Virtual Delay Unit Based Digital $nk \pm m$-order Harmonic Repetitive Controller for PWM Converter

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***Abstract*—Repetitive control (RC) scheme presents an attractive solution to achieve excellent steady-state tracking error and low total harmonic distortion (THD) for periodic signals. RC can produce extremely large gains at fundamental and each harmonic frequency of reference signal to achieve all harmonics suppression. However, a DC-AC inverter always has uneven THD distribution, e.g. THD concentrates at $4k \pm 1$ orders for signal-phase inverter, and $6k \pm 1$ orders for three-phase inverter. Furthermore, a digital RC requires a integral ratio of the sampling frequency and the reference frequency, whereas the digital control system cannot always meet this requirement. For example, (e.g. 60 Hz reference signal with a 5 kHz sampling frequency, or grid-connected converter under grid frequency fluctuation, etc.). In this paper, virtual delay unit (VDU) based digital $nk \pm m$-order harmonic RC is presented to solve the problems above. The VDU produces a different virtual RC sampling frequency from the system sampling frequency. The virtual sampling frequency for digital RC can be flexibly adjusted based on the integral ratio requirement. The advantage of VDU is that it does not vary the system sampling frequency and it is easy to be realized. Furthermore, $nk \pm m$-order harmonic repetitive controller is selected to provide a selective harmonic compensation (SHC). Experimental results of VDU based $nk \pm m$-order harmonic RC for 60 Hz single-phase DC/AC inverter with 5 kHz system sampling frequency are provided to show the effectiveness of the proposed VDU-based SHC.**

***Index Terms*—Virtual delay unit, $nk \pm m$-order harmonic suppression, repetitive control, phase lead compensation, fractional delay, DC/AC inverter, total harmonic distortion**

I. Introduction

The constant voltage constant frequency (CVCF) PWM inverters are widely used in many industrial facilities [1]–[3]. Repetitive control, producing extremely large gain at fundamental and each harmonic frequency of reference signal, is an effective control strategy to decrease THD and improve AC voltage tracking performance [4]–[10].

In practice, power harmonics caused by power inverters usually concentrate on some particular frequencies, e.g. single-phase H-bridge inverters mainly produce $4k \pm 1 (k = 1, 2, \ldots)$ order power harmonics; three-phase H-bridge inverters mainly produce on $6k \pm 1 (k = 1, 2, \ldots)$ order power harmonics [11]–[15]. Thus, the selective harmonics compensation controller will be more effective to suppress output distortion.

Digital repetitive control with the form of $z^{-N}/(1 - z^{-N})$ has a one-reference-period delay that is caused by delay module z^{-N}, where N should equal to the ratio of sampling frequency and reference fundamental frequency [16]. In most existing RC works, N is required to be an integer. However, there is no guarantee that the ratio is an integer in some situations such as: (1) The sampling frequency is not an integral multiple of the reference fundamental frequency [17]. For example, 60 Hz reference AC voltage signal requires an 166.67 delay units when the sampling frequency of the system, which should be an integral division of micro-control unit's clock frequency, is chosen as 10 kHz; (2) For distributed power generation system, it is difficult or even impossible to maintain a constant gird frequency, in which the grid frequency is also the reference voltage or current frequency of grid-connected active power filter or converter [1], [18], [19]. For instance, in a 12 kHz sampling frequency system, if the 60 Hz grid frequency fluctuates in the range from 59.5 Hz to 60.5 Hz, the ratio of the sampling frequency and the grid frequency is changing in the range from 198.35 to 201.68 accordingly; It is difficult to maintain a constant integral delay unit in digital RC. Inaccurate delay unit N will deviate the peak gains of digital RC away from the fundamental and harmonic frequencies, which will degrade harmonics suppression and deteriorate output performance.

