

A Low-Complexity Gaussian Message Passing Iterative Detector for Massive MU-MIMO Systems

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Abstract—This paper considers a low-complexity Gaussian Message Passing Iterative Detection (GMPID) method over a pairwise graph for a massive Multiuser Multiple-Input Multiple-Output (MU-MIMO) system, in which a base station with M antennas serves K Gaussian sources simultaneously. Both K and M are large numbers and we consider the cases that $K < M$ in this paper. The GMPID is a message passing algorithm based on a fully connected loopy graph, which is well known that it is not convergent in some cases. In this paper, we first analyse the convergence of GMPID. Two sufficient conditions that the GMPID converges to the Minimum Mean Square Error (MMSE) detection are proposed. However, the GMPID may still not converge when $K/M > (\sqrt{2} - 1)^2$. Therefore, a new convergent GMPID with equally low complexity called SA-GMPID is proposed, which converges to the MMSE detection for any $K < M$ with a faster convergence speed. Finally, numerical results are provided to verify the validity and accuracy of the proposed theoretical results.

I. INTRODUCTION

Multiuser Multiple-Input and Multiple-Output (MU-MIMO) has become a key technology for wireless communication standards like IEEE 802.11n (Wi-Fi), IEEE 802.11ac (Wi-Fi), WiMAX (4G) and Long Term Evolution (4G). Recent research investigations [1], [2] show that MU-MIMO will play a vital role in the future wireless systems. Recently, the massive MU-MIMO, where the Base Station (BS) has a very large number of antennas (e.g., hundreds or even more), has attracted more and more attention [1]–[5]. For instance, several researchers prove that the massive MU-MIMO can bring huge improvement both in throughput and energy efficiency [3]–[5].

One of the current challenging problems in massive MU-MIMO is the low-complexity signal detection in the up-link [2]. In the case of Gaussian sources, it is well known that Minimum Mean Square Error (MMSE) detection is optimal. However, the complexity is significantly high for its unfavorable matrix inversion [6]. Another kind of MU-MIMO detection is graph based detection called message passing algorithm [7], [8]. There are two kinds message passing algorithms, One of which is Gaussian Belief Propagation (GaBP) algorithm based on a graph that consisted by variable

nodes [9]–[14] and the other is Gaussian Message Passing Iterative Detection (GMPID) based on a pairwise graph that consists of variable nodes and sum nodes [15]–[20]. Both of them are efficient distributed message passing algorithms for Gaussian graphical models. Moreover, the GMPID has also been studied for equalization in the inter-symbol interference channel [15] and the decoding of modern channel codes, such as turbo codes and low density parity check codes [16].

It is proved that if the factor graph is of tree structure, the means and variances of the message passing algorithm converge to the true marginal means and approximate marginal variances respectively [7], [8]. However, if the graph has cycles, the message passing algorithm may fail to converge. Most previous works of the message passing algorithm focus on the convergence of the GaBP algorithm. Three general sufficient conditions for convergence of GaBP in loopy graphs are known: diagonal-dominance [10], [11], convex decomposition [9] and walk-summability [12]. Recently, a necessary and sufficient variances convergence condition of the GaBP is given in [13]. For the GMPID based on the pairwise graph, a sufficient condition of the means convergence is given in [17]. However, the posterior density matrixes of each sum node are needed to calculate [17], which introduces the matrix inversion operation and a much higher computational complexity during the message updating. Montanari [18] has proved the GMPID algorithm converges to the optimal MMSE solution for any arbitrarily loaded randomly-spread CDMA system, but this only works for CDMA MIMO system with binary channels. In general, the GMPID has lower computational complexity and a better Mean Square Error (MSE) performance than the GaBP algorithm. To the best of our knowledge, the convergence of GMPID based on a pairwise graph is far from completion.

In this paper, we analyse the convergence of the existing GMPID and propose a new convergent detection method for massive MU-MIMO systems with K users and M antennas. Let $\beta = K/M$ and $\beta < 1$. The contributions of this paper are summarized as follows.

