Quadrotor LPV Control using Static Output Feedback

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***Abstract*—This paper addresses the problem of attitude/altitude control of a quadrotor. The main contribution consists in developing a simple Linear Parameter Varying model which includes the motor dynamics and weight variations. Afterwards, a reference model is introduced and an error model is derived. An integral action is thus naturally included in the loop. The proposed controller takes the form of a static output feedback which is synthesised using the Linear Matrix Inequalities framework. Thanks to a relaxation method the nonlinear terms are removed from the matrix inequalities. The controller in then reconstructed as a combination of the integral of the error, the actual output and the preview reference signal.**

Simulations are conducted for a scenario showing the ability of the design method to handle different performance objectives.

***Index Terms*—UAV, LMI, Preview Control, Takagi-Sugeno**

I. Introduction

UAVs had a tremendous development in the last decade. This concerns both fun, professional and industrial applications. Among them, drones are expected to be useful in rescue missions and operations in hostile environments. Their use is also beginning to be democratized for autonomous missions in the context of precision agriculture. A drone can carry different loads according to the needs of analysis and the duration of the mission. Its mass and inertial parameters can then undergo strong variations.

The control of the UAVs has at the same time experienced several developments implementing different types of control laws. In [13], the problem of attitude and altitude control was addressed. An error model simplifies the problem and a Linear Parameter Varying (LPV) controller has been proposed under the form of a state feedback. Model predictive robust controller is developed in [14] to solve the path following problem for a quadrotor [6]. More recently stochastic model predictive control is used in [17].

In addition, LMI based robust control methods have been for example considered in [8]. Adaptive control has been treated in [9]. Different nonlinear control methods have been applied for the control of quadrotors. In [15], a high order sliding mode control is used after performing a feedback linearization. In [5], an integral backstepping control is proposed for controlling the attitude, altitude and lateral modes of the quadrotor. This has been also considered in [4]. In [7], model uncertainties and disturbances are handled with sliding mode observer. In addition, fault estimation and fault tolerant control have been considered in [16] and [10].

The aim of this paper is to propose a simple design procedure of static output feedback for quadrotor. The use of static output feedback controller simplifies greatly control law reading and implementation [1], [11]. However, the synthesis is known as a hard problem but some relaxation procedures allow to reduce conservatism [12]. In our context, the main objective of the static output feedback synthesis is to handle the mass and rotors velocity variations which are assumed to be measured. This is achieved using a Takagi-Sugeno (TS) formalism which allows to obtain a linear parameter varying model in the form of a convex sum of submodels [19]. The controller is then synthesized on the basis of a 4 submodels system.

The remainder of the paper is organized as follows. Section II presents the classical quadrotor model which is simplified in order to obtain a TS model suitable for control synthesis. Section III is devoted to the reference model tracking problem setting and model adaptation. Thus, the controller is synthesized in section IV while simulation results are presented in section V. Finally conclusions and some future work proposals wrap-up the paper.

II. System model and problem statement

This paper aims the development of a static output feedback controller for a quadrotor. The vehicle has six degrees of freedom and only four actuators [2]. It is thus underactuated. It has to pitch in order to move forward and to roll in order to move laterally. The vertical movements is ensured by equal thrust on the propellers. The three rotational movement are governed by differential thrust between left and right motors for the roll, the front and the rear for the pitch and differential torque clockwise and counter clockwise for the yaw.

