Optimization method based on simplex algorithm for current control of modular multilevel converters

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***Abstract*— The paper proposes an optimization formulation of the control problem for modular multilevel converter (MMC). The main control stage computes arm voltages on average over a fixed switching period by minimizing control errors in order to satisfy as best as possible the desired references of input, circulating and output currents, while taking into account arm voltage limits. Then, mean-values of required arm voltages are achieved by phase shift pulse-width modulation (PSPWM) by computing duty cycles for each submodule while taking into account the issue of active balancing of the capacitor voltages in a secondary control stage. The proposed optimization problems are solved by using a numerical method based on the simplex algorithm and simulation results are shown in order to support the validity of the approach.**

Keywords*— *Modular multilevel converter, current control, control allocation, numerical optimization, simplex algorithm.

I. Introduction

Multilevel converters are now recognized as a major solution for medium-voltage high power applications [1], [2]. Multilevel conversion makes it possible to reach high voltage ranges by using low or medium-voltage switches while improving considerably the harmonic quality of output currents and voltages.

Among multilevel conversion topologies is the MMC [3]. A 3-phase modular multilevel inverter is illustrated in Fig. 1. Thanks to its modular structure, MMC is appreciated for its simple scalability, flexibility in design, its simple assembling in construction, its excellent harmonic performances, and improved availability in practice. The main control objectives of the MMC are the obtaining of the output currents, the correct choice and regulation of the input current in order to provide the needed output power and to keep the converter energy constant, and the balancing of the submodule capacitor voltages. Dynamics and control of the MMC have been the topic of several studies [4]–[9]. Control strategies are mainly based on the use of linear controllers and multilevel modulation. Due to the high number of available control variables, constraints to respect and objectives to satisfy, control of MMC is quite complex and is still a wide axis of research.

In this paper, we propose a new approach for the control of the MMC. The main aim is to achieve an optimized control of the converter thanks to the computation of simple linear variables by taking into account existing ranges of operation. Such approach has been already illustrated for the 4-leg 2-level inverter and the flying capacitor inverter under the control allocation framework [10]–[13]. We propose an optimization problem of the control of the three-phase modular multilevel inverter to be solved in real time.

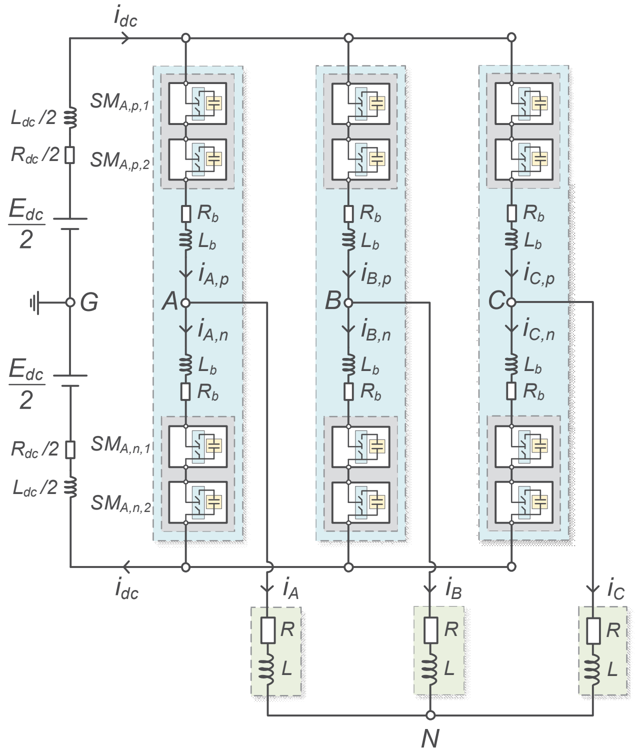

Fig. 1. Illustration of a 3-phase modular multilevel DC-AC converter.

The main problem does not depend on the number of submodules per arm as control variables are the mean-values of arm voltages over the control period. The proposed numerical optimization method is based on the use of the simplex algorithm. Additionally to the main control problem, in the same way, a second optimization problem is developed on a per-arm basis for the determination of duty cycles satisfying required arm voltages while enabling an active balancing of capacitor voltages. Computed values of duty cycles are realized by Phase Shift Pulse-Width Modulation (PSPWM). Input current reference is basically determined in order to control the mean- value of arm energies to a common reference value. The control strategy is evaluated in simulation and results consolidate the validity of the two control stages.

In section 2, the modelling of the MMC is detailed. In section 3, we develop the proposed optimization formulation for the control of the MMC currents. Finally, the control strategy is evaluated in simulation in section 4.