- 1) We prove that the variances of GMPID definitely converge to the MSE of MMSE detection, which also gives an alternative way to estimate the MSE performance for the MMSE detection.
- 2) Two sufficient conditions are derived to guarantee that the

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means of GMPID converge to that of MMSE detection for $\{\beta : 0 < \beta < (\sqrt{2} - 1)^2\}$.

- 3) A new convergent GMPID called SA-GMPID is proposed, which converges and converges faster than GMPID to the MMSE detection for any $\{\beta : 0 < \beta < 1\}$.

II. SYSTEM MODEL AND MMSE DETECTOR

In this section, the massive MU-MIMO system model and some preliminaries about the optimal MMSE detector for the massive MU-MIMO systems will be introduced.

A. System Model

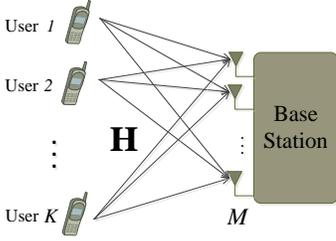


Fig. 1. MU-MIMO system model: K autonomous single-antenna terminals simultaneously communicate with an array of M antennas of the base station..

Fig. 1 shows the system model. we consider a uplink MU-MIMO system with K users and one BS with M antennas [2]–[4]. For massive MIMO, the K and M are very large (be hundreds or thousands), e.g., $M = 600$ and $K = 100$. The $M \times 1$ received signal vector \mathbf{y} at the BS is represented by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

where \mathbf{H} denotes the $M \times K$ channel matrix, $\mathbf{n} \sim \mathcal{N}^M(0, \sigma_n^2)$ is an $M \times 1$ independent Additive White Gaussian Noise (AWGN) vector and \mathbf{x} is the message vector sent from K users. We assume that the channels only suffer from the small-scale fading without large-scale fading, in which \mathbf{H} denotes the Rayleigh fading channel matrix whose entries are independently and identically distributed (i.i.d.) with zero mean and unit variance, i.e., normal distribution $\mathcal{N}(0, 1)$. Each component of \mathbf{x} is Gaussian distributed, i.e., $x_k \sim \mathcal{N}(0, \sigma_{x_k}^2)$, $k \in \{1, 2, \dots, K\}$. The task of multi-user detection at the BS is to estimate the transmitted signal vector \mathbf{x} from the received signal vector \mathbf{y} . Noting that the channel matrix \mathbf{H} can be usually obtained by time-domain and/or frequency-domain training pilots [5], we assume that the BS knows the Channel State Information (CSI) \mathbf{H} . In this paper, we only consider the real MU-MIMO system because the complex case can be easily extended from the real case [14].

B. MMSE Detector

It is well known that MMSE detection is optimal under MSE measure when the sources are gaussian distributed [21]. Let \mathbf{V}_x denote the covariance matrix of the sources \mathbf{x} . Then, the MMSE detector [6] is given by

$$\hat{\mathbf{x}} = \sigma_n^{-2} \mathbf{V}_x \mathbf{H}^T \mathbf{y} = \sigma_n^{-2} (\sigma_n^{-2} \mathbf{H}^T \mathbf{H} + \mathbf{V}_x^{-1})^{-1} \mathbf{H}^T \mathbf{y}, \quad (2)$$

where $\mathbf{V}_x = (\sigma_n^{-2} \mathbf{H}^T \mathbf{H} + \mathbf{V}_x^{-1})^{-1}$, which denotes the deviation of the estimation to the initial sources. Moreover, the k th diagonal element v_{kk} of the covariance matrix \mathbf{V}_x denote the deviation of the estimation to the source x_k .

The following is given by the random matrix theory [21].

Proposition 1: When $\beta = K/M < 1$ is fixed, $K \rightarrow \infty$ and the sources are i.i.d. with $\mathcal{N}(0, \sigma_x^2)$, the MSE performance of the MMSE detection is described by

$$MSE = \sigma_x^2 \left(1 - \frac{1}{4s\beta M} F(sM, \beta) \right) \rightarrow \frac{\sigma_n^2}{M - K}, \quad (3)$$

where $s = \sigma_x^2 / \sigma_n^2$ is signal-to-noise ratio.