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A. Quadrotor model

The three rotational motions are the pitch, roll and yaw where angles are denoted θ, ϕ and ψ respectively. The global position coordinates of the center of mass are denoted x_c for the longitudinal motion, y_c for the lateral motion and z_c for the vertical motion. This position is given in the global frame. Expanding the rigid body motion of the quadrotor leads to six equations of motion given in the inertial frame and described by

- Translation

$$\left\{\begin{array}{lcl} \ddot{x}_c & = & \left(s_\phi s_\psi + c_\phi s_\theta c_\psi\right)\frac{T_z}{m} \\ \ddot{y}_c & = & \left(s_\phi c_\psi + c_\phi s_\theta s_\psi\right)\frac{T_z}{m} \\ \ddot{z}_c & = & -g + \left(c_\phi c_\theta\right)\frac{T_z}{m} \end{array}\right. \tag{1}$$

- Rotation

$$\left\{\begin{array}{lcl} \ddot{\phi} & = & \dot{\theta}\dot{\psi}\left(\frac{J_y - J_z}{J_x}\right) - \frac{J_r}{J_x}\dot{\theta}\Omega_r + l\frac{T_\phi}{J_x} \\ \ddot{\theta} & = & \dot{\phi}\dot{\psi}\left(\frac{J_z - J_x}{J_y}\right) - \frac{J_r}{J_y}\dot{\phi}\Omega_r + l\frac{T_\theta}{J_y} \\ \ddot{\psi} & = & \dot{\phi}\dot{\theta}\left(\frac{J_x - J_y}{J_z}\right) + l\frac{T_\psi}{J_z} \end{array}\right. \tag{2}$$

where c_x and s_x stand for $\cos x$ and $\sin x$ respectively, with $x = \{\phi, \theta, \psi\}$. m is the mass, g is the gravity while J_x, J_y and J_z are the moments of inertia of the vehicle. J_r is the moment inertia of the propellers and Ω_r is the sum of the propellers velocities.

$$\Omega_r = \Omega_2 + \Omega_4 - \Omega_1 - \Omega_3 \tag{3}$$

where Ω_i denotes the i-th rotor velocity.

The force T_z is the body frame vertical thrust. It is obtained from the equation

$$T_z = k_f\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) = \sum_{i=1}^{4} T_i \tag{4}$$

and thrusts appearing in the equations are also expressed in the body frame and are denoted T_ϕ, T_θ and T_ψ. They are given by

$$\begin{array}{lcl} T_\phi & = & k_f\left(\Omega_4^2 - \Omega_2^2\right) = T_4 - T_2 \\ T_\theta & = & k_f\left(\Omega_3^2 - \Omega_1^2\right) = T_3 - T_1 \\ T_\psi & = & k_z\left(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2\right) = (T_2 + T_4) - (T_1 + T_3) \end{array}$$

As considered in [13], we restrict the purpose of the paper to the altitude and attitude tracking [3]. Thus, the equations related to the longitudinal and lateral translation motions are removed.

B. Actuator model

Adopting an actuator model has twofold. First of them is to reflect the low pass filtering of each actuator with a time constant τ_i. The second allows us to prevent the B matrix of the obtained state-space representation to be parameter dependent. Each actuator thrust Laplace transform is given by

$$\mathcal{T}_i(s) = \frac{K_i}{1 + \tau_i s} V_i(s), \quad i = 1, 2, 3, 4 \tag{5}$$

$\mathcal{T}_i(s)$ is the Laplace transform of the thrust $T_i(t)$, V_i is the pulse width modulation (PWM) voltage applied to rotor i, K_T is the armature gain. The corresponding differential equation is

$$\dot{T}_i = -\frac{1}{\tau_i} T_i + \frac{K_i}{\tau_i} v_i \tag{6}$$

Notice that the time constant and the armature gain can be considered as different for each rotor in order to handle potential actuator fault.

