PALETTE REORDERING UNDER AN EXPONENTIAL POWER DISTRIBUTION MODEL OF PREDICTION RESIDUALS

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ABSTRACT

Palette reordering is one of the most effective approaches for improving the compression of color-indexed images. Recently, a theoretically motivated modification of a reordering technique proposed by Zeng *et al.* was suggested, based on an exponential distribution model of the prediction residuals. In this paper, we develop this theoretical analysis further, exploiting a broader model based on exponential power distributions.

1. INTRODUCTION

The compression of color-indexed images is very demanding for most general purpose continuous-tone image coding techniques. Specialized approaches for coding color-indexed images do exist (see, for example, [1, 2, 3, 4]). However, it remains an important topic to ensure that general purpose image coding techniques, such as JPEG-LS [5, 6] or lossless JPEG 2000 [7, 8], do produce acceptable results with this class of images.

Color-indexed images are represented by a matrix of indexes (the index image) and by a color-map or palette. The indexes in the matrix address positions in the color-map and, therefore, establish the colors of the corresponding pixels. For a particular image, the mapping between index values and colors is not unique. In fact, it can be arbitrarily permuted, under the condition that the corresponding index image is changed accordingly.

Palette reordering is a class of preprocessing methods, having the goal of finding a permutation of the color palette such that the resulting image of indexes is more suitable for compression. If the optimal configuration is sought, then the computational complexity involved can be high. In fact, the number of possible configurations for a table of M colors corresponds to the number of permutations of M objects, which equals $M!$. Therefore, exhaustive search is impractical for most of the interesting cases, which motivated several sub-optimal, lower complexity, proposals [9, 10, 11, 12, 13, 14, 15, 16, 17].

Recently, a theoretically motivated modification of a reordering method proposed by Zeng *et al.* [14] was suggested [17]. By assuming that the first order prediction residuals are frequently well modeled by an exponential distribution, a new set of parameters was proposed, leading to important improvements in the lossless compression of the index images [17]. In this paper, we develop the theory further, by broadening the underlying distribution model and by studying, in practice, how appropriate is the exponential distribution that was assumed in [17]. This is done through the adoption of an exponential power distribution model which, as a particular case, includes the exponential distribution.

2. ZENG'S METHOD

The palette re-indexing method proposed by Zeng *et al.* [14] is based on an one-step look-ahead greedy approach, which aims at increasing the lossless compression efficiency of color-indexed images.

The algorithm starts by finding the index that is most frequently located adjacent to other (different) indexes, and the index that is most frequently found adjacent to it. This pair of indexes is the starting base for an ordered set, $\mathcal{S}$, that will be constructed, one index at a time, during the operation of the re-indexing algorithm. We denote by v_i the indexes already assigned to the ordered set (i indicates the position of the index in the ordered set and, therefore, its distance to the left end side of the set) and by u those still unassigned. Therefore, just before starting the iterations, $\mathcal{S} = \{v_1, v_2\}$, where v_1 and v_2 are the two indexes mentioned above. New indexes can only be attached to the left or to the right extremity of the ordered set.

The algorithm then proceeds as follows. For each iteration, compute u_L and u_R according to:

$$u_L = \arg\max_{u \notin \mathcal{S}} D_L(u), \quad (1)$$

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where

$$D_L(u) = \sum_{v_i \in \mathcal{S}} \alpha_i \, C(u, v_i), \tag{2}$$

and

$$u_R = \arg\max_{u \notin \mathcal{S}} D_R(u), \tag{3}$$

where

$$D_R(u) = \sum_{v_i \in \mathcal{S}} \alpha_{|\mathcal{S}|-i+1} \, C(u, v_i). \tag{4}$$

The function $C(i, j) = C(j, i)$ denotes the number of occurrences (measured in the initial index image) corresponding to pixels with index i that are spatially adjacent to pixels with index j. The α_k are weights controlling the impact of the $C(u, v_k)$ on $D_L(u)$ and $D_R(u)$ and, originally [14], were proposed to be given by

$$\alpha_k = \log_2 \left(1 + \frac{1}{k}\right).$$

The new set is given by $\{u_L, v_1, \ldots, v_{|\mathcal{S}|}\}$, if $D_L(u_L) > D_R(u_R)$, or by $\{v_1, \ldots, v_{|\mathcal{S}|}, u_R\}$, otherwise. This iterative process continues until assigning all indexes. Finally, the re-indexed image is constructed by applying the mapping $v_i \mapsto (i-1)$ to all image pixels, and changing the color-map accordingly.