When $\beta = K/M < 1$, $K \rightarrow \infty$, the MSE of the MMSE detection is determined by the variance of the Gaussian noise and $M - K$, but independent of the variances of the sources.

Remark: The complexity of MMSE detector is $\mathcal{O}(K^3 + MK^2)$, where $\mathcal{O}(K^3)$ is for the inverse calculation and $\mathcal{O}(MK^2)$ is for the matrix multiplication $\mathbf{H}^T \mathbf{H}$. As this complexity is very high, it motivates us to use a low complexity detection method.

III. GAUSSIAN MESSAGE PASSING ITERATIVE DETECTION

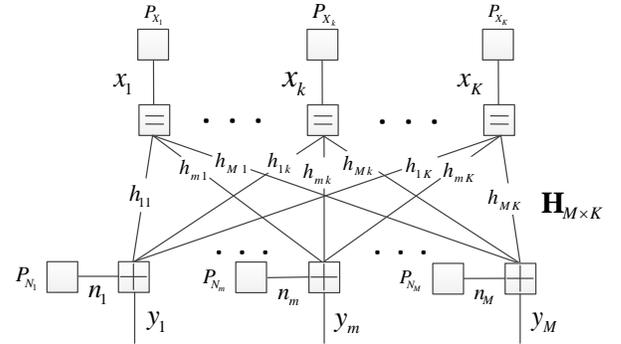


Fig. 2. Gaussian message passing iterative detection for MU-MIMO systems. The channel parameter from user k to antenna m is h_{mk} , the distribution constraints of each source and noise are denoted by P_{x_k} and P_{N_m} .

In this paper, we consider the GMPID based on a pairwise factor graph for the MU-MIMO system. Fig. 2 gives the factor graph of MU-MIMO system. The process is very similar to the Belief Propagation (BP) decoding process of LDPC code [16]. The differences are: 1) the different passing messages on each edge (the mean and variance of a Gaussian distribution); 2) the different message update functions.

A. Message Update at Sum Nodes

Each sum node can be seen as a multiple-access process and the message update at the sum nodes is given by

$$\begin{cases} e_{m \rightarrow k}^s(t) = y_m - \sum_{i \neq k} h_{mi} v_{i \rightarrow m}^v(t-1), \\ v_{m \rightarrow k}^s(t) = \sum_{i \neq k} h_{mi}^2 v_{i \rightarrow m}^v(t-1) + \sigma_n^2, \end{cases} \quad (4)$$

where $i, k \in \{1, 2, \dots, K\}$, $m \in \{1, 2, \dots, M\}$, y_m is the m -th element of the received vector \mathbf{y} , h_{mi} is the element

of channel matrix \mathbf{H} and σ_n^2 is the variance of the Gaussian noise. What's more, $e_{k \rightarrow m}^v(t)$ and $v_{k \rightarrow m}^v(t)$ denote the mean and variance passed from k th variable node to m th sum node respectively, $e_{m \rightarrow k}^s(t)$ and $v_{m \rightarrow k}^s(t)$ denote the mean and variance passed from m th sum node to k th variable node respectively. The initial value $\mathbf{v}^v(0)$ equals to $+\infty$ and $\mathbf{e}^v(0)$ equals to $\mathbf{0}$, where $\mathbf{v}^v(t)$ and $\mathbf{e}^v(t)$ are vectors containing the elements $v_{k \rightarrow m}^v(t)$ and $e_{k \rightarrow m}^v(t)$ respectively.