In order to suppress selective order harmonics, $nk \pm m$-order harmonic repetitive controller scheme is selected in this paper. A delay module $z^{-N/n}$ needs to be produced inside $nk \pm m$-order RC [12]. To achieve the purpose of integral ratio of the sampling frequency and reference frequency for digital RC, this paper proposes a VDU-based $nk \pm m$-order RC scheme. The VDU, which is built by a 2nd-order FIR filter [20], can set a flexible virtual sampling frequency for digital RC according to system requirement in which the ratio should also be the integral multiple of n. With such a flexible virtual sampling frequency, digital RC can maintain a special selected ratio of virtual sampling frequency and reference frequency N_v by adjusting coefficients in the virtual delay unit even when the reference frequency is under disturbance. In this way, the VDU provides a fundamental solution to solve the integral delay unit problem in digital RC.

The reminder of this paper is organized as follows: Section II introduces conventional RC (CRC), $nk \pm m$-order harmonic RC; and proposes the VDU-based $nk \pm m$ RC in details based

on the improvement of conventional RC (CRC) scheme; To verify the effectiveness of the proposed method, a VDU-based $nk \pm m$ RC for single-phase 60 Hz DC/AC inverter in section III, which is followed by concluding remarks in Section IV.

II. Virtual Delay Unit based RC

A. Conventional RC

Fig. 1 shows a typical digital control system with plug-in RC controller [21], where $R(z)$ is the periodic reference input with frequency f_r, $G_c(z)$ is the conventional feedback controller, $G_p(z)$ is the plant model, $D(z)$ is the disturbance, and $Y(z)$ is the system output. Plug-in RC controller is a feed-forward controller that consists of RC gain K_r, robustness filter $Q(z)$, stability filter $G_f(z)$, and period delay z^{-N} in which $N = f_s/f_r$ is the ratio of sampling frequency f_s and the fundamental frequency f_r of reference signal $R(z)$.

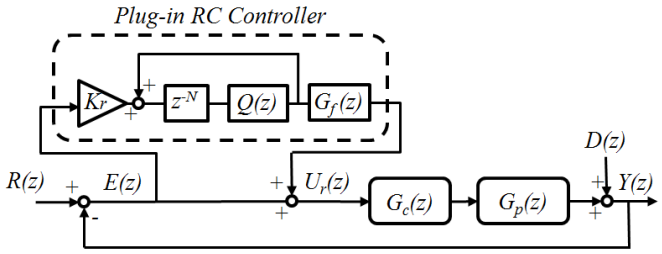

Fig. 1. Control system with plug-in RC controller

The transfer function of the RC G_{rc} in Fig. 1 is:

$$G_{rc}(z) = \frac{U_r(z)}{E(z)} = K_r \frac{z^{-N}Q(z)}{1 - z^{-N}Q(z)} G_f(z) \tag{1}$$

where $E(z) = R(z) - Y(z)$ is the system tracking error.

In the frequency domain, RC transfer function (1) has infinity magnitude gain at reference signal fundamental and all harmonic frequencies when $Q(z) = 1$. Therefore, RC can achieve zero steady-state error tracking of periodic signals.

B. Digital $nk \pm m$ RC

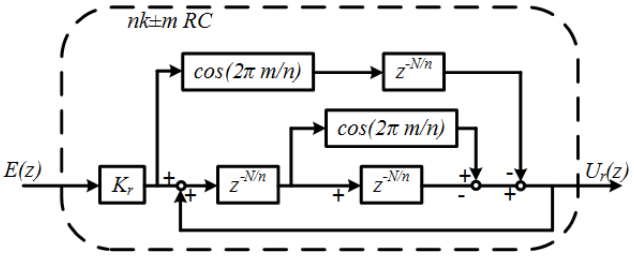

Fig. 2. Digital $nk \pm m$ RC

For $nk \pm m$-order SHC, the compensated harmonic is only selected towards those interested harmonics, and the SHC will increase compensation effectiveness and RC response speed [12]. The transfer function of digital $nk \pm m$ RC shown in Fig. 2 is:

$$G_{nk\pm mRC}(z) = K_r \frac{cos(2\pi m/n) \cdot z^{N/n} - 1}{z^{2N/n} - 2cos(2\pi m/n) \cdot z^{N/n} + 1} \tag{2}$$

where $n, m \in \mathbb{N}$, with $n > m \geq 0$. Compared with CRC, it occupies less data memory and can offer much faster error convergence rate at $nk \pm m$-order harmonic frequencies.