3. GENERALIZED ZENG'S METHOD

In [17], a modification of Zeng's algorithm was proposed, relying on an exponential model for the distribution of first order prediction residuals, and on the assumption that the entropy of the absolute differences between neighboring pixels is a good indicator of the degree of compressibility of an image. In what follows, we extend the work reported in [17] in order to accommodate an exponential power distribution model of the prediction residuals.

According to the greedy strategy of Zeng's algorithm, the next index, $\bar{u}$, that should integrate $\mathcal{S}$ is the one that implies the largest increase in code length if its choice is postponed to the next iteration. It is well-known that, for a memoryless source, the number of bits required to represent the occurrence of a given symbol s is given by $-\log_2 P(s)$, where $P(s)$ denotes the probability of occurrence of s.

We start by defining the estimated code length implied by placing index u on the left end side of $\mathcal{S}$

$$l_L(u) = -\sum_{v_i \in \mathcal{S}} C(u, v_i) \, \log_2 P(i), \tag{5}$$

by placing it one position farther away

$$l_L^+(u) = -\sum_{v_i \in \mathcal{S}} C(u, v_i) \, \log_2 P(i+1), \tag{6}$$

by placing it on the right end side of $\mathcal{S}$

$$l_R(u) = -\sum_{v_i \in \mathcal{S}} C(u, v_i) \, \log_2 P(|\mathcal{S}| - i + 1), \tag{7}$$

and by placing it one position farther away from the right end side

$$l_R^+(u) = -\sum_{v_i \in \mathcal{S}} C(u, v_i) \, \log_2 P(|\mathcal{S}| - i + 2). \tag{8}$$

The new index, $\bar{u}$, should satisfy

$$\bar{u} = \arg\max_{u \notin \mathcal{S}} \Delta l(u), \tag{9}$$

with

$$\Delta l(u) = \begin{cases} l_L^+(u) - l_L(u), & \text{if} \quad l_R(u) - l_L(u) > 0 \\ l_R^+(u) - l_R(u), & \text{otherwise.} \end{cases} \tag{10}$$

In words, for each candidate index, u, its best position (left or right) is chosen, i.e., the one that minimizes the code length. Then, among all those indexes, we pick the one producing the largest increase in code length if its choice is postponed to the next iteration.

Now, we can write

$$l_L^+(u) - l_L(u) = \sum_{v_i \in \mathcal{S}} \log_2 \frac{P(i)}{P(i+1)} C(u, v_i) = \sum_{v_i \in \mathcal{S}} \alpha_i \, C(u, v_i), \tag{11}$$

if the best position for index u is the left end side, or

$$\begin{aligned} l_R^+(u) - l_R(u) &= \sum_{v_i \in \mathcal{S}} \log_2 \frac{P(|\mathcal{S}| - i + 1)}{P(|\mathcal{S}| - i + 2)} C(u, v_i) = \\ &= \sum_{v_i \in \mathcal{S}} \alpha_{|\mathcal{S}|-i+1} \, C(u, v_i), \end{aligned} \tag{12}$$

if the best position is the right end side, where

$$\alpha_k = \log_2 \frac{P(k)}{P(k+1)}, \tag{13}$$

and where $P(k)$ denotes the probability of occurrence of a difference of k units between two neighboring pixels.

