BLIND DECONVOLUTION OF NOISY BLURRED IMAGES VIA DISPERSION MINIMIZATION

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ABSTRACT

In linear image restoration, the point spread function of the degrading system is assumed known even though this information is usually not available in real applications. As a result, both blur identification and image restoration must be performed from the observed noisy blurred image. This paper presents a computationally simple linear adaptive finite impulse response filter for blind image deconvolution. This is essentially a two-dimensional version of the Constant Modulus Algorithm that is well known in the field of blind equalization. The two-dimensional extension is shown capable of reconstructing noisy blurred images using partial *a priori* information about the true image and the point spread function. The method is applicable to minimum as well as mixed phase blurs. Experimental results are provided.

1. INTRODUCTION

The purpose of image restoration is to reconstruct an unobservable *true* image from a degraded observation. Blur and observation noise are the main sources of degradation. An observed image can be written, ignoring additive noise, as the Two-Dimensional (2-D) convolution of the true image with a Linear Shift-Invariant (LSI) blur, known as the Point Spread Function (PSF). Restoration in the case of known blur, assuming the linear degradation model, is called *linear image restoration* and it has been investigated extensively in the last three decades giving rise to a variety of solutions [1] [2]. In many practical situations, however, the blur is unknown. Hence, both blur identification and image restoration must be performed from the degraded image. Restoration in the case of unknown blur is called *blind image restoration (deconvolution)*.

Kundur and Hatzinakos [3] [4] provide excellent tutorials which divide blind image deconvolution methods into two major groups: i) those which estimate the PSF *a priori* independent of the true image, and ii) those which estimate the PSF and the true image simultaneously. Algorithms belonging to the first class are computationally simple, but they are limited to situations in which the PSF has a special parametric form, and the true image has certain features. Algorithms belonging to the second class, which are computationally more complex, must be used for more general situations.

In this paper, a new iterative blind image deconvolution method that belongs to the second class is proposed. The method is based on linear adaptive Finite Impulse Response (FIR) filtering and is most applicable to six or less-bit gray scale images. The proposed method utilizes a cost function like all other iterative linear adaptive filtering methods in order to update coefficients of the adaptive filter. The Constant-Modulus (CM) cost [5] [6], which is one of the most studied and implemented methods of blind adaptive equalization for data communications over dispersive channels, is used as the cost function. First, it is shown how the method can be extended to the 2-D case. Then, this 2-D extension is applied to the blind image deconvolution problem.

This paper is organized as follows. The problem is described in section 2. The proposed method is explained in detail in section 3, which also discusses the CM cost. Computer simulation results are provided in section 4. Finally, section 5 concludes the paper.

2. PROBLEM STATEMENT

Consider the Single-Input Single-Output (SISO) discrete-time LSI system depicted in Fig. 1, in which $f(m,n)$, $h(m,n)$, $v(m,n)$, and $g(m,n)$ represent the the true image, the PSF of the degrading system, additive Gaussian noise that is independent of $f(m,n)$, and the degraded image, respectively.

It is clear from Fig. 1 that the observed $M \times N$ noisy blurred image $g(m,n)$ can be written as

$$\begin{aligned} g(m,n) &= f(m,n) * h(m,n) + v(m,n) \\ &= \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} f(k,l) h(m-k, n-l) + v(m,n) \end{aligned}$$

for $m = 0, \ldots, M-1$, $n = 0, \ldots, N-1$, where $*$ denotes the 2-D linear convolution operator and $h(m,n)$ assumes non-zero values only over the support S_h. Since blurs are usually modeled as 2-D Finite Impulse Response (FIR) filters, S_h is a proper subset of the set of 2-D integers.

The PSF of the degrading system, $h(m,n)$, is usually unknown in most real applications. Hence, the true image must be estimated directly from the noisy blurred image using only partial information about the true image and the PSF. This process is called *blind image deconvolution*. As shown in [7], ambiguities in both gain and

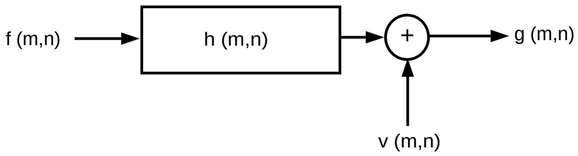

Fig. 1. Linear image degradation model.

delay are inherent to blind image deconvolution. Hence, the problem to be solved can be stated more precisely as follows: *Obtain an estimate of the form* $\hat{f}(m,n) \approx \alpha f(m-m_0, n-n_0)$ *for some real* $\alpha \neq 0$ *and for some integers* m_0, n_0 *from the observed image* $g(m,n)$*, using partial or no information about the true image* $f(m,n)$ *and the PSF* $h(m,n)$.