II. MODELLING OF THE MMC

A. Converter arms

Each leg is comprised of two arms [4], denoted here as positive and negative, respectively. Each arm is constituted by a series-connection of n_{SM} submodules. As illustrated in Fig. 2, each submodule has a capacitor that plays the role of a DC-source and that can be inserted in the arm x of the leg K thanks to the associated j-th switching cell of state $S_{K,x,j}$. The switching state is 0 for by-pass, and 1 for insertion of the capacitor. Dead-times are neglected and ideal operation of switching cells is assumed. High voltage levels can be reached by adjusting the number of submodules inserted.

B. DC source

The DC-side voltage is denoted E_{dc} and is assumed constant here. The line impedance is modelled by inductance L_{dc} and resistance R_{dc}. The ground reference is taken to be the mid-point of the DC-bus.

C. Load

The load is assumed to be balanced and with star-connection, as illustrated in Fig. 1. The load impedance is modelled by inductance L and resistance R.

D. Dynamics of currents

Positive and negative arm currents $\mathbf{i_p}$ and $\mathbf{i_n}$ are often decomposed [4]–[9] similarly to the form

$$\mathbf{i_p} = \begin{pmatrix} i_{Ap} & i_{Bp} & i_{Cp} \end{pmatrix}^T = \frac{i_{dc}}{3} + \frac{\mathbf{i_o}}{2} + \frac{\mathbf{i_c}}{2} \tag{1}$$

$$\mathbf{i_n} = \begin{pmatrix} i_{An} & i_{Bn} & i_{Cn} \end{pmatrix}^T = \frac{i_{dc}}{3} - \frac{\mathbf{i_o}}{2} + \frac{\mathbf{i_c}}{2} \tag{2}$$

where i_{dc} is the input current, satisfying

$$i_{dc} = i_{Ap} + i_{Bp} + i_{Cp} = i_{An} + i_{Bn} + i_{Cn} \tag{3}$$

where $\mathbf{i_o}$ is the output current vector of the 3 load currents,

$$\mathbf{i_o} = \begin{pmatrix} i_A & i_B & i_C \end{pmatrix}^T \tag{4}$$

and where $\mathbf{i_c}$ is the circulating current vector,

$$\mathbf{i_c} = \begin{pmatrix} i_{Ac} & i_{Bc} & i_{Cc} \end{pmatrix}^T \tag{5}$$

The input current provides active power to the load and compensates for losses in the converter. Circulating currents are inner currents flowing through the legs of the converter, and do not affect the AC side or the DC side. Therefore, a control possibility is to minimize circulating currents in order to reduce losses.

By defining the following parameters and matrices

$$R_o = R + R_b/2 \qquad L_o = L + L_b/2 \tag{6}$$

$$R_s = R_{dc} + 2R_b/3 \qquad L_s = L_{dc} + 2L_b/3 \tag{7}$$

$$R_c = R_b \qquad L_c = L_b \tag{8}$$

$$\mathbf{H_{DM}} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \tag{9}$$

$$\mathbf{H_{CM}} = 1/3\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \tag{10}$$

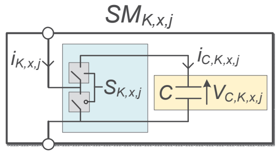

Fig. 2. Illustration of the leg-K arm-x j-th half-bridge submodule.

the dynamics of output, input and circulating currents can be described by

$$L_o \frac{d}{dt}\mathbf{i_o} = -R_o \mathbf{i_o} + \mathbf{v_{o,mmc}} \tag{11}$$

$$L_c \frac{d}{dt}\mathbf{i_c} = -R_c \mathbf{i_c} + \mathbf{v_{c,mmc}} \tag{12}$$

$$L_s \frac{d}{dt} i_{dc} = -R_s i_{dc} + v_{s,mmc} \tag{13}$$

where

$$\mathbf{v_{o,mmc}} = -\frac{1}{2}\mathbf{H_{DM}}\left(\mathbf{v_p} - \mathbf{v_n}\right) \tag{14}$$

$$\mathbf{v_{c,mmc}} = -\frac{1}{2}\mathbf{H_{DM}}\left(\mathbf{v_p} + \mathbf{v_n}\right) \tag{15}$$

$$v_{s,mmc} = E_{dc} - \mathbf{H_{CM}}\left(\mathbf{v_p} + \mathbf{v_n}\right) \tag{16}$$

