Regularization of Transportation Maps for Color and Contrast Transfer

Julien Rabin, Julie Delon, Yann Gousseau

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Julien Rabin

CEREMADE, Univ. Paris Dauphine
Place du Maréchal De Lattre De Tassigny
75775 Paris cedex 16

Julie Delon, Yann Gousseau

LTCI, Télécom ParisTech
46, rue Barrault
75634 Paris cedex 13

ABSTRACT

In this paper, we take interest in the process of assigning a given color distribution to a given image. Two examples of such an image modification are histogram equalization (or specification) and color transfer, in which the color palette of a *style* image is assigned to a *source* image. Classical methods for gray level specification, as well as more recent methods for color transfer, can be defined as optimal transportation problems. The corresponding image modifications are known to produce visually unpleasing effects such as the removal of details and texture, as well as the enhancement of noise or compression patterns. In this paper, a new method is proposed for the suppression of these artifacts. The method relies on a non local regularization of the *transportation map*, defined as the difference between the original image and the modified one. The interest of using this method is demonstrated on the aforementioned applications: contrast adjustment and color transfer.

Index Terms— color transfer, contrast adjustment, histogram specification, optimal transportation, image regularization.

1. INTRODUCTION

Modifying the gray level or color distribution of images is one of the most common operation for image enhancement. Beside the classical histogram equalization or specification [1] to be found in any reference book on image processing, more involved techniques have been developed to correct the flicker to be found in old motion pictures [2], to combine characteristics between images acquired in different lighting conditions, *e.g.* photographs taken with and without flash [3], to compute intermediate histograms between images [4] or to transfer color characteristics from one image to another one [5, 6].

A large proportion of these enhancement techniques may be interpreted in the mathematical framework of optimal transportation, also known as the Monge-Kantorovich problem [7]. The color (or luminance) transfer corresponds to an optimal flow, which minimizes the transportation cost between two histograms. For instance, the well-known *histogram equalization* is obtained as the solution of an optimal transportation problem. The same is true for histogram specification, as we will see in the next section. Recently, this approach has been extended to color images: see [4, 2] for contrast and color adjustment of image sequences and [6] for color transfer between image pairs.

One of the known drawback of histogram manipulation methods is a tendency to produce unpleasant visual artifacts, as illustrated by Figure 2(c). This results from the definition of the transportation flow, which does not take into account the spatial regularity of images. In this paper, we introduce a new method to regularize the *transportation map*, defined as the difference between the original image and the one obtained after histogram modification. This method, drawing its inspiration from works on edge preserving filters [8, 9], makes it possible to remove noise or compression artifacts, while preserving the original image details and textures.

In Section 2, histogram adjustment and color transfer are presented from the perspective of optimal transportation, and corresponding artifacts are detailed. Then, a method for the regularization of the transportation map is proposed in Section 3 as a way to suppress these artifacts. We conclude with experiments and comparisons in Section 4.

2. COLOR AND CONTRAST TRANSFER

2.1. Color and contrast transfer between images - Recall.

Let $u : \Omega \mapsto \mathbb{R}^n$ be a discrete **source image**, with $n = 1$ for a gray level image, $n = 3$ for a color image, and where $\Omega \subset \mathbb{Z}^2$ is the bounded image domain. We denote by h_u its gray level or color distribution, defined as the finite sum $h_u = \sum h_\lambda \delta_\lambda$, where $h_\lambda = \frac{1}{|\Omega|} \#\{x \in \Omega;\ u(x) = \lambda\}$.

Assigning a **target** distribution f to u amounts to find a mapping T (a contrast change) such that the distribution of $T(u)$ is close to f in some sense

$$h_{T(u)} \simeq f. \quad (1)$$ [1]

[1] Most of the time, this equality cannot be exactly satisfied.

Among mappings T satisfying (1), one generally looks for a mapping minimizing a global transportation cost, for instance defined as

$$\int_{\mathbb{R}^n} |T(\lambda) - \lambda|^2 h_u(d\lambda). \tag{2}$$

The study of this problem is called *optimal transportation* [7].

If $n = 1$, one can show that an application T satisfying (1) and minimizing (2) is $F^{-1} \circ H_u$, where F and H_u are the respective cumulative distribution functions of f and h_u, and where F^{-1} is defined as $F^{-1}(t) = \min\{\lambda \in \mathbb{R};\ F(\lambda) \geq t\}$. If f is a constant distribution on the range of u, one gets the well known *histogram equalization*. More generally, if f is the gray level distribution h_v of a **style image** v, then $T = H_v^{-1} \circ H_u$ is called *histogram specification*.

