Optimized current reference generation for system-level harmonic mitigation in a diesel-electric ship using non-linear model predictive control

Espen Skjong
Center for Autonomous Marine Operations and Systems,
Department of Engineering Cybernetics,
Norwegian University of Science and Technology
Email: espen.skjong@itk.ntnu.no

Marta Molinas
Department of Engineering Cybernetics,
Norwegian University of Science and Technology
Email: marta.molinas@ntnu.no

Tor Arne Johansen
Center for Autonomous Marine Operations and Systems,
Department of Engineering Cybernetics,
Norwegian University of Science and Technology
Email: tor.arne.johansen@itk.ntnu.no

***Abstract*—Non-linear optimal control using collocation and multiple shooting is investigated in this paper to generate the current reference signal for the lower level control of an active filter in a marine vessel with diesel-electric propulsion. The optimization objective is aimed at using the active filter for system-level minimization of Total Harmonics Distortion with the minimum rating of the active filter. The investigation of the different algorithms is oriented to the search of a solution that can offer a good compromise between accuracy and real-time implementation abilities. Results indicate that linear problem formulations are more suitable for real time implementation as they require less computational costs with minimal loss of flexibility. Non-linear problem formulations provide higher flexibility at the cost of higher computational efforts.**

I. Introduction

Harmonics propagation due to the presence of non-linear loads is a major power quality issue in marine vessels [1, 2]. The use of active filtering is an attractive alternative to the mitigation of harmonics that can offer fast current tracking performance. However, active filtering techniques were originally developed to locally provide harmonic currents to non-linear loads so that the current supplied by generators will only contain the fundamental component and by that will not be a source of harmonics pollution for the rest of the power system [3, 4]. Local load compensation has therefore been the main application of active harmonic filtering and along with the principle of perfect compensation of the load harmonics, there was no scope for further optimization. In a marine vessel, power generation and loads are dispersely located through an electrical distribution system whose impedances will determine the propagation of the harmonics generated by the non-linear loads. When there are several non-linear loads being the source of harmonics in the system, local load compensation will no-longer provide the perfect filtering of the system harmonics. The harmonics generated by all loads will affect the Total Harmonic Distortion (THD) at the system buses and locally filtering one load's harmonics will not eliminate the harmonic propagation in the system. System-level wide spectrum harmonics mitigation approaches have not been extensively reported in the literature [5]. Some authors have proposed wide spectrum mitigation solutions based on passive filters [6, 7]. Nonetheless, these solutions will generally require a retuning when the harmonic pollution changes substantially from the design conditions, since they are passive and are not able to optimally follow the changing spectrum conditions [4]. Under changing spectrum conditions and due to their inherent fast harmonic current tracking performance, active filtering can benefit from system-level optimization algorithms to generate in real-time the current reference signal to be actuated by the active filter [8, 9]. This harmonic current reference, when generated by the system-level optimization algorithm, will induce in the system, the optimal harmonic current distribution that will be required to minimize the THD on all buses of the system. At the same time, the optimization algorithm can be designed to do that with the minimum active filter power rating.

This paper explores non-linear programming methods for testing optimization algorithms that will exploit the active filter capabilities for generating in real-time the optimal harmonic current waveform (using the phase and amplitude of all harmonic components) that will minimize the overall system THD with the minimum filter power rating.

The optimization problem is formulated as a Non-linear Model Predictive Control (NMPC) with both time varying and constant controls. The NMPC is solved as a Non-Linear Programming (NLP) problem using collocation and multiple shooting. As will be discussed, collocation and multiple shooting have different requirements when it comes to the number of discretization steps within a fixed time horizon. The number of discretization steps is directly tied to both accuracy and computational costs, hence an increase in number of discretization steps may lead to better accuracy at the cost of higher computational costs. It is also showcased that the optimization problem can be formulated as a linear problem which requires less computational effort at the cost of reduced flexibility when working close to the active filter's physical limits. Accuracy and flexibility are important to reduce the harmonic pollution within allowed bounds, however some accuracy and flexibility may be compromised to achieve a real-time implementation.

In this paper the focus is on generating an optimized filter current reference that will result in a power system THD conditioning better than what local filtering techniques can achieve and within the requirements of the standards for electric ships [10]. Multiple shooting and collocation techniques are tested and discussed for NMPC, and a linear problem is formulated which has better real-time properties than the non-linear optimization formulation.

II. Power Distribution Grid Model

A marine vessel's power distribution grid is designed to handle loss of power caused by e.g. short circuited distribution lines, faults related to different loads and tripping of generators. This is often done by advanced power grid designs using bus-tie breakers and protection relays. A simplified model of a vessel AC distribution grid is assumed in figure 1, including two generators, two loads and an active harmonic filter, where robustness given by redundant buses and bus-tie breakers are disregarded. We refer to [11] for robust shipboard electrical power system designs.