In (14)–(16), $\mathbf{v_{o,mmc}}$ are MMC output voltages that drive the output currents, $\mathbf{v_{c,mmc}}$ are MMC inner circulating voltages that drive the circulating currents, $v_{s,mmc}$ is MMC input voltage that drives the input current, and $\mathbf{v_p} = (v_{Ap}\ v_{Bp}\ v_{Cp})^T$ and $\mathbf{v_n} = (v_{An}\ v_{Bn}\ v_{Cn})^T$ are the positive and negative arm voltages, respectively. Arm voltages are functions of switching states S_{Kxj} and capacitor voltages V_{CKxj}:

$$v_{K,x} = \sum_j S_{K,x,j} V_{C,K,x,j} \tag{17}$$

MMC voltages are components of the positive and negative arm voltages. The control of MMC currents by MMC voltages is achieved by the selection of suitable arm voltages.

E. Dynamics of capacitor voltages

The evolution of the voltage V_{CKxk} of the leg-K arm-x j-th capacitor of is given by

$$\frac{d}{dt}V_{C,K,x,j} = \frac{1}{C} S_{K,x,j} i_{K,x} \tag{18}$$

F. Arm energies

The energy E_{Kx} of the leg-K arm-x is given by

$$E_{K,x} = \frac{1}{2} C \sum_j V_{C,K,x,j}{}^2 \tag{19}$$

Let assume that output current references are achieved,

$$i_{o,ref}(t) = \begin{pmatrix} i_{A,ref} \\ i_{B,ref} \\ i_{C,ref} \end{pmatrix}(t) = I_o \begin{pmatrix} \sin 2\pi f t \\ \sin 2\pi f t - 2\pi/3 \\ \sin 2\pi f t + 2\pi/3 \end{pmatrix} \tag{20}$$

where I_o is the desired peak amplitude and f is the fundamental frequency. The power provided to the arm on average

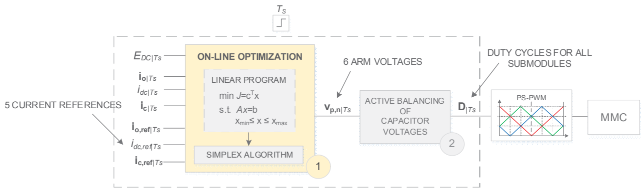

Fig. 3. Diagram of the proposed general control strategy.

over the fundamental period $T = 1/f$ can be derived from (1), (2), and (14)–(16), and is given by

$$
\begin{aligned}
&< \frac{d}{dt} E_{K,x} >_T = < P_{K,x} >_T = < v_{K,x} i_{K,x} >_T \\
&= -\frac{R_s}{6} {i_{dc}}^2 + \frac{E_{dc}}{6} i_{dc} - \frac{R_o}{4} {I_o}^2
\end{aligned}
\tag{21}
$$

The first term corresponds to losses due to the input current, the second term is the contribution of input power and the third term corresponds to the output active power consumption in the arm. So, ideally, in order to keep constant energies in the arms, the input power must compensate for losses and for the active power required by the load.

III. PROPOSED OPTIMIZATION FORMULATION FOR THE CONTROL OF THE MMC

A. General control strategy

A diagram of the general control strategy is available in Fig. 3. The control of the MMC is divided into two stages. The main control stage is dedicated to the computation of required arm voltages in order to satisfy the given current references while automatically taking into account the limits of arm voltages. Then, the secondary control stage is responsible for the realization of the previously computed arm voltages by means of PSPWM. This is done on an arm-per-arm basis. Duty cycles are determined primarily in order to obtain the desired arm voltage and secondarily to minimize the differences of capacitor voltages for keeping them close to their mean value.

For both the control stages, linear optimization problems are derived for each control problem, respectively, in order to find a feasible solution while respecting the existing constraints. The linear programs are to be solved for each control period thanks to a numerical optimization method based on the use of the simplex algorithm.

B. Discrete state-space model of currents

The main objective is to control the output, input and circulating currents to their references values. The control variables are the mean-values of positive and negative arm voltages over the control period T_S. In the following of this paper, the mean-value notation is omitted for simplicity.