C. More simplified model

The previous model still exhibits too much parameters and its polytopic representation will involve 2^9 submodels. If one considers control synthesis using Linear Matrix Inequalities methods, the resolvability of the resulting LMI conditions in this case is quite compromised due to conservativeness of conditions which will request the common stabilization of a huge number of submodels. In order to reduce this number, we adopt here a more simplified model. First of all, the simplification makes each of the modes decoupled from the product of the velocities, allowing to write:

$$\left\{\begin{array}{lcl} \ddot{\phi} & = & -\frac{J_r}{J_x}\dot{\theta}\Omega_r + l\frac{(T_4 - T_2)}{J_x} \\ \ddot{\theta} & = & -\frac{J_r}{J_y}\dot{\phi}\Omega_r + l\frac{(T_1 - T_3)}{J_y} \\ \ddot{\psi} & = & l\frac{(T_2 + T_4) - (T_1 + T_3)}{J_z} \\ \ddot{z}_c & = & -g + \frac{T_1 + T_2 + T_3 + T_4}{m} \end{array}\right. \tag{7}$$

Gathering all the equations, one can establish the LPV model

$$\dot{x}(t) = A(m, \Omega_r)\, x(t) + Bv(t) + Ed(t) \tag{8}$$

where the state vector is has twelve components: $x = \left(\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, z_c, \dot{z}_c, T_1, T_2, T_3, T_4\right)^T$, the control input vector is composed of the four motor voltages $v = (v_1, v_2, v_3, v_4)^T$, the disturbance input is the Earth gravity $d = g$, and the matrix $A(m, \Omega_r)$ is given by

$$A(m, \Omega_r) = \left(\begin{array}{ccc} A_{11} & A_{12} & A_{13} \end{array}\right)$$

$$A_{11} = \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{J_r}{J_x}\Omega_r \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{J_r}{J_y}\Omega_r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right), A_{12} = \left(\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$A_{13} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{l}{J_x} & 0 & \frac{l}{J_x} \\ 0 & 0 & 0 & 0 \\ \frac{l}{J_y} & 0 & -\frac{l}{J_y} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{l}{J_z} & \frac{l}{J_z} & -\frac{l}{J_z} & \frac{l}{J_z} \\ 0 & 0 & 0 & 0 \\ \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\ -\frac{1}{\tau_1} & 0 & 0 & 0 \\ 0 & -\frac{1}{\tau_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_4} \end{pmatrix}$$

while

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{K_1}{\tau_1} & 0 & 0 & 0 \\ 0 & \frac{K_2}{\tau_2} & 0 & 0 \\ 0 & 0 & \frac{K_3}{\tau_3} & 0 \\ 0 & 0 & 0 & \frac{K_4}{\tau_4} \end{pmatrix}, \quad E = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The main objective of the control design procedure is to synthesize a controller that could be scheduled according to mass, inertia and rotor speed variation. As it can be seen from the simplified quadrotor model, it is linear of both parameters. In addition, parametrizing the moment of inertia in the form $J_i = \kappa_i m$, $(i = \{x, y, z\})$, one can thus obtain a LPV model depending on two parameters, the mass $m \in [m_m, m_M]$ and the velocity $\Omega_r \in [\Omega_m, \Omega_M]$.

Thus a Takagi-Sugeno model with fours submodels could be obtained. depending on the extremal values of the parameters. This representation is called nonlinear sector approximation [19]. In fact, defining $\rho_1 = m \in [m_m, m_M]$ and $\rho_2 = \Omega_r/m \in \left[\frac{\Omega_m}{m_M}, \frac{\Omega_M}{m_m}\right]$, a four submodels TS system is achieved

$$\dot{x}(t) = \left(\sum_{i=1}^{4} \mu_i \bar{A}_i\right) x(t) + \bar{B}u(t) + \bar{E}d(t) \tag{9}$$

where $\mu_i \geq 0$, $1 \leq i \leq 4$, $\sum_{i=1}^{4} \mu_i = 1$ and

$$\begin{aligned} \mu_1 &= m_{11}m_{21}, \quad \mu_2 = m_{11}m_{22} \\ \mu_3 &= m_{12}m_{21}, \quad \mu_4 = m_{12}m_{22} \end{aligned}$$

with

$$\begin{aligned} m_{11} &= \frac{\rho_{1max} - \rho_1}{\rho_{1max} - \rho_{1min}}, \quad m_{12} = 1 - m_{11} \\ m_{21} &= \frac{\rho_{2max} - \rho_2}{\rho_{2max} - \rho_{2min}}, \quad m_{22} = 1 - m_{21} \end{aligned}$$