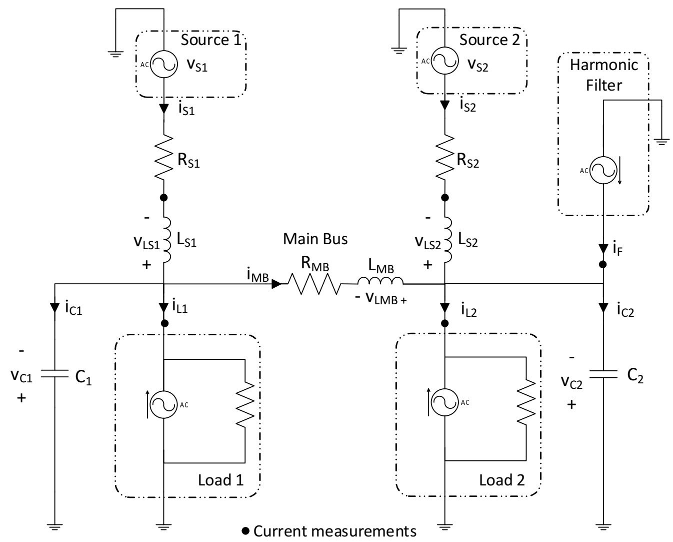

Fig. 1: Simplified model of an AC power distribution grid.

TABLE I: Power distribution grid parameters used in case study.

Parameter	Value
L_{S1}	1 mH
L_{S2}	1 mH
L_{MB}	1 mH
R_{S1}	$(0.1 \cdot L_{S1} \cdot \omega)$ Ω
R_{S2}	$(0.1 \cdot L_{S2} \cdot \omega)$ Ω
R_{MB}	$(0.1 \cdot L_{MB} \cdot \omega)$ Ω
C_1	0.1 μF
C_2	0.1 μF

A. Model formulation

The mathematical model for the power distribution grid illustrated in figure 1 can be derived using Kirchhoff's current and voltage laws, and can be stated as

$$\begin{aligned} L_{S1}\frac{di_{S1}}{dt} &= v_{S1} - R_{S1}i_{S1} - v_{C1} \\ C_1\frac{dv_{C1}}{dt} &= i_{S1} - i_{MB} - i_{L1} \\ L_{MB}\frac{di_{MB}}{dt} &= v_{C1} - v_{C2} - R_{MB}i_{MB} \\ C_2\frac{dv_{C2}}{dt} &= i_{MB} + i_{S2} - i_{L2} + i_F \\ L_{S2}\frac{di_{S2}}{dt} &= v_{S2} - R_{S2}i_{S2} - v_{C2}, \end{aligned} \tag{1}$$

One should note that capacitors are inserted in the power grid model for modeling purposes, and should be given small values with the only purpose of having a numerically robust solution to the equations.

The generators are modeled as ideal voltage sources with a voltage phase shift ϕ_V,

$$v_S(t) = \sqrt{2}V_{rms}\sin(\omega t + \phi_V), \tag{2}$$

while the non-linear loads are modeled as current sources with harmonic components of order 5,7,11 and 13 with phase shifts $\phi_{L,i}$ and peak values (amplitudes) $I_{L,i}$,

$$i_L(t) = \sum_i I_{L,i}\sin\left(i\left(\omega t + \phi_{L,i}\right)\right), \quad \forall i \in \{1, 6k \pm 1 | k = 1,2\}. \tag{3}$$

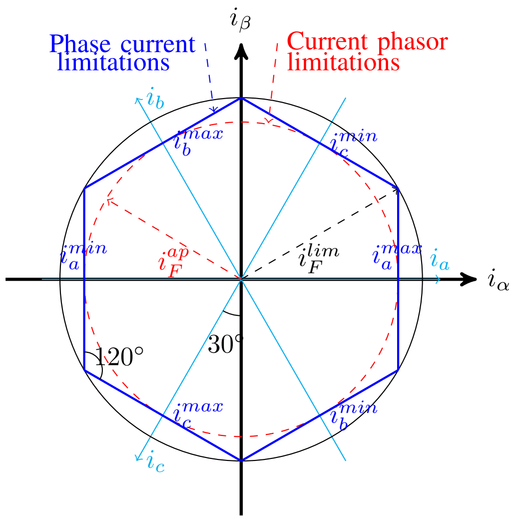

Fig. 2: Harmonic filter constraints: Three-phase three-wire represented by the $\alpha\beta$ and abc frames [12].

The harmonic filter is also modeled as a current source and it is designed to supply higher order harmonic components to mitigate harmonic propagation in the vessel distribution system. The filter model can be expressed as

$$i_F(t) = \sum_i I_{F,i}\sin\left(i\left(\omega t + \phi_{F,i}\right)\right), \quad \forall i \in \{6k \pm 1 | k = 1,2\}, \tag{4}$$

where $I_{F,i}$ is the peak value (amplitudes) of the filter's harmonic current components. The fundamental frequency can experience excursions from the rated value due to unbalances between production and consumption of power. Those frequency excursions can be modelled as disturbances for the purpose of simulation as

$$f(t) = f_f + A_p \sin(2\pi f_t t), \tag{5}$$

where f_f is the fundamental frequency, A_p is the frequency variation peak and f_t is the rate of the frequency variations. The rate of the frequency variations, f_t, is often within 0.1-1 Hz, and the frequency peak, A_p, is assumed to be 1-5 Hz. Hence, the time varying angular frequency is given by $\omega := \omega(t) = 2\pi f(t)$.

