Optimization Based Rate Allocation and Scheduling in TDMA Based Wireless Mesh Networks

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***Abstract*—Wireless mesh networking is a promising technology for building broadband wireless access networks. However, wireless mesh networks based on CSMA/CA MAC protocols suffer from unfairness and poor QoS support. Using TCP as a rate control mechanism in such networks further exacerbates the problem. Efficient rate allocation and scheduling algorithms that handle both *multicast and unicast* traffic in wireless mesh networks are needed with the increasing popularity of multicast and multimedia applications.**

In this paper, we propose a framework that performs both *rate allocation and scheduling* for unicast and multicast traffic in TDMA-based wireless mesh networks. The rate allocation algorithm is based on Network Utility Maximization. The graph coloring-based scheduling algorithm achieves the allocated rates. Simulation results show that our framework provides guaranteed throughput and low delay for both multicast and unicast traffic. Furthermore, our framework significantly outperforms a previously published framework that has a similar objective.

I. INTRODUCTION

Wireless mesh networking (WMN) is an emerging technology that uses multi-hop communications to provide cost-efficient broadband Internet access for community or enterprise users. A typical wireless mesh network consists of mesh routers and mesh clients [1]. Mesh routers are connected to form a static multi-hop backbone. Mesh routers that are connected to the Internet serve as Internet gateways. Mesh clients, such as laptops and PDAs, connect to mesh routers to access the Internet. In wireless mesh networks, inter-router and client-router communications usually use different radio technologies to reduce interference. For example, IEEE 802.16 [2] (commonly known as WiMAX) can be used for inter-router communication while client-router communication uses IEEE 802.11 [3].

Multicasting is a key technology for content distribution over wireless mesh networks. However, the introduction of multicast traffic in the network should not starve the existing unicast traffic [4]. It is important to consider the coexistence of multicast and unicast traffic in the network when allocating network resources.

In this paper, we propose a framework for both *rate allocation and scheduling for multicast and unicast sessions* in wireless mesh networks. Our rate allocation algorithm is based on Network Utility Maximization (NUM) [5]. We first establish the notion of *transmissions* as a generalization of *links* and construct contention graphs based on the contention relationships among transmissions. The contention graph is used to model the rate allocation constraints. By using receiver-oriented utility functions and maximizing the total utility of the network, our rate allocation gives incentive to users to join multicast sessions, while preventing extreme unfairness to unicast sessions [4]. Our scheduling algorithm is based on graph coloring of the contention graph, and it assumes the spatial TDMA MAC protocol [6]. We choose TDMA rather than CSMA/CA because the random nature of CSMA/CA MAC protocols makes QoS support inherently difficult. Spatial TDMA enables collision-free communications and fine control of the throughput and delay of network traffic. Given the contention relationship of the transmissions in the network, our scheduling algorithm produces optimal schedules that achieve the allocated rates.

Our work contains several major contributions: First, our framework provides efficient and fair rate allocation for both unicast and multicast traffic in wireless mesh networks. Second, our framework effectively schedules the allocated rates which results in guaranteed throughput and bounded delay for session recipients. Third, our framework is very efficient and is suitable for a centralized implementation.

The rest of the paper is organized as follows: In Section II we review the related work and highlight our contributions. Section III introduces the system model for the rate allocation and scheduling. Section IV first introduces a previously published framework and then gives a detailed description of our proposed framework including the rate allocation and the scheduling algorithms. We provide a simulation evaluation of our framework in Section V. Conclusions and future work are given in Section VI.

II. RELATED WORK

There is little prior work that considers both rate allocation and scheduling for multicast traffic in TDMA based wireless mesh networks. In [7], the authors proposed an interference-aware fair scheduling algorithm named LOF for multicast in wireless mesh network. The major focus of their work is the scheduling of a single multicast session where each receiver gets the same throughput. For networks consisting of multiple multicast sessions, LOF allocates an equal rate to each session. In contrast, we strive to optimize the total utility of the network by allocating different rates to different sessions. However, we still guarantee that different receivers of a single session get the same throughput. In [8], the authors

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proposed schemes for multicast tree construction and multicast scheduling. However, they did not define how to allocate rates to different multicast sessions. In [9], the authors proposed a throughput maximization framework for multicast traffic in wireless mesh networks. The maximization problem is decomposed into a routing subproblem and a power control subproblem.

Rate allocation for unicast traffic in wireless networks has been extensively studied. Some of the papers [10], [11] assume that a CSMA/CA MAC protocol is used while others [12], [13] jointly consider rate allocation with MAC protocol design. All the above papers use Network Utility Maximization [5] as a modeling tool. NUM was originally proposed for Internet congestion control [14], [15] and has become an important tool for modeling and designing resource allocation algorithms [5]. Our rate allocation algorithm differs from the work mentioned above in several important ways. First, we consider rate allocation for both multicast and unicast sessions. Second, our rate allocation takes variable link capacities into consideration. Third, by using perfect contention graphs to model rate allocation constraints, we guarantee that the allocated rates can be supported by an efficient scheduling algorithm proposed in this paper. The previous work mentioned above either do not provide scheduling algorithms that guarantee the allocated rates [10]–[12] or the proposed scheduling algorithm is not efficient enough for a practical implementation [13].