From (6)–(16), a continuous state-space model of MMC currents is given by

$$
\frac{d}{dt}\begin{pmatrix} \mathbf{i_o} \\ \mathbf{i_c} \\ i_{dc} \end{pmatrix} = \mathbf{A_s}\begin{pmatrix} \mathbf{i_o} \\ \mathbf{i_c} \\ i_{dc} \end{pmatrix} + \mathbf{B_s}\begin{pmatrix} \mathbf{v_p} \\ \mathbf{v_n} \end{pmatrix} + \mathbf{H_s} \tag{22}
$$

where

$$
\mathbf{A_s} = \begin{pmatrix} -R_o/L_o \times \mathbf{I_{3\times3}} & & \\ & -R_c/L_c \times \mathbf{I_{3\times3}} & \\ & & -R_s/L_s \end{pmatrix} \tag{23}
$$

$$
\mathbf{B_s} = \begin{pmatrix} -1/(2L_o)\times \mathbf{H_{DM}} & 1/(2L_o)\times \mathbf{H_{DM}} \\ -1/(2L_c)\times \mathbf{H_{DM}} & -1/(2L_c)\times \mathbf{H_{DM}} \\ -1/L_s \times \mathbf{H_{CM}} & -1/L_s \times \mathbf{H_{CM}} \end{pmatrix} \tag{24}
$$

$$
\mathbf{H_s} = \begin{pmatrix} 0 & 0 & E_{dc}/L_s \end{pmatrix}^T \tag{25}
$$

and where $\mathbf{I_{3\times3}}$ denotes the order-3 identity matrix. As the sum of output currents is zero, as well as the sum of circulating currents (deductible from (1)–(5)), quantities of interest can be chosen as the first two output currents, the first two circulating currents and the input current. So, define the following matrix $\mathbf{C_s}$ as

$$
\mathbf{C_s} = \begin{pmatrix} \mathbf{C_o} & & \\ & \mathbf{C_c} & \\ & & C_{dc} \end{pmatrix} = \left(\begin{array}{ccc|ccc|c} 1 & 0 & 0 & & & & \\ 0 & 1 & 0 & & & & \\ \hline & & & 1 & 0 & 0 & \\ & & & 0 & 1 & 0 & \\ \hline & & & & & & 1 \end{array}\right) \tag{26}
$$

Interesting quantities to be controlled are now given by the product $\mathbf{C_s}(\mathbf{i_o}\ \mathbf{i_c}\ i_{dc})^T$.

A first-order Euler prediction over the control period T_S gives the discrete equations

$$
\begin{pmatrix} \mathbf{i_o} \\ \mathbf{i_c} \\ i_{dc} \end{pmatrix}[k+1] = (\mathbf{I_{7\times7}} + T_S \mathbf{A_s})\begin{pmatrix} \mathbf{i_o} \\ \mathbf{i_c} \\ i_{dc} \end{pmatrix}[k] + T_S \mathbf{B_s}\begin{pmatrix} \mathbf{v_p} \\ \mathbf{v_n} \end{pmatrix}[k] + T_S \mathbf{H_s} \tag{27}
$$

C. Common mode voltage equation

An idea is to exploit available degrees of freedom in order to choose preferred value of neutral voltage or common mode voltage, v_{NG}. This voltage is given by

$$
v_{NG} = -1/2 \times \mathbf{H_{CM}}(\mathbf{v_p} - \mathbf{v_n}) \tag{28}
$$

We will choose a preferred value of $v_{NG,ref} = 0$ V.

D. Constraints

Arm voltages must be positive and are limited by the per-leg sums of capacitor voltages

$$
0 \le \mathbf{v_x} \le \mathbf{v_{x,max}} = \begin{pmatrix} \cdots & \sum_j V_{C,K,x,j} & \cdots \end{pmatrix}^T \tag{29}
$$

E. Input and circulating current references

The input current reference is basically determined in order to keep the mean-value E_m of arm energies

$$
E_m = \frac{1}{6}\sum E_{K,x} = \frac{1}{6}\sum\left[\frac{1}{2} C {V_{C,K,x,j}}^2\right] \tag{30}
$$

at a constant reference value $E_{m,ref}$

$$
E_{K,x,ref} = E_{m,ref} = n_{SM} \cdot C/2 \cdot {V_{C,ref}}^2 \tag{31}
$$

where

$$V_{C,ref} = E_{dc} / n_{SM} \tag{32}$$

Ideally, in order to bring the mean-value of arm energies to its reference value over a given fixed time T_B, from (21), the ideal reference of input current $i_{dc,ref}$ is

$$i_{dc,ref} = \frac{\frac{E_{dc}}{2} - \left(\left(\frac{E_{dc}}{2} \right)^2 - 6R_s \left(\frac{R_o}{4} I_o^{\ 2} + \frac{E_{m,ref} - E_m}{T_B} \right) \right)^{1/2}}{R_s} \tag{33}$$

This reference will be held during T_B before being updated. The circulating currents should be ideally zero.