B. Message Update at Variable Nodes

Each variable node can be seen as a broadcast process and the message update at the variable nodes is denoted by

$$\begin{cases} v_{k \rightarrow m}^v(t) = \left(\sum_i h_{ik}^2 v_{i \rightarrow k}^{s^{-1}}(t) + \sigma_{x_k}^{-2} \right)^{-1}, \\ e_{k \rightarrow m}^v(t) = v_{k \rightarrow m}^v(t) \sum_i h_{ik} v_{i \rightarrow k}^{s^{-1}}(t) e_{i \rightarrow k}^s(t). \end{cases} \quad (5)$$

where $k \in \{1, 2, \dots, K\}$, $i, m \in \{1, 2, \dots, M\}$ and $\sigma_{x_k}^2$ denotes the variance of the source x_k .

After the given number of iterations between (4) and (5), we output \hat{x}_k as the estimation of x_k and its MSE $\sigma_{\hat{x}_k}^2$.

$$\sigma_{\hat{x}_k}^2 = v_{k \rightarrow m}^v(t), \quad \hat{x}_k = e_{k \rightarrow m}^v(t), \quad (6)$$

where $k \in \{1, 2, \dots, K\}$.

It is easy to calculate the computational complexity for each iteration. In each iteration, it needs about $8KM$ multiplications. Therefore, the complexity is low ($O(KMN_{ite})$), where N_{ite} is the number of iterations.

C. Variances Convergence of GMPID (New Results)

In this subsection, we give the variances convergence of GMPID. The inequations in this paper correspond to the component-wise inequality. From (4)(5), we have

$$v_{k \rightarrow m}^v(t) = \left(\sum_i h_{ik}^2 \left(\sum_{j \neq k} h_{ij}^2 v_{j \rightarrow i}^v(t-1) + \sigma_n^2 \right) + \sigma_{x_k}^{-2} \right)^{-1}. \quad (7)$$

As the initial value $\mathbf{v}^v(0)$ is equal to $+\infty$, it is easy to see that $\mathbf{v}^v(t) > 0$ for any $t > 0$ during the iteration. So $\mathbf{v}^v(t)$ has a lower bound $\mathbf{0}$. From (7), we can see that $\mathbf{v}^v(t)$ is a monotonically non-decreasing function with respect to $\mathbf{v}^v(t-1)$. Moreover, we can get $\mathbf{v}^v(1) < \mathbf{v}^v(0) = +\infty$ for the first iteration. Therefore, it can be shown that $\mathbf{v}^v(t) \leq \mathbf{v}^v(t-1)$ with $\mathbf{v}^v(1) \leq \mathbf{v}^v(0)$ from the monotonicity of the iteration function. These mean that $\{\mathbf{v}^v(t)\}$ is a monotonic decreasing sequence but lower bounded. Thus, sequence $\{\mathbf{v}^v(t)\}$ converges to a certain value, i.e., $\lim_{t \rightarrow \infty} \mathbf{v}^v(t) = \mathbf{v}^*$.

To simplify the calculation, we assume $\mathbf{V}_x = \sigma_x^2 \mathbf{I}_K$, i.e., $\sigma_{x_k}^2 = \sigma_x^2$. With the symmetry of all the elements of \mathbf{v}^* , we can get $v_{k \rightarrow m}^* = \hat{\sigma}^2$, $k \in \{1, \dots, K\}$ and $m \in \{1, \dots, M\}$. Thus, from (7), the convergence point $\hat{\sigma}^2$ can be solved by

$$\hat{\sigma}^2 = \left(\sum_i h_{ik}^2 \left(\hat{\sigma}^2 \sum_{j \neq k} h_{ij}^2 + \sigma_n^2 \right) + \sigma_x^{-2} \right)^{-1}. \quad (8)$$

As the channel parameters h_{ik}^2 and h_{ij}^2 are independent with each other, the above expression can be rewritten as

$$\sigma_x^{-2} \sum_{j \neq k} h_{ij}^2 \hat{\sigma}^4 + (\sigma_n^2 \sigma_x^{-2} + \sum_i h_{ik}^2 - \sum_{j \neq k} h_{ij}^2) \hat{\sigma}^2 - \sigma_n^2 = 0. \quad (9)$$

