# DISSIMILARITY MEASURES FOR CONTENT-BASED IMAGE RETRIEVAL 

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#### Abstract

Dissimilarity measurement plays a crucial role in contentbased image retrieval. In this paper, 16 core dissimilarity measures are introduced and evaluated. We carry out a systematic performance comparison on three image collections, Corel, Getty and Trecvid2003, with 7 different feature spaces. Two search scenarios are considered: single image queries based on the Vector Space Model, and multi-image queries based on $k$-Nearest Neighbours search. A number of observations are drawn, which will lay a foundation for developing more effective image search technologies.


Index Terms- dissimilarity measure, feature space, content-based image retrieval

## 1. INTRODUCTION

Content-based Image Retrieval (CBIR) provides users with a way to browse or retrieve images from large image collections based on visual similarity. Visual feature extraction and dissimilarity measures are the key issues for any CBIR system. The combination of these two attributes determines the overall effectiveness of the system. Therefore, given the visual features generated in a CBIR system, it is crucial to choose the most appropriate dissimilarity measure to achieve the best possible mean average precision.

There have been some attempts to theoretically summarise existing dissimilarity measures [1] and to evaluate dissimilarity measures for texture [2] and shape based image search [3]. Our own previous work [4] gives a description of 14 dissimilarity measures on six feature spaces, but only single-image queries are conducted on one image collection (Corel).

In this paper, we conduct a systematic investigation on this issue, with a view to generalize our previous preliminary work over three collections under two different retrieval scenarios. Firstly, based on [4] we introduce and categorize 16 typical dissimilarity measures theoretically. Then, experiments are carried out on three image collections, with

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seven different typical feature spaces, using both single image queries and multi-image queries. Our empirical evaluation provides evidence and insights on which dissimilarity measure works well on which feature spaces.

## 2. DISSIMILARITY MEASURES

Dissimilarity measures are classified into three categories according to their theoretical origins. Details can be found in [4].

Geometric Measures treat objects as vectors. Let $v$ and $w$ be two vectors in a $n$-dimensional real vector space, i.e. $v, w \in \mathbb{R}^{n}$. The distance between $v$ and $w$ can be measured by the following functions:

Minkowski Family: $\left(\sum_{i=1}^{n}\left|v_{i}-w_{i}\right|^{p}\right)^{\frac{1}{p}}, p>0$
Cosine: $1-\frac{v \cdot w}{|v| \cdot|w|}$, where $|\cdot|$ is the Euclidean norm
Canberra: $\sum_{i=1}^{n} \frac{\left|v_{i}-w_{i}\right|}{\left|v_{i}\right|+\left|w_{i}\right|}$
Squared Chord: $\sum_{i=1}^{n}\left(\sqrt{v_{i}}-\sqrt{w_{i}}\right)^{2}$
Partial-Histogram Intersection [5]: $1-\frac{\sum_{i=1}^{n}\left(\min \left(v_{i}, w_{i}\right)\right)}{\min (|v|,|w|)}$
Remarks: The Minkowski distance is a general form of a series of distance measures, such as Euclidean ( $\mathrm{p}=2$ ), City Block $(\mathrm{p}=1)$, Chebyshev $(p=\infty)$ and fractional distances (i.e., $0<p<1$ ) [6]. In this paper we studied fractional distances with three different parameters $p=0.25,0.5,0.75$. Note that the fractional distances are not metric because they violate the triangle inequality. Furthermore, the Squared Chord distance is only defined for non-negative components.

Information Theoretic Measures are derivatived from Shannon's entropy theory and treat objects as probabilistic distributions, i.e., $v_{i} \geq 0, \Sigma v_{i}=1$.

Kullback-Leibler (K-L) Divergence [7]: $\sum_{i=1}^{n} v_{i} \log \frac{v_{i}}{w_{i}}$
Jeffrey Divergence: $\quad \sum_{i=1}^{n}\left(v_{i} \log \frac{v_{i}}{m_{i}}+w_{i} \log \frac{w_{i}}{m_{i}}\right)$, where $m_{i}=\frac{v_{i}+w_{i}}{2}$