Before proceeding with the discussion of harmonic filter constraints, the distribution system model is extended to a three-phase three-wire configuration.

B. Three-phase three-wire configuration

The electrical distribution system model described in the previous section (single phase) can easily be extended to three-phase three-wire configuration. Due to the properties of three-phase three-wire the $\alpha\beta 0$ frame is simplified to $\alpha\beta$, where the β current is lagging the α current by 90°. This is due to the assumption of balanced sources, hence the neutral wire in a three-phase four-wire configuration is excessive. The three-phase three-wire model for the voltage sources, assuming balanced sources, can be written in the $\alpha\beta$ frame as

$$\mathbf{v}_S(t) = \begin{bmatrix} v_{S,\alpha}(t) \\ v_{S,\beta}(t) \end{bmatrix} = \begin{bmatrix} \sqrt{3}V_{rms}\sin(\omega t + \phi_V) \\ \sqrt{3}V_{rms}\sin(\omega t + \phi_V + \frac{\pi}{2}) \end{bmatrix}. \tag{6}$$

In the same way, the load and filter models can be extended to three-phase three-wire using the $\alpha\beta$ frame, however, for the filter model (eq. (4)) the phase $\phi_{F,i}$, $\forall i \in \{6k \pm 1 | k = 1, 2\}$ for each harmonic component should be equal for the α and β phases with a 90° phase shift. Also the filter amplitudes in the α and β frame are kept equal for each harmonic component when considering balanced loads to ensure a balanced filter.

In the rest of this paper subscript α and β are used to denote the α and β phases for each voltage and current component, and the vectors (phasors) $\mathbf{v}$ and $\mathbf{i}$ are used to represents voltages and currents, respectively, given in the $\alpha\beta$ frame. We refer to [4] for details regarding the $\alpha\beta$ frame in three-phase three-wire configurations.

C. Active Filter (AF) constraints

The harmonic filter model should be constrained due to the physical limitations. Figure 2 illustrates the limitations in both the abc and the $\alpha\beta$ frames. In abc frame the phases are restricted by

$$i_j^{min} \leq i_j \leq i_j^{max}, \quad \forall j \in \{a, b, c\}, \tag{7}$$

which forms the hexagon given in figure 2. These restrictions can be expressed in the $\alpha\beta$ frame by

$$-i_F^{lim} \leq -i_{F,\beta} + \frac{\sqrt{3}}{3} i_{F,\alpha} \leq i_F^{lim} \tag{8a}$$

$$-i_F^{lim} \leq i_{F,\beta} + \frac{\sqrt{3}}{3} i_{F,\alpha} \leq i_F^{lim} \tag{8b}$$

$$-i_F^{ap} \leq i_{F,\alpha} \leq i_F^{ap} \tag{8c}$$

$$-i_F^{lim} \leq i_{F,\beta} \leq i_F^{lim}, \tag{8d}$$

where the hexagon's apothem is given by

$$i_F^{ap} = \sqrt{\left(i_F^{lim}\right)^2 - \left(\frac{i_F^{lim}}{2}\right)^2} = \frac{\sqrt{3}}{2} i_F^{lim}, \tag{9}$$

and i_F^{lim} is a design variable representing the filter's phase current limitations. Eq. (8) gives a set of linear constraints, however the current phasor constraint, which is given by the red dotted circle in figure 2, is assumed to be the real physical limitation in the $\alpha\beta$ frame and is given by

$$(i_{F,\alpha})^2 + (i_{F,\beta})^2 \leq (i_F^{ap})^2. \tag{10}$$

The current phasor constraint given by eq. (10) is non-linear and may increase the required calculation costs to find an optimal solution to the problem. Both sets of constraints, eq. (8) and (10), will be discussed and simulated in section IV. For notational simplicity we define the feasible region for the filter's current vector $\mathbf{i}_F \in \mathbb{S}$.

III. Optimization Methods

The overall goal of the optimization is to minimize the total harmonic distortions at all buses of the vessel distribution system by generating an optimal harmonic current reference signal for the active filter. One approach to optimize filter currents is the non-linear Model Predictive Control (NMPC). The NMPC approach is based on simplified plant models and re-optimizes the controls after each completed optimization horizon using new measurements as initial values to the optimization problem. In this paper we address the NMPC problem where the filter current amplitudes and phases are both treated as i) *constant controls* and ii) *time varying controls* during one optimization horizon. A NLP solver using *Collocation* and *Multiple shooting* are considered to solve the problem. Before discussing collocation and multiple shooting methods, the problem is formulated in standardized form.