C. Virtual Delay Unit based RC

For $nk \pm m$ RC, delay number N, which is the ratio of system sampling frequency and reference frequency, in periodic delay $z^{-N/n}$ should be not only an integer but also an integral multiple of n. Otherwise, the performance of $nk \pm m$ RC can be greatly affected. However, it's difficult or even impossible to have the ratio always as an integral multiple of n when sampling frequency f_s or fundamental frequency f_r are varying or under certain disturbance.

To address this problem, a virtual sampling frequency f_v is proposed as an approximated varying sampling frequency solution in this paper. While the system sampling frequency f_s is a fixed value, the RC sampling frequency f_v will be a varying one approximated by a FIR filter. And it can be set based on the selection of the integral number N_v which is the multiple of n in VDU-based $nk \pm m$ RC:

$$f_v = N_v \times f_r \tag{3}$$

From the perspective of RC accuracy, the delay number N_v can be chosen as any integer that leads to f_v between $0.5f_s$ and $2f_s$. In most cases, the virtual sampling frequency f_v will be a different value from the system sampling frequency f_s. And the ratio γ of f_s and f_v is:

$$\gamma = \frac{f_s}{f_v} = 1 + F \tag{4}$$

where F, $-0.5 < F < 1$, is the fractional part of γ. Hence, the relationship between system sampling interval T_s and virtual sampling interval T_v is:

$$T_v = \gamma T_s = (1 + F) \times T_s \tag{5}$$

Therefore, the virtual delay unit z_v is derived as:

$$z_v^{-1} = e^{sT_v} = e^{s(1+F)T_s} = z^{-(1+F)} \tag{6}$$

The virtual delay unit z_v^{-1} can be approximated by establishing a 2nd-order FIR filter of the system delay unit z^{-1}, which is implemented by Lagrange interpolation method [22] as follow:

$$z_v^{-1} = z^{-(1+F)} \approx \begin{cases} |F| + (1 - |F|)z^{-1}, -0.5 < F < 0 \\ (1 - |F|)z^{-1} + |F|z^{-2}, 0 \leq F < 1 \end{cases} \tag{7}$$

Thus, the virtual sampling frequency can approximate any sampling rate between $f_s/2$ and $2f_s$. When the system sampling frequency f_s is large enough, the approximation of Lagrange interpolation can get a very desirable accuracy.

The 2nd-order FIR low pass filter approximated virtual delay unit z_v^{-1} in (7) will cause a change of magnitude. Therefore, an offset gain K_v is required to compensate the magnitude loss at the fundamental frequency of the reference signal.

$$K_v = \frac{1}{\| z_v^{-N_v/n} \|}\Bigg|_{\omega = 2\pi f_r} \tag{8}$$

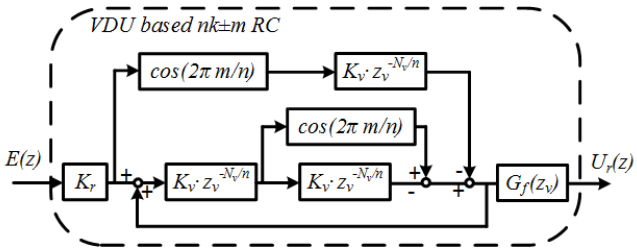

Fig. 3. VDU based $nk \pm m$ RC

With these considerations, the VDU-base $nk \pm m$ RC scheme has a structure shown in Fig. 3 and its transfer function can be written as:

$$G_{nk\pm mRC}(z_v) = K_r \frac{K_v \cdot cos(2\pi m/n) \cdot z_v^{N_v/n} - 1}{K_v^2 \cdot z_v^{2N_v/n} - 2K_v \cdot cos(2\pi m/n) \cdot z_v^{N_v/n} + 1} \quad (9)$$

For this VDU-based $nk \pm m$ RC system, the input and output sampling frequencies are still the system sampling frequency f_s. Not like multi-rate sampling [3], aliasing does not exist. There is no anti-aliasing between VDU-based $nk \pm m$ RC in virtual sampling frequency and digital control system in system sampling frequency.

