# Missile Tracking Using Knowledge-Based Adaptive Thresholding

*Steven Haker*
Department of Radiology
Brigham and Women's Hospital
Boston, MA 02115

*Guillermo Sapiro*
Dept. of Electrical Engineering
University of Minnesota
Minneapolis, MN 55455

*Allen Tannenbaum*
Dept. of Electrical Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0250

*Don Washburn*
Phillips Labs
Kirtland Air Force Base
Albuquerque, New Mexico 87131

**Abstract**

In this paper, we apply a knowledge-based segmentation method developed for still and video images to the problem of tracking missiles and high speed projectiles. Since we are only interested in segmenting a portion of the missile (namely, the nose cone), we use our segmentation procedure as a method of adapting thresholding. The key idea is to utilize a priori knowledge about the objects present in the image, e.g. *missile* and *background*, introduced via Bayes' rule. Posterior probabilities obtained in this way are anisotropically smoothed, and the image segmentation is obtained via MAP classifications of the smoothed data. When segmenting sequences of images, the smoothed posterior probabilities of past frames are used as prior distributions in succeeding frames.

**Key words:** Tracking, anisotropic diffusion, Bayesian statistics, knowledge-based segmentation.

## 1 Introduction

In this note, we apply our knowledge-based segmentation method to a tracking problem connected with missiles in the atmosphere and high speed projectiles. This method has already been successfully applied to the segmentation of SAR and MRI medical imagery [2, 6].

Since noise is in general non-additive, anisotropic diffusion [4] and related techniques directly applied to the image do not produce satisfactory results. Moreover, these techniques do not introduce prior information about the number of objects present in the scene when directly applied in the image space. In our approach, we combine Bayes' rule with anisotropic diffusion, introducing a priori knowledge into the segmentation process and solving the non-additivity problem of the noise. We also extend this approach to the segmentation of video data, incorporating basic learning capabilities to the knowledge.

In the case of tracking missiles, we are interested in tracking the location of the nose of the missile. Thus we only need to separate the relevant portion of the missile from the background. Because of the noisy nature of the images due to atmospheric effects simpler thresholding techniques (e.g., histogramming the pixel distributions and trying to separate the peaks) do not work very well for the missile videos. On the other hand, we have found that the knowledge-based approach of [2, 6] applied to two classes gives a type of adaptive thresholding which is shown here to work quite well with the data sets that we tried.

## 2 Basic Set-up

Our set-up begins with the assumption that the image is composed of $n$ classes of objects. For sequences of images, this value $n$ is assumed constant. In this paper we will assume two classes, corresponding to the missile and the background. Thus in this setting we will see that we have a form of *adaptive thresholding.* The technique however is general and can be applied to any number of classes. In [2, 6], this number of classes was three. The goal of our segmentation is to determine to which class each pixel in each image belongs.

Our basic model assumes that the value of each pixel in a given class can be thought of as a random variable with a known distribution, and that these variables are independent across pixels (this just simplifies the exposition, but any distribution can be used). Thus, for example for the case of normal distributions the likelihood of a particular pixel $i$ having a certain value $v$ given that it is in class $c \in \{\text{missile}, \text{background}\}$ is:

$$\Pr(V_i = v | C_i = c) = \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{1}{2}\frac{(v-\mu_c)^2}{\sigma_c^2}\right) \quad (1)$$

where $i$ is an index ranging over all pixels in the missile image, $V_i$ is the value of the pixel, and $C_i$ is its class. As usual, $\mu_c$ and $\sigma_c$ denote the mean and standard deviation of class $c$; these are assumed known. In practice, these parameters are estimated from a set of sample images or learned from past frames for video data.

Next, we assume that there is some known prior probability that a particular pixel will belong to a certain class. For single-image data sets, we assume a homogeneous prior, i.e., that $\Pr(C_i = c)$ is the same over all spatial indices $i$. It is, however, possible to incorporate a priori knowledge about the image here, for example if it were known that the missile is more likely to be near the center of the image than near the edge. For sequences of images, we have used a learned prior, as described below.

