Spatially-Adaptive Regularized Pel-Recursive Motion Estimation Based on Cross-Validation

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Extended Abstract:

This paper addresses the problem of recursively estimating the motion or displacement vector field (DVF) due to the optical flow in images sequences. Some of the sources of difficulty involved in DVF calculation are the nonstationarity of the field, the ill-posed nature of the problem and the presence of noise in the data.

The developed pel-recursive algorithms minimize the displaced frame difference (DFD) function defined by

$$\Delta(\mathbf{r}, \mathbf{r}-\mathbf{d}(\mathbf{r})) = f_k(\mathbf{r}) - f_{k-1}(\mathbf{r}-\mathbf{d}(\mathbf{r})) \tag{1}$$

where $\mathbf{r} = [\mathrm{x}, \mathrm{y}]^T$ is the pixel location, $\mathbf{d}(\mathbf{r}) = [dx, dy]^T$ is the corresponding (true) displacement vector at the working point r in the current frame f_k and f_{k-1} is the previous frame. The DFD is a function of the intensity differences and it represents the error due to the nonlinear temporal prediction of the intensity field through the displacement vector. It is apparent from (1) that the relationship between the DVF and the intensity field is nonlinear. An estimate of $\mathbf{d}(\mathbf{r})$ is obtained by means of the Taylor series expansion of $f_{k-1}(\mathbf{r}-\mathbf{d}(\mathbf{r}))$ about the location $(\mathbf{r}-\mathbf{d}_0(\mathbf{r}))$, with $\mathbf{d}_0(\mathbf{r})$ representing a prediction of $\mathbf{d}(\mathbf{r})$. This results in

$$\Delta(\mathbf{r}, \mathbf{r}-\mathbf{d}_0(\mathbf{r})) = \nabla^T f_{k-1}(\mathbf{r}-\mathbf{d}_0(\mathbf{r}))\mathbf{u}(\mathbf{r}) + e(\mathbf{r}, \mathbf{d}_0(\mathbf{r})) \tag{2}$$

where the displacement update vector $\mathbf{u}(\mathbf{r}) = [u_x, u_y]^T = \mathbf{d}(\mathbf{r}) - \mathbf{d}_0(\mathbf{r})$, $e(\mathbf{r}, \mathbf{d}_0(\mathbf{r}))$ represents the error resulting from the truncation of the higher order terms and $\nabla = [\delta/\delta_x, \delta/\delta_y]^T$ is the spatial gradient vector. By applying Eq. (2) to all points in a small area around the working point, and assuming constant image intensity along the motion trajectory gives the following matrix-vector form expression:

$$z = Gu + n \tag{3}$$

The observation vector $\mathbf{z}$ contains all temporal gradients $\Delta(\mathbf{r}, \mathbf{r}-\mathbf{d}_0(\mathbf{r}))$ stacked, the matrix $\mathbf{G}$ is obtained by stacking the spatial gradient operators at each observation, and the error terms have formed vector $\mathbf{n}$.

The solution $\hat{\mathbf{u}}_0(\lambda)$ that minimizes the MSE of Eq. (3) is the pseudoinverse estimate. Since it can be quite poor if the noise term is significant and/or $\mathbf{G}$ is singular. In order to improve $\hat{\mathbf{u}}_0(\lambda)$, regularization was used. This technique deals with instability and non-uniqueness in inverse problems. It imposes the assumption that the true DVF is smooth, so that its solution is better than $\hat{\mathbf{u}}_0(\lambda)$.

$$\hat{\mathbf{u}}(\lambda) = (\mathbf{G}^{\mathbf{T}}\mathbf{G} + \lambda\mathbf{Q}^{\mathbf{T}}\mathbf{Q})^{-1}\mathbf{G}^{\mathbf{T}}\mathbf{z} \tag{4}$$

where λ controls the degree to which the solution will be regularized, and $\mathbf{Q}$ is a regularization operator controlling the kind of smoothing (correction) used.

This paper explores two scenarios: λ is a scalar and $\lambda = [\lambda_1, \lambda_2]$. In both cases, the estimate of the regularization parameter λ used for computing Eq. (4) is computed according to the cross-validation criterion.

The GCV concept provides a way of testing the validity of the regularization parameter estimate $\bar{\lambda}$ based on a posteriori knowledge (the available noisy images). A GCV function $V(\lambda)$ is defined for each location $\mathbf{r}$. The GCV criterion is expressed as a nonlinear function whose minimum cannot be determined analytically. Hence, the optimal values of λ must be determined by means of numerical algorithms.

The major contributions of this paper are: the extension of the cross-validation function principle to DVF estimation, and the use of DFD-based adaptive schemes for calculation of the estimates of the regularization parameter.

Results for the two above-mentioned cases are compared to the Wiener filter solution

$$\hat{\mathbf{u}}_{Wiener}(\lambda) = (G^T G + \mu I)^{-1} G^T \mathbf{z} \tag{5}$$

where $\mu = \sigma_n^2/\sigma_u^2$. It should be pointed out that both the observation noise $\mathbf{n}$ and the update vector $\mathbf{u}$ are considered as being zero-mean and white with $E[\mathbf{n}\mathbf{n}^{\mathbf{T}}] = \sigma_n^2\mathbf{I}$ and $E[\mathbf{u}\mathbf{u}^{\mathbf{T}}] = \sigma_u^2\mathbf{I}$, respectively.

Some experimental results can be found in Table 1 and Fig. 2. They were obtained by processing frames 16 e 17 of the Mother and Daughter sequence. Fig. 1 shows frame 16 of this sequence. The results from the proposed algorithm are compared to the ones obtained by means of the Wiener filter (Eq. (5)).

Table 1: Comparison between the Cross-Validation algorithm and the Wiener filter.

Noise(dB)	Wiener		GCV	
	$DFD(dB)$	$IMC(dB)$	$DFD(dB)$	$IMC(dB)$
∞	7.63	8.14	5.36	9.42
30	9.10	7.90	8.03	8.63
20	14.55	5.34	12.98	5.84
15	19.55	4.06	18.91	4.26

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Figure 1: Original frame 16.

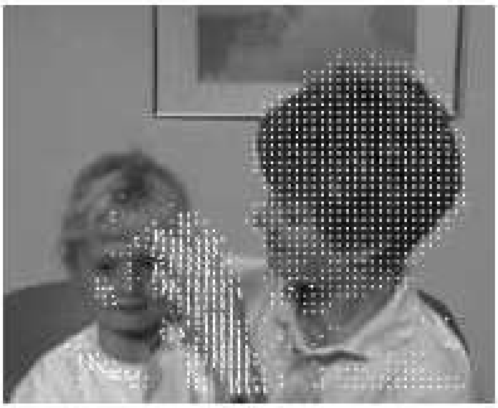

Figure 2: Displacement vector field between frames 16 and 17.