In the case of color images, the previous 1D modifications may classically be applied to the luminance channel of images. More generally, one consider three-dimensional histograms within the same formalism, with $n = 3$. An application T satisfying (1) is then called *color transfer* and can be seen as a way of recoloring u by using a prescribed color palette f. Finding an optimal T for equation (2) in this case is more tedious. T can be estimated numerically, for instance by using the simplex algorithm. Most of the time, this estimation leads to expansive computations. In practice, a satisfying application T can be computed by estimating iteratively 1D optimal transportation flows on random axes, as proposed in [6]. The resulting transportation is fast to compute, although not optimal in the sense of (2).

2.2. Visual artifacts

Three important visual artifacts can be caused by contrast or color transfers:

- ▷ ***Noise enhancement:*** this happens if the variance of the noise in u increases after the application of T to u, as illustrated for instance in Figure 1(b).
- ▷ ***Compression artifacts:*** these artifacts appear when the image u is highly compressed and when pixels with a very similar color are mapped to different colors (see Figure 2(g)).
- ▷ ***Detail loss:*** this results from a reduction of contrast between u and $T(u)$.

Observe that these three artifacts are due to local irregularities of the mapping T. Noise enhancement and compression artifacts happen when T increases the distances between neighboring colors or gray levels in u, whereas detail loss happen when T decreases these distances.

A fourth problem, inherent to color transfer between images, is the one of ***color proportion***. Ideally, the mapping should be defined in such a way that pixels having similar colors in the source image should be mapped to similar colors. However, this is unfeasible if the proportions of colors are very different in the source and the target distributions, as illustrated by Figures 2(c) and 2(k).

3. A NEW REGULARIZATION APPROACH FOR TRANSPORTATION MAPS

The solution we propose in order to regularize the transportation map is inspired from *non-local filters* [9]. This concept has been introduced for image denoising by Yaroslavsky [8]. Similar filters have been independently defined, as *SUSAN* [10] or the *Bilateral Filter* [11]. Recently, a somehow radical extension of this approach, the so-called "Non-Local Mean" filter [9] has been shown to outperform many edge preserving smoothing approaches to image denoising. In what follows, we will make use of the Yaroslavsky filter to regularize transportation maps.

3.1. Non Local Map Regularization

In the following, by a slight abuse of language, we call *transportation map* the image of differences $\mathcal{M}(u) = T(u) - u$. We propose to regularize this map thanks to Yaroslavsky filters, that is, we define the following operator:

$$[\mathrm{Y}_u\, \mathcal{M}(u)](x) = \frac{1}{C(x)} \int_{y \in \mathcal{N}(x)} [\mathcal{M}(u)](y) \cdot e^{-\frac{\|u(x)-u(y)\|^2}{\sigma^2}} dy \tag{3}$$

where $\|.\|$ stands for the Euclidean distance in $\mathbb{R}^n$, $\mathcal{N}(x) \subset \Omega$ is a spatial neighborhood of the pixel x, σ is a tuning parameter of the method and $C(x)$ is the normalization constant

$$C(x) = \int_{y \in \mathcal{N}(x)} e^{-\frac{\|u(x)-u(y)\|^2}{\sigma^2}} dy \ .$$

The resulting regularization of the image $T(u)$, referred to as *Transportation Map Regularization* (**TMR**), is then defined as $\mathrm{TMR}_u(T(u)) :=\ u + \mathrm{Y}_u\, \mathcal{M}(u)$. Now, observe that this formulation can be divided in two terms :

$$\mathrm{TMR}_u(T(u)) = \underbrace{\mathrm{Y}_u\big(T(u)\big)}_{\text{filtering of image } T(u)} + \underbrace{u - \mathrm{Y}_u(u)}_{\text{source image detail}} . \tag{4}$$

First, the image $T(u)$ is filtered by a non-local mean operator Y_u, following the regularity of the source image u. This operation attenuates noise, compression and color proportion artifacts but also the details of the image $T(u)$. The second operation performed by the TMR filter consists in adding the quantity $\Delta u = u - \mathrm{Y}_u(u)$, which can be considered as details of the image source (*e.g.* texture and fine structures). We will see in the experimental section that these two steps are very important to obtain a natural rendering of the image.