Moreover, we can also write

$$\begin{aligned} l_R(u) - l_L(u) &= \sum_{v_i \in \mathcal{S}} \left(\log_2 P(i) - \log_2 P(|\mathcal{S}| - i + 1)\right) C(u, v_i) = \\ &= \sum_{v_i \in \mathcal{S}} \beta_i \, C(u, v_i), \end{aligned} \tag{14}$$

where

$$\beta_k = \log_2 \frac{P(k)}{P(|\mathcal{S}| - k + 1)}. \tag{15}$$

For exponentially power distributed residuals, i.e., considering

$$P(k) = A\theta^{k^\gamma}, \quad 0 < \theta < 1, \quad 0 \leq k < M, \quad \gamma > 0 \tag{16}$$

Eq. (13) reduces to

$$\alpha_k = \log_2 \frac{A\theta^{k^\gamma}}{A\theta^{(k+1)^\gamma}} = \left(k^\gamma - (k+1)^\gamma\right) \log_2 \theta, \qquad (17)$$

and (15) to

$$\beta_k = \log_2 \frac{A\theta^{k^\gamma}}{A\theta^{(|S|-k+1)^\gamma}} = \left(k^\gamma - (|S|-k+1)^\gamma\right) \log_2 \theta. \quad (18)$$

Finally, we note that the $\log_2 \theta$ term can be eliminated, since it is a constant factor, although bearing in mind that it is always negative.

4. EXPERIMENTAL RESULTS AND DISCUSSION

In this Section, we provide experimental results showing the compression gain that can be obtained if an exponential power model is used, in comparison to the exponential model (i.e., for $\gamma = 1.0$). We give compression results for the same collection of color-indexed images that have been used in [17]. These are images both from synthetic and natural origins and of various sizes and number of colors. We provide results not only for a JPEG-LS encoder, as in [17], but also for a JPEG 2000 lossless encoder. Table 1 presents the compression results that have been obtained (both in terms of number of bytes and bits per pixel), including the size of the color-maps. The best value of γ was determined for each image / encoder pair, and the compression gain in relation to the exponential distribution ($\gamma = 1.0$) is presented.

From the results presented in Table 1, we observe that, in fact, the exponential model seems to be a reasonable choice for most of the images. From the 30 test images included in Table 1, 17 (18 for JPEG 2000) of them had compression improvements of less than one percent. However, for some of the images (6 for JPEG-LS and 8 for JPEG 2000) the lossless compression gain was over 3%.

A somewhat surprising observation is that, for 17 images, the best value of γ is the same for both encoders, and for 6 others they differ only by 0.1. This means that, apparently, for most of the images, the same distribution model is well suited for encoding engines so different as those of JPEG-LS (prediction-based) and JPEG 2000 (transform-based). In our opinion, this observation deserves further study, because it may contribute for establishing links between these two coding principles.

Finally, although currently it is not very practical to use the exponential power model for palette reordering (due to the need of searching for the best γ for each image), it may be so if a low complexity way of guessing it from the image is found. This is a possibility that we plan to exploit in a near future.

