from C_1 and $\gamma'_{j,h}$ from C_2 , Bob computes the j^{th} nearest neighboring record to Q, as $t'_{j,h} = \gamma'_{j,h} - r_{j,h} \mod N$, for $1 \le j \le k$ and $1 \le h \le m$.

4.3 Security Analysis

Here we analyze the security guarantees of the proposed protocols. First, due to the encryption of Q and by semantic security of the Paillier cryptosystem, Bob's input query Q is protected from Alice, C_1 and C_2 in both protocols.

In the $SkNN_b$ protocol, the decryption operations at step 3(b) of Algorithm 5 reveal d_i values to C_2 . In addition, since C_2 generates the top k index list (at step 3(c) of Algorithm 5) and sends it to C_1 , the data access patterns are revealed to C_1 and C_2 . Therefore, our basic $SkNN_b$ protocol is secure under the assumption that d_i values can be revealed to C_2 and data access patterns can be revealed to C_1 and C_2 .

On the other hand, the security analysis of $SkNN_m$ is as follows. At step 2 of Algorithm 6, the outputs of SSED and SBD are in encrypted format, and are known only to C_1 . In addition, all the intermediate results decrypted by C_2 in SSED are uniformly random in \mathbb{Z}_N . Also, as mentioned in [21], the SBD protocol is secure. Thus, no information is

Algorithm 6 SkNN_m $(E_{pk}(T),Q) \rightarrow \langle t'_1,\ldots,t'_k \rangle$

Require: C_1 has $E_{pk}(T)$ and π ; C_2 has sk; Bob has Q

- 1: Bob sends $E_{pk}(Q) = \langle E_{pk}(q_1), \dots, E_{pk}(q_m) \rangle$ to C_1
- 2: C_1 and C_2 :
 - (a). for i = 1 to n do:
 - $E_{pk}(d_i) \leftarrow \text{SSED}(E_{pk}(Q), E_{pk}(t_i)) \text{ and } [d_i] \leftarrow \text{SBD}(E_{pk}(d_i))$
- 3: **for** s = 1 to k **do:**
 - (a). C_1 and C_2 :
 - $[d_{\min}] \leftarrow SMIN_n([d_1], \dots, [d_n])$
 - (b). C_1 :
 - $E_{pk}(d_{\min}) \leftarrow \prod_{\gamma=0}^{l-1} E_{pk}(d_{\min,\gamma+1})^{2^{l-\gamma-1}}$
 - if $s \neq 1$ then, for $1 \leq i \leq n$

$$- E_{pk}(d_i) \leftarrow \prod_{\gamma=0}^{l-1} E_{pk}(d_{i,\gamma+1})^{2^{l-\gamma-1}}$$

• for i = 1 to n do:

$$- \tau_i \leftarrow E_{pk}(d_{\min}) * E_{pk}(d_i)^{N-1}$$
$$- \tau_i' \leftarrow \tau_i^{r_i}, \text{ where } r_i \in_R \mathbb{Z}_N$$

- $\beta \leftarrow \pi(\tau')$; send β to C_2
- (c). C_2 :
 - $\beta'_i \leftarrow D_{sk}(\beta_i)$, for $1 \le i \le n$
 - Compute U, for $1 \le i \le n$:

- if
$$\beta'_i = 0$$
 then $U_i = E_{pk}(1)$
- else $U_i = E_{pk}(0)$

- Send U to C_1
- (d). C_1 :
 - $V \leftarrow \pi^{-1}(U)$
 - $V'_{i,j} \leftarrow \text{SM}(V_i, E_{pk}(t_{i,j}))$, for $1 \leq i \leq n$ and $1 \leq j \leq m$
 - $E_{pk}(t'_{s,i}) \leftarrow \prod_{i=1}^n V'_{i,i}$, for $1 \le j \le m$
 - $E_{pk}(t'_s) = \langle E_{pk}(t'_{s,1}), \dots, E_{pk}(t'_{s,m}) \rangle$
- (e). C_1 and C_2 , for $1 \le i \le n$:
 - $E_{pk}(d_{i,\gamma}) \leftarrow \text{SBOR}(V_i, E_{pk}(d_{i,\gamma})), \text{ for } 1 \leq \gamma \leq l$

The rest of the steps are similar to steps 4-6 of $SkNN_b$

revealed during step 2 of Algorithm 6. In each iteration, the output of SMIN_n is known only to C_1 and no information is revealed to C_2 . Also, C_1 and C_2 do not know which record belongs to current global minimum. Thus, data access patterns are protected from both C_1 and C_2 . At step 3(c) of Algorithm 6, a component-wise decryption of β reveals the tuples that satisfy the current global minimum distance to C_2 . However, due to permutation by C_1 , C_2 cannot trace back to the corresponding data records. Also, note that decryption of β gives either encryptions of 0's or random numbers in \mathbb{Z}_N . Similarly, since U is an encrypted vector, C_1 cannot know which tuple corresponds to current global minimum distance. Thus, data access patterns are further protected at this step from C_1 . In addition, the update process at step 3(e) of Algorithm 6 does not leak any information to C_1 and C_2 . In summary, C_1 and C_2 do not know which data records correspond to the output set $\langle t'_1, \ldots, t'_k \rangle$.