3. IMAGE DECONVOLUTION VIA DISPERSION MINIMIZATION

This section explains the proposed method in detail. In the remainder of the paper, the true image pixels are assumed to have odd integer gray levels $\pm 1, \pm 3, \dots, \pm(L-1)$, where L is the number of gray levels in the true image, unless otherwise stated[1]. The CM cost on which the proposed method depends are explained next before the proposed method is described.

3.1. The CM Cost

Even though traditional uses of the CM cost have all been One-Dimensional (1-D), the CM cost can be extended for use in 2-D. The CM term was introduced for blind equalization of communication signals over dispersive channels by Godard [5] and Treichler and Agee [6]. The reader is referred to [8] for a comprehensive introduction to the CM cost in the context of adaptive equalization. This section generalizes the CM cost for use in 2-D by reformulating the cost for a real-valued zero-mean true image $f(m,n)$ and a real-valued PSF $h(m,n)$. It is assumed that each gray level of the true image is equally likely (*a suitable preprocessing of the degraded image such as histogram equalization may be necessary to satisfy this condition*). The CM cost is given by

$$\begin{aligned} J_{CM} &:= E[(\hat{f}^2(m,n) - \gamma)^2] \\ &= E[\hat{f}^4(m,n)] - 2\sigma_f^2 \kappa_f E[\hat{f}^2(m,n)] + \sigma_f^4 \kappa_f^2 \end{aligned}$$

where γ and κ_f are the dispersion constant and normalized kurtosis of the true image, respectively. They are defined by

$$\kappa_f := \frac{E[f^4(m,n)]}{(E[f^2(m,n)])^2}$$

$$\gamma := \frac{E[f^4(m,n)]}{E[f^2(m,n)]}.$$

[1]Most of real images are 8-bit having non-zero gray levels. These images can be transformed to have gray levels $\pm 1, \pm 3, \dots, \pm(L-1)$ by a uniform or non-uniform thresholding based on the distribution of pixels in the true image.

Gray levels	γ	κ_f
2	1	1
4	8.2	1.64
8	37	1.716
16	152.2	1.790
32	613	1.797
64	2456.2	1.799
128	9829	1.799
256	39320	1.800

Table 1. Dispersion constant and normalized kurtosis for a zero mean uniformly distributed image having different gray levels.

It is evident from its definition that the CM cost penalizes the deviations of $\hat{f}^2(m,n)$ from constant γ. This is why the proposed method is called *blind image deconvolution using dispersion minimization*. Table 1 gives the dispersion constant and normalized kurtosis of a zero mean uniformly distributed gray scale image for various gray levels.

Gradient Descent (GD) methods are generally used to solve for CM estimators (dispersion minimizers) because closed form expressions do not usually exist. Since exact GD requires statistical knowledge of the degraded image, which is not available in real applications, stochastic GD methods are utilized. The algorithm that performs a stochastic GD minimization of J_{CM} is referred to as the *Constant Modulus Algorithm or CMA*. Plotting the CM cost versus the adaptive filter parameters results in a surface called *the CM cost surface*. CMA attempts to minimize the CM cost by starting at some location on the surface and following the trajectory of the steepest descent.

3.2. Proposed Algorithm

The proposed method is shown in Fig. 2, where the observed image $g(m,n)$ is applied to a 2-D adaptive FIR filter $w(m,n)$ with support $[-C,C] \times [-D,D]$ which tries to remove the blur. Thus, output of the adaptive filter at the jth iteration $\hat{f}_j(m,n)$ is an estimate of the true image given by

$$\hat{f}_j(m,n) = \sum_{k=-C}^{C} \sum_{l=-D}^{D} w_j(k,l) g(m-k, n-l) \quad (1)$$

for $m = 0, \dots, M-1$, $n = 0, \dots N-1$, where $w_j(k,l)$ are the adaptive filter coefficients at the jth iteration for $-C \leq k \leq C$, $-D \leq l \leq D$. Initially, since the adaptive filter is not a good approximation to the inverse of the blur, $\hat{f}_j(m,n)$ is not reliable enough. However, it may be used in an adaptive scheme to obtain a better estimate for the next spatial location. If the true image $f(m,n)$ were known, then the difference between $\hat{f}_j(m,n)$ and $f(m,n)$ could be used to provide an efficient update of the filter parameters. In blind image deconvolution, however, the true image is unavailable. As in adaptive equalization, one possibility is to attempt to minimize the dis-