The matrices $\bar{A}_i$, $1 \leq i \leq 4$ are obtained from

$$\begin{aligned} \bar{A}_1 &= A(m_m, \Omega_m) \\ \bar{A}_2 &= A(m_m, \Omega_M) \\ \bar{A}_3 &= A(m_M, \Omega_m) \\ \bar{A}_4 &= A(m_M, \Omega_M) \end{aligned}$$

Notice that the obtained model is an exact representation of the model of equation (8). The obtained model is a continuous time model. However for implementation aspects and when considering preview information, it is more convenient to consider it in discrete time domain. In fact the preview information is only available at some sensor sample times. Knowing that, the model is discretized using a simple Euler method, this leads to the discrete-time state-space model with sample time of $T = 0.05\,\text{sec}$.

$$\begin{cases} x(k+1) = \left(\sum_{i=1}^{4} \mu_i A_i\right) x(k) + Bu(k) + Ed(k) \\ y(k) = Cx(k) \end{cases} \tag{10}$$

where $(A_i, B, E) = \left(I_{12} + T\bar{A}_i, T\bar{B}, T\bar{E}\right)$.

The output vector is constituted by the quadrotor altitude and attitude position which are obtained from the matrix

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

III. Controller design

The considered model has the gravity a constant disturbance input. In order to cancel it in the control design and obtaining a zero steady state error, one can take the difference operator on both sides of equation (10) which leads to:

$$\begin{cases} \Delta x(k+1) = \left(\sum_{i=1}^{4} \mu_i A_i\right) \Delta x(k) + B\Delta u(k) \\ \Delta y(k) = C\Delta x(k) \end{cases} \tag{11}$$

We consider in this paper the problem of reference signal tracking with preview information. The output signal $y(k)$ has to follow the predefined reference signal $r(k)$. Defining the error signal $e(k)$ as

$$e(k) = y(k) - r(k) \tag{12}$$

and writing the error dynamics, one can define the augmented plant with the augmented state vector $\tilde{x} = \left[e^T(k), \Delta x^T(k)\right]^T$ which reads

$$\begin{cases} \tilde{x}(k+1) = \sum_{i=1}^{4} \mu_i \left(\tilde{A}_i \tilde{x}(k) + \tilde{B}\Delta u(k) + G_p \Delta r(k)\right) \\ e(k) = \tilde{C}\tilde{x}(k) \end{cases} \tag{13}$$

where

$$\tilde{A}_i = \begin{bmatrix} I_4 & C \\ 0 & A_i \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}, G_p = \begin{bmatrix} -I_4 \\ 0 \end{bmatrix},$$

$$\tilde{C} = \begin{bmatrix} I_4 & 0 \end{bmatrix}$$

Suppose now that the reference signal values are known n_p samples ahead. Let us define the vector

$$x_r(k) = \left[\begin{array}{ccc} \Delta r^T(k) & \ldots & \Delta r^T(k+n_p) \end{array} \right]^T$$

and the matrix

$$A_r = \left[\begin{array}{ccccc} 0 & I_4 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & I_4 \\ 0 & \cdots & \cdots & 0 & 0 \end{array} \right]$$

The objective now is to embed the reference signal appearing in equation (13) into a state vector. This is achieved by defining the state vector

$$\hat{x}(k) = \left[\begin{array}{cc} \tilde{x}^T(k) & x_r^T(k) \end{array} \right]^T$$

which allows to obtain the augmented system

$$\left\{ \begin{array}{l} \hat{x}(k+1) = \sum_{i=1}^4 \mu_i \left(\hat{A}_i \hat{x}(k) + \hat{B} \Delta u(k) \right) \\ y_p(k) = \hat{C} \hat{x}(k) \end{array} \right. \tag{14}$$

where

$$\hat{A}_i = \left[\begin{array}{cc} \tilde{A}_i & G_p \\ 0 & A_r \end{array} \right], \hat{B} = \left[\begin{array}{c} \tilde{B} \\ 0 \end{array} \right], \hat{C} = \left[\begin{array}{ccc} I_q & & \\ & C & \\ & & I_{(n_p q+q)} \end{array} \right]$$