The objective of the problem will be to minimize, by control, the total harmonic currents in the distribution system, and prevent harmonic distortions from propagating through the power distribution system. Assuming that the higher order harmonics (5th, 7th, 11th and 13th) generated by a 6-pulse diode rectifier are known and given by

$$\mathbf{i}_L^{hh} = \begin{bmatrix} \sum_i I_{L,\alpha,i}\sin\left(i\left(\omega t + \phi_{L,i}\right)\right) \\ \sum_i I_{L,\beta,i}\sin\left(i\left(\omega t + \phi_{L,i} + \frac{\pi}{2}\right)\right) \end{bmatrix}, \quad \forall i \in \{6k \pm 1 | k = 1, 2\}, \tag{11}$$

and the algebraic state vector represented by

$$\mathbf{z} = \left[\mathbf{v}_{S1}^\top, \mathbf{v}_{S2}^\top, \mathbf{i}_{L1}^\top, \mathbf{i}_{L2}^\top, \mathbf{i}_F^\top, (\mathbf{i}_{L1}^{hh})^\top, (\mathbf{i}_{L2}^{hh})^\top\right]^\top, \tag{12}$$

where the loads $\mathbf{i}_{L1}$ and $\mathbf{i}_{L2}$ are the three-phase three-wire extension of eq. (3) given in the $\alpha\beta$ frame, the filter current $\mathbf{i}_F$ is the three-phase three-wire extension of eq. (4) given in the $\alpha\beta$ frame, and the harmonic components of each load, $\mathbf{i}_{L1}^{hh}$ and $\mathbf{i}_{L2}^{hh}$, are given by eq. (11). The dynamic state vector is given by

$$\mathbf{x} = \left[\mathbf{i}_{S1}^\top, \mathbf{i}_{S2}^\top, \mathbf{i}_{MB}^\top, \mathbf{v}_{C1}^\top, \mathbf{v}_{C2}^\top\right]^\top . \tag{13}$$

A. Time varying and constant controls problem formulation

If constant controls are assumed throughout the optimization horizon, the controls can be treated as parameters. Hence, the parameter vector is given by

$$\mathbf{p} = \left[I_{F,\alpha,i}, I_{F,\beta,i}, \phi_{F,i}\right]^\top , \quad \forall i \in \{6k \pm 1 | k = 1, 2\} , \tag{14}$$

and the control vector is given by $\mathbf{u} = 0$. If time-varying parameters are to be used to solve the problem we define the parameters as controls where the control vector is given by

$$\mathbf{u} = \left[I_{F,\alpha,i}, I_{F,\beta,i}, \phi_{F,i}\right]^\top , \quad \forall i \in \{6k \pm 1 | k = 1, 2\} \tag{15}$$

and the parameter vector is given by $\mathbf{p} = 0$. The NMPC problem can now be stated as

$$\begin{aligned}
\min_{\mathbf{p},\mathbf{u}} \quad & V(\mathbf{x}, \mathbf{z}, \mathbf{u}, \mathbf{p}) = \sum_{n=1}^{N} l(\mathbf{x}_n, \mathbf{z}_n, \mathbf{u}_{n-1}, \mathbf{p}) \\
\text{s.t.} \quad & \\
& \dot{\mathbf{x}}_n = \mathbf{f}(\mathbf{x}_n, \mathbf{z}_n, \mathbf{u}_{n-1}, \mathbf{p}) \quad \forall n \in \{1, \ldots, N\} \\
& \mathbf{h}(\mathbf{x}_n, \mathbf{z}_n, \mathbf{u}_{n-1}, \mathbf{p}) = 0, \quad \forall n \in \{1, \ldots, N\} \\
& \mathbf{g}(\mathbf{x}_n, \mathbf{z}_n, \mathbf{u}_{n-1}, \mathbf{p}) \le 0 | i_{F,n} \in \mathbb{S}, \quad \forall n \in \{1, \ldots, N\} \\
& \text{given initial values } \mathbf{x}_0, \mathbf{z}_0 | \text{initial value } \mathbf{i}_{F,0} \in \mathbb{S},
\end{aligned} \tag{16}$$

where index n denotes a control point, $\mathbf{h}(\cdot)$ and $\mathbf{g}(\cdot)$ represents equality and inequality constraints, respectively, and the index n denotes a control point. The stage cost function $l(\cdot)$ for constant controls minimizing the harmonic distortions close to the loads, with constant weights q_1, q_2, is

$$\begin{aligned}
l(\mathbf{x}, \mathbf{z}, \mathbf{u}, \mathbf{p}) = {} & q_1 \left(i_{F,\alpha} - i_{L1,\alpha}^{hh}\right)^2 + q_1 \left(i_{F,\beta} - i_{L1,\beta}^{hh}\right)^2 \\
& + q_2 \left(i_{F,\alpha} - i_{L2,\alpha}^{hh}\right)^2 + q_2 \left(i_{F,\beta} - i_{L2,\beta}^{hh}\right)^2 .
\end{aligned} \tag{17}$$