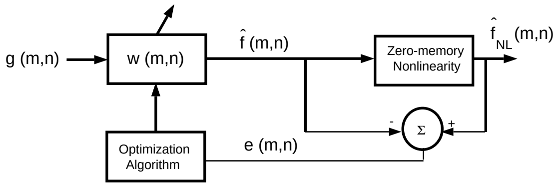

Fig. 2. Block diagram of the proposed method.

persion of $\hat{f}_j(m,n)$ using the CM cost J_{CM}. Since it is not possible to minimize an expected value directly, the method uses an instantaneous estimate of J_{CM} given by

$$J := \frac{1}{4}\left(\hat{f}^2(m,n) - \gamma\right)^2. \tag{2}$$

Note that the function of the zero-memory nonlinearity (the rightmost term in Fig. 2) is to produce an artificially generated desired image $\hat{f}_{NL,j}(m,n)$ for the algorithm so that an error between $\hat{f}_{NL,j}(m,n)$ and the output of the adaptive filter $\hat{f}_j(m,n)$ can be obtained to update the adaptive filter coefficients. The zero-memory nonlinearity is chosen such that this difference is equal to negative of the gradient of J.

The stochastic GD minimization is used to update the adaptive filter parameters. The derivative of J with respect to the adaptive filter parameters is needed in order to implement the stochastic GD minimization. Let $\mathbf{w}_j$, $\mathbf{g}(m,n)$ denote the following lexicographically ordered adaptive filter vector at the j the iteration and the regressor vector for the (m,n)th pixel:

$$\mathbf{w}_j := \begin{bmatrix} w_j(-P,-Q) \\ w_j(-P,-Q+1) \\ w_j(-P,-Q+2) \\ \vdots \\ w_j(P,Q) \end{bmatrix} \tag{3}$$

$$\mathbf{g}(m,n) := \begin{bmatrix} g(m+P,n+Q) \\ g(m+P,n+Q-1) \\ g(m+P,n+Q-2) \\ \vdots \\ g(m-P,n-Q) \end{bmatrix} \tag{4}$$

Using vectors $\mathbf{w}_j$ and $\mathbf{g}(m,n)$, the output of the adaptive filter for the (m,n)th pixel at the jth iteration $\hat{f}_j(m,n)$ can be written as

$$\hat{f}_j(m,n) = \mathbf{w}_j^T \mathbf{g}(m,n) \tag{5}$$

where T denotes vector transposition. Now, the derivative of J with respect to $\mathbf{w}_j$ can be evaluated, which is given by

$$\frac{dJ}{d\mathbf{w}_j} = \left(\hat{f}_j^2(m,n) - \gamma\right)\hat{f}_j(m,n)\mathbf{g}(m,n). \tag{6}$$

Hence, the adaptive filter is updated according to

$$\begin{aligned} \mathbf{w}_{j+1} &= \mathbf{w}_j - \mu\frac{dJ}{d\mathbf{w}_j} \\ &= \mathbf{w}_j - \mu\phi(\hat{f}_j(m,n))\mathbf{g}(m,n) \end{aligned} \tag{7}$$

where μ is a small positive step-size that guarantees stability of the algorithm and

$$\phi(\hat{f}_j(m,n)) := \left(\hat{f}_j^2(m,n) - \gamma\right)\hat{f}_j(m,n) \tag{8}$$

is called the *prediction error function*. The prediction error function $\phi(\cdot)$ has some interesting and important properties when the coefficients of the adaptive filter are near the global minimum of J_{CM}. The static and dynamic convergence analysis of the proposed method in the vicinity the global minimum of J_{CM} can be performed by using important features of $\phi(\cdot)$ which were presented in detail in [7].

Equations (5)-(7) constitute the proposed *blind image deconvolution via dispersion minimization* method. When convergence occurs, $\hat{f}(m,n)$ provides an estimate of the true image $f(m,n)$, and the lexicographically ordered adaptive filter vector $\mathbf{w}$ is an approximate inverse of the PSF.

