EXPERIMENTS ON GEOMETRIC IMAGE ENHANCEMENT

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ABSTRACT

In this paper we experiments with geometric algorithms for image smoothing. Examples are given for MRI and ATR data. The algorithms are based on the results in [2, 22, 25, 26, 29]. Here we emphasize experiments with the affine invariant geometric smoother or affine heat equation, originally developed for binary shape smoothing, and found to be efficient for gray-level images as well. Efficient numerical implementations of these flows give anisotropic diffusion processes which preserve edges.

1. INTRODUCTION

In this paper, we apply a geometric smoothing technique based on invariant curve evolutions to MRI and ATR data. The theory of planar curve evolution has been considered in a variety of fields such as differential geometry [9, 11, 24], parabolic equations theory [4], numerical analysis [18], computer vision [13, 23, 25], viscosity solutions [6, 8], and image processing [2, 3, 19, 26, 29]. One of the most important of such flows is derived when the planar curve deforms in the direction of the Euclidean normal, with speed equal to the Euclidean curvature. Formally, let $\mathcal{C}(p,t): S^1 \times [0,\tau) \rightarrow \mathbf{R}^2$ be a family of smooth embedded curves in the plane (boundaries of planar shapes), where $p \in S^1$ parametrizes the curve, and $t \in [0,\tau)$ parametrizes the family. Assume that this family of curves evolves according to the evolution equation

$$\left\{ \begin{array}{l} \frac{\partial \mathcal{C}(p,t)}{\partial t} = \frac{\partial^2 \mathcal{C}(p,t)}{\partial v^2} = \kappa(p,t)\vec{\mathcal{N}}(p,t), \\ \mathcal{C}(p,0) = \mathcal{C}_0(p), \end{array} \right. \tag{1}$$

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where $v(p)$ is the *Euclidean arc-length*, κ the *Euclidean curvature*, and $\vec{\mathcal{N}}$ the *inward unit normal* [12]. Gage and Hamilton [9] proved that a simple and smooth convex curve evolving according to (1), converges to a round point. Grayson [11] proved that an embedded planar curve converges to a simple convex one when evolving according to (1). The flow (1), is denoted as the *Euclidean geometric heat flow*. It has been used for the definition of a geometric, Euclidean invariant, multiscale representation of planar shapes [1, 13]. As we will show below, this flow is also important for image enhancement applications. Note than in contrast with the classical heat flow, the Euclidean geometric heat flow is intrinsic to the curve, that is, only depends on the geometry of the curve and not on its parametrization. This flow, as well as the other presented below, can be used also to solve the common shrinking problem of smoothing processes [28].

Recently, we introduced a new curve evolution equation, the *affine geometric heat flow* [24]:

$$\left\{ \begin{array}{l} \frac{\partial \mathcal{C}(p,t)}{\partial t} = \frac{\partial^2 \mathcal{C}(p,t)}{\partial s^2}, \\ \mathcal{C}(p,0) = \mathcal{C}_0(p), \end{array} \right. \tag{2}$$

where s is the *affine arc-length*, i.e., the simplest affine invariant parametrization [12, 24]. This evolution is the affine analog of equation (1), and admits affine invariant solutions, i.e., if a family $\mathcal{C}(p,t)$ of curves is a solution of (2), the family obtained from it via an unimodular affine mapping, is a solution as well. We proved that any simple and smooth convex curve evolving according to (2), converges to an ellipse [24]. Since the affine normal $\mathcal{C}_{ss}$ exists just for non-inflection points, we presented the natural extension of the flow (3) for non-convex initial curves in [27]:

$$\frac{\partial \mathcal{C}(p,t)}{\partial t} = \left\{ \begin{array}{ll} 0, & p \text{ inflection point}, \\ \mathcal{C}_{ss}(p,t), & p \text{ non-inflection point}. \end{array} \right. \tag{3}$$

In this case, we proved (see also [5]) that the curve first becomes convex, as in the Euclidean case, and after that it converges into an ellipse according to the results of [24]. The flow (3) defines a geometric, affine invariant, multiscale representation of planar shapes [25].

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In [25] we proved that the flow holds all the required properties of scale-spaces, as for example causality and order preserving. See the mentioned reference for planar shapes smoothing examples.