Statistic Measures compare two objects in a distributed manner, and basically assume that the vector elements are samples.
$\chi^{2}$ Statistics [8]: $\sum_{i=1}^{n} \frac{\left(v_{i}-m_{i}\right)^{2}}{m_{i}}$, where $m_{i}=\frac{v_{i}+w_{i}}{2}$
Pearson's Correlation Coefficient: $1-|p|$,
where $p=\frac{n \sum_{i=1}^{n} v_{i} w_{i}-\left(\sum_{i=1}^{n} v_{i}\right)\left(\sum_{i=1}^{n} w_{i}\right)}{\sqrt{\left[n \sum_{i=1}^{n} v_{i}^{2}-\left(\sum_{i=1}^{n} v_{i}\right)^{2}\right]\left[n \sum_{i=1}^{n} w_{i}^{2}-\left(\sum_{i=1}^{n} w_{i}\right)^{2}\right]}}$
Kolmogorov-Smirnov [9]: $\max _{1 \leq i \leq n}\left|F_{v}(i)-F_{w}(i)\right|$
Cramer/von Mises Type: $\quad \sum_{i=1}^{n}\left(F_{v}(i)-F_{w}(i)\right)^{2}$, where $F_{v}(i)$ and $F_{w}(i)$ are the probability distribution functions of the object vectors [9].

## 3. VISUAL FEATURES

We applied seven typical image features including HSV, $\operatorname{margRGB}-H, \operatorname{margRGB}-\mathrm{M}$ for color; Gabor, Tammura for texture; konvolution for structure and thumbnail.

Colour: HSV is a three-dimensional joint colour histogram in the cylindrical colour space. MargRGB-H creates a onedimensional histogram for each component individually. MargRGB-M records the first four central moments of each colour channel distribution.

Texture: Gabor is a texture feature generated using Gabor wavelets. Here, we decompose each image into two scales and four directions. Tamura is a three-dimensional texture feature composed by measures of image coarseness, contrast and directionality [10].

Structure: Konvolution discriminates between low level structures in an image, and is designed to recognize horizontal, vertical and diagonal edges at several scales [11].

Thumbnail: This is a feature created from the pixel intensity values of a scaled down image. Here we use a size of 40 by 30 resulting in a dense vector of length 1,200 .

## 4. RETRIEVAL METHODS

In the single-image-query model, a database of images is searched to find images similar to the given query image. In a multi-image-query model, more than one query examples are given; the system aims to find images similar to the positive examples. In this papaer we use the vector space model for single-image queries and $k$-nearest neighbours with additional negative examples for multi-image queries.

Vector Space Model (VSM). The images are represented as vectors in a multi-dimensional feature space and then ranked according to their distances to the query vector.
$\boldsymbol{k}$-Nearest Neighbours ( $k$-NN) [11, 6]. We use a variation of the distance-weighted $k$-Nearest Neighbours approach. Positive examples are supplied as the queries, and negative examples are selected from the training set excluding the categories that any positive query image belongs to. Test images are then ranked according to their dissimilarity to these examples according to

$$
\begin{equation*}
R(i)=\frac{\sum_{\mathrm{neg} \in N}(\operatorname{dist}(i, \operatorname{neg}))^{-1}}{\sum_{\operatorname{pos} \in P}(\operatorname{dist}(i, \operatorname{pos}))^{-1}} \tag{1}
\end{equation*}
$$

where $P$ and $N$ are the sets of positive and negative examples, from the $k$ nearest neighbours of the test image respectively. $\operatorname{dist}(i, \mathrm{neg})$ is the distance between the test image $i$ and the negative example neg; $\operatorname{dist}(i, \operatorname{pos})$ is the distance between $i$ and the positive example pos. A value of $k=40$ is used for our experiments.

## 5. EXPERIMENTS

We conducted a comprehensive empirical performance study, using both VSM based single-image queries and $k$-NN based multi-image queries, on three databases including Corel, Getty and Trecvid2003.

### 5.1. Data Sets

Corel. We use a subset of Corel dataset, which was created by Pickering and Rüger [11]. It consists of 6192 images, belonging to 63 categories. We randomly split the collection into $25 \%$ training data and $75 \%$ test data. For single image queries, we use every image in the training set as a query. Multi-image queries are conducted for each category with the number of positive examples varying from 1 to $6 ; 100$ negative examples are selected from the training set per query. As there are 63 categories we generate 378 multi-image queries for each dissimilarity measure and feature space combination.