4. SIMULATION RESULTS

A computer simulation result is provided in this section to demonstrate usefulness of the proposed method. The classical 8-bit gray-scale *Lena* image was chosen as a test image. Histogram equalization was performed on the test image, which results in an approximately uniformly distributed image. Then, its mean was subtracted from the histogram equalized image resulting in a zero-mean uniformly distributed image. Finally, a uniform quantization was applied to the zero mean uniformly distributed image to obtain a 4-bit true image that fulfills most of the assumptions made about the true image.

In order to obtain artificially generated blurred image, a 5×5 scatter blur was applied to the true image. Zero-mean Gaussian noise was added to the blurred image to get the observed noisy blurred image at 70 dB-Blurred Signal-to-Noise Ratio (BSNR). Performance was measured in terms of the frequently used Improvement in Signal-to-Noise Ratio (ISNR) metric.

Note that the CM cost is non-convex. Hence, the method may converge to a local minimum instead of the global minimum of J_{CM} depending on how it is initialized. If there is no *a priori* information about the PSF, the adaptive filter is initialized using a 2-D spike characterized by a non-zero coefficient usually located somewhere in the central portion of the adaptive filter. If there is *a priori* information about the PSF, this information may aid in initializing the adaptive filter in a better way.

Blind deconvolution result using the proposed method is depicted in Fig. 3. Support of the adaptive filter and the value of step-size μ were 5×5 and 0.0001. The method was successful in estimating the true image. Magnitude of the 32×32-point 2-D Discrete Fourier Transform (DFT) of the blur and the adaptive filter at convergence are shown in Fig. 4. Observe that the adaptive filter has approximately converged to the inverse of the blur (this may be a local minimum since the CM cost is non-convex and 2-D impulse function initialization may not produce the global minimum of the CM cost).

Fig. 3. Blind deconvolution result for a 4-bit true image at 70 dB BSNR. (left) True image; (middle) Degraded image; (right) Estimated true image, ISNR 6.12 dB.

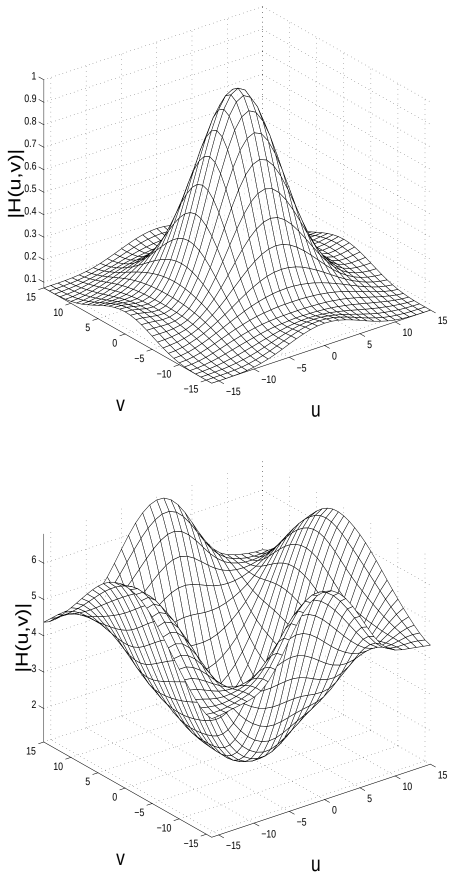

Fig. 4. Magnitude of (top) blur; (bottom) adaptive filter at convergence.

5. CONCLUSIONS

A new method which is based on linear adaptive FIR filtering for blind deconvolution of noisy blurred images was proposed in this paper. The method is essentially a 2-D extension of the CMA. An important aspect of the method is that images which were blurred by minimum or mixed-phase blurs can be recovered. This is due to the fact the method does not impose constraints on the phase of the blur. Another important aspect is that the method is computationally simple, which makes its implementation easy for real applications. Performance of the method depends on the kurtosis (number of gray levels) of the true image and BSNR. As the true image kurtosis and BSNR increase performance worsens. The method is most likely to work for six or less-bit images up to 30 dB BSNR. Finally, the CM cost could be minimized using an adaptive Auotoregresive (AR) filter instead of an adaptive FIR filter ,which provides better results, but whose implementation and analysis are more difficult. See [7] for complete details.

6. REFERENCES

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