Given a set of intensity distributions $\Pr(V_i = v|C_i = c)$ and priors $\Pr(C_i = c)$, we can apply Bayes' Rule from elementary probability theory to calculate the posterior probability that a given pixel belongs to a particular class, given its intensity:

$$\Pr(C_i = c|V_i = v) = \frac{\Pr(V_i = v|C_i = c)\ \Pr(C_i = c)}{\sum_\gamma \Pr(V_i = v|C_i = \gamma)\ \Pr(C_i = \gamma)}. \quad (2)$$

Our proposed approach is to calculate the posteriors $P_i^c := \Pr(C_i = c|V_i = v)$ using the given distributions and (2) above, and then to apply anisotropic smoothing to each $P^c$ (note that the denominator is just a normalization constant that can be "ignored"). Specifically, we have chosen to smooth by evolving $P^c$ according to a discretized version of the partial differential equation

$$\frac{\partial P^c}{\partial t} = ((P_y^c)^2 P_{xx}^c - 2P_x^c P_y^c P_{xy}^c + (P_x^c)^2 P_{yy}^c)^{1/3}. \quad (3)$$

This equation defines the affine geometric heat flow, under which the level sets of $P^c$ undergo affine curve shortening. This particular diffusion equation was chosen because of its affine invariance, because it preserves edges well, and because of its numerical stability and ease of computation. See [1, 5] for details and other applications of this filter. The goal of this process is to diffuse information from one pixel to the other, making then a region-based (adaptive) decision and not just a local pixel-wise one.

The segmentation is then obtained using the maximum a posteriori probability estimate after anisotropic smoothing. That is,

$$C_i^* = \underset{c \in \{\text{missile, background}\}}{\arg\max} \Pr{}^*(C_i = c|V_i = v) \quad (4)$$

where $\Pr^*(C_i = c|V_i = v)$ is the smoothed posterior probability.

## 3 Video Data

When segmenting sequences of images, we have extended the model so that information from one frame is used in the segmentation of the next, and in this way have introduced a kind of learning into the method. There are a number of ways in which this can be done. By far the most effective way we have found is to modify our assumption of homogeneous priors. In particular, we have used the smoothed posteriors $P^c$ from one frame as priors $\Pr(C_i = c)$ in the segmentation of the next frame. We have also tested relaxing our assumption that the pixel intensities are distributed according to fixed normal distributions. We estimated the distribution parameters of the normal distributions from frame to frame by calculating new sample means and variances based on the segmentation of earlier images. Finally, we completely removed the assumption that the intensities are normally distributed. This was done by calculating the sample distribution of intensities within each class as images were segmented, and then using this distribution as $\Pr(V_i = v|C_i = c)$ in (1) when segmenting succeeding frames.

## 4 Examples

The image data which we used is stored as 16 bit floating point data. The data was scaled to range between 0.0 and 255.0. The data was created using a the wave optics simulation package which simulates the distortion of tracking a high speed projectile through the atmosphere.

In order to get initial estimates for $\mu_c$ and $\sigma_c$, a few images were segmented by hand. Once areas of each image were identified as either missile, or background, the sample mean and standard deviation of the values of the pixels in these areas were calculated. These values were then used for $\mu_c$ and $\sigma_c$ in (1) and (2). We found that a single set of values for the parameters $\mu_c$ and $\sigma_c$ worked well for many different image sets. The values used in the segmentations below were:

$$(\mu_{missile}, \mu_{background}) = (100, 4.3) \tag{5}$$
$$(\sigma_{missile}, \sigma_{background}) = (20, 2.1) \tag{6}$$

Next, values for $\Pr(C_i = c)$ were chosen. We have found that the segmentation process is quite robust with respect to these values. In fact, $\Pr(C_i = c) \equiv \frac{1}{2}$ provided satisfactory results. However, when segmenting sequences of images, significant gains in speed are possible through the use of adaptive priors, as described above.

To segment the initial image, the data was read and scaled. The image itself was then smoothed directly by applying (3) for a small number of iterations, typically three. Next, the posterior probabilities were calculated using (1) and (2), and the parameter estimates (5),(6) above. The posterior probabilities $P^c$ were then smoothed using (3) for a number of iterations. In general, after the smoothing, the probabilities need to be scaled back to add to one. That is, the diffusion process does not guarantee to preserve the $L^1 1$ norm of the vector. On the other hand, this is not necessary for the present case of just two classes, the process guarantees that the images remain legal posterior probability functions, meaning they are positive and add to one. For more than two classes, if needed, we can replace the particular anisotropic diffusion process here used by a slight modification of it that also guarantees the preservation of the vector as a probability one, without the need for an explicit re-normalization; see [3]. Four iterations was the average number required to produce a good result. Whenever (3) was applied, the maximum time step which ensures numerical stability was used. The final step in the calculation was to use (4) to determine the class of each pixel. The results were saved as images so that they could be compared visually to the original.