The aim now is to design a static output feedback controller of the form

$$\Delta u(k) = \left(\sum_{i=1}^4 \mu_i K_i \right) y_p(k) \tag{15}$$

making the closed-loop system

$$\hat{x}(k+1) = \sum_{i=1}^4 \mu_i \left(\hat{A}_i + \hat{B} K_i \hat{C} \right) \hat{x}(k) \tag{16}$$

robustly asymptotically stable.

Following the results provided in [18], given a scalar β and matrices Q, W, if there exist $P_i > 0$, U_i, L_i, G_i, $1 \leq i \leq 4$ such that

$$\Pi_i < 0 \qquad (1 \leq i \leq 4) \tag{17}$$

where

$$\Pi_i = \left(\begin{array}{ccc} -G_i - G_i^T + P_i & * & * \\ \hat{A}_i G_i + \hat{B}_i L_i Q & -P_i & * \\ \hat{C} G_i - UQ & \beta W^T L_i^T \hat{B}^T & \Theta \end{array} \right) \tag{18}$$

and $\Theta = -\beta UW - \beta W^T U^T$, then the system (16) is robustly asymptotically stable, and the gain matrix can be obtained by

$$K_i = L_i U^{-1} \tag{19}$$

One can notice that this matrix gain involves the following components

$$K_i = \left(\begin{array}{ccccc} K_{ei} & K_{yi} & K_{ri}(0) & \ldots & K_{ri}(n_p) \end{array} \right) \tag{20}$$

thus the command increment is given by

$$\Delta u(k) = K_e e(k) + K_y \Delta y(k) + \sum_{j=0}^{n_p} K_r(j) \Delta r(k+j) \tag{21}$$

where

$$K_e = \sum_{i=1}^4 \mu_i K_{ei}, \quad K_y = \sum_{i=1}^4 \mu_i K_{yi},$$

$$K_r(j) = \sum_{i=1}^4 \mu_i K_{ri}(j)$$

and finally the control input is obtained from

$$u(k) = K_e \sum_{j=0}^{n_p} e(k) + K_y y(k) + \sum_{j=0}^{n_p} K_r(j) r(k+j) \tag{22}$$

IV. Practical controller design

The Takagi-Sugeno model is considered assuming that the mass varies in the interval $[m_m, m_M]$ with $m_M = 1$ kg and $m_M = 2$ kg. The total propeller velocity is varying in the interval $[\Omega_m, \Omega_M]$ with $\Omega_m = -0.2$ and $\Omega_M = 0.2$. The controller is designed using the procedure developed above. The preview horizon is taken up to $n_p = 5$ points.

V. Testing scenario

As stated in the introduction, this paper is concerned with the attitude/altitude control. It is then assumed that the reference values for the pitch, roll, yaw and attitude are provided by an external loop. The quadrotor is requested to follow a circular path while increasing its altitude for 5 m to 10 m. The path is shown in Figure 1. One can see that the reference path is well followed. This can be verified in the figure 2 where the absolute value does not exceed 0.1 m. The altitude reference is also well followed as presented in figure 3. This is also the case for the pith angle (Figure 4). The reference value set for the yaw angle is 0. This is reached in less than one second (Figure 6).