When M is large, taking an expectation for (9) with respect to the channel parameters h_{ik}^2 and h_{ij}^2 , we get

$$K \sigma_x^{-2} \hat{\sigma}^4 + (\sigma_n^2 \sigma_x^{-2} + M - K) \hat{\sigma}^2 - \sigma_n^2 = 0. \quad (10)$$

Then $\hat{\sigma}^2$ is the positive solution of (10), i.e.,

$$\hat{\sigma}^2 = \frac{\sqrt{(\sigma_n^2 \sigma_x^{-2} + M - K)^2 + 4K \sigma_x^{-2} \sigma_n^2} - (\sigma_n^2 \sigma_x^{-2} + M - K)}{2K \sigma_x^{-2}}. \quad (11)$$

With (11), (5) and (6), we can get the following proposition.

Proposition 2: When $\beta = K/M < 1$ is fixed, $K \rightarrow \infty$ and the sources are i.i.d. with $\mathcal{N}(0, \sigma_x^2)$, the variances of GMPID converge to

$$\sigma_{\hat{x}}^2 = \hat{\sigma}^2 \approx \frac{\sigma_n^2}{M - K + s^{-1}}. \quad (12)$$

Comparing (3) and (12), we can see that it gets the same results as performance analysis by the random matrix theory. Thus, the following theorem can be given.

Theorem 1: When $\beta = K/M < 1$ is fixed, $K \rightarrow \infty$ and the sources are i.i.d. with $\mathcal{N}(0, \sigma_x^2)$, the variances of GMPID converge to the exact MSE of the MMSE detection.

It should be pointed out that the above analysis provides an alternative method to estimate the MSE performance of the MMSE detection. The variances convergence analysis here is simpler and even suitable for the irregular channel matrix.

Similarly, sequence $\{\mathbf{v}^s(t)\}$ also converges to a certain value, i.e., $v_{m \rightarrow k}^s \rightarrow \tilde{\sigma}^2$. From (4), we can get

$$\tilde{\sigma}^2 \approx K \hat{\sigma}^2 + \sigma_n^2. \quad (13)$$

Let $\gamma = \hat{\sigma}^2 / \tilde{\sigma}^2$, from (12) and (13), we get

$$\gamma = \frac{1}{K + \sigma_n^2 / \hat{\sigma}^2} \approx (M + s^{-1})^{-1}. \quad (14)$$

D. Means Convergence of GMPID (New Results)

In this subsection, the means convergence of GMPID will be given. Unlike the variances, the means are not always convergent. Two sufficient conditions for the means convergence are given as follows.

Theorem 2: When $\beta = K/M < 1$ is fixed and $K \rightarrow \infty$, the GMPID converges to the MMSE estimation if any of the following conditions holds.

1. The matrix $\mathbf{I}_K + \gamma (\mathbf{H}^T \mathbf{H} - \mathbf{D}_{\mathbf{H}^T \mathbf{H}})$ is strictly or irreducibly diagonally dominant,
2. $\rho(\gamma (\mathbf{H}^T \mathbf{H} - \mathbf{D}_{\mathbf{H}^T \mathbf{H}})) < 1$.

Where $\gamma = \hat{\sigma}^2 / \tilde{\sigma}^2$ and $\rho(\mathbf{A})$ is the spectral radius of \mathbf{A} .

Proof: The proof is omitted due to the page limit. ■

As $\gamma \rightarrow \frac{1}{M+s^{-1}}$ and $K \rightarrow \infty$ with $\beta < 1$, from random matrix theory, we have

$$\rho(\gamma (\mathbf{H}^T \mathbf{H} - \mathbf{D}_{\mathbf{H}^T \mathbf{H}})) \rightarrow \beta + 2\sqrt{\beta}, \quad (15)$$

for a finite s . Then, from the second condition of Theorem 2, we have the following corollary.

Corollary 1: When $\beta = K/M < 1$ is fixed and $K \rightarrow \infty$, the GMPID converges to the MMSE estimation if $\beta < (\sqrt{2} - 1)^2$.