F. Formulation of the current control optimization problem

In order to find a feasible solution that will satisfy both constraints and current references and result in an optimized operation of the converter, an idea is to formulate an optimization problem. Define a criterion J that has to be minimized. Control errors are defined for each objective as

$$\mathbf{e_o} = \mathbf{C_o i_o}[k+1] - \mathbf{C_o i_{o,ref}}[k+1] \tag{34}$$

$$\mathbf{e_c} = \mathbf{C_c i_c}[k+1] - \mathbf{C_c i_{c,ref}}[k] = \mathbf{C_c i_c}[k+1] \tag{35}$$

$$e_{dc} = C_{dc} i_{dc}[k+1] - C_{dc} i_{dc,ref}[k] \tag{36}$$

$$e_{vn} = v_{NG}[k+1] - v_{NG,ref}[k] = v_{NG}[k+1] \tag{37}$$

and are gathered into the criterion J. Thus, an associated optimization problem is

$$\begin{aligned} &\min_{\mathbf{v_p},\mathbf{v_n}} J, \quad J = \varepsilon_o \|\mathbf{e_o}\| + \varepsilon_c \|\mathbf{e_c}\| + \varepsilon_{dc} \|\mathbf{e_{dc}}\| + \varepsilon_{vn} \|\mathbf{e_{vn}}\| \\ &\text{u.c.} \quad 0 \le \mathbf{v_p} \le \mathbf{v_{p,max}} \quad 0 \le \mathbf{v_n} \le \mathbf{v_{n,max}} \end{aligned} \tag{38}$$

where u.c. means "under constraints", where || || is a norm, and ε's are weighting factors that make it possible to define priorities between control error terms. A possibility is to tune these factors by a trial & error process such that $\varepsilon_o >> \varepsilon_c >> \varepsilon_{dc} >> \varepsilon_{vn}$. The choice of the $\ell 1$ norm in the expression of the criterion J enables the formulation of a linear program (LP) and the use of the simplex algorithm [10].

As a LP requires positive decision variables, add new *artificial variables* that are the negative part e_x^- and the positive part e_x^+ of any variable e_x that may be negative as

$$e_x = e_x^{\ +} - e_x^{\ -} \qquad 0 \le e_x^{\ +} \qquad 0 \le e_x^{\ -} \tag{39}$$

Finally, a linear program in standard form is defined from the optimization problem as

$$\begin{aligned} &\min_{\mathbf{x}} J, \quad J = \mathbf{c}^T \mathbf{x} \\ &\text{u.c.} \quad \mathbf{Ax} = \mathbf{b}, \ 0 \le \mathbf{x} \le \mathbf{x_{max}} \end{aligned} \tag{40}$$

All matrices and vectors of the LP are given at the end of the paper. **A** is of size 6×18, **b** is of size 6×1 (column vector), and **c**, **x**, and $\mathbf{x_{max}}$ are of size 18×1 (column vectors).

G. Active balancing of the capacitor voltages

Additionally to the main control objective, it is proposed here to realize the reference arm voltages, computed previously by the current control stage, by PSPWM while taking into account the issue of the active balancing of the voltages of the submodule capacitors. For simplicity, this control stage is performed arm by arm, as the 6 resulting control problems are independent.

First of all, duty cycles denoted $D_{K,x,j}$ are defined to be the mean value of the switching states $S_{K,x,j}$ over a switching period T_S equal to the control period:

$$D_{K,x,j} = < S_{K,x,j} >_{T_S} \tag{41}$$

In order to obtain the desired arm voltage, duty cycles of the corresponding arm must satisfy

$$\sum_{j=1}^{n_{SM}} V_{C,K,x,j} D_{K,x,j} = v_{K,x} \tag{42}$$

Then, a first-order prediction of the capacitor voltages gives

$$V_{C,K,x,j}[k+1] = V_{C,K,x,j}[k] + i_{K,x}[k] \cdot T_S / C \cdot D_{K,x,j} \tag{43}$$

Ideally, the active balancing should regulate the capacitor voltages to their reference value, which is E_{dc}/n_{SM}. However, in practice, voltage ripple occurs inevitably, due to at least the output current component in the arm current. Note that the available duty cycles give n_{SM} control variables but there are $n_{SM}+1$ control equations: the global problem is overdetermined. Consequently, here, the objective is to bring the capacitor voltages close to their mean value, so all the capacitors contain nearly the same amount of energy at each time. For this purpose, it is proposed to compute duty cycles that minimize the differences of the predicted capacitor voltages,

$$\begin{aligned} &\forall j, \ 1 \le j \le n_{SM} - 1, \quad V_{C,K,x,j+1}[k+1] - V_{C,K,x,j}[k+1] = \\ &V_{C,K,x,j+1}[k] - V_{C,K,x,j}[k] + i_{K,x}[k] \cdot T_S / C \cdot \left(D_{K,x,j+1} - D_{K,x,j} \right) \end{aligned} \tag{44}$$

while primarily satisfying the reference arm voltage.