Several scheduling algorithms [16]–[18] have been proposed for IEEE 802.16 wireless mesh networks. An interference-aware scheduling algorithm is proposed in [18]. The proposed algorithm aims to find the maximum number of concurrent transmissions to achieve high throughput. Djukic, et al. propose the Bellman-Ford TDMA scheduling algorithm in [17]. The major advantage of Bellman-Ford scheduling algorithm over the interference-aware scheduling algorithm is that it takes the scheduling delay into consideration while taking advantages of spatial reuse. All of these papers only consider unicast traffic, and they assume that traffic rate requirements are given for the scheduling algorithm. Our framework jointly considers rate allocation and scheduling for both unicast and multicast traffic.

III. System Model

A. Routing Tree and Transmission Set

Our framework only deals with the aggregated traffic between the mesh routers and the gateway; we assume that mesh routers employ appropriate methods to distribute traffic to their clients. For simplicity, we only consider downlink traffic from the gateway node to mesh routers. However, our framework can be applied to uplink traffic as well.

In this paper, we assume that the routing information is given to our framework. Since we only consider traffic from the Internet gateway to mesh routers, we use the Weighted Connected Dominating Set (WCDS) algorithm proposed in [8] to construct a broadcast tree and then prune the tree to accommodate all the multicast and unicast sessions. We denote the resulting tree structure as the *routing tree*. Note that our framework can work with other routing schemes. For example, we can also accommodate using a separate multicast tree for each multicast session as was done in [7].

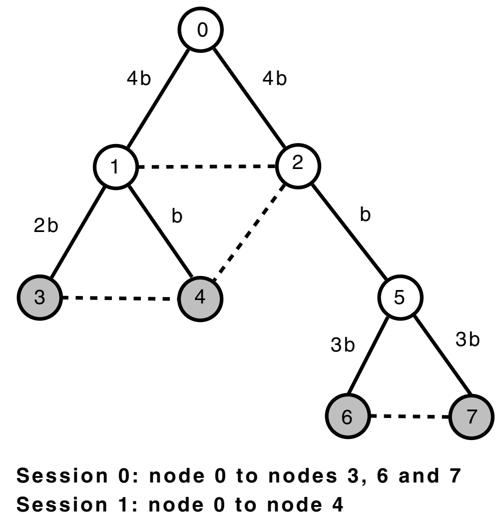

Fig. 1. An Example Scenario

Figure 1 shows an example scenario we use to describe our framework. In this figure, solid lines depict links belonging to the routing tree while dashed lines depict other potential links. We denote the set of network sessions as S. For this example scenario, two network sessions s_0 and s_1 are defined. The source of s_0 is node 0; the recipients of s_0 are nodes 3, 6 and 7. The source of s_1 is node 0; the recipient of s_1 is node 4. We also specify the rate of each link in the routing tree, where symbol b represents the minimum link rate supported by the network.

In order to model multicast sessions, we define the notion of *transmissions*, which is a generalization of *links*. A transmission consists of the following attributes: (1) A sender; (2) A list of recipients; (3) The session carried by this transmission; (4) Transmission rate used. Note that we can also allow a transmission to carry multiple sessions. However, in order to accommodate scenarios where multiple multicast trees are constructed, a transmission is only associated with a single session in this paper.

MAC protocols such as IEEE 802.11 do not support multi-rate multicast and mandate that all link-layer broadcasts use the same rate. In [7], all link-layer broadcasts proceed with the rate of 2Mbps. However, recently proposed MAC protocols such as IEEE 802.16 begin to support multi-rate mulicast at the link layer [8]. In this paper, we assume a MAC protocol that supports multi-rate multicast by employing different transmission rates for link-layer broadcasting. The rate of a transmission is the minimum rate of all the links between the sender and the recipients [8]. The link rate depends on the modulation and coding scheme used for the link, which is determined by the Signal to Noise Ratio (SNR) at the receiver side of the link.

We denote the set of transmissions as T. Given the routing

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tree R and the session set S, our algorithm to construct transmission set T is shown in Figure 2. For routing schemes where multiple multicast trees are constructed, we can execute the above algorithm for each multicast tree and aggregate the resulting transmissions. For the example scenario shown in Figure 1, the algorithm outputs 6 transmissions as shown in Table I.

TABLE I
AN EXAMPLE TRANSMISSION SET

Transmission ID	0	1	2	3	4	5
Sender	0	0	1	1	2	5
Recipient List	1, 2	1	3	4	5	6,7
Session List	0	1	0	1	0	0
Transmission Rate	4b	4b	2b	b	b	3b

B. Transmission Contention Graph

In [19], Jain, et al. defined two contention models: Protocol Model and Physical Model. In [8], Chou, et al. proposed and justified a contention model for multi-rate multicasting based on the Protocol Model. Under this model, every link-layer transmission rate corresponds to a fixed transmission range and interference range. In addition, transmission i conflicts with transmission j if (1) Both transmissions share the same sender, or (2) Any recipient of transmission j is within the interference range of the sender of transmission i or vice versa.

According to the above contention model, we can define a *transmission contention (conflict) graph* $G_c(V_c, E_c)$ based on the contention relationship among different transmissions. Contention graphs have been used in several papers [13], [19] to model network resource constraints. The vertex set V_c contains all the transmissions in the network. An edge in set E_c indicates that two vertices (transmissions) contend with each other. Figure 3 shows the contention graph of our example scenario. Nodes in the figure represent the transmissions defined in Table I.