Getty. We use a subset of Getty dataset, which was created by Yavlinsky and Rüger [12]. We randomly split the dataset into 2,560 training and 5,000 test images. We use each image in the training set as a query. The groundtruth is generated by considering the images in the test set, that share at least one common keyword (the same 184 keywords as in [12]) with a query as relevant to the query. For the $k$-nearest neighbours method we use each image in the training set as a query; 100 negative images are selected per query. There are 2560 multi-image queries for each dissimilarity measure and feature space combination.

TRECVID2003. It comprises 32,318 key-frames from the Trecvid 2003 video collection. The search task consists of 25 real-world query topics [13] as query images. For multiimage queries the number of positive examples per query

Table 1. Recommended Dissimilarity Measures

|  | VSM | $k$-NN |
| :--- | :--- | :--- |
| HSV | Squared Chord, $\chi^{2}, ~ H i s-~$ <br> togram, City Block | Squared Chord, $\chi^{2}$, Frac- <br> tional (p=0.75) |
| margRGB-H | Fractional (p=0.5) | Squared Chord, $\chi^{2}$ |
| margRGB-M | Euclidean, City Block | Squared Chord, City Block, <br> Euclidean |
| konv | Squared Chord, $\chi^{2}, ~ C i t y ~$ <br> Block, Jeffrey | Squared Chord, $\chi^{2}, ~ C i t y ~$ <br> Block |
| Gabor | Fractional (p=0.25), Frac- <br> tional (p=0.5) | Fractional (p=0.5), Can- <br> berra, $\chi^{2}$, Squared Chord |
| Tamura | Fractional (p=0.5), Frac- <br> tional (p=0.75) | Canberra, <br> (p=0.75) |
| thumbnail | City Block, Jeffrey | Canberra, Fractional (p=0.5) |

ranges from 1 to $3 ; 100$ negative images are selected per query. There are 75 multi-image queries for each dissimilarity measure and feature space combination. We expect a lower mean average precision on this dataset, owing to the large size of the collection and the difficulty of the queries.

### 5.2. Experimental Results and Analysis

We compute mean average precision (MAP), which has been extensively used by the Text REtrieval Conference (TREC) community [14] as the performance measure.

Results on the three datasets are listed in Table 2-4. The MAP for single-image and multi-image queries are shown, respectively, at the left hand side and right hand side of each cell.

We observe that for each feature space the effects of different dissimilarity measures follow a similar trend on different datasets. In general, the Squared Chord, Fractional ( $p=0.5$ ), $\chi^{2}$ and Cityblock usually get a better performance than the other measures. For each feature space and dissimilarity measures, we list the top five MAP values for all the three datasets in Table 1. We recommend them for future use.

## 6. CONCLUSION

A comprehensive study has been conducted for 16 dissimilarity measures on seven typical feature spaces with both single and multi image queries on three collections including the real-world image collection TRECVID2003.

We have shown that Squared Chord, Fractional ( $p=0.5$ ), $\chi^{2}$ and Cityblock usually get a better performance than the widely used Euclidean distance. For each feature space we recommend dissimilarity measures that give the top five mean average precision values on all the three collections, for two scenarios separately. The findings from this investigation can be a foundation for developing more effective contentbased image retrieval systems. Surprisingly, Squared Chord distance shows superior performance with almost all feature spaces, but it should be noted that it can only deal with features with non-negative components.

## 7. REFERENCES

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Table 2. Mean Average Precision for the Corel dataset