For time varying controls the stage cost function is given by

$$\begin{aligned}
l(\mathbf{x}, \mathbf{z}, \mathbf{u}, \mathbf{p}) = {} & q_1 \left(i_{F,\alpha} - i_{L1,\alpha}^{hh}\right)^2 + q_1 \left(i_{F,\beta} - i_{L1,\beta}^{hh}\right)^2 \\
& + q_2 \left(i_{F,\alpha} - i_{L2,\alpha}^{hh}\right)^2 + q_2 \left(i_{F,\beta} - i_{L2,\beta}^{hh}\right)^2 \\
& + q_u \left(\mathbf{u}_{I_F}^\top \mathbf{u}_{I_F}\right)^2 ,
\end{aligned} \tag{18}$$

with constant weights q_1, q_2, q_u, where $\mathbf{u}_{I_F} \subset \mathbf{u}$ that includes the filter's amplitudes. The last part in eq. (18) is added to minimize the filter amplitudes, hence minimizing the filter's power rating, and also provides stability and robustness with regards to modeling errors. $q_u < q_1, q_2$ as minimizing the power rating is of lesser importance than decreasing the harmonic distortions in the power grid.

B. Direct Collocation

In a direct collocation scheme the state trajectories satisfying the ODE (16) are approximated by polynomials on each control interval within the optimization horizon. Each polynomial is parametrized by interpolating points, which have the same dimension as the state space formulation and are extra decision variables in the NLP scheme. In this paper we use the *Gauss-Radau* collocation points τ of degree d. For one control interval, where t_0 denotes the start of the control interval, $n \in N$ number of control intervals and $d = 5$, we have that

$$\begin{aligned}
\tau &= [0, 0.057104, 0.276843, 0.583590, 0.860240, 1] \\
t_{nj} &:= t_{n0} + \Delta\tau_j, \quad \forall j = 0, \ldots, d+1.
\end{aligned}$$

By letting $\mathbf{x}_j$ denote the dynamic state vector at time point t we define a Lagrangian polynomial basis,

$$\begin{aligned}
& L_j(\tau) = \prod_{r=0, r\neq j}^{d} \frac{\tau - \tau_r}{\tau_j - \tau_r} \\
& \text{such that} \qquad\qquad , \\
& L_j(\tau_r) = \begin{cases} 1, & \text{if } j = r \\ 0, & \text{otherwise} \end{cases}
\end{aligned} \tag{19}$$

and approximate the state trajectories as

$$\tilde{\mathbf{x}}(t_{nj}) = \sum_{r=0}^{d} L_r \left(\frac{t_{nj} - t_{n0}}{\Delta\tau_j}\right) \mathbf{x}_r . \tag{20}$$

In particular, for the control intervals $n = 1, \ldots, N$ we have

$$\dot{\tilde{\mathbf{x}}}(t_{nj}) = \frac{1}{\Delta\tau_j} \sum_{r=0}^{d} \dot{L}_r(\tau_j) \mathbf{x}_{nr} := \frac{1}{\Delta\tau_j} \sum_{r=0}^{d} C_{r,j} \mathbf{x}_{nr} \tag{21}$$

$$\tilde{\mathbf{x}}_{n,d+1} = \sum_{r=0}^{d} L_r(1) \mathbf{x}_{nr} := \sum_{r=0}^{d} D_r \mathbf{x}_{nr} , \tag{22}$$

which gives the collocation equations

$$\Delta\tau_j \mathbf{f}(\mathbf{x}_{nj}, \mathbf{z}_{nj}, \mathbf{u}_{n-1,j}, \mathbf{p}) - \sum_{r=0}^{d} C_{r,j} \mathbf{x}_{nr} = 0, \quad j = 1, \ldots, d \tag{23}$$

$$\mathbf{x}_{n,d+1} - \sum_{r=0}^{d} D_r \mathbf{x}_{nr} = 0, \quad n = 1, \ldots, N-1 \tag{24}$$

which must be satisfied for every control interval. The collocation and continuity equations, eq. (23)-(24), are added to the NLP formulation defined in eq. (16). Multiple intermediate collocation points for each control interval are usually used to improve the accuracy of the collocation scheme [13].

Despite the fact that the derivative of the differential equations are approximated by polynomials and an increase in the number of decision variables and equality constraint, the collocation scheme has shown to be advantageous in complex NLP formulations. We refer to [13] for more details regarding collocation methods.

C. Direct Multiple Shooting

In the multiple shooting method the time domain is divided into smaller time intervals and the DAE (Differential Algebraic Equation) models are integrated separately in each interval [13]. In direct multiple shooting the controls are discretized on the coarse grid given by each interval. To provide continuity of the states across intervals, equality constraints are added to the NLP. If we denote the dynamic state vector $\mathbf{x}$ at the start of interval n as $\mathbf{x}_n(t_{n,0})$ and end of the interval as $\mathbf{x}_n(t_{n,1})$, the equality constraints which binds the intervals $n \in N$ are

$$\mathbf{x}_{n-1}(t_{n-1,1}) - \mathbf{x}_n(t_{n,0}) = 0, \quad n = 2, \dots, N, \tag{25}$$

where $\mathbf{x}_1(0) = \mathbf{x}_0$, $\mathbf{x}_0$ initial value. The equality constraints, eq. (25), are added to the NLP formulation defined in eq. (16). The integration scheme used in the implementation of the multiple shooting method in this paper is the explicit *Runge-Kutta 4* (RK4) [13, ch. 9].