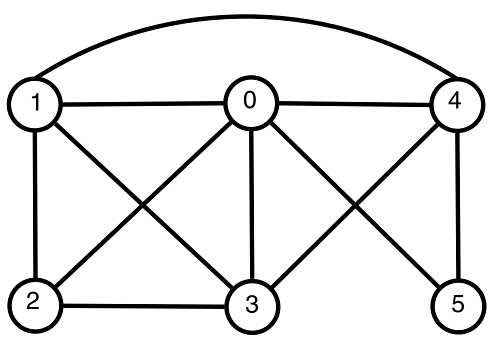

Fig. 3. An Example Contention Graph

We are interested in *maximal cliques* and *maximal independent sets* of the contention graph. A set of nodes is a clique if its induced subgraph is complete. A maximal clique is defined as a clique that is not contained in any other clique. We denote the set of maximal cliques of a contention graph as C. In the transmission contention graph given in Figure 3, we have three maximal cliques, $C_1 = \{0, 1, 2, 3\}, C_2 = \{0, 1, 3, 4\}$ and $C_3 = \{0, 4, 5\}$. An independent set is a set of nodes that are not connected by any edges, and a maximal independent set is an independent set not contained by any other independent set.

C. Clique Feasibility and Scheduling Feasibility

For contention graph G_c, let I be the family of all independent sets of this graph. Schedule D can be defined as an infinite sequence of independent sets, $I_1, I_2, ..., I_k, ...$, where $I_k \in I$ [12], [20]. The frequency of transmission i in schedule D is then defined as:

$$f_i = \lim_{t \to \infty} \frac{\sum_{k=1}^{k=t} D(i,k)}{t}$$

where $D(i, k)$ is an indicator function such that, $D(i, k) = 1$ if $i \in I_k$, and $D(i, k) = 0$ otherwise. We say that a schedule D is periodic with period N, if $I_j = I_{N+j} = I_{2N+j} = \cdots$ for all $1 \leq j \leq N$ [20].

For a transmission contention graph with M transmissions, a vector of frequencies $\boldsymbol{f} = (f_1, ..., f_M)$ is feasible if there exists a schedule D such that the frequency of the ith transmission in schedule D is at least f_i for $1 \leq i \leq M$ [20]. The frequency of transmission i can be treated as the normalized bandwidth allocated to this transmission in schedule D [12]. A vector of frequencies $\boldsymbol{f}$ is *clique feasible* [20] for the contention graph G_c if

$$\sum_{i \in C_j} f_i \leq 1, \forall C_j \in C$$

It has been shown that if the contention graph is a perfect graph, clique feasibility is equivalent to *scheduling feasibility* of the frequency vector [20], [21].

In graph theory, a *perfect graph* is a graph in which every induced subgraph can be colored with p colors where p is the size of the largest clique in the subgraph. A *chordal graph*, a graph that does not contain an induced k-cycle for $k \geq 4$, is a perfect graph. Chordal graphs are sometimes called *triangulated graphs*. An alternative definition for chordal graph is related to simplicial elimination ordering. A vertex v of a graph G is called *simplicial* if its neighbors in G form a clique. A *simplicial elimination ordering* of G is a vertex order $v_1, ..., v_i, ..., v_n$ such that every vertex v_i is a simplicial vertex in the subgraph induced by $\{v_1, ..., v_i\}$ [22]. A graph is a chordal graph if and only if it has a simplicial elimination ordering [22].

We can use the lexicographic BFS algorithm [23] for recognizing a chordal graph and outputting a simplicial elimination ordering with time complexity $O(m + n)$, where m is the number of edges and the n is the number of vertices of the input graph. We can transform a general graph to a chordal graph in $O(mn)$ time using the LEX M minimal triangulation algorithm [24]. Recently, an $O(n^{2.69})$ implementation of LEX M has been proposed in [25]. In this paper, in order to maintain the scheduling feasibility of the frequency vector, we apply the LEX M algorithm to transform a general contention graph to

$T = \emptyset$
for each session s in S **do**
  for each node n in R that is a recipient of s **do**
    Add session s to node n's session list
  end for
end for
Use the Breadth First Search (BFS) algorithm to partition the nodes in R into different levels $l_0, \ldots, l_m$
for $l = l_m$ to l_0 **do**
  for ech node n at level l **do**
    Add all the sessions in the session lists of nodes n's children to node n's session list
  end for
end for
for each node n in R **do**
  if n has children **then**
    for each session s in the session list of node n **do**
      Construct a new transmission t
      Add each children of n which has session s in its session list into the recipient list of t
      Add transmission t into T
    end for
  end if
end for
return T

Fig. 2. Algorithm - ConstructTransmissionSet(R, S)

a chordal graph. Since the transformation only adds edges to the contention graph, the resulting contention graph will not introduce any violation of the existing contention relationship. In fact it adds unnecessary contention relationships, which results in more conservative rate allocation and less spatial reuse. However the benefits of maintaining chordal graph lie in several aspects: First, a perfect graph enables our algorithm to produce rate allocations that are schedulable. Second, a chordal graph facilitates the optimal scheduling algorithm, which will be introduced in Section IV-C. Third, a perfect graph enables $O(m+n)$ time algorithm for obtaining the maximal clique set [23]. For general graphs, algorithms for obtaining the maximal clique set has exponential time complexity [26].