Both the objectives of realization of the arm voltage and active balancing of the capacitor voltages can be gathered into the following control problem:

$$\left(\begin{array}{cccccc} V_{C,K,x,1}[k] & \cdots & V_{C,K,x,j}[k] & \cdots & V_{C,K,x,n_{SM}}[k] & \\ \hline -1 & 1 & & & & \\ & \ddots & \ddots & & & \\ & & \ddots & \ddots & & \\ & & & -1 & 1 & \end{array} \right) \times \begin{pmatrix} D_{K,x,1} \\ \vdots \\ D_{K,x,j} \\ \vdots \\ D_{K,x,n_{SM}} \end{pmatrix} = \left(\begin{array}{c} v_{K,x,ref} \\ \hline C \dfrac{V_{C,K,x,1}[k] - V_{C,K,x,2}[k]}{i_{K,x}[k] T_S} \\ \vdots \\ C \dfrac{V_{C,K,x,n_{SM}-1}[k] - V_{C,K,x,n_{SM}}[k]}{i_{K,x}[k] T_S} \end{array} \right) \tag{45}$$

Here, the problem is fully determined, but the constraints must be respected:

$$0 \le D_{K,x,j} \le 1 \tag{46}$$

Moreover, when the arm current is close to zero, the capacitor voltages cannot be controlled anymore. In order to keep the differences of duty cycles small for obtaining regular switching patterns, a maximum threshold value is indicated. For example, it is possible to limit the absolute values of the differences of duty cycles up to 0.1.

TABLE I. SIMULATION PARAMETERS

Symbol	Meaning	Values
T_S	*Control period*	0.5 ms
T_B	*Control period for E_m, see (33)*	$10 \cdot T_S = 5$ ms
f	*Fundamental frequency*	50 Hz
R, L	*Load resistance and inductance*	10 Ω, 1.3 mH
R_b, L_b	*Arm resistance and inductance*	10 mΩ, 100 µH
R_{dc}, L_{dc}	*DC-bus resistance and inductance*	100 mΩ, 2 mH
E_{dc}	*DC-bus voltage*	1000 V
C	*Submodules capacitances*	5 mF
n_{SM}	*Number of submodules per arm*	3
T_{step}	*Simulation step*	$T_S/1000 = 0.5$ µs

Finally, in order to find a feasible solution of duty cycles that reaches the desired arm voltages while minimizing differences of capacitor voltages as best as possible, a LP is formulated similarly to the previous one, and more particularly the one derived in [13]. In practice, this LP is to be solved in real time thanks to the simplex algorithm.

IV. SIMULATION RESULTS

Simulations were carried out thanks to the MATLAB-Simulink environment. Simulation parameters are given in Table I. The first reference value of the output current amplitude I_o is 25 A. At 400 ms, a step change occurs and the reference value becomes 38 A. Circulating current references are 0 A. In all following figures, dashed lines correspond to reference values of shown quantities.

Fig. 4 shows output voltages and currents. Desired output currents are correctly obtained. The algorithm responds quickly to the step of reference amplitude. Arm voltages are illustrated in Fig. 5. Output voltages depend on differences between positive and negative arm voltages. Input current results from MMC input voltage that is increased when the mean value of the sum of positive and negative arm voltages decreases. When the step of current reference amplitude occurs, input current reference has to increase to furnish sufficient active power to the load, see Fig. 7. Thanks to the optimization formulation, limits of arm voltages are intrinsically taken into account, and optimized waveforms of arm voltages are flattened to achieve the desired output voltages and input current while keeping arm voltages positive.

In practice, as shown in Fig. 6, it is difficult to avoid circulating currents because of the ripple of capacitor voltages mainly due to the output current component in arm currents. Moreover, the control problem is formulated only on average over T_S, and due to the instantaneous differences between the real arm voltages and their computed mean values, due to the low impedance represented by the parameters R_c and L_c, and due to the low number of submodules per arm used in this simulation (for high performance and quality of conversion, n_{SM} is usually much higher), high harmonics are present in the circulating currents. However, circulating currents are maintained the nearest to 0 A thanks to the corresponding control error in the criterion to be minimized.