IV. A NEW FAST CONVERGE DETECTOR SA-GMPID

From Corollary 1, we know that when $(\sqrt{2} - 1)^2 \leq \beta < 1$ the GMPID may not converge. So, in this subsection, we will give a *scaled-and-added* GMPID called SA-GMPID to fix the convergence of GMPID. In the following, let $\mathbf{H}' = \sqrt{w}\mathbf{H}$ and $\mathbf{y}' = \sqrt{w}\mathbf{y}$, where $h'_{mk} = \sqrt{w}h_{mk}$ is an element of matrix \mathbf{H}' and w is a relaxation parameter. We let $\gamma = \hat{\sigma}^2/\tilde{\sigma}^2$ and assume $v_{k \rightarrow m}^v(t)$ and $v_{m \rightarrow k}^s(t)$ converges to $\hat{\sigma}^2$ and $\tilde{\sigma}^2$ respectively. We deal with the case when $\beta = K/M < 1$.

A. SA-GMPID Algorithm

The message update of SA-GMPID of variable node (5) is

$$\begin{cases} v_{k \rightarrow m}^v(t) = \left(\sum_i h_{ik}^2 v_{i \rightarrow k}^{s-1}(t) + \sigma_{x_k}^{-2} \right)^{-1}, \\ e_{k \rightarrow m}^v(t) = v_{k \rightarrow m}^v(t) \sum_i h'_{ik} v_{i \rightarrow k}^{s-1}(t) e_{i \rightarrow k}^s(t) - (w-1)e_{k \rightarrow m}^v(t-1). \end{cases} \quad (16)$$

The message update at the sum node (4) is changed as

$$\begin{cases} e_{m \rightarrow k}^s(t) = \mathbf{y}'_m - \sum_{i \neq k} h'_{mi} e_{i \rightarrow m}^v(t-1), \\ v_{m \rightarrow k}^s(t) = \sum_{i \neq k} h_{mi}^2 v_{i \rightarrow m}^v(t-1) + \sigma_n^2. \end{cases} \quad (17)$$

After the iteration between (16) and (17), output

$$\begin{cases} \hat{\sigma}_{x_k}^2 = \left(\sum_m h_{mk}^2 v_{m \rightarrow k}^{s-1}(t) + \sigma_{x_k}^{-2} \right)^{-1}, \\ \hat{x}_k = \hat{\sigma}_{x_k}^2 \sum_m (h'_{mk} v_{m \rightarrow k}^{s-1}(t) e_{m \rightarrow k}^s(t) - \frac{w-1}{M} e_{k \rightarrow m}^v(t-1)), \end{cases} \quad (18)$$

where $k \in \{1, 2, \dots, K\}$. Next, its convergence will be proved.

B. Convergence Analysis of SA-GMPID

Theorem 3: When $\beta = K/M < 1$ is fixed and $K \rightarrow \infty$, the SA-GMPID converges to the MMSE detector if the relaxation parameter w satisfies $0 < w < 2/\lambda_{max}^{\mathbf{A}}$, where $\lambda_{max}^{\mathbf{A}}$ is the largest eigenvalue of $\mathbf{A} = \gamma(\mathbf{H}^T \mathbf{H} - \mathbf{D}_{\mathbf{H}^T \mathbf{H}}) + \mathbf{I}_K$.

Proof: The proof is omitted due to the page limit. ■

The optimal relaxation parameter w is given by

$$w = 2/(\lambda_{min}^{\mathbf{A}} + \lambda_{max}^{\mathbf{A}}). \quad (19)$$

By the random matrix theory [24], when $\beta = K/M$ is fixed and $K \rightarrow \infty$, the smallest and largest eigenvalues $\lambda_{max}^{\mathbf{A}}$ and $\lambda_{min}^{\mathbf{A}}$ of matrix \mathbf{A} are estimated given by

$$\lambda_{min}^{\mathbf{A}} = 1 + \gamma M \left[(1 - \sqrt{\beta})^2 - 1 \right], \lambda_{max}^{\mathbf{A}} = 1 + \gamma M \left[(1 + \sqrt{\beta})^2 - 1 \right]. \quad (20)$$