To formally define clique feasibility, we first define matrix $\boldsymbol{Q} = \{Q_{nm}\}$:

$$Q_{nm} = \begin{cases} 1, & \text{if clique } n \text{ contains transmission } m \\ 0, & \text{otherwise} \end{cases}$$

For the example scenario, with maximal clique set $C = \{C_1, C_2, C_3\}$ and transmission set $T = \{0,1,2,3,4,5\}$, we have

$$\boldsymbol{Q} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Thus we can express the clique feasibility of a contention graph G_c as follows:

$$\sum_{f:Q_{nm}=1} f_m \leq 1, \quad \forall n \in C$$

In the following, we express the clique feasibility in terms of the session rate vector. We assume that a fixed-length scheduling period consists of several time slots. Each time slot has a fixed duration. We use $\boldsymbol{x} = (x_s, s \in S)$ to denote the *rate allocation vector* of network sessions where x_s is expressed in bits per scheduling period. Assume that transmission m carries session s and the transmission rate is b_m, expressed in bits per time slot. Also denote the total number of time slots in a scheduling period as N. Then we can express the transmission frequency f_m needed to support session rate x_s as follows:

$$f_m = \frac{\lceil x_s/b_m \rceil}{N}$$

In fact, $\lceil x_s/b_m \rceil$ is the number of time slots needed for transmission m to support the session rate x_s. In order to simplify the rate allocation algorithm, we ignore the ceiling function and express f_m as

$$f_m = \frac{x_s/b_m}{N}$$

As a result, the allocated rate x_s may need to be scaled down to x'_s so that $\frac{\lceil x'_s/b_m \rceil}{N} \leq \frac{x_s/b_m}{N}$.

We define a matrix $\boldsymbol{R} = \{R_{ms}\}$:

$$R_{ms} = \begin{cases} \frac{1}{b_m * N}, & \text{if transmission } m \text{ carries session } s \\ 0, & \text{otherwise} \end{cases}$$

By using the matrix $\boldsymbol{P} = \boldsymbol{QR}(P_{ns} = Q_{nm}R_{ms})$, we can write the scheduling feasibility in the following form:

$$\boldsymbol{Px} < 1 \tag{1}$$

IV. Rate Allocation and Scheduling Framework

In [7], the authors proposed a scheduling algorithm named Least Overlapped First (LOF) and a rate allocation strategy for multiple multicast sessions. Thereafter, we use LOF to denote the proposed framework including both the rate allocation and the scheduling algorithms. We choose LOF for comparison since, to the best of our knowledge, it is the only framework that considers both rate allocation and scheduling for multicast traffic in TDMA based wireless mesh networks. Note that LOF does not take multi-rate multicast into consideration. For the purpose of comparison, we extend the LOF framework in the following subsection to accommodate multi-rate multicast.

In contrast to the LOF framework, our framework first derives the rate allocation using the Network Utility Maximization where the allocation constraint is introduced in Section III-C. We then propose a scheduling algorithm based on Optimal Graph Coloring (OGC). Thereafter, we use OGC to denote our proposed framework including both the rate allocation and the scheduling algorithms.

A. Least Overlapped First

LOF first enumerates all the independent sets in the contention graph. Note that LOF works with the original contention graph without triangulation. For a successful schedule, all transmissions need to be included and each transmission can be included only once. To maximize spatial reuse, LOF tries to construct a schedule using few independent sets as possible.

Each independent set is assigned a rank, which is equal to the number of common transmissions this set has with all other independent sets of the same size. At each step of the scheduling algorithm, LOF selects the independent set which has the maximum number of transmissions that do not intersect with the members of the existing schedule. If there is more than one independent set with the same number of non-intersecting transmissions, LOF selects the one with the lowest rank to be added to the schedule breaking ties arbitrarily. LOF terminates when all transmissions are included in the schedule.

In [7], all sessions are implicitly assigned the same rate, which is determined by the length of the scheduling period and the duration of a time slot. We generalize LOF to make it work with different link rates and a fixed length scheduling period. For every independent set added into the schedule, define its rate as the lowest rate among all the transmissions in the set. Denote a schedule as $I = \{I_1, ..., I_K\}$, where I_i represents the independent sets that are included in the schedule. For each independent set I_i in the schedule, denote its rate as b_i. The session rate r is then determined by solving the following equation

$$\sum_{i=1}^{K} \frac{r}{b_i} = N$$

where N is the total number of available time slots in a scheduling period.

For the example scenario in Figure 1 and its corresponding contention graph in Figure 3, the set of all independent sets is: { {0}, {1}, {2}, {3}, {4}, {5}, {2,4}, {2,5}, {3,5}, {1,5} }. Independent set {2,4} has rank 1; independent sets {3,5} and {1,5} have rank 2 and independent set {2,5} has rank 3. LOF will construct the schedule {{3,5},{2,4},{0}, {1}}. Assuming that there are 100 time slots in a scheduling period, the session rate vector is $(40b, 40b)$. The schedule of a period is shown in Figure 4.