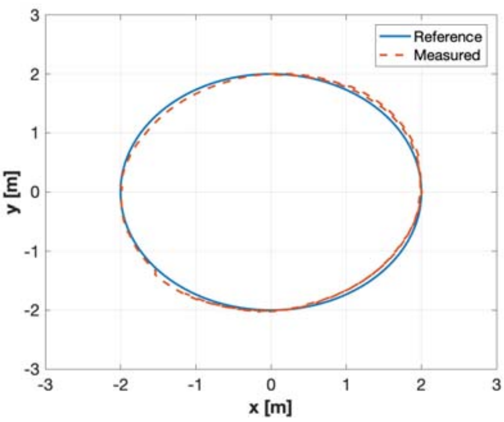

Fig. 1. Measured and reference path

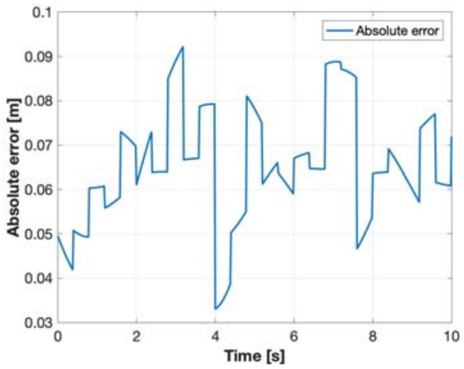

Fig. 2. Absolute error on path following

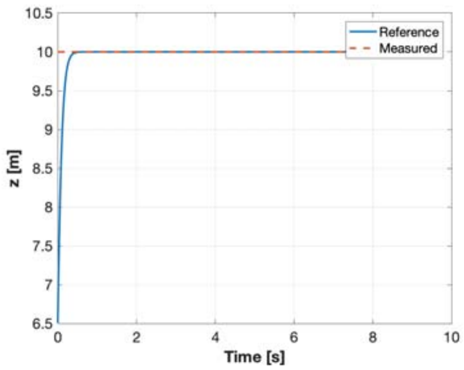

Fig. 3. Measured and reference value for the altitude

The same simulation is now conducted by increasing the quadrotor mass until 2 kg. The system responses in this case are shown in figures 7 to 10. The responses become more oscillatory but remain acceptable.

VI. Conclusion

This paper addresses the problem of attitude/altitude control of an UAV. The focus is put on handling mass variation of the UAV according to the specific application of transporting

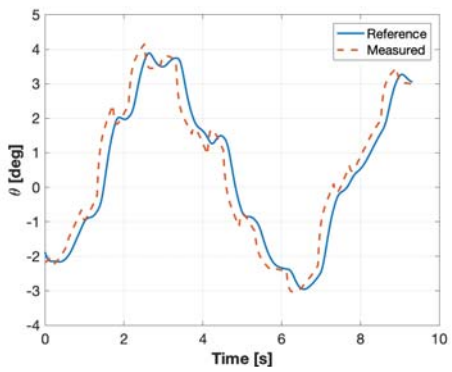

Fig. 4. Measured and reference values for the pitch angle

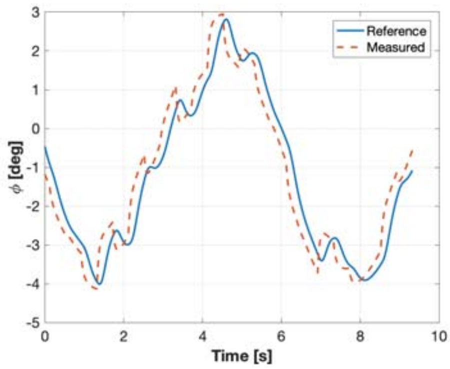

Fig. 5. Measured and reference values for the roll angle

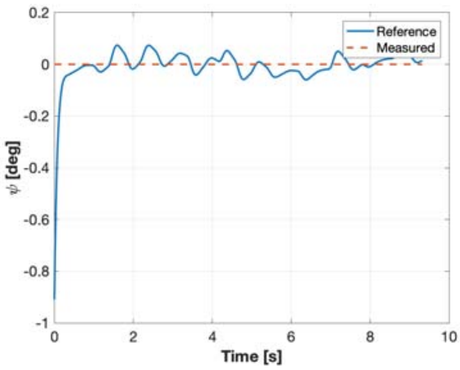

Fig. 6. Measured and reference value for the pitch angle

different device types. Following an incremental model simplification, a simple LPV model is obtained for the selected motions. Afterwards, a 4 submodels Takagi-Sugeno model is derived. Thus the problem of reference tracking is formulated for static output feedback. It is solved using LMI conditions framework. The obtained controller is found to be able to follow the prescribed trajectory with high level of performance. Future works will both concern the exploration of reactive path planning realization and fault tolerant control.