Based on the above discussions, it is clear that the proposed $SkNN_m$ protocol protects the confidentiality of the data, privacy of user's input query, and hides the data access patterns.

4.4 Complexity Analysis

The computation complexity of $SkNN_b$ is bounded by O(n*m+k) encryptions, decryptions and exponentiations. In practice $k \ll n*m$; therefore, the computation complexity of $SkNN_b$ is bounded by O(n*m) encryptions and exponentiations (assuming that encryption and decryption operations under Paillier cryptosystem take similar amount of time).

In the $SkNN_m$ protocol, the computation complexity is bounded by O(n) instantiations of SBD and SSED, O(k) instantiations of $SMIN_n$, and O(n*l) instantiations of SBOR. We emphasize that the computation complexity of the SBD protocol proposed in [21] is bounded by O(l) encryptions and O(l) exponentiations. Also, the computation complexity of SSED is bounded by O(m) encryptions and O(m) exponentiations. In addition, the computation complexity of $SMIN_n$ is bounded by $O(l*n*log_2n)$ encryptions and $O(l*n*log_2n)$ exponentiations. Since SBOR utilizes SM as a sub-routine, the computation cost of SBOR is bounded by (small) constant number of encryptions and exponentiations. Based on the above analysis, the total computation complexity of the $SkNN_m$ protocol is bounded by $O(n*(l+m+k*l*log_2n))$ encryptions and exponentiations.

5 Empirical Results

In this section, we discuss the performances of the proposed protocols in detail under different parameter settings. We used Paillier cryptosystem [18] and implemented the proposed protocols in C. Various experiments were conducted on a Linux machine with an Intel® Xeon® Six-CoreTM CPU 3.07 GHz processor and 12GB RAM running Ubuntu 10.04 LTS.

Since it is difficult to control the parameters in a real dataset, we randomly generated synthetic datasets depending on the parameter values in consideration. Using these synthetic datasets we can perform a more elaborated analysis on the computation costs of the proposed protocols under different parameter settings. We encrypted these datasets attribute-wise, using the Paillier encryption whose key size is varied in our experiments, and the encrypted data were stored on our machine. Based on the protocols protocols, we then executed a random query over this encrypted data. For the rest of this section, we do not discuss about the performance of Alice since it is a one-time cost. Instead, we evaluate and analyze the performances of $SkNN_b$ and $SkNN_m$ separately. In addition, we compare the two protocols. In all our experiments, the Paillier encryption key size, denoted by K, is set to either 512 or 1024 bits.

5.1 Performance of $SkNN_b$

In this sub-section, we analyze the computation costs of $SkNN_b$ by varying the number of data records (n), number of attributes (m), number of nearest neighbors (k), and encryption key size (K). Note that $SkNN_b$ is independent of the domain size of attributes (l).

First, by fixing k=5 and K=512, we evaluated the computation costs of $SkNN_b$ for varying n and m. As shown in Figure 2(a), the computation costs of $SkNN_b$ grows linearly with n and m. For example, when m=6, the computation time of $SkNN_b$ increases from 44.08 to 87.91 seconds when n is varied from 2000 to 4000. A similar

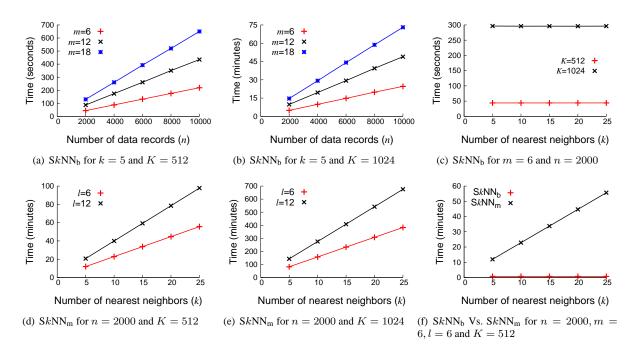


Figure 2: Time complexities of $SkNN_b$ and $SkNN_m$ for varying values of n, m, l, k and encryption key size K

trend is observed for K = 1024 as shown in Figure 2(b). For any fixed parameters, we observed that the computation time of $SkNN_b$ increases almost by a factor of 7 when K is doubled.

Next, by fixing m=6 and n=2000, we evaluated the running times of $SkNN_b$ for varying k and K. The results are shown in Figure 2(c). Irrespective of K, the computation time of $SkNN_b$ does not change much with varying k. This is because most of the cost in $SkNN_b$ comes from the SSED protocol which is independent of k. E.g., when K=512 bits, the computation time of $SkNN_b$ changes from 44.08 to 44.14 seconds when k is changed from 5 to 25. Based on the above discussions, it is clear that the running time of $SkNN_b$ mainly depends on (or grows linearly with) k0 and k1 which further justifies our complexity analysis in Section 4.4.