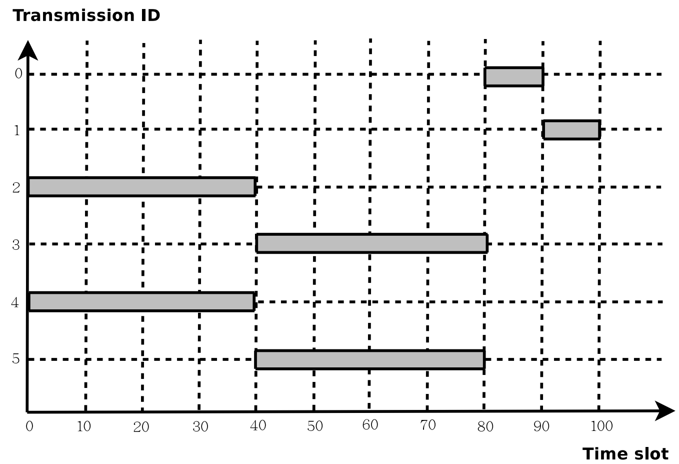

Fig. 4. The Schedule Obtained by LOF

B. Utility Maximization Based Rate Allocation

We develop our rate allocation algorithm based on Network Utility Maximization (NUM) [5]. We associate a utility function $U_s(x_s)$ to every session $s \in S$. Similar to [4], we cast the problem of rate allocation as that of maximizing the total utility of the network. When we consider both unicast and multicast sessions, it has been pointed out that it is appropriate to use *receiver-oriented* utility functions such as $U_s(x_s) = r_s u(x;\alpha)$, where r_s is the number of recipient of session s and $u(x;\alpha)$ is defined as follows [4]:

$$u(x;\alpha) = \begin{cases} \ln(x), & \alpha = 0 \\ -\frac{x^{-\alpha}}{\alpha}, & \alpha > 0 \end{cases}$$

The parameter α provides a mechanism to tune the unfairness to unicast while retaining the incentive to use multicast [4]. In this paper, we choose $\alpha = 0$ for the following reasons: First, it provides incentive to use multicast since some amount of unfairness to unicasts is achieved [4]. Second, when our framework is applied to networks containing only unicast traffic, it achieves proportional fairness among unicast traffic [27]. However, our framework can also accommodate other values of α if the resulting utility function is concave.

The utility maximization objective can be formally written as

$$\textbf{P:} \max_{x_s \geq 0} \sum_{s \in S} U_s(x_s), \text{ subject to } \boldsymbol{Px} \leq 1 \qquad (2)$$

The above problem is referred to as the system primal problem. It is a typical convex optimization problem [28], which can be solved by using the Lagrange duality [14], [15]. The dual problem is defined as

$$\textbf{D:}\ \min_{\boldsymbol{\lambda} \geq 0} D(\boldsymbol{\lambda}) \tag{3}$$

where

$$\begin{aligned} D(\boldsymbol{\lambda}) &= \max_{x_s \geq 0} \left(\sum_{s \in S} U_s(x_s) - \boldsymbol{\lambda}^T (\boldsymbol{P}\boldsymbol{x} - 1) \right) \\ &= \sum_{s \in S} \max_{x_s \geq 0} (U_s(x_s) - x_s \sum_{n \in C} \lambda_n P_{ns}) + \sum_{n \in C} \lambda_n \end{aligned}$$

Let us define

$$\mu_s = \sum_{n \in C} \lambda_n P_{ns}$$

In fact, the Lagrange multiplier λ_n can be interpreted as the shadow price [14] of clique n. Also $x_s \mu_s$ may be interpreted as the total price charged for session s for transmitting at rate x_s.

To solve the dual problem, we use an iterative algorithm with the help of the gradient projection method [10]. The algorithm is described in Figure 5. A similar algorithm is used in [10]. The algorithm takes the inputs of the maximal clique set C, session set S, initial vector $\boldsymbol{x}$ and $\boldsymbol{\lambda}$.

By choosing an appropriate step size γ, starting from any initial vector $\boldsymbol{x}$ and $\boldsymbol{\lambda}$, the above iterative algorithm will converge to the optimal solution $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ [10], [13]. Furthermore, the solution is primal-dual optimal, which means $\boldsymbol{x}^*$ is also the optimal rate vector for the primal problem. The optimal solution can be obtained in worst-case polynomial-time complexity [28], [29].

C. Optimal Graph Coloring Based Scheduling

Vertex coloring of a graph assigns different colors to vertices so that no two adjacent vertices have the same color. The greedy coloring algorithm [22] is a heuristic for graph coloring with $O(m+n)$ time complexity. If we give greedy coloring a simplicial elimination ordering of the vertices, then the greedy algorithm yields an optimal coloring [30]. In other words, greedy graph coloring algorithm is optimal for chordal graphs [30].

Based on the above discussion, we propose a scheduling algorithm based on a greedy vertex coloring of the contention graph. The algorithm is described in Figure 6 where time slots are considered as colors to be assigned to different transmissions. Denote the set of time slots in a scheduling period as $P = \{1, ..., N\}$. The algorithm takes the inputs of the triangulated contention graph G_c, the transmission set T and the set of time slots P.