As can be seen, the multiple shooting method requires an explicit integration scheme, while the collocation method is derived based on an implicit integration scheme and does not require a *stand-alone* integrator due to the trajectory approximations.

D. Implementation Aspects

The implementation of the collocation and multiple shooting methods are realized in Python using the `CasAdi` framework, which is a symbolic framework for algorithmic differentiation and numeric optimization [14]. The NLP algorithm used is IPOPT. One should note that Python does not provide a real-time framework suitable for this problem, and is only used for the purpose of proof of concept.

IV. Results

To illustrate the benefit of using optimization to provide filter current references to an active harmonic filter we propose one test case with asymmetric non-linear loads. Before presenting the test case, some general findings regarding the different constraints (hexagon and circular constraints) and constant controls versus time varying controls have to be discussed.

A. General findings

When constant controls are used to define the filter currents it can be shown that the filter current phases $\phi_{F,i}$ will never be utilized. This means the equality constraints defining the filter currents could be made linear by removing the phase controls. Moreover, if the hexagon constraints are used in conjunction with constant controls the filter current will never violate the circular constraint, due to zero filter current phase controls. Hence, when constant controls are used in the optimization, the filter phases can be removed and the linear hexagon constraints can be used resulting in a convex quadratic program.

When time varying controls are used and the filter currents are close to the filter's physical limits defined by the non-linear circular constraint, the use of the linear hexagon constraints will violate the physical limits of the filter due to the utilization of the filter current phases. Hence, the hexagon constraints cannot be used when operating with time varying controls close to the filter's physical limits. A solution to this could be to approximate the circular constraint by a regular polygon with more sides than the hexagon, e.g. a dodecagon. However, this is not discussed in this paper.

B. Test case with asymmetric non-linear loads

The test case to be discussed is based on asymmetric loads, where load 1 provides more harmonic pollution to the distribution grid than load 2. The test case may represent a situation where the filter is installed in the wrong part of the vessel grid, or the loads represents an operation where sometimes the harmonic pollution from load 1 are higher than the pollution from load 2. The test case configuration is summarized in table II. As can be seen, the number of discretization steps are much higher for the multiple shooting than the collocation method. This is because the multiple shooting method needs more discretization steps to provide an accurate and sufficient solution to the optimization problem. The time horizon is chosen to be just above one period defined by the fundamental frequency, due to assumed frequency variations according to eq. (5).

TABLE II: Test case configuration.

Load 1 amplitudes [1st, 5th, 7th, 11th, 13th]	$I^{\alpha}_{L1} = I^{\beta}_{L1}$ $= [0.9, 0.3, 0.2, 0.1, 0.1]$ $[pu]$
Load 2 amplitudes [1st, 5th, 7th, 11th, 13th]	$I^{\alpha}_{L2} = I^{\beta}_{L2}$ $= [0.9, 0.2, 0.1, 0.1, 0]$ $[pu]$
Load 1 phases [1st, 5th, 7th, 11th, 13th]	ϕ_{L1} $= [0, 0, 0, 0, 0]$ $[pu]$
Load 2 phases [1st, 5th, 7th, 11th, 13th]	ϕ_{L2} $= [0, 0, 0, 0, 0]$ $[pu]$
Discretization	Collocation: $N = 35$, $d = 5$, $NICP$[1] = 2 Multiple Shooting: $N = 110$
Time horizon	$T = 0.025$ [s]
Frequency model	$f_f = 50$ [Hz], $A_p = 2$ [Hz], $f_t = 0.5$ [Hz]

Figure 3 shows the results after optimization, using both constant controls and time varying controls, for both the multiple shooting and the collocation method. As showcased, time varying controls tend to be more effective than constant controls when it comes to reducing the Total Harmonic Distortion (THD) when working close to the filter's physical limits, given by i_f^{lim}. This is because when using time varying controls the filter current phases are utilized, hence providing more filtering within the filter's physical limitations. However, when increasing the filter limit the solutions using constant and time varying controls converge, which indicates that the solutions are well within the filter's constraints. This means that if a small filter is to be used in situations with high levels of harmonic pollution an optimization scheme using time varying controls will give slightly lower THDs due to the utilization of the filter phases $\phi_{F,i}$.

Another point to be discussed is that multiple shooting gives lower THD values than collocation. This is caused by

[1]Number of intermediate collocation points

the approximations of the dynamic states in the collocation method. By increasing the number of discretization steps the collocation will not converge towards the solution using multiple shooting, it will only increase the computational costs which will be discussed later on.