Capacitor voltages of leg A are shown in Fig. 8. They are kept near to their reference thanks to the second optimization problem derived similarly to the one introduced in section 3 and in [13]. Neutral potential is shown in Fig. 9. The controlled quantity is the average value over the switching period, represented by the dashed line. It is a relevant illustration of the ability of the method to automatically exploit degrees of freedom in order to satisfy the primary current references.

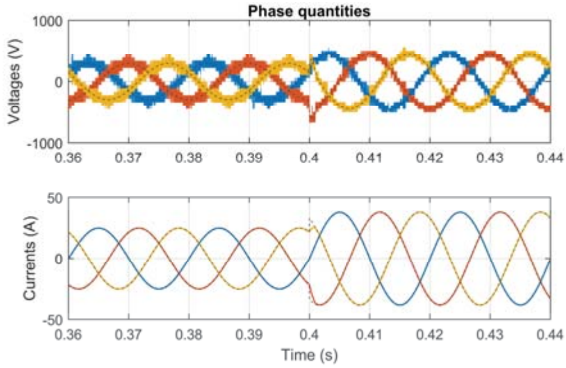

Fig. 4. Phase voltages and currents.

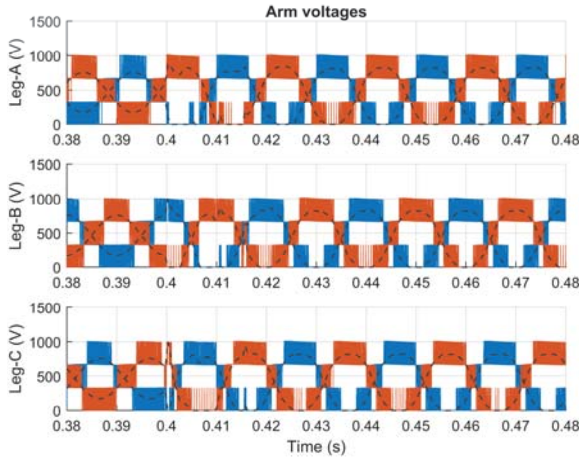

Fig. 5. Arm voltages.

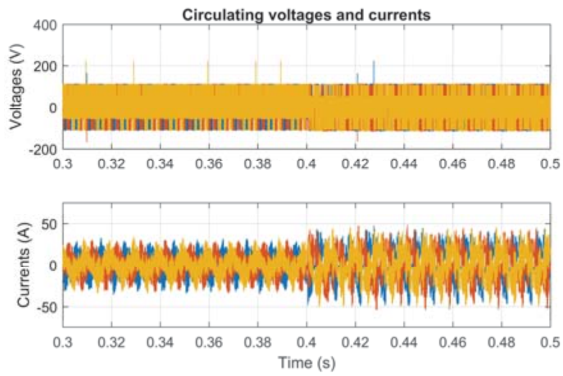

Fig. 6. Circulating voltages and currents.

When the amplitude step occurs at t = 0.4 s, the neutral potential is used to overcome arm voltage limits for achieving the needed MMC voltages. Due to PSPWM, the instantaneous values are different from the computed mean values. Nevertheless, current references are tracked efficiently.

V. CONCLUSION

In this paper, a control strategy based on online optimization is proposed for control of MMC that feeds an inductive three phase balanced load. The main control stage is dedicated to the tracking of references of output currents, input current and circulating currents. An optimization formulation is developed in order to automatically compute mean-values of arm voltages on average over the control period that satisfy

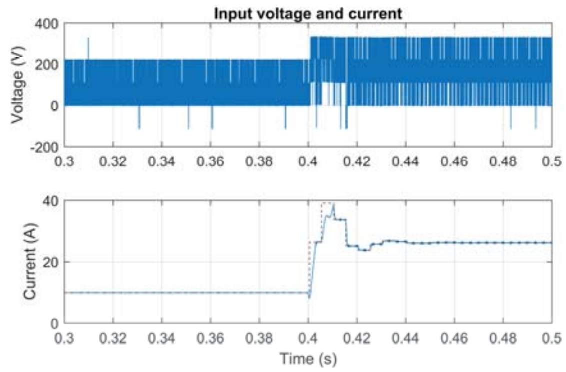

Fig. 7. Input voltage and current.