If the scheduling period contains N time slots, we can prove that graph G_e can be colored using at most N colors, which means that all transmissions can be scheduled in at most N time slots. In this case, we say that the scheduling algorithm realizes the rate allocation. The proof is given as follows:

According to the definition of perfect graph given by Claude Berge, perfect graph G can be colored with $\omega(G)$ colors, where $\omega(G)$ is the size of the largest clique in G [21]. Our rate allocation satisfies the clique feasibility constraint defined in Section III-C:

$$\sum_{i \in C_j} f_i \leq 1, \forall C_j \in C$$

where C is the set of maximal cliques of graph G_c. According to the construction of G_e, every maximal clique of G_e corresponds to a maximal clique of G_c. Assume that the largest clique of G_e is constructed from maximal clique C_j of G_c. The size of the largest clique of G_e can be calculated as

$$\sum_{i \in C_j} S_i$$

where S_i is the number of time slots allocated to the transmission that corresponds to vertex i in G_c. Since $S_i \leq f_i * N$ as shown in Section III-C, we have

$$\sum_{i \in C_j} S_i \leq N$$

Thus we have proved that the size of the largest clique of G_e is equal to or smaller than N. Therefore, all transmissions can be scheduled in at most N time slots.

Assume that a scheduling period contains 100 time slots. For the example scenario in Figure 1 and its corresponding contention graph in Figure 3, our rate allocation algorithm yields the session rate vector $(60b, 20b)$. The schedule obtained by OGC is shown in Figure 7.

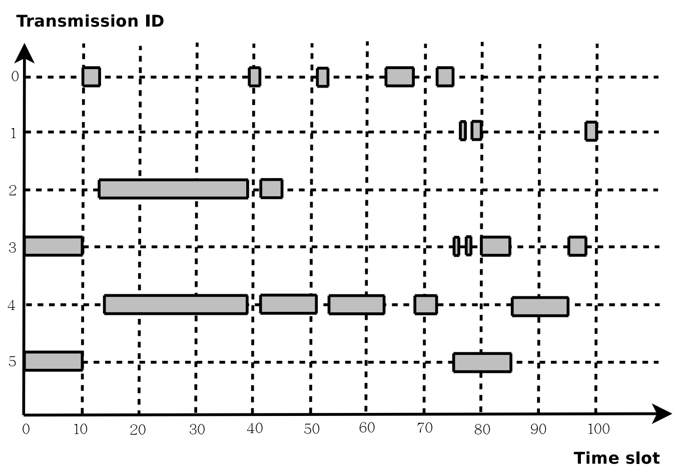

Fig. 7. The Schedule Obtained by OGC

V. Performance Evaluation

A. Simulation Setup

We use the ns-2 [31] simulator for performance evaluation. We simulate a flat area of 2000m by 2000m with 30 randomly positioned stationary mesh routers. A gateway node is placed in the center of the topology. We generate 100 random topologies, and our results represent the average performance over these 100 random topologies. Each simulation run lasts 10 seconds. In our simulation, we use a TDMA MAC protocol similar to IEEE 802.16. A MAC frame consists of a control

$t = 0$ *// t is the iteration number*
$\boldsymbol{x}(t) = \boldsymbol{x}$
$\boldsymbol{\lambda}(t) = \boldsymbol{\lambda}$
while $\boldsymbol{x}$ does not converge **do**
  for each clique $c \in C$ **do**
    Update λ_n as $\lambda_n(t+1) = [\lambda_n(t) - \gamma \frac{\partial D(\boldsymbol{\lambda}(t))}{\partial \lambda_n}]^+$ *// γ is the step size*
    In our case, $\lambda_n(t+1) = [\lambda_n(t) - \gamma(1 - \sum_{s \in S} x_s(t) P_{ns}]^+$
  end for
  for each session $s \in S$ **do**
    Update x_s by solving the problem $\max_{x_s \geq 0} (U_f(x_s) - x_s \mu_s)$
    In our case, $x_s(t+1) = \frac{r_s}{\mu_s}$
  end for
  $t = t + 1$
end while

Fig. 5. Algorithm - RateAllocation(C, S, $\boldsymbol{x}$, $\boldsymbol{\lambda}$)

$G_e = \emptyset$
for each vertex v in G_c **do**
  // Denote t_v as the ID the transmission that v corresponds to
  // Denote n_v as the number of time slots needed for transmission t_v
  $n_v = \frac{\lceil x_s / b_m \rceil}{N}$, where x_s is the rate of the session carried by transmission t_v
  Add n_v vertices to G_e, and their corresponding transmission is t_v
  // Denote this group of vertices as O_v
  Add edges between every pair of these vertices
end for
for each edge $e : (v_i, v_j)$ in G_c **do**
  Add edges between nodes in O_{v_i} and O_{v_j}
end for
// G_e is still a chordal graph based on its construction
Get a simplicial elimination ordering ν of G_e using the lexicographic BFS algorithm
// The following is the greedy graph coloring algorithm
for $i = 0$ to $|\nu|$ **do**
  $Q(\nu(i)) =$ nodes that are neighbors of $\nu(i)$
  $p =$ lowest time slot in P that is not used in $Q(\nu(i))$
  Assign time slot p to vertex $\nu(i)$
  Set transmission $t_{\nu(i)}$ to be active at time slot p
end for

Fig. 6. Algorithm - OptimalGraphColoring(G_c, T, P)

subframe and a data subframe, as shown in Figure 8. The duration of a frame corresponds to the scheduling period defined in our scheduling algorithm. A frame is divided into fixed-length time slots.

Similar to the centralized scheduling scheme used in IEEE 802.16 mesh networks, we implement our framework in a centralized manner. The benefits of a centralized implementation of the NUM based rate allocation has been discussed in [32].