Observe that the pixel-based comparisons in the TMR filter could be replaced by patch comparisons, following the NL-means filter [9]. However, while Yaroslavsky filter is less robust than the NL-means for denoising purposes, it is particularly adapted in our case, where the image u is regular. Besides, it permits a faster approach and a better preservation of edges.

3.2. Properties

Observe that this filter leaves all the images $u + \lambda$, $\lambda \in \mathbb{R}^n$, unchanged. If the application T consists in a multiplication by a positive constant α, then $\mathrm{TMR}_u(\alpha u) = \alpha u + (1 - \alpha) \cdot \Delta u$. If $\alpha > 1$, the transfer T increases the contrast. In that case, the TMR filter reduces the noise contained in the image difference Δu. If $\alpha < 1$ the transfer T decreases the contrast and the TMR filter restore the lost details contained in Δu.

3.3. Implementation

Iteration of TMR filter In practice, more than one iteration of the TMR filter may be required to remove all the aforementioned artifacts. The image $T(u)$, after k iterations of the TMR filter, can be written as follows:

$$\mathrm{TMR}_u^k(T(u)) := \mathrm{Y}_u^k\big(T(u)\big) + u - \mathrm{Y}_u^k(u) ,$$

where Y_u^k refers to the recursive use of the Y_u operator.

Convergence map One can show that $\mathrm{Y}_u^k \mathcal{M}(u)$ slowly tends towards a constant map when $k \to \infty$. In order to stop automatically the iterations of the filter and to speed up the computation of the iterated TMR filter, we compute for each iteration a *convergence map*, written $\mathcal{C}$ and defined as follows:

$$\mathcal{C}(x) = \left\| \, [\mathrm{Y}_u^k \, \mathcal{M}(u)](x) - [\mathrm{Y}_u^{k-1} \, \mathcal{M}(u)](x) \, \right\| \ .$$

We consider that there is numerical convergence in pixel x when $\mathcal{C}(x) < t$. The TMR filter is hence only applied to pixels for which the convergence map is greater than the threshold t. It also allows us to decide when the iteration of TMR filter should be stopped. In practice, the convergence threshold has been set equal to $t = 1$ (for $n \times 8$-bit images).

Note that the TMR filter could also benefit from other acceleration techniques (see *e.g.* [12, 13]).

4. EXPERIMENTS

Observe that the TMR filter relies on two different parameters. The most important one is σ, which is used to compute the weighting terms in the computation of the regularized map (Formula (3)). The second parameter is the radius ρ of the disk used here to define the neighborhood $\mathcal{N}$. In all the following experiments, we have used $\sigma = 10$ and $\rho = 10$ pixels.

Figure 1 shows the result of one iteration of the TMR filter when the mapping T is the gray level histogram equalization (denoted by EQ) of the image u shown in Figure 1(a). The resulting image, $\mathrm{TMR}_u(\mathrm{EQ}(u))$, shown on Figure 1(d), can be compared with the result obtained by applying the regularization *before* the equalization, displayed on Figure 1(e). This alternative approach follows the spirit of the two-scale decomposition approach of [13] in the case of *tone mapping* (contrast reduction for high dynamic range images). In their approach, the image u is first decomposed into a base layer (here represented by $\mathrm{Y}_u(u)$) and a detail layer $\Delta u = u - \mathrm{Y}_u(u)$. A contrast reduction is then applied to the base layer, and Δu is added to the result to obtain the final image. Figure 1(e) demonstrates that this approach, well suited for dynamic reduction, is not adapted to histogram equalization.

Figure 2 shows the results of the *iterated* TMR filter on several examples of color transfer. In these examples, we use the efficient color transfer algorithm proposed in [6], which is both fast and easy to implement. These examples show how the TMR filter is able to remove or greatly reduce the artifacts listed in Section 2.2 and due to the limitations of color transfer. Observe that another regularization scheme for color transfer has been proposed by the authors of [6]. This scheme, called *regraining*, relies on a variational approach to find an image with the same colors as $T(u)$, while restoring the details of the source image u. However, we see on Figure 2(k) that this variational approach fails at removing completely artifacts such as inconsistencies in color proportions.