|  | HSV |  | margRGB-H |  | margRGB-M |  | konvolution |  | Gabor |  | Tamura |  | thumbnail |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Geometric Measures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Fractional ( $\mathrm{p}=0.25$ ) | 0.1059 | 0.1807 | 0.1294 | 0.1912 | 0.0823 | 0.1339 | 0.0677 | 0.0801 | 0.1566 | 0.1605 | 0.1437 | 0.1448 | 0.1329 | 0.1375 |
| Fractional ( $\mathrm{p}=0.5$ ) | 0.1506 | 0.2953 | 0.1269 | 0.1964 | 0.0871 | 0.1461 | 0.0731 | 0.1086 | 0.1490 | 0.1882 | 0.1286 | 0.01773 | 0.1289 | 0.1503 |
| Fractional ( $\mathrm{p}=0.75$ ) | 0.1733 | 0.2747 | 0.1236 | 0.1911 | 0.0898 | 0.1489 | 0.0850 | 0.1383 | 0.1416 | 0.1811 | 0.1097 | 0.1626 | 0.1238 | 0.1445 |
| City Block ( $\mathrm{p}=1$ ) | 0.1682 | 0.2532 | 0.1207 | 0.1877 | 0.0912 | 0.1495 | 0.0951 | 0.1481 | 0.1350 | 0.1791 | 0.0949 | 0.1538 | 0.1176 | 0.1398 |
| Euclidean (p=2) | 0.1289 | 0.1969 | 0.1128 | 0.1855 | 0.0917 | 0.1476 | 0.0761 | 0.1043 | 0.1161 | 0.1789 | 0.0678 | 0.1024 | 0.0929 | 0.1293 |
| Chebyshev ( $\mathrm{p}=\infty$ ) | 0.1094 | 0.1559 | 0.1013 | 0.1591 | 0.0886 | 0.1412 | 0.0555 | 0.0772 | 0.0615 | 0.1205 | 0.0358 | 0.0536 | 0.0332 | 0.0592 |
| Cosine | 0.1345 | 0.1577 | 0.1204 | 0.1595 | 0.0778 | 0.1403 | 0.0716 | 0.0702 | 0.1057 | 0.1202 | 0.0671 | 0.0544 | 0.0756 | 0.0585 |
| Canberra | 0.1568 | 0.2779 | 0.1333 | 0.2016 | 0.0824 | 0.1396 | 0.0709 | 0.1104 | 0.1496 | 0.2296 | 0.1267 | 0.1880 | 0.1211 | 0.1593 |
| Squared Chord | 0.1876 | 0.2894 | 0.1294 | 0.2044 | 0.0967 | 0.1607 | 0.0984 | 0.1597 | 0.1259 | 0.1898 | 0.0880 | 0.1507 | 0.0904 | 0.1170 |
| Histogram | 0.1682 | 0.1586 | 0.1207 | 0.1566 | 0.0720 | 0.1382 | 0.0551 | 0.0779 | 0.0680 | 0.1178 | 0.0319 | 0.0539 | 0.0486 | 0.0588 |
| Information-Theoretic Measures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Kullback-Leibler | 0.1779 | 0.1052 | 0.1113 | 0.1888 | 0.0893 | 0.1443 | 0.0528 | 0.1444 | 0.1019 | 0.1205 | 0.0948 | 0.0672 | 0.0467 | 0.0828 |
| Jeffrey | 0.1555 | 0.2345 | 0.1185 | 0.1808 | 0.0902 | 0.1470 | 0.0960 | 0.1473 | 0.1353 | 0.1782 | 0.0950 | 0.1562 | 0.1196 | 0.1404 |
| Statistic Measures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\chi^{2}$ Statistics | 0.1810 | 0.2754 | 0.1282 | 0.2010 | 0.0832 | 0.1352 | 0.0897 | 0.1597 | 0.1303 | 0.1966 | 0.0984 | 0.1573 | 0.0940 | 0.1198 |
| Pearson | 0.1307 | 0.1825 | 0.1182 | 0.1832 | 0.0818 | 0.1417 | 0.0692 | 0.1240 | 0.1035 | 0.1663 | 0.0763 | 0.1010 | 0.0665 | 0.0933 |
| Kolmogorov | 0.0967 | 0.1477 | 0.1041 | 0.1687 | 0.0750 | 0.1132 | 0.0426 | 0.0878 | 0.0575 | 0.0383 | 0.0598 | 0.0583 | 0.0618 | 0.0769 |
| Cramer | 0.0842 | 0.1352 | 0.1077 | 0.1699 | 0.0724 | 0.1088 | 0.0406 | 0.0675 | 0.0529 | 0.0766 | 0.0516 | 0.0439 | 0.0564 | 0.0513 |