In a control subframe, via control message exchange, mesh routers join multicast sessions or establish a unicast session between the gateway node and itself. The gateway node then constructs the routing tree, performs rate allocation and establishes the network wide schedule. The schedule is also propagated to mesh routers during the control subframe. Our rate allocation algorithm (including transmission set and perfect contention graph construction) is a polynomial-time algorithm while our scheduling algorithm is a linear time algorithm. In addition our framework operates on aggregated traffic between the mesh routers and the gateway. Thus our framework is very efficient and is suitable for a centralized implementation. Since our framework can be executed at the beginning of every frame, it can accommodate dynamic scenarios where network traffic changes frequently.

The detailed simulation parameters related to the physical and MAC layer properties are given in Table II. Table III shows how the type of modulation and link rate are related to the received SNR [16]. CBR is used as the traffic source. We use aggregated throughput, average delay and total utility as the evaluation metrics. We compare the performance of LOF and OGC under different network scenarios.

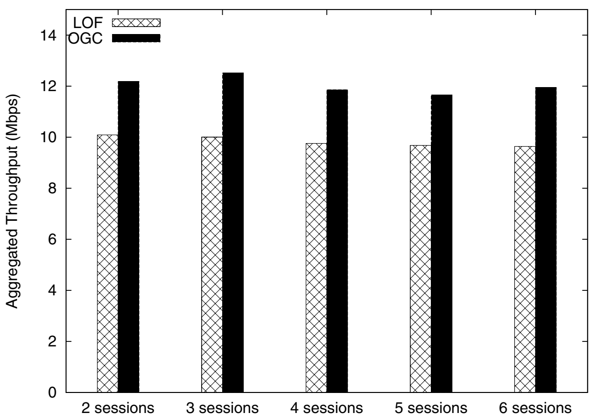

Fig. 9. Aggregated Throughput Comparison as a Function of Number of Sessions

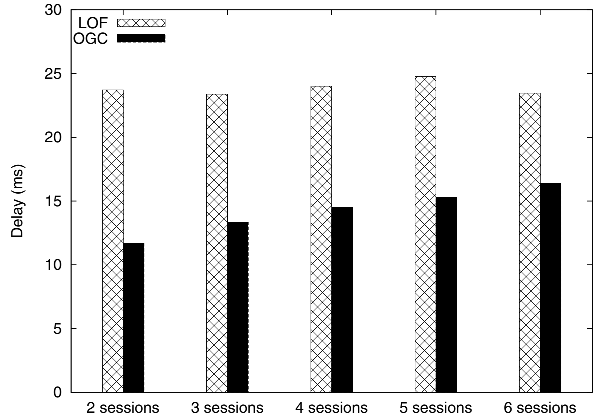

Fig. 10. Average Delay Comparison as a Function of Number of Sessions

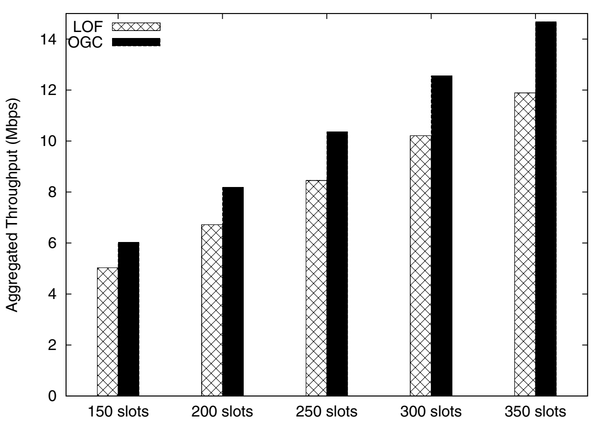

Fig. 11. Aggregated Throughput Comparison as a Function of Number of Time Slots

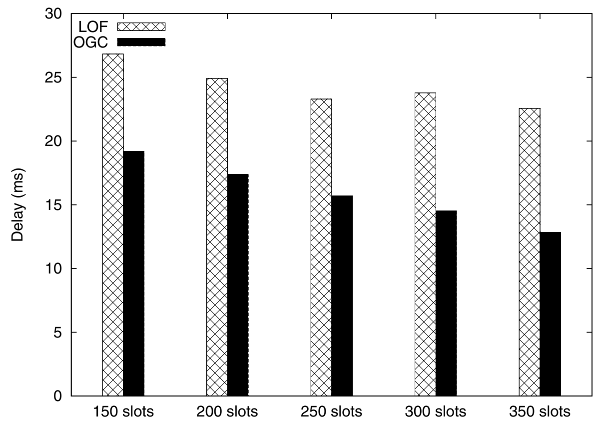

Fig. 12. Average Delay Comparison as a Function of Number of Time Slots

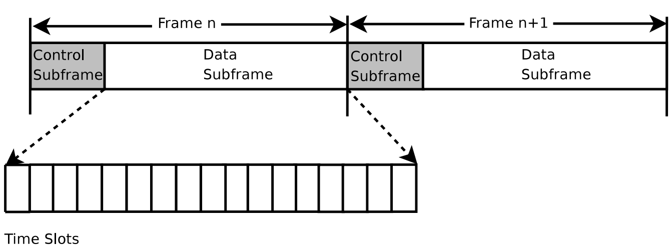

Fig. 8. MAC Frame Structure

TABLE II
SIMULATION PARAMETERS

Parameter	Value
Frequency band (GHz)	5
Channel bandwidth (MHz)	20
Frame duration (ms)	20
Time slot duration (μs)	50
Transmit power (dBm)	20
Path Loss Model	28.3 log(d) + 41.9, d is the distance
Noise Level (dBm)	bandwidth$*\frac{4*10^{-12}}{10^{9}}$, according to [33]