To segment a sequence of images, the first image in the sequence was segmented as above. The smoothed posterior probabilities $P^c$ were then used as prior probabilities in the segmentation of the second image, and similarly for all succeeding images. Some of the results are given below in figures (1) through (6). In each case, the position of the nose (and several other key points) are indicated on the original data for comparison with the segmented data. Since the original data is synthetic, these points can be determined exactly from the undistorted images. These are taken from a data set of 300 images. We were able to find the tip (nose) of the missile to within the specified tolerance on the whole data set.

We also tried using adaptive intensity distributions from frame to frame. We did this by calculating new sample means and variances $\mu_c$ and $\sigma_c$ based on the segmentations of earlier images. As a further generalization, we tried relaxing the assumption of normally distributed intensities. This was done by keeping track of the actual distribution of intensities within each segmented class as frames were processed, and then using this distribution as $\Pr(V_i = v | C_i = c)$ in (1) when segmenting succeeding frames. In general, we did not see a marked improvement over the static distribution model when using either of these methods. We believe that this is another indication that our basic method is robust.

## 5 Concluding Remarks

In this paper we have used an adaptive thresholding method based on knowledge-based segmentation for the segmentation of missile video data. The result is a fast and reliable algorithm that segments such data based both on prior and learned information.

Simple prior distributions and adaptation techniques were used in this paper, since the results obtained were already satisfactory. For more difficult data, it is possible to introduce more sophisticated multi-scale texture models for the likelihood of the background. Another possible extension will be to consider that $n$, the number of classes in the image, is not given and needs to be estimated as well. This can be done for example via EM-type algorithms. Note though that since the scheme here described is extremely fast, especially for video data where the number of smoothing steps is dramatically reduced, a brute-force search for $n$ in a given range might be good enough for a number of applications.

## References

[1] L. Alvarez, F. Guichard, P. L. Lions, and J. M. Morel, "Axioms and fundamental equations of image

processing," *Arch. Rational Mech. Anal.* **123**, pp. 3–41, 1993.

[2] S. Haker, G. Sapiro, and A. Tannenbaum, "Knowledge-based segmentation of SAR data with learned priors," *IEEE Trans. Image Processing* **9**, pp. 298–302, 2000.

[3] A. Pardo and G. Sapiro, "Vector probability diffusion," to appear in *IEEE Signal Processing Letters*, 2001.

[4] P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion," *IEEE-PAMI* **12**, pp. 629–639, 1990.

[5] G. Sapiro and A. Tannenbaum, "On affine plane curve evolution," *Journal of Functional Analysis* **119**, pp. 79-120, 1994.

[6] P. Teo, G. Sapiro, and B. Wandell, "Creating connected representations of cortical gray matter for functional MRI visualization," *IEEE Trans. Medical Imaging* **21**, pp. 65–72, 1998.

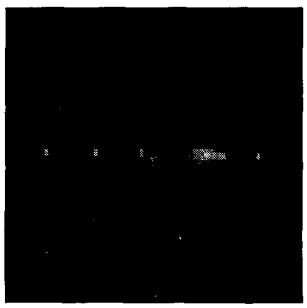

**Figure 1:** Video Sequence Frame 67

**Figure 2:** Segmentation of Figure 1

**Figure 3:** Video Sequence Frame 69

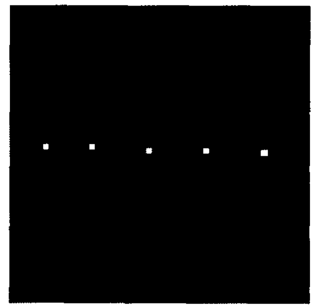

**Figure 4:** Segmentation of Figure 3

**Figure 5:** Video Sequence Frame 71

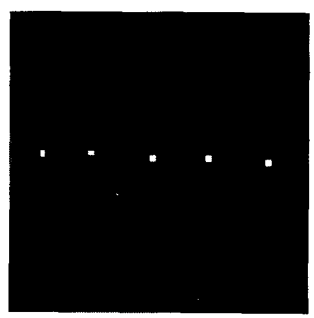

**Figure 6:** Segmentation of Figure 5