III. VDU-based $4k \pm 1$ RC for Single-phase DC/AC Inverter

A. Modelling of the PWM inverter

Fig. 5 shows a plug-in VDU-based $4k \pm 1$ RC controlled single-phase DC/AC inverter, where E_n is the DC bus voltage; L and C are inductor-capacitor (LC) filter; R is linear load; C_r, L_r, and R_r are capacitor, inductor, and resistor in rectifier load, respectively. The output voltage V_{out} and inductor current I_L are two system states for state feedback controller; V_{in} is the input PWM voltage with the following definition:

$$v_{in} = \begin{cases} E_n, \text{ if } S_1 \text{ and } S_2 \text{ are on, } S_3 \text{ and } S_4 \text{ are off} \\ -E_n, \text{ if } S_1 \text{ and } S_2 \text{ are off, } S_3 \text{ and } S_4 \text{ are on} \end{cases} \quad (10)$$

With a sampling period of $T_s = 1/f_s$, the discrete-time state space of the single-phase inverter system in Fig. 5 can be written as :

$$\begin{bmatrix} v(k+1) \\ i(k+1) \end{bmatrix} = \begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix} \begin{bmatrix} v(k) \\ i(k) \end{bmatrix} + \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} u(k) \quad (11)$$

where

$\varphi_{11} = 1 - T_s/(RC) + T_s^2/(2R^2C^2) - T_s^2/(2LC)$,
$\varphi_{12} = T_s/C - T_s^2/(2RC^2)$,
$\varphi_{21} = -T_s/L + T_s^2/(2RLC)$,
$\varphi_{22} = 1 - T_s^2/(2LC)$,
$g_1 = E_n T_s^2/(2LC)$,
$g_2 = E_n T_s/L$.

TABLE I
Single-phase PWM System parameters

Parameter	Value	Parameter	Value
DC bus voltage E_n	200 V	Linear load R	200 Ω
Inductor filter L_f	3 mH	Capacitor filter C_f	60 μF
Rectifier capacitor C_r	60 μF	Rectifier inductor L_r	3 mH
Rectifier resistance R_r	200 Ω	PWM frequency	15 kHz
Sampling frequency f_s	5 kHz	Feedback controller k_1	0.4
Feedback controller k_2	7	Feedback controller g	1.4
Reference voltage RMS	110V	Reference voltage f	60 Hz

B. State feedback controller design

The state feedback controller based on system (11) has the form as:

$$u = -KX(k) + gv_{ref}(k) = -k_1 v(k) - k_2 i(k) + gv_{ref}(k) \quad (12)$$

where k_1, k_2 and g are controller parameters, v_{ref} is the reference sinusoidal voltage.

With the state feedback controller (12), closed-loop system becomes:

$$\begin{bmatrix} v(k+1) \\ i(k+1) \end{bmatrix} = \begin{pmatrix} \varphi_{11} - g_1 k_1 & \varphi_{12} - g_1 k_2 \\ \varphi_{21} - g_2 k_1 & \varphi_{22} - g_2 k_2 \end{pmatrix} \begin{bmatrix} v(k) \\ i(k) \end{bmatrix} + \begin{pmatrix} g_1 g \\ g_2 g \end{pmatrix} v_{ref}(k) \quad (13)$$

The poles of feedback control system can be assigned by adjusting coefficients k_1 and k_2. The transfer function can be rewritten as:

$$H(z) = \frac{m_1 z + m_2}{z^2 + p_1 z + p_2} \quad (14)$$

where

$p_1 = -(\varphi_{22} - g_2 k_2) - (\varphi_{11} - g_1 k_1)$,
$p_2 = (\varphi_{11} - g_1 k_1)(\varphi_{22} - g_2 k_2) - (\varphi_{12} - g_1 k_2)(\varphi_{21} - g_2 k_1)$,
$m_1 = g_1 k$,
$m_2 = g_2 k - g_1 k(\varphi_{22} - g_2 k_2)$.

With the parameters in Table I, the closed loop transfer function for the system can be derived as:

$$G(z) = \frac{1.481z + 0.0105}{z^2 - 0.5318z} \quad (15)$$

The feedback control system is stable with the poles at 0.5318 and 0 in the unit cycle. With only feedback controller, Fig. 4 shows a obvious phase lag in tracking and voltage waveform distortion under no load, linear load and rectifier load conditions. The major THD components appear on the $3rd$ and $5th$ harmonic frequencies.