We should also add that in [27], we give a general method for writing down invariant flows with respect to any Lie group action on $\mathbf{R}^2$. This was formalized, together with uniqueness results, in [15], and extended to surfaces in [16].

Recently, algorithms for image smoothing were developed based on the Euclidean shortening flow (1) and related equations. In this paper we experiment with the affine flow. This point is motivated by the fact that the implementation of the Euclidean-type version of (3) seems to be more stable than the original Euclidean flow (see Section 3 and [25, 29]).

2. EUCLIDEAN IMAGE PROCESSING

In this section, we present algorithms for image processing which are related to the Euclidean shortening flow (1). In general, $\Phi_0 : \mathbf{R}\times\mathbf{R} \to \mathbf{R}$ represents a gray-level image, where $\Phi_0(x, y)$ is the gray-level value. The algorithms that we describe are based on the formulation of partial differential equations, with Φ_0 as initial condition. The solution $\Phi(x, y, t)$ of the differential equation gives the processed image.

Rudin *et al.* [22] presented an algorithm for noise removal, based on the minimization of the total first variation of Φ. The minimization is performed under certain constraints and boundary conditions. Note that κ, the Euclidean curvature of the level sets, is exactly the Euler-Lagrange derivative of this total variation. In [30] we investigate an stochastic approach for the efficient computation of the Lagrange multiplier for this variational problem.

Alvarez *et al.* [2] proposed an algorithm for image selective smoothing and edge detection. In this case, the image evolves according to

$$\Phi_t = g(\| G * \nabla\Phi \|)\ \| \nabla\Phi \| \operatorname{div}\left(\frac{\nabla\Phi}{\| \nabla\Phi \|}\right), \qquad (4)$$

where G is a smoothing kernel, and $g(r)$ is a nonincreasing function which tends to zero as $r \to \infty$. The terms of equation (4) have the following natural interpretation [2]. First the expression $\| \nabla\Phi \| \operatorname{div}\left(\frac{\nabla\Phi}{\|\nabla\Phi\|}\right)$, is equal to $\Phi_{\xi\xi}$, where ξ is the direction normal to $\nabla\Phi$. Thus it diffuses Φ in the direction orthogonal to the gradient $\nabla\Phi$, and does not diffuse in the direction of $\nabla\Phi$. It can be shown that the evolution $\Phi_t = \| \nabla\Phi \| \operatorname{div}\left(\frac{\nabla\Phi}{\|\nabla\Phi\|}\right)$ is identical to

$$\Phi_t = \frac{1}{\Phi_x^2 + \Phi_y^2}(\Phi_y^2\Phi_{xx} - 2\Phi_x\Phi_y\Phi_{xy} + \Phi_x^2\Phi_{yy}) = \kappa \| \nabla\Phi \|, \qquad (5)$$

which implies that the level sets of Φ move according to the Euclidean shortening flow given by equation (1) [2, 18]. For general results concerning the evolution of level sets, see [6, 8, 18].

Next the term $g(\| G * \nabla\Phi \|)$ is used for the enhancement of the edges. If $\|\nabla\Phi\|$ is "small", then the diffusion is strong. If $\|\nabla\Phi\|$ is "large" at a certain point (x, y), this point is considered as an edge point, and the diffusion is weak.

Consequently, equation (4) gives an anisotropic diffusion, extending the ideas first proposed by Perona and Malik [20]. The equation looks like the level sets of Φ are moving according to (1), with the velocity value "altered" by the function $g(\cdot)$. Other approaches for anisotropic diffusion, derived from variations of [20] as well, can be found in [10, 21].

Alvarez and Mazorra [3] proposed an algorithm which combines the shock filter [19] with the anisotropic diffusion (4). Actually, in their experiments, they used the directional smoothing operator proposed in [2] (equation (4)), but others, such as the one proposed in Section 3 below, can be used as well.

Note that in the image enhancement algorithm given in [22] the steady state solution of the evolution gives the enhanced image, while in the algorithm given by (4) a stopping condition must be added. Possible automatic stopping conditions can be the achievement of a certain difference between the original image and the processed one, achievement of certain smoothness level, etc.

3. AFFINE SMOOTHING

As we saw in previous section, there is a close relationship between the curve evolution flow (1), and recently developed image enhancement and smoothing algorithms (see equation (5)). In this section, we explain why we propose the use of the affine shortening flow (3) instead of the Euclidean one.