B. Simulation Results

In the first set of simulations, we vary the number of sessions in the networks from 2 to 6. In our simulations, a frame consists of 400 time slots where 300 of them are allocated to downlink data traffic. The source of these sessions is the gateway node, and we randomly choose 1 to 5 nodes as the recipients for each session. The aggregated throughput of session recipients achieved by using LOF and OGC under different number of sessions are compared in Figure 9. The performance improvement of OGC over LOF varies from 25% to 30%. LOF fails to take advantage of multi-rate multicast and guarantees equal throughput to all receivers. OGC achieves higher aggregated throughput by maximizing the total utility of the network and using a link rate aware scheduling algorithm. In our simulations, we observe that for both LOF and OGC, the throughput of a recipient is always equal to the allocated rate of its corresponding session. This demonstrates that both LOF and OGC effectively schedule the allocated rates.

Figure 10 compares the average delay of recipients achieved

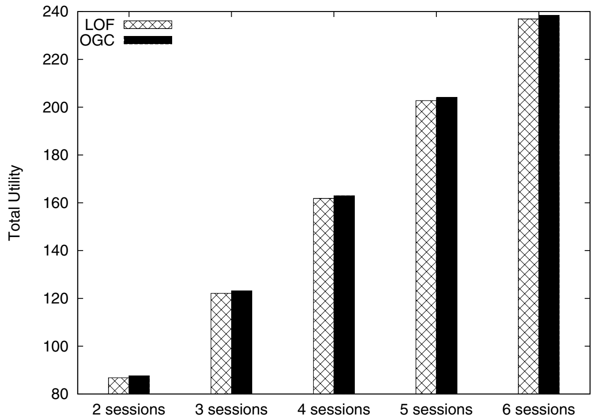

Fig. 13. Total Utility Comparison as a Function of Number of Sessions

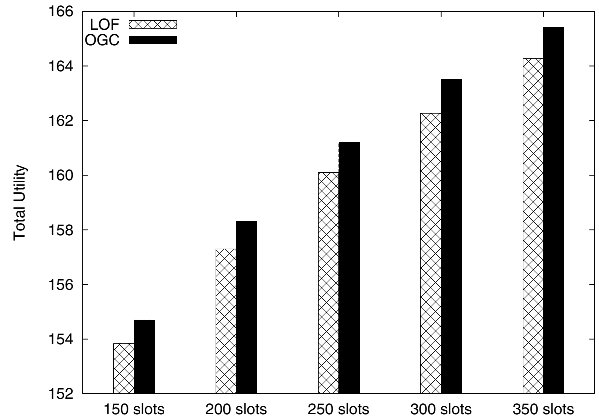

Fig. 14. Total Utility Comparison as a Function of Number of Time Slots

TABLE III
MODULATION TYPE AND LINK RATE VS. SNR

Modulation	Bit-rate (Mbps)	Received SNR (dB)
BPSK 1/2	7.68	3
QPSK 1/2	15.36	6
QPSK 3/4	23.04	8.5
16QAM 1/2	30.72	11.5
16QAM 3/4	46.08	15
64QAM 2/3	61.44	19
64QAM 3/4	69.12	21

by using LOF and OGC. OGC results in a much smaller average delay than LOF. Neither LOF nor OGC considers the ordering of transmissions in a frame to reduce the chances that mesh routers transmit before receiving data from the parent node in the routing tree. However, since OGC schedules a transmission multiple times in a frame while LOF schedules a transmission only once in a frame, transmission order has much less impact on OGC than on LOF. Figure 13 shows the total utility achieved by using LOF and OGC in different scenarios. OGC consistently achieves higher utility than LOF since the rate allocation in OGC is based on maximizing total utility.

We conduct another set of simulations by varying the number of time slots allocated for downlink data traffic from 150 to 350 time slots. In this set of simulations, we fix the number of sessions in the network to be 4. The aggregated throughput of session recipients achieved by using LOF and OGC under different numbers of time slots are compared in Figure 11. The performance improvement of OGC over LOF varies from 20% to 25%. In addition, aggregated throughput increases with the number of available time slots. Similar to the previous set of simulations, for both LOF and OGC, the throughput of a recipient is always equal to the allocated rate of its corresponding session.

Figures 12 and 14 show the average delay and total utility achieved by using LOF and OGC under different numbers of time slots. Similar to the previous set of simulations, OGC consistently achieves lower delay and higher utility than LOF.

VI. CONCLUSION

In this paper we proposed a framework for rate allocation and scheduling in TDMA based wireless mesh networks. By using transmission contention graphs to model the resource allocation constraints and by maximizing the total utility of the network, our framework provides fair and efficient rate allocation to multicast and unicast sessions. The proposed graph coloring based scheduling algorithm produces an optimal schedule to support the allocated rates. By maintaining perfect contention graphs, our framework enables rate allocation and scheduling with low time complexity. Our framework significantly outperforms LOF in terms of throughput and delay in our simulation results. In this paper, we assume that routing is considered separately from our framework. We plan to investigate the joint optimization of routing, rate allocation and scheduling in TDMA based wireless mesh networks in the future.

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