Table 3. Mean Average Precision for the Getty dataset

|  | HSV |  | margRGB-H |  | margRGB-M |  | konvolution |  | Gabor |  | Tamura |  | thumbnail |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Geometric Measures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Fractional ( $\mathrm{p}=0.25$ ) | 0.1408 | 0.1546 | 0.1505 | 0.1501 | 0.1441 | 0.1454 | 0.1414 | 0.1431 | 0.1527 | 0.1526 | 0.1544 | 0.1531 | 0.1458 | 0.1526 |
| Fractional ( $\mathrm{p}=0.5$ ) | 0.1482 | 0.1724 | 0.1499 | 0.1555 | 0.1465 | 0.1518 | 0.1427 | 0.1509 | 0.1509 | 0.1584 | 0.1502 | 0.1582 | 0.1459 | 0.1539 |
| Fractional ( $\mathrm{p}=0.75$ ) | 0.1575 | 0.1743 | 0.1487 | 0.1559 | 0.1484 | 0.1541 | 0.1448 | 0.1531 | 0.1492 | 0.1571 | 0.1469 | 0.1551 | 0.1458 | 0.1536 |
| City Block ( $\mathrm{p}=1$ ) | 0.1628 | 0.1740 | 0.1475 | 0.1557 | 0.1497 | 0.1557 | 0.1472 | 0.1554 | 0.1479 | 0.1561 | 0.1445 | 0.1531 | 0.1455 | 0.1534 |
| Euclidean (p=2) | 0.1503 | 0.1586 | 0.1449 | 0.1551 | 0.1523 | 0.1581 | 0.1442 | 0.1518 | 0.1445 | 0.1541 | 0.1396 | 0.1485 | 0.1445 | 0.1528 |
| Chebyshev ( $\mathrm{p}=\infty$ ) | 0.1510 | 0.1514 | 0.1426 | 0.1531 | 0.1520 | 0.1575 | 0.1396 | 0.1474 | 0.1392 | 0.1486 | 0.1311 | 0.1408 | 0.1391 | 0.1462 |
| Cosine | 0.1561 | 0.1565 | 0.1498 | 0.1512 | 0.1507 | 0.1553 | 0.1420 | 0.1473 | 0.1341 | 0.1442 | 0.1298 | 0.1412 | 0.1324 | 0.1409 |
| Canberra | 0.1484 | 0.1629 | 0.1421 | 0.1501 | 0.1451 | 0.1506 | 0.1420 | 0.1503 | 0.1445 | 0.1599 | 0.1434 | 0.1572 | 0.1408 | 0.1521 |
| Squared Chord | 0.1657 | 0.1788 | 0.1484 | 0.1586 | 0.1489 | 0.1577 | 0.1480 | 0.1563 | 0.1470 | 0.1574 | 0.1408 | 0.1519 | 0.1435 | 0.1524 |
| Histogram | 0.1628 | 0.1661 | 0.1475 | 0.1504 | 0.1432 | 0.1494 | 0.1319 | 0.1502 | 0.1253 | 0.1420 | 0.1218 | 0.1385 | 0.1222 | 0.1364 |
| Information-Theoretic Measures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Kullback-Leibler | 0.1140 | 0.1243 | 0.1391 | 0.1525 | 0.1422 | 0.1428 | 0.1448 | 0.1419 | 0.1329 | 0.1388 | 0.1285 | 0.1390 | 0.1351 | 0.1398 |
| Jeffrey | 0.1582 | 0.1772 | 0.1466 | 0.1584 | 0.1493 | 0.1499 | 0.1472 | 0.1563 | 0.1480 | 0.1575 | 0.1454 | 0.1519 | 0.1458 | 0.1525 |
| Statistic Measures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\chi^{2}$ Statistics | 0.1640 | 0.1760 | 0.1482 | 0.1579 | 0.1453 | 0.1500 | 0.1479 | 0.1563 | 0.1471 | 0.1574 | 0.1415 | 0.1520 | 0.1438 | 0.1526 |
| Pearson | 0.1517 | 0.1614 | 0.1447 | 0.1501 | 0.1500 | 0.1602 | 0.1433 | 0.1525 | 0.1339 | 0.1493 | 0.1296 | 0.1455 | 0.1337 | 0.1404 |
| Kolmogorov | 0.1433 | 0.1452 | 0.1513 | 0.1612 | 0.1386 | 0.1436 | 0.1391 | 0.1479 | 0.1398 | 0.1478 | 0.1369 | 0.1450 | 0.1389 | 0.1368 |
| Cramer | 0.1415 | 0.1434 | 0.1552 | 0.1629 | 0.1381 | 0.1431 | 0.1378 | 0.1459 | 0.1391 | 0.1436 | 0.1372 | 0.1448 | 0.1381 | 0.1427 |