Annex

The quadrotor parameters are listed in the following table.

TABLE I
Quadrotor model parameters

Parameter	Value
J_x	0.03 kg.m^2
J_y	0.03 kg.m^2
J_z	0.04 kg.m^2
$\frac{1}{\tau_i}, (1 \leq i \leq 4)$	15 rad/sec
l	0.2 m
m	1.4 kg

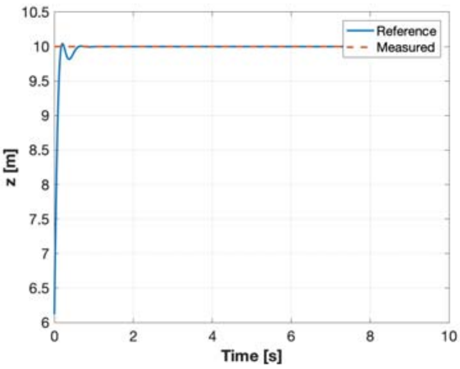

Fig. 7. Measured and reference value for the altitude for added mass

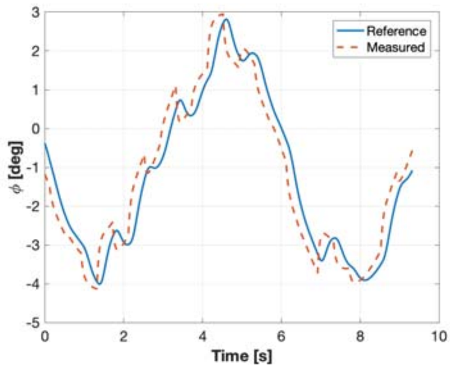

Fig. 9. Measured and reference values for the roll angle for added mass

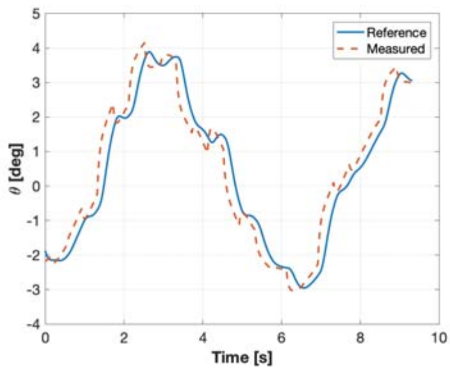

Fig. 8. Measured and reference values for the pitch angle for added mass

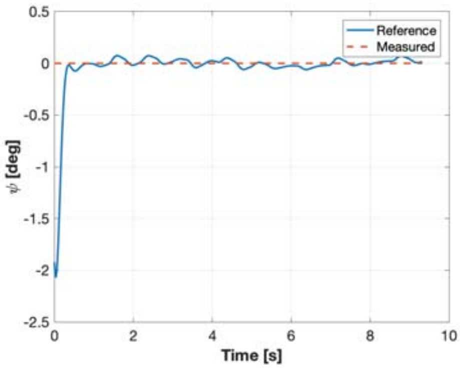

Fig. 10. Measured and reference value for the pitch angle for added mass

REFERENCES

[1] X.H. Chang, and G.H. Yang, "New results on output feedback H_∞ control for linear discrete-time systems,"IEEE Trans. Autom. Control, 2014, 59, (5), pp. 1355–1359.

[2] A. Honglei, L. Jie, M. Hongxu, and C. Kewei, "Quaternion based geometric control of quadrotor UAV," Proc. 24th Chinese Control Decis. Conf., Taiyuan, China, pp. 1366–1370 (2012).