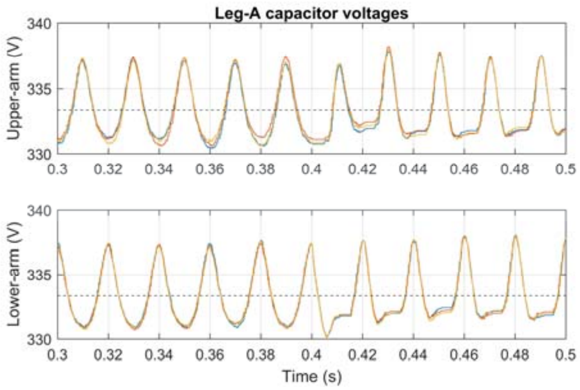

Fig. 8. Leg-A capacitor voltages.

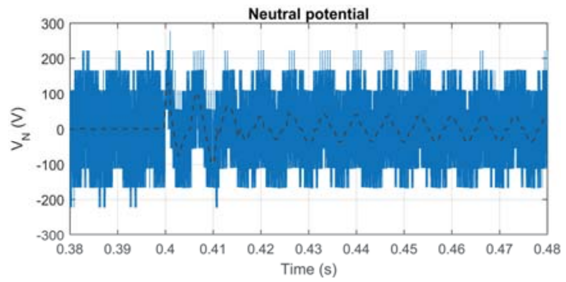

Fig. 9. Neutral potential.

the desired references while taking into account the voltage limits. The linear program derived is to be solved in real time by a numerical optimization method based on the simplex algorithm. Thanks to this control method, when saturation occurs, waveforms of arm voltages are adapted and current references are still well tracked. In the same way, an additional optimization formulation is proposed for a secondary control stage, which is responsible for the computation of optimized duty cycles of submodules in order to realize the required arm voltages by PSPWM while achieving active balancing of capacitor voltages. Thanks to this strategy, differences of capacitor voltages of a given arm are minimized to maintain balanced energies for all submodules over time, and capacitor voltages are kept close to their reference values over a fundamental period.

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Appendix

$$\mathbf{x} = \left(\mathbf{v_p} \quad \mathbf{v_n} \;\vdots\; \mathbf{e_o}^+ \quad \mathbf{e_c}^+ \quad e_{dc}^+ \quad e_{vn}^+ \;\vdots\; \mathbf{e_o}^- \quad \mathbf{e_c}^- \quad e_{dc}^- \quad e_{vn}^-\right)^T \tag{47}$$

$$\mathbf{x_{max}} = \left(\mathbf{v_{p,max}} \quad \mathbf{v_{n,max}} \;\vdots\; |i_{A,ref}| \quad |i_{B,ref}| \quad E_{dc}/(2R_c)\cdot(1 \;\; 1) \quad E_{dc}/(2R_s) \quad E_{dc}/2 \;\vdots\; |i_{A,ref}| \quad |i_{B,ref}| \quad E_{dc}/(2R_c)(1 \;\; 1) \quad E_{dc}/(2R_s) \quad E_{dc}/2\right)^T \tag{48}$$

$$\mathbf{c}^T = \left((0 \;\cdots\; 0) \quad (0 \;\cdots\; 0) \;\vdots\; \varepsilon_o\cdot(1 \;\; 1) \quad \varepsilon_c\cdot(1 \;\; 1) \quad \varepsilon_{dc} \quad \varepsilon_{vn} \;\vdots\; \varepsilon_o\cdot(1 \;\; 1) \quad \varepsilon_c\cdot(1 \;\; 1) \quad \varepsilon_{dc} \quad \varepsilon_{vn}\right) \tag{49}$$

$$\mathbf{A} = \left(\begin{array}{cc|ccc|ccc} & & -\mathbf{I}_{2\times 2} & & & \mathbf{I}_{2\times 2} & & \\ \multicolumn{2}{c|}{\mathbf{C_s B_s} T_S} & & -\mathbf{I}_{2\times 2} & & & \mathbf{I}_{2\times 2} & \\ & & & & -1 & & & 1 \\ \hline -1/2\,\mathbf{H_{CM}} T_S & 1/2\,\mathbf{H_{CM}} T_S & & & -1 & & & 1 \end{array}\right) \tag{50}$$

$$\mathbf{b} = \left(\begin{array}{c} \mathbf{C_s}\left(\mathbf{i_{o,ref}}[k+1] \quad \mathbf{i_{c,ref}}[k] \quad i_{dc,ref}[k]\right)^T - \mathbf{C_s}\left(\mathbf{I}_{7\times 7} + T_S \mathbf{A_s}\right)\left(\mathbf{i_o}[k] \quad \mathbf{i_c}[k] \quad i_{dc}[k]\right)^T - \mathbf{C_s} T_S \mathbf{H_s} \\ \hline v_{NG,ref} \end{array}\right) \tag{51}$$