C. Plug-in VDU-based $4k \pm 1$ RC

In order to improve the state feedback performance, the plug-in VDU-based $4k \pm 1$ RC controller is designed as Fig. 3. The delay period N_v is selected as 60 which meets the integral multiple requirement.

The virtual delay unit is obtained from (7) as:

$$z_v^{-1} = z^{-\frac{f_s}{N \times f_r}} = z^{-1.389} \approx 0.611z^{-1} + 0.389z^{-2} \quad (16)$$

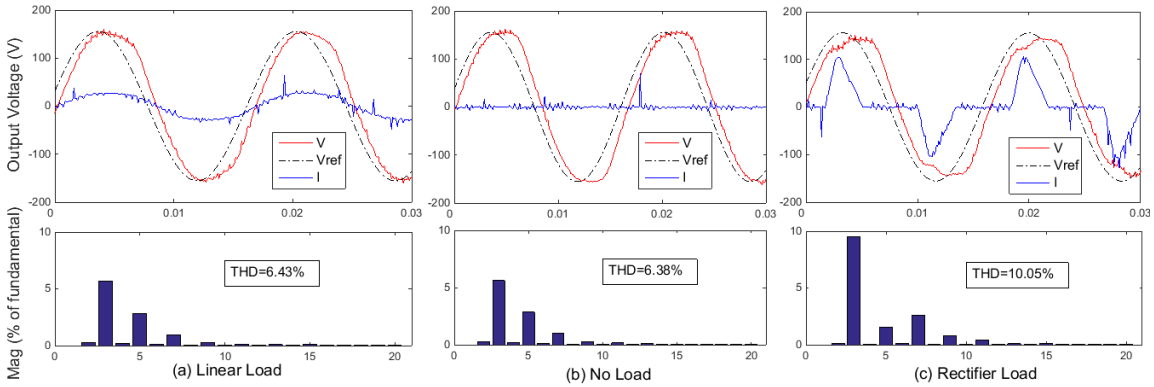

Fig. 4. Output performance under feedback control: (a)linear load, (b)no load, (c)rectifier load

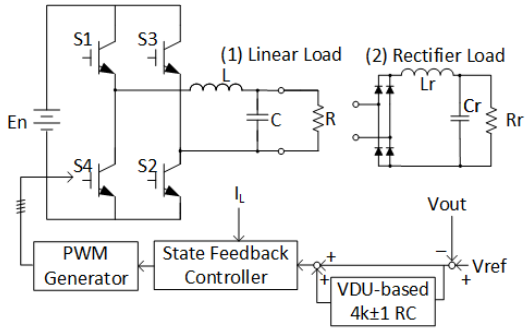

Fig. 5. Plug-in VDU-based RC controlled inverter system

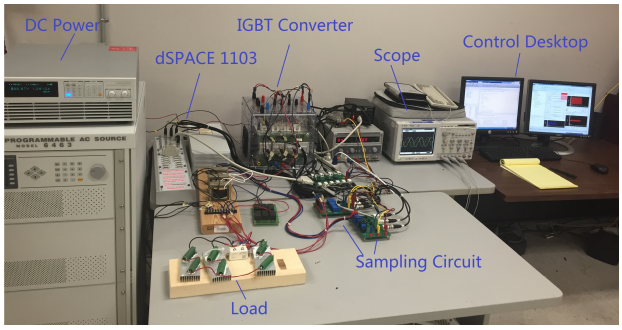

Fig. 6. Experiment setups

The gain offset K_v is calculated from (8) as:

$$K_v = \frac{1}{\| z_v^{-15} \|}\Big|_{\omega=2\pi\times 60} = 1.0102 \tag{17}$$

The next step is to build the linear phase lead filter $G_f(z_v)$. Note that $G_f(z_v)$ will be used to compensate not only the phase lag of system in (15), but also some unmodelled delay in the system. Therefore, the phase lead of $G_f(z_v)$ is determined by following the analysis in [4]. The final phase lead is determined as

$$G_f(z_v) = z_v^5 \tag{18}$$

And RC gain K_r is selected as 0.8 for this case.