[3] A. Tayebi, and S. McGilvray, "Attitude stabilization of a VTOL quadrotor aircraft," IEEE Trans. Control Syst. Technol., Vol. 14, No. 3, pp. 562–571 (2006).

[4] T. Madani, and A. Benallegue, "Control of a quadrotor mini-helicopter via full state backstepping technique," Proc. 45th IEEE Conf. Decis. Control, San Diego, CA, USA, pp. 1515–1520 (2006).

[5] S. Bouabdallah, and R. Siegwart, "Full control of a quadrotor," Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst., San Diego, CA, USA, pp. 153–158 (2007).

[6] G.V. Raffo, M.G.Ortega, and F.R. Rubio, "An integral predictive/nonlinear H_∞ control structure for a quadrotor helicopter," Automatica, Vol. 46, No. 1, pp. 29–39 (2010).

[7] L. Besnard, Y. B. Shtessel, and B. Landrum, "Quadrotor vehicle control via sliding mode controller driven by sliding mode disturbance observer," J. Frankl. Inst.-Eng. Appl.Math., Vol. 349, No. 2, pp. 658–684 (2012).

[8] T. Ryan, and H. J. Kim, "LMI-based gain synthesis for simple robust quadrotor control," IEEE Trans. Autom. Sci. Eng., Vol. 10, No. 4, pp. 1173–1178 (2013).

[9] W. Zeng, B. Xian, C. Diao, Q. Yin, H. Li, and Y. Yang, "Nonlinear adaptive regulation control of a quadrotor unmanned aerial vehicle," Proc. IEEE Int. Conf. Control Applicat. (CCA), Denver, CO, USA, pp. 133–138 (2011).

[10] Y. Zhang, and J. Jiang, "Bibliographical review on reconfigurable fault-tolerant control systems," Annu. Rev. Control, Vol. 32, No. 2, pp. 229–252 (2008).

[11] J.X. Dong, and G.H. Yang, "Static output feedback H_∞ control of a class of nonlinear discrete-time systems,"Fuzzy Sets Syst, 2009, 160, (19), pp. 2844–2859

[12] J.X. Dong, and G.H. Yang, "Robust static output feedback control synthesis for linear continuous systems with polytopic uncertainties,"Automatica, 2013, 49, (6), pp. 1821–1829.

[13] D. Rotondo, F. Nejjari, and V. Puig, "Model Reference Quasi-LPV Control of a Quadrotor UAV,"2014 IEEE Conference on Control Applications (CCA), October 8-10, Antibes, France.

[14] K. Alexis, G. Nikolakopoulos, and A. Tzes, "Model predictive quadrotor control: attitude, altitude and position experimental studies,"IET Contr. Theory Appl., Vol. 6, No. 12, pp. 1812-1827 (2012).

[15] A. Benallegue, A. Mokhtari, and L. Fridman, "High-order sliding-mode observer for a quadrotor UAV,"Int. J. Robust Nonlinear Control, Vol. 18, No. 4-5, pp. 427-440, 2008.

[16] K. Búukkabasakal, B. Fidan, and A. Savran, "Mixing adaptive fault tolerant control of quadrotor UAV,"Asian Journal of Control, Vol. 19, No. 5, pp. 1-14, 2017.

[17] Y. Yang, Y. Chen, Tang, C., and L. Y. Chai, "Quadrotor helicopters trajectory tracking with stochastic model predictive control,"2017 Australian and New Zealand Control Conference (ANZCC), pp. 166-171, 2017.

[18] L. Li, and F. Liao, "Parameter-dependent preview control with robust tracking performance,"IET Control Theory & Application, Vol. 11 Iss. 1, pp. 38-46, 2017.

[19] K. Tanaka, and H.O. Wang, "Fuzzy Control Systems Design and Analysis,"Wiley Inc, New York, 2001.