The spectral radius of $\mathbf{I}_K - w\mathbf{A}$ is $\rho_{min}(\mathbf{I}_K - w\mathbf{A}) = \frac{\lambda_{max}^{\mathbf{A}} - \lambda_{min}^{\mathbf{A}}}{\lambda_{max}^{\mathbf{A}} + \lambda_{min}^{\mathbf{A}}} < 1$. From (20), we have $w = 1/(1 + \gamma M\beta)$. When $\beta < 1$, from (14), $M\gamma \rightarrow 1$. Thus,

$$w = 1/(1 + \beta), \quad \rho_{min}(\mathbf{I}_K - w\mathbf{A}) \approx \frac{2\sqrt{\beta}}{1 + \beta} < 1. \quad (21)$$

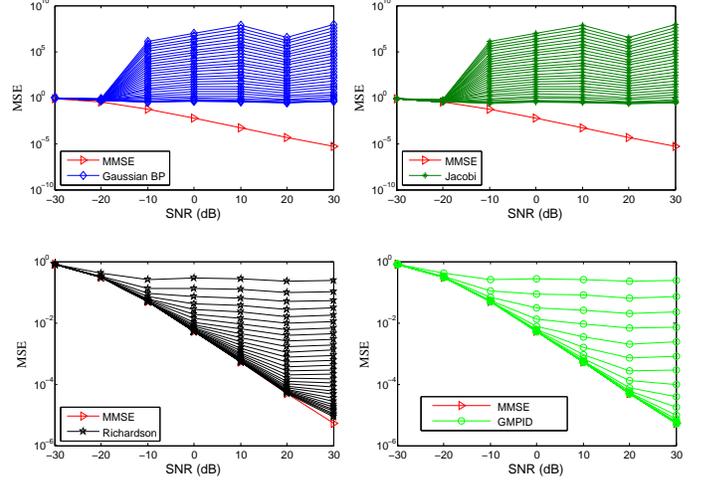


Fig. 3. Performance comparison between GMPID and the other iterative methods: Gaussian BP, Jacobi and Richardson method. The simulations are for 100×300 MU-MIMO system with $\beta = 1/3$ and $1 \sim 30$ iterations.

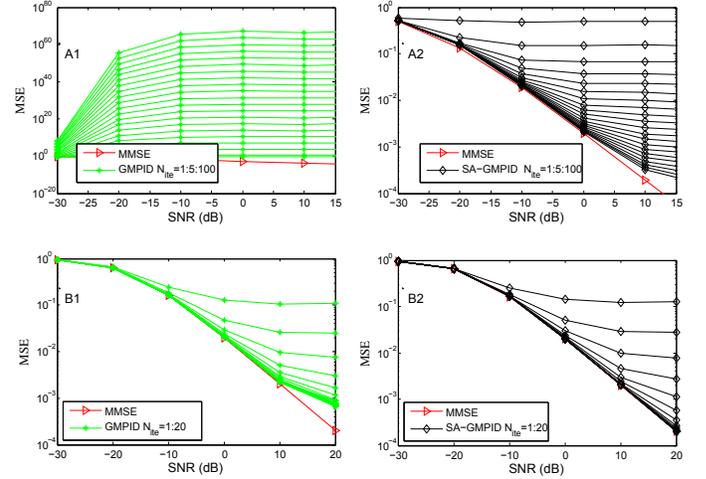


Fig. 4. Performance comparison between the GMPID and SA-GMPID. Figures A1 and A2 are for 1000×1500 MU-MIMO system with $1 \sim 100$ iterations and $\beta = 2/3$. Figures B1 and B2 are for 10×60 MU-MIMO system with 20 iterations and $\beta = 1/6$.