Table 4. Mean Average Precision for the Trecvid2003 dataset

|  | HSV |  | margRGB-H |  | margRGB-M |  | konvolution |  | Gabor |  | Tamura |  | thumbnail |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Geometric Measures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Fractional (p=0.25) | 0.0105 | 0.0126 | 0.0090 | 0.0140 | 0.0069 | 0.0132 | 0.0115 | 0.0264 | 0.0263 | 0.0290 | 0.0187 | 0.0210 | 0.0192 | 0.0280 |
| Fractional ( $\mathrm{p}=0.5$ ) | 0.0137 | 0.0168 | 0.0097 | 0.0142 | 0.0077 | 0.0132 | 0.0120 | 0.0172 | 0.0259 | 0.0290 | 0.0208 | 0.0210 | 0.0204 | 0.0260 |
| Fractional (p=0.75) | 0.0161 | 0.0180 | 0.0100 | 0.0143 | 0.0081 | 0.0136 | 0.0133 | 0.0172 | 0.0254 | 0.0262 | 0.0210 | 0.0222 | 0.0215 | 0.0240 |
| City Block ( $\mathrm{p}=1$ ) | 0.0149 | 0.0176 | 0.0101 | 0.0136 | 0.0084 | 0.0140 | 0.0139 | 0.0176 | 0.0249 | 0.0262 | 0.0209 | 0.0238 | 0.0223 | 0.0228 |
| Euclidean ( $\mathrm{p}=2$ ) | 0.0106 | 0.0164 | 0.0101 | 0.0139 | 0.0090 | 0.0140 | 0.0115 | 0.0168 | 0.0233 | 0.0250 | 0.0189 | 0.0230 | 0.0229 | 0.0236 |
| Chebyshev ( $\mathrm{p}=\infty$ ) | 0.0086 | 0.0144 | 0.0088 | 0.0137 | 0.0084 | 0.0144 | 0.0107 | 0.0136 | 0.0169 | 0.0238 | 0.0093 | 0.0170 | 0.0079 | 0.0168 |
| Cosine | 0.0120 | 0.0121 | 0.0104 | 0.0132 | 0.0101 | 0.0116 | 0.0135 | 0.0116 | 0.0255 | 0.0154 | 0.0177 | 0.0162 | 0.0219 | 0.0152 |
| Canberra | 0.0118 | 0.0132 | 0.0087 | 0.0136 | 0.0083 | 0.0136 | 0.0118 | 0.0180 | 0.0257 | 0.0274 | 0.0165 | 0.0242 | 0.0207 | 0.0232 |
| Squared Chord | 0.0160 | 0.0180 | 0.0104 | 0.0145 | 0.0096 | 0.0140 | 0.0143 | 0.0176 | 0.0264 | 0.0278 | 0.0183 | 0.0242 | 0.0221 | 0.0272 |
| Histogram | 0.0149 | 0.0127 | 0.0101 | 0.0129 | 0.0062 | 0.0116 | 0.0072 | 0.0116 | 0.0059 | 0.0150 | 0.0067 | 0.0182 | 0.0059 | 0.0140 |
| Information-Theoretic Measures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Kullback-Leibler | 0.0058 | 0.0120 | 0.0076 | 0.0140 | 0.0071 | 0.0132 | 0.0105 | 0.0128 | 0.0155 | 0.0278 | 0.0097 | 0.0174 | 0.0139 | 0.0136 |
| Jeffrey | 0.0133 | 0.0178 | 0.0101 | 0.0146 | 0.0083 | 0.0132 | 0.0138 | 0.0128 | 0.0246 | 0.0274 | 0.0209 | 0.0174 | 0.0219 | 0.0136 |
| Statistic Measures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\chi^{2}$ Statistics | 0.0157 | 0.0181 | 0.0104 | 0.0145 | 0.0091 | 0.0152 | 0.0143 | 0.0176 | 0.0265 | 0.0274 | 0.0190 | 0.0234 | 0.0223 | 0.0272 |
| Pearson | 0.0119 | 0.0145 | 0.0105 | 0.0140 | 0.0091 | 0.0136 | 0.0120 | 0.0192 | 0.0201 | 0.0266 | 0.0176 | 0.0242 | 0.0166 | 0.0276 |
| Kolmogorov | 0.0065 | 0.0.131 | 0.0077 | 0.0128 | 0.0058 | 0.0124 | 0.0078 | 0.0132 | 0.0056 | 0.0166 | 0.0060 | 0.0174 | 0.0074 | 0.0156 |
| Cramer | 0.0064 | 0.0124 | 0.0089 | 0.0146 | 0.0057 | 0.0124 | 0.0065 | 0.0128 | 0.0052 | 0.0158 | 0.0064 | 0.0174 | 0.0068 | 0.0252 |