5.2 Performance of $SkNN_m$

We also evaluated the computation costs of $SkNN_m$ for varying values of k, l and K. Throughout this sub-section, we fix m=6 and n=2000. However, we observed that the running time of $SkNN_m$ grows linearly with n and m.

For K=512 bits, the computation costs of $SkNN_m$ for varying k and l are as shown in Figure 2(d). Following from Figure 2(d), for l=6, the running time of $SkNN_m$ varies from 11.93 to 55.65 minutes when k is changed from 5 to 25 respectively. Also, for l=12, the running time of $SkNN_m$ varies from 20.68 to 97.8 minutes when k is changed from 5 to 25 respectively. In either case, the cost of $SkNN_m$ grows almost linearly with k and k.

A similar trend is observed for K=1024 as shown in Figure 2(e). In particular, for any given fixed parameters, we identified that the computation cost of $SkNN_m$ increases by almost a factor of 7 when K is doubled. For example, when k=10, $SkNN_m$ took 22.85 and 157.17 minutes to generate the 10 nearest neighbors of Q under K=512 and 1024 bits respectively. Furthermore, when k=5, we observed that around 69.7% of cost in $SkNN_m$ is accounted due to $SMIN_n$ which is initiated k times in $SkNN_m$ (once in each iteration). Also, the cost incurred due to $SMIN_n$ increases from 69.7% to at least 75% when k is increased from 5 to 25.

In addition, by fixing n=2000, m=6, l=6 and K=512, we compared the running times of both protocols for varying values of k. As shown in Figure 2(f), the running time of $SkNN_b$ remains to be constant at 0.73 minutes since it is almost independent of k. However, the running time of $SkNN_m$ changes from 11.93 to 55.65 minutes as we increase k from 5 to 25.

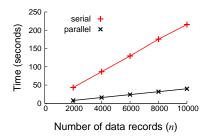


Figure 3: Comparison of parallel and serial versions of SkNN_b for m = 6, k = 5 and K = 512 bits

Based on the above results, it is clear that the computation costs of $SkNN_m$ are significantly higher than that of $SkNN_b$. However, we emphasize that $SkNN_m$ is more secure than $SkNN_b$; therefore, the two protocols act as a trade-off between security and efficiency. Also, it is important to note that Bob's computation cost is mainly due to the encryption of his input query record. As an example, for m=6, Bob's computation costs are 4 and 17 milliseconds when K is 512 and 1024 bits respectively. This further shows that our proposed protocols are very efficient from end-user's perspective.

5.3 Towards Performance Improvement

At first, it seems that the proposed protocols are costly and may not scale well for large datasets. However, in both protocols, we emphasize that the computations involved on each data record are independent of others. Therefore, we can parallelize the operations on data records for efficiency purpose. To further justify this claim, we implemented a parallel version of our $SkNN_b$ protocol using OpenMP programming and compared its computation costs with its serial version. As mentioned earlier, our machine has 6 cores which can be used to perform parallel operations on 6 threads. For m = 6, k = 5 and K = 512 bits, the comparison results are as shown in Figure 3. The observation is that the parallel version of $SkNN_b$ is roughly 6 times more efficient than its serial version. This is because of the fact that the parallel version can execute operations on 6 data records at a time (i.e., on 6 threads in parallel). For example, when n = 10000, the running times of parallel and serial versions of $SkNN_b$ are 40 and 215.59 seconds respectively.

We believe that similar efficiency gains can be achieved by parallelizing the operations in $SkNN_m$. Based on the above discussions, especially in a cloud computing environment where high performance parallel processing can easily be achieved, we claim that the scalability issue of the proposed protocols can be eliminated or mitigated. In addition, using the existing map-reduce techniques, we can drastically improve the performance further by executing parallel operations on multiple nodes. We will leave this analysis to future work.

6 Conclusion

Query processing is an important task in database management systems. In particular, k-nearest neighbors is one of the commonly used query in many data mining applications such as detection of fraud by credit card companies and prediction of tumor cells levels in blood. With the recent growth of cloud computing as a new IT paradigm, data owners are more interested to outsource their databases as well as DBMS functionalities to the cloud. Under an outsourced database environment, where encrypted data are stored in the cloud, secure query processing over encrypted data becomes challenging. To protect user privacy, various secure k-nearest neighbor (SkNN) techniques have been proposed in the literature. However, the existing SkNN techniques over encrypted data are not secure.

Along this direction, we proposed two novel SkNN protocols over encrypted data in the cloud. The first protocol, which acts as a basic solution, leaks some information to the cloud. On the other hand, our second protocol is fully secure, that is, it protects the confidentiality of the data, user's input query, and also hides the data access patterns. However, the second protocol is more expensive compared to the basic protocol. Also, we evaluated the performance of our protocols under different parameter settings. As a future work, we will investigate and extend our research to other complex conjuctive queries over encrypted data.

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