D. Experiment Validation

In the experiments, the plug-in VDU-based $4k \pm 1$ RC and state feedback controller are generated in MATLAB Simulink and implemented by dSPACE controller board. The output voltage and inductor current are recorded via ControlDesk for system control and performance analysis.

Fig. 6 shows the experimental setup, which consists of a dSPACE-controlled PWM IGBT full-bridge device, LC filter, DC power source, load resistances, and sampling circuit. The feedback and RC controllers are designed in Matlab Simulink and implemented by a dSPACE DS1103 control board to control H-bridge IGBT converter.

The VDU-based $4k\pm1$ RC experiments are conducted under three different loads shown in Fig. 5: linear load, no load, and rectifier load. VDU-based $4k \pm 1$ RC with phase lead compensation is applied when system reaches a steady state under state feedback control for three different kinds of loads. The steady-state, THDs and transient responses under three different loads are shown in Fig. 7 from (a) to (c). From output performance in Fig. 7, output voltage under VDU-based $4k \pm 1$ RC can excellently tracking the reference with very little tracking error. Comparing state feedback control, $4k \pm 1$ order harmonics gets obvious suppression. Thus, the THDs get as low as 1.67%, 1.64%, 4.47% for three kinds of loads. The figures in Fig. 7 third line show the transient tracking error after applying VDU-based $4k\pm1$ RC. The output performances get very quick response: the tracking errors converge in less than one reference period.

Fig. 8 shows that the VDU-based $4k \pm 1$ RC operates under sudden step load switches between linear and no load in (a) and (b), and between rectifier and no load in (c) and (d). It is clear from the diagrams that output voltages do not vary too much and recover from the sudden step load changes within two cycles. The experiments prove that VDU-based $4k\pm1$ RC is robust to sudden load changes. Moreover, it is noticed from Fig. 8 that VDU-based $4k\pm1$ RC and state feedback controller are complementary: state feedback control offers instantaneous dynamic response to the sudden load change, but its tracking