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Image	Colors	JPEG-LS						JPEG 2000					
		mZeng ($\gamma = 1.0$)		Proposed			Gain	mZeng ($\gamma = 1.0$)		Proposed			Gain
		Size	bpp	Size	bpp	γ	%	Size	bpp	Size	bpp	γ	%
pc	6	320,479	0.745	320,479	0.745	1.0	0.0	322,560	0.749	322,560	0.749	1.0	0.0
books	7	10,458	1.469	10,345	1.453	1.3	1.1	11,392	1.601	11,392	1.601	1.0	0.0
music	8	1,620	1.051	1,620	1.051	1.0	0.0	2,117	1.374	2,117	1.374	1.0	0.0
winaw	10	16,569	0.450	16,569	0.450	1.0	0.0	22,505	0.611	22,505	0.611	1.0	0.0
party8	12	5,993	0.318	5,959	0.316	2.3	0.6	7,782	0.413	7,626	0.405	2.3	2.0
netscape	32	13,405	1.752	13,322	1.741	0.8	0.6	18,285	2.390	18,215	2.381	0.1	0.4
sea_dusk	46	3,732	0.189	3,732	0.189	1.0	0.0	2,700	0.137	2,604	0.132	0.4	3.6
benjerry	48	3,977	1.137	3,974	1.137	1.1	0.1	6,186	1.769	6,145	1.758	0.2	0.7
gate	84	19,543	2.566	19,489	2.559	0.9	0.3	23,124	3.037	23,119	3.036	0.9	0.0
descent	122	22,834	2.854	22,745	2.843	0.7	0.4	27,395	3.424	27,188	3.398	0.7	0.8
sunset	204	88,610	2.307	88,126	2.294	1.2	0.6	125,416	3.266	124,929	3.253	1.2	0.4
yahoo	229	6,072	1.789	5,995	1.767	0.5	1.3	7,646	2.253	7,513	2.214	0.4	1.7
airplane	256	145,657	4.445	142,963	4.362	1.8	1.9	155,603	4.748	152,647	4.658	1.9	1.9
anemone	256	211,103	4.966	204,085	4.801	1.5	3.3	241,394	5.678	233,559	5.494	1.5	3.2
arial	256	280,074	6.183	280,074	6.183	1.0	0.0	294,979	6.512	293,722	6.484	0.6	0.4
baboon	256	212,881	6.496	211,123	6.442	1.5	0.8	219,469	6.697	218,264	6.660	1.5	0.5
bike3	256	372,720	4.154	372,720	4.154	1.0	0.0	434,896	4.847	434,896	4.847	1.0	0.0
boat	256	190,834	5.823	184,279	5.623	2.0	3.4	199,142	6.077	191,779	5.852	2.0	3.7
clegg	256	488,553	5.456	482,503	5.388	1.3	1.2	548,572	6.126	543,946	6.074	1.3	0.8
cwheel	256	172,718	2.878	171,273	2.854	0.8	0.8	194,616	3.243	192,291	3.204	0.8	1.2
fractal	256	282,417	5.828	273,932	5.653	1.6	3.0	292,557	6.038	283,206	5.845	2.2	3.2
frymire	256	521,446	3.376	514,140	3.329	0.5	1.4	651,322	4.217	629,749	4.078	0.6	3.3
ghouse	256	272,465	4.541	269,666	4.494	1.3	1.0	305,487	5.091	302,681	5.044	1.3	0.9
girl	256	172,202	5.255	170,288	5.196	1.1	1.1	181,728	5.545	180,209	5.499	1.2	0.8
house	256	39,767	4.854	39,447	4.815	1.4	0.8	42,106	5.139	41,791	5.101	1.4	0.8
lena	256	165,457	5.049	159,620	4.871	1.9	3.5	178,097	5.435	168,597	5.145	1.9	5.3
monarch	256	192,548	3.917	192,548	3.917	1.0	0.0	222,169	4.520	222,169	4.520	1.0	0.0
peppers	256	164,481	5.019	155,068	4.732	1.6	5.7	176,898	5.398	166,909	5.093	1.7	5.6
serrano	256	204,369	3.273	198,111	3.173	0.7	3.1	269,893	4.323	260,911	4.179	0.6	3.3
tulips	256	198,226	4.032	198,226	4.032	1.0	0.0	228,854	4.656	228,755	4.654	1.8	0.0
Average	—	—	3.122	—	3.077	—	1.4	—	3.521	—	3,460	—	1.7

Table 1. Lossless compression results, using JPEG-LS and lossless JPEG 2000 encoders, of a number of synthetic and natural color-indexed images. The “mZeng” values refer to the technique proposed in [17], whereas “Proposed” refers to the reordering method based on the exponential power distribution model addressed in this paper. “Gain” indicates the percentage of compression of the “Proposed” in relation to the corresponding “mZeng”. The γ columns indicate the best value of this parameter for each image / encoder pair. All compression values include the size of the color-map.

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