When designing an optimization scheme it is important that the discretization does not hide any information about the load currents, meaning that the frequency of the discretization must be at least two times larger than the highest harmonic frequency in the loads (Nyquist-Shannon theorem). In this case the highest harmonic frequency is given by $\max\{13 \cdot (50 \pm 2\text{Hz})\} = 676$ Hz. The discretization frequency used in the collocation is $\frac{35}{0.025\text{s}} = 1400$ Hz. If the solution of the whole optimization horizon is to be used by the filter it is quite important that the number of discretization steps does not violate the active filter's bandwidth. In such case the discretization from the multiple shooting will result in a bandwidth of $\frac{110}{0.025\text{s}} = 4400$ Hz which in most cases is too high for an active filter to handle.

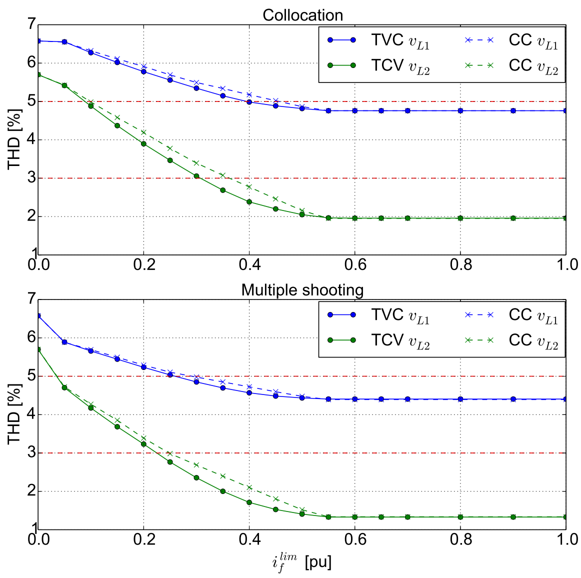

Fig. 3: THD values for different filter current limits using collocation and multiple shooting with Time Varying Controls (TVC) and Constant Controls (CC).

Figure 4 shows a comparison between the different methods using both time varying and constant controls. A local filtering method (not based on optimization) which only tries to remove harmonic pollution from load 2 is included as reference. As can be seen, the solution of the optimization (including both multiple shooting and collocation using both time varying and constant controls) is better than the local filtering method. The local filtering only considers load 2, and since the harmonic pollution from load 1 is greater than the one from load 2, the local filtering is not optimal. Hence, harmonic conditioning using optimization could be beneficial when it comes to asymmetric loads due to the ability to consider more sources of harmonic pollution at once and find the optimal filter current which gives the lowest (and best) overall THD.

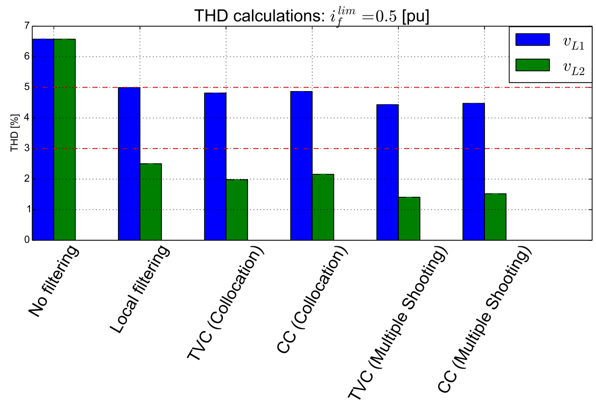

Fig. 4: Comparison of multiple shooting and collocation with Time Varying Controls (TVC) and Constant Controls (CC).

Computational cost is quite important when it comes to a real-time implementation of the optimization scheme. Figure 5 shows the average time consumption (warm start) when using both collocation and multiple shooting with time varying and constant controls for different filter current limits, i_f^{lim}. The optimization was performed on a laptop (Intel Core i7-4600U CPU 2.10GHz $\times$ 4).

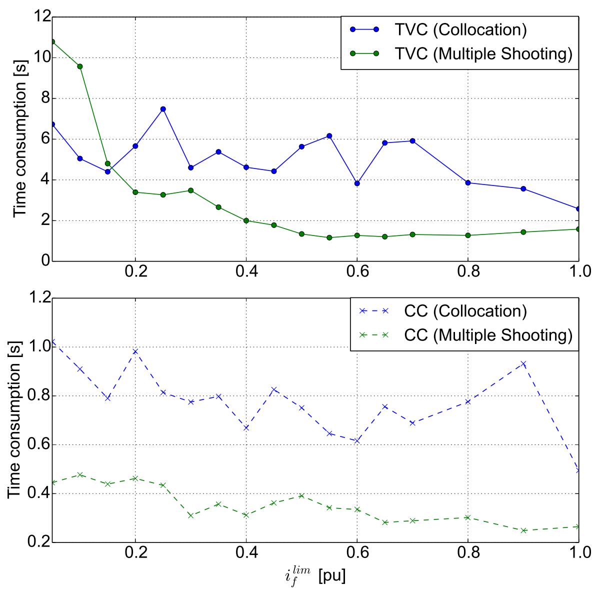

Fig. 5: Average time consumption for different filter current limits using collocation and multiple shooting with Time Varying Controls (TVC) and Constant Controls (CC).