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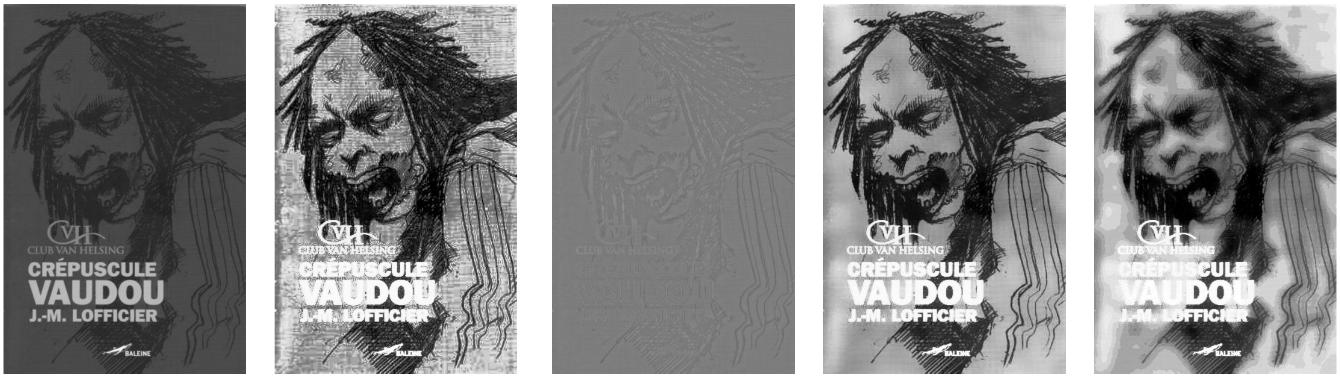
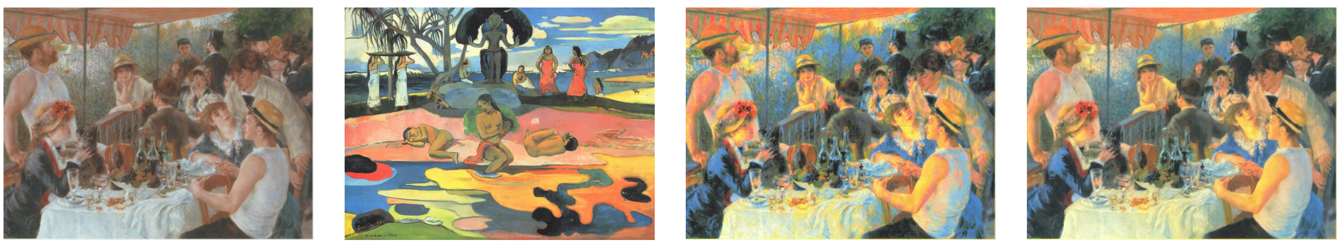


(a) Source image u. (b) Histogram equalization: $\mathrm{EQ}(u)$. (c) Detail layer: $\Delta u = u - \mathrm{Y}_u(u)$. (d) $\mathrm{TMR}_u(\mathrm{EQ}(u)) = \mathrm{Y}_u \circ \mathrm{EQ}(u) + \Delta u$. (e) Alternative method [13]: $\mathrm{EQ} \circ \mathrm{Y}_u(u) + \Delta u$.

Fig. 1. **Transportation Map Regularization (*TMR*) principle.** *(d)* Application of the TMR filter to regularize the transportation map obtained by histogram equalization (Figure (b)). *(e)* The two-scale decomposition approach used in [13] is not adapted in this framework.



(a) Source image (Auguste Renoir, *Le déjeuner des Canotiers*, 1881). (b) Style image (Paul Gauguin, *Mahana no atua – le jour de Dieu*, 1894). (c) Color transfer. (d) Iterated TRM filter.

(e) Source image. (f) Style image. (g) Color transfer. (h) Iterated TRM filter.

(i) Source image. (j) Style image. (k) Color transfer + regraining with approach of [6]. (l) Iterated TRM filter.

Fig. 2. **Illustration of color transfer regularization with the iterated TMR filter.** On each line, the first two columns represent images with different color palettes. The third column is obtained by transferring the colors of the second image to the first one, using the transfer method from [6]. This operation (as any such transfer procedure) produces artifacts: noise enhancement, compression artifacts, poor color proportions, and detail loss. The fourth column shows the regularization of the color transfer obtained by the iterated TMR filter.