It is well-known in the theory of curve evolution, that if the velocity $\vec{\mathcal{V}}$ of the evolution is a geometric function of the curve, then the geometric behavior of the curve is affected only by the normal component of this velocity, i.e., by $< \vec{\mathcal{V}}, \vec{\mathcal{N}} >$. The tangential velocity component only affects the parametrization of the evolving curve [7, 25]. Therefore, instead of looking at (3), we can consider a Euclidean-type formulation of it. In [24], we proved that $< C_{ss}, \vec{\mathcal{N}} > = \kappa^{1/3}\vec{\mathcal{N}}$. Since $\kappa = 0$ at inflection points, and the inflection points are affine invariant, we obtain that the evolution given by

$$C_t = \kappa^{1/3}\vec{\mathcal{N}}, \qquad (6)$$

is geometric equivalent to the affine shortening flow (3). Then the trace (or image) of the solution to (6) is affine invariant.

It is interesting to note that the affine invariant property of (6) was also pointed out by Alvarez *et al.* [1], based on a completely different approach. For existence of the Euclidean and affine heat flow in the viscosity framework, see [1, 6, 8]. The existence of the Euclidean and affine geometric heat flows for Lipschitz functions is obtained from the results in [4, 5] as well.

The process of embedding a curve in a 3D surface, and looking at the evolution of the level sets, is frequently used for the digital implementation of curve evolution flows [18]. Let us consider now what occurs when the level sets of Φ evolve according to (6). It is easy to show that the corresponding evolution equation for Φ is given by

$$\Phi_t = (\Phi_y^2 \Phi_{xx} - 2\Phi_x \Phi_y \Phi_{xy} + \Phi_x^2 \Phi_{yy})^{1/3}. \qquad (7)$$

This equation was used in [25] for the implementation of the novel affine invariant scale-space for planar curves mentioned in the Introduction. If we compare (5) with (7), we observe that the denominator is eliminated. This not only makes the evolution (6) affine invariant [1, 25], it also can make the numerical implementation more stable [18]. This is the main reason why we proposed in [26] to research the use of the affine shortening flow in the place of the Euclidean one for the algorithms presented in the previous section.

4. EXPERIMENTAL RESULTS

Before presenting our simulation results, we should point out that recently Niessen *et al.* [14] compared experimentally the classical Gaussian filtering with the Euclidean heat flow (5) and the affine one (7), obtaining the best results for (7) as expected. We now present results for (7) with functions g different for the unity (in contrast with the experiments in [14]). The selection of $g \neq 1$ will better preserve edges, since points with high gradient will move with less velocity.

In Figure 1, we present several steps of the affine based image smoothing (equation (7) with $g = 1/r$ as in (4)). Note the preservation of salient edges. Instead of using Gaussian filtering for smoothing the image before the gradient computation for g in (4), the flow (7) itself can be used, obtaining a completely geometric flow. If we are interested in obtaining a completely affine invariant image smoothing process (the Euclidean gradient is not affine invariant), the "affine gradient" proposed in [17] can be used for the computation of g in (4).

In Figure 2 we show the results of the affine based smoother for ATR data. The original image is presented first, then the degraded one, and two steps of the algorithm follow.

5. CONCLUDING REMARKS

The importance of the Euclidean shortening flow for image smoothing and edge detection has been amply demonstrated by the works of Alvarez *et al.* [1, 2, 3], and by Osher and Rudin [19, 22]. We have developed the affine analogue of the Euclidean flow in [24, 25, 27], and have shown that the numerical implementation of this flow can be more stable. Further, in [26] we proposed to replace the Euclidean flow by this novel affine one for image enhancement as well.

In this work we used the affine based flows for enhancement of MRI and ATR data. More examples on MRI, as well as details on the algorithms, can be found in [29]. We tested the anisotropic diffusion based on the affine heat flow. In this case a stopping condition should be added to the algorithm. Note than since the right hand of the affine flow (7) (i.e., $\kappa^{1/3} \parallel \nabla\Phi \parallel$) is a potential function, it is the Euler-Lagrange of a variational problem. Therefore, adding constraints as in [22], the algorithm can made to stop automatically as well.

As demonstrated by the examples presented here, salient edges are preserved during the proposed smoothing processes. These algorithms can also be combined in different ways with segmentation algorithms, in order to perform local geometric smoothing or to improve the segmentation itself.

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