For time varying controls it is clear that multiple shooting is faster to converge than collocation as long as the filter is not too small. It is also evident that the time consumption from multiple shooting is more stable than for the collocation when increasing the filter current limit. Despite the difference in time consumption between collocation and multiple shooting, for time varying controls, they are quite high compared to when using constant controls. Both collocation and multiple shooting gives much lower time consumptions when using constant controls due to the linear problem formulation. As can be seen, the time consumption for multiple shooting (with constant controls) is much better than for collocation. Increasing the number of discretization steps for collocation to try to get a better solution may introduce an additional computational cost.

Figure 6 shows the number of iterations used to provide the solution of the optimization problem by using collocation and multiple shooting with both time varying and constant controls. Again, for time varying controls, multiple shooting requires less iterations to converge toward a solution than collocation as long as the filter is not too small. Multiple shooting is also more stable with regards to number of iterations than collocation. For constant controls multiple shooting uses less iterations compared to collocation. Hence, multiple shooting gives a solution faster with lower THD values compared to collocation which is both slower and gives a solution with higher THD values.

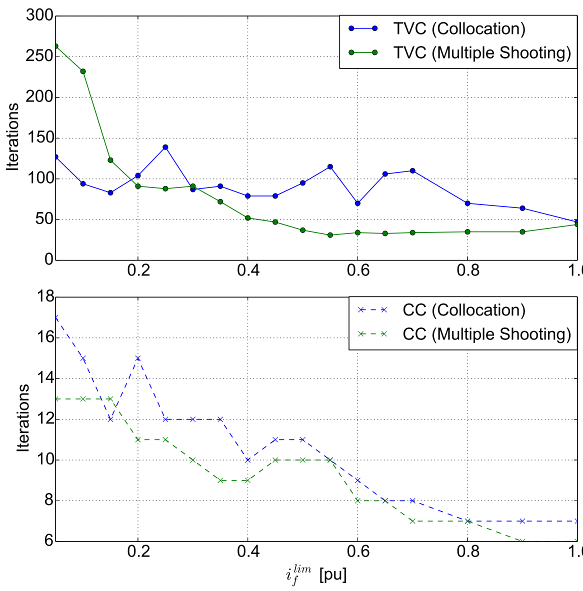

Fig. 6: Average number of iterations for different filter current limits using collocation and multiple shooting with Time Varying Controls (TVC) and Constant Controls (CC).

V. Discussion

As earlier discussed, when using constant controls and the hexagon constraints the optimization problem becomes linear. The computational costs related to the linear problem represented in figures 5 and 6 may be lowered when using a quadratic programming method. Hence, an effective and optimized linear solver, such as the qpOASES [15] or CVXGEN [16], embedded in a suitable hardware platform could provide the computational power needed to solve this optimization problem in real-time. In the literature several linear MPC and linear optimization problem implementations, such as [17, 18, 19, 20, 21], are reported with good real-time properties. However, when using constant controls, flexibility is lost due to not being able to alter the filter currents multiple times within one horizon. Therefore, time varying controls are sought for and a linear problem representation using time varying controls would be ideal with regards to computational costs.

If working close to the active filter's physical limits the NMPC formulation could be beneficial due to the reduced THDs when utilizing the filter current amplitudes and phases as time varying controls. In the literature it has been reported many efficient NMPC implementations with great real-time properties, such as [22], which could be used to bring the computational costs of the NMPC discussed in this paper to acceptable levels. FPGAs have also been reported to give sufficient computational effort for solving NMPC problems in real-time, [23, 24]. However, one solution to reduce the computational costs would be to model the filter currents as linear second order standing fluctuations and use a polygon approximation for the circular constraint.

VI. Conclusion

Two different harmonic filter controls have been outlined using non-linear programming with collocation and multiple shooting. The active filter control problem was formulated as a NMPC problem using both time varying and constant controls. It was shown that the constant controls had lower computational costs compared to time varying controls, but time varying controls utilized the filter current phases to provide better filtering when working close to the active filter's physical limits. A local filtering method was included for comparison, and both collocation and multiple shooting gave better THD values for the asymmetric loads.

The multiple shooting required a higher number of discretization steps than the collocation method, but gave overall better THD values with less computational costs.

Two sets of constraint, a non-linear circular constraint and linear hexagon constraints were proposed. The circular constraint, which was derived on the basis of the filter's current vector's (phasor's) bounds, reflected the true physical limits of the filter, whereas the hexagon constraints gave linear approximations derived on the basis of the phase constraints. It was shown that the hexagon constraints were suitable for the constant control NMPC problem and gave lower computational costs than the NMPC with time varying controls. The linear hexagon constraint was proven to violate the filter's physical limits for the NMPC with time varying controls when working close to the filter's physical limits.

The filter phases were never utilized when constant controls were used, hence removing the filter phases from the optimization problem and using the hexagon constraints the problem was shown to be linear. However, the linear problem was in this paper solved by a non-linear solver which may not give the lowest computational costs for linear problems. Given this limitation, future efforts will be directed to the implementation in a real-time environment by solving the optimization as a linear problem with constant controls and the linear hexagon constraints with the appropriate solvers.

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