TABLE I
CONVERGENCE COMPARISON BETWEEN SA-GMPID, GMPID, JACOBI, GABP AND RICHARDSON METHOD. THE ‘‘C’’ AND ‘‘D’’ DENOTE CONVERGENT AND DIVERGENT RESPECTIVELY, AND ‘‘+’’ AND ‘‘-’’ DENOTE THE RIGHT LIMIT AND THE LEFT LIMIT RESPECTIVELY.

Figure	β	Jacobi & GaBP	GMPID	Richardson & SA-GMPID
Fig. 4	$\beta < (\sqrt{2} - 1)^2$	C	C	C
Fig. 5	$\beta \rightarrow (\sqrt{2} - 1)^2_+$	D	C	C
Fig. 6	$\beta \rightarrow 1_-$	D	D	C

Comparing with (15), we get the following corollary.

Corollary 2: The SA-GMPID converges faster than the GMPID when $\beta = K/M < 1$ is fixed and $K \rightarrow \infty$.

V. SIMULATION RESULTS

In this section, we give the numerical results of the proposed GMPID for the MU-MIMO system with Gaussian sources.

Assume that the sources are i.i.d. with $x_k \sim \mathcal{N}(0, 1)$ and the entries of the channel matrix \mathbf{H} are i.i.d. with normal distribution $\mathcal{N}(0, 1)$. In the following simulations, N_{ite} denotes the number of iterations, $SNR = \frac{1}{\sigma_n^2}$ and $MSE = \frac{1}{K}(\mathbf{x} - \hat{\mathbf{x}})^2$ denotes the average mean squared error. All the simulations are repeated 500 times to get the results.

Fig. 3 gives the average MSE performance and convergence comparison between the SA-GMPID and the other iterative methods: Jacobi method, Gaussian BP method and Richardson method [14], where $K = 100$ and $M = 300$. We can see that the GMPID converges faster (although $\beta = 1/3 > (\sqrt{2} - 1)^2$) to the MMSE detection than the other three methods. Furthermore, the SA-GMPID converges even when the Jacobi method and Gaussian BP method diverge. It should be noted that SA-GMPID has the lowest complexity.

Fig. 4 gives the average MSE performance comparison between the GMPID and SA-GMPID for the cases that $\beta = 2/3$, $K = 1000$, $M = 1500$ with 100 iterations (figures A1 and A2) and $\beta = 1/6$, $K = 10$, $M = 60$ with 20 iterations (figures B1 and B2), respectively. We can see that the GMPID is diverging when $\beta \rightarrow 1$ with figure A1. In particular, figure A2 shows the SA-GMPID converges to the MMSE detection with the increase number of iterations. This verifies our analysis result in Theorem 3. Furthermore, figures B1 and B2 show that 1) SA-GMPID converges faster to the MMSE detection than GMPID (Corollary 2) and 2) the proposed theoretical results are also suitable for MU-MIMO systems with a small number of antennas and users.

Table I concludes the convergence comparison of the different detection methods, where “C” (or “D”) denotes convergent (divergent) and “+” (or “-”) denotes right limit (left limit). It shows that 1) all the iterative methods are convergent when $\beta < (\sqrt{2} - 1)^2$, 2) the Jacobi and GaBP methods are divergent when β larger than $(\sqrt{2} - 1)^2$, 3) GMPID is still convergent when β close to $(\sqrt{2} - 1)_+^2$ and 4) Richardson method and SA-GMPID are convergent when β close to 1.

VI. CONCLUSION

A low-complexity detection method GMPID has been discussed, in which the means and variances are transmitted between the variable nodes and sum nodes. The convergence of GMPID has been analysed. It is proved that the variances of GMPID converge to the MSE of MMSE detection. Two sufficient conditions that the GMPID converges to the MMSE detection have also been presented. As the GMPID does not converge when $\beta \rightarrow 1$, SA-GMPID has been proposed, which converges to MMSE detection in any case that $K < M$. Numerical results are provided to verify the proposed theoretical results. It should be noted that our simulations show that the proposed theories are also suitable for MU-MIMO systems with a limited number of antennas and users.

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