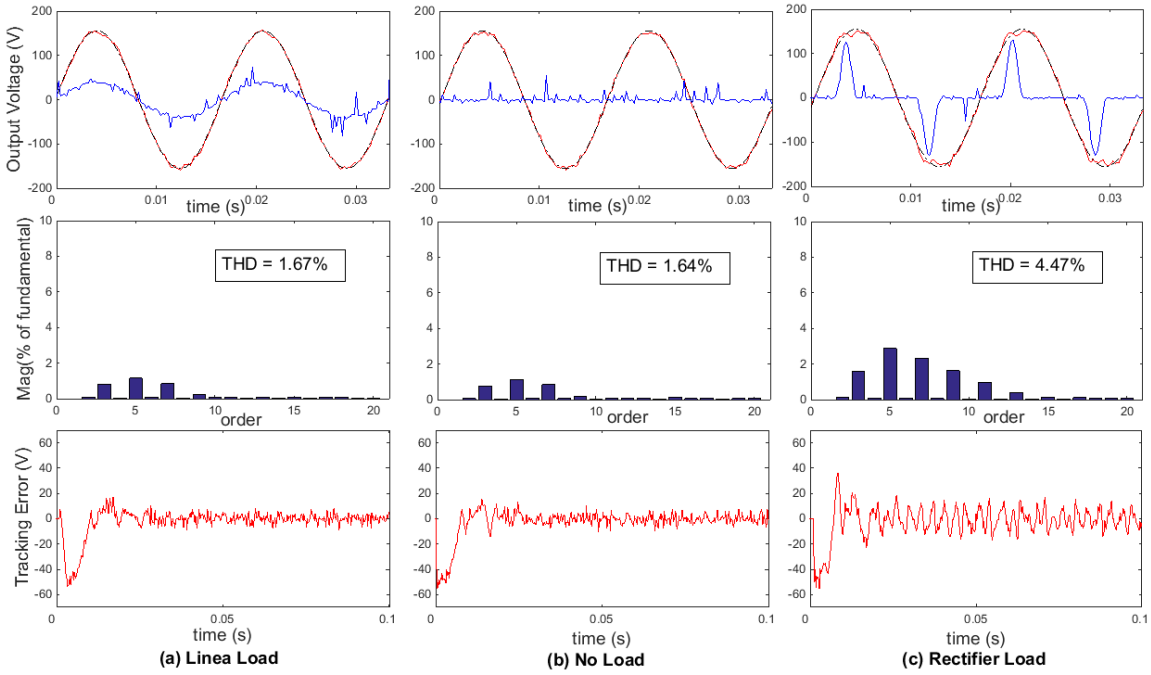

Fig. 7. VDU-based $4k \pm 1$ RC experiment results: (a) linear load, (b) no load, (c) rectifier load

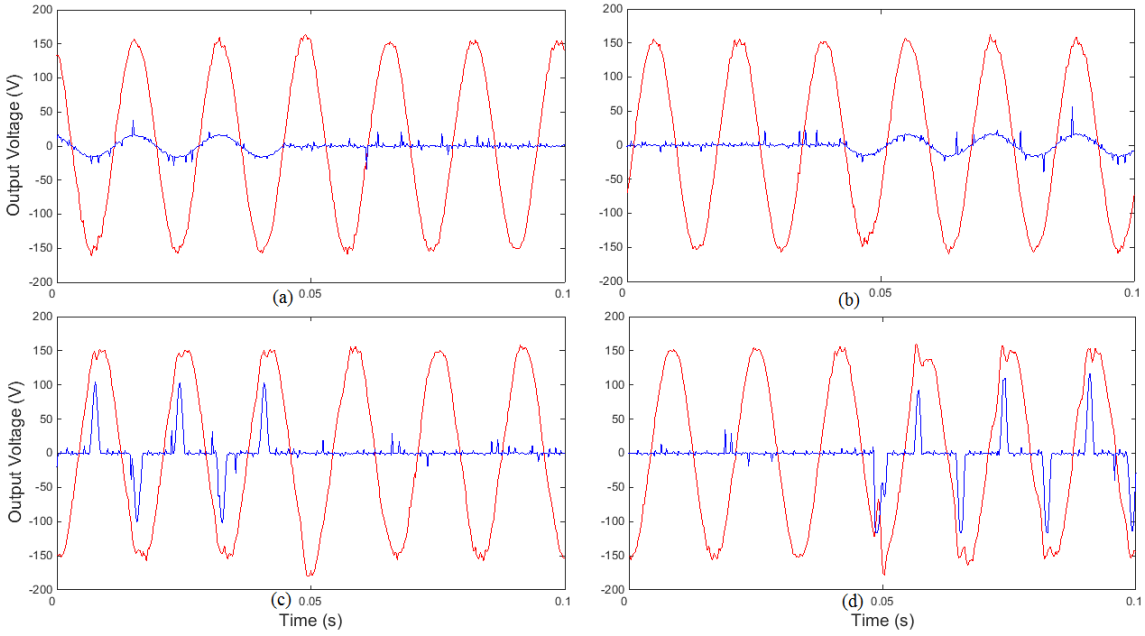

Fig. 8. VDU-based $4k \pm 1$ RC under sudden load change: (a) load change from linear load to no load (b) load change from no load to linear load (c) load change from rectifier load to no load (d) load change from no load to rectifier load

accuracy is relatively low; the plug-in VDU-based $4k \pm 1$ RC controller can significantly reduce the tracking errors with a fast convergence speed.

IV. Conclusion

In this paper, $nk \pm m$-order RC scheme is selected to suppress specific order harmonic distortion based on experiment result under conventional controller. The VDU-based RC is proposed to solve the issue of degradation with the fractional ratio of the sampling frequency and the fundamental reference frequency. With the virtual delay unit, RC can be implemented with a flexible virtual sampling frequency which can be different from the system sampling rate. With VDU-based $nk \pm m$-order RC, system get very quick tracking convergence speed and excellent steady-state performance from experiments in two system: single-phase DC/AC inverter and three-phase DC/AC inverter. The proposed RC provides a high-performance but low-cost control solution to CVCF PWM inverters.

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