Efficient Distributed Data Structures for Future Many-core Architectures

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Abstract

We study general techniques for implementing distributed data structures on top of future many-core architectures with **non cache-coherent or partially cache-coherent memory**. With the goal of contributing towards what might become, in the future, the concurrency utilities package in Java collections for such architectures, we end up with a comprehensive collection of data structures by considering different variants of these techniques. To achieve scalability, we study a generic scheme which makes all our implementations *hierarchical*. We consider a collection of known techniques for improving the scalability of concurrent data structures and we adjust them to work in our setting. We have performed experiments which illustrate that some of these techniques have indeed high impact on achieving scalability. Our experiments also reveal the performance and scalability power of the hierarchical approach. We finally present experiments to study energy consumption aspects of the proposed techniques by using an energy model recently proposed for such architectures.

1 Introduction

The dominant parallelism paradigm used by most high-level, high productivity languages such as Java, is that of threads and cache-coherent shared memory among all cores. However, cachecoherence does not scale well with the number of cores [1]. So, future many-core architectures are not expected to support cache-coherence across all cores. They would rather feature multiple coherence islands, each comprised of a number of cores that share a coherent view of a part of the memory, but no hardware cache-coherence will be provided among cores of different islands. Instead, the islands will be interconnected using fast communication channels. In recent literature, we meet even more aggressive approaches with Intel having proposed two fully non cache-coherent architectures, Runnemede [2] and SSC [3]. Additionally, [4] presents the Formic board, a 512-core non cache-coherent prototype.

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Such architectures impose additional effort in programming them, since they require to explicitly code all communication and synchronization using messages between processors. Previous works [5, 6, 7] indicate the community's interest in bridging the gap between non cachecoherent or distributed architectures, and high-productivity programming languages by implementing runtime environments, like the Java Virtual Machine (JVM), for such architectures, which maintain the shared-memory abstraction.

The difficulty in parallelizing many applications comes from those parts of the computation that require communication and synchronization via data structures. Thus, the design of effective concurrent data structures is crucial for many applications. On this avenue, java.util.concurrent, which is Java's concurrency utilities package (JSR 166) [8, 9] provides a wide collection of concurrent data structures [8, 10, 11, 12]. To run Java-written programs on a non cache-coherent architecture, Java's VM must be ported to that architecture, and some fundamental communication and synchronization primitives (such as CAS, locks, and others) must be implemented. Normally, once this is done, it will be possible to execute applications that employ the concurrency utilities package without any code modification. However, the algorithms provided in the package have been chosen to perform well on shared memory architectures. So, they take no advantage of the communication and synchronization features of non cache-coherent architectures and do not cope with load balancing issues or with the distribution of data among processors. Thus, they are expected to be inefficient when executing through JVMs ported for such an architecture, even if optimized implementations of locks, CAS, and other primitives are provided on top of the architecture. Therefore, there is an urgent need to develop novel data structures and algorithms, optimized for non cache-coherent architectures.

We study general techniques for implementing distributed data structures, such as stacks, queues, dequeues, lists, and sets, on many-core architectures with non or partially cache-coherent memory. With the goal of contributing to what might become, in the future, the concurrency utilities package in Java collections for such architectures, we end up, by considering different variants of these techniques, ending up with a comprehensive collection of data structures, richer than those provided in java.util.concurrent. Our collection, which is based on message-passing to achieve the best of performance, facilitates the execution of Java-written code on non-cache coherent architectures, without any modification, in a highly efficient way.

To achieve *scalability*, that is, maintaining good performance as the number of cores increases [13], we study a generic scheme, which can be used to make all our implementations hierarchical [14, 15]. The *hierarchical* version of an implementation exploits the memory structure and the communication characteristics of the architecture to achieve better performance. Specifically, just one core from each island, the *island master*, participates in the execution of the implemented distributed algorithm, whereas the rest submit their requests to this core. To execute the implemented distributed algorithm, the island masters must exploit the communication primitives provided for fast communication among different islands. In architectures with thousands of cores, we could employ a more advanced hierarchical structure, e.g. a tree-like hierarchy, of intermediate masters for better scalability. Depending on the implemented data structure, the island master employs elimination [16], batching and other techniques to enhance scalability and performance. In the partially cache-coherent case, the cores of the same island may synchronize by employing combining [15, 17].

For efficiency, in some of our implementations, we employ a highly scalable distributed hash table (DHT) which uses a simple standard technique [18, 19, 20] to distribute the data on different nodes. Based on it and by employing counting networks [21, 22], we can come up with fully decentralized, scalable implementations of queues and stacks. We also present implementations of (sorted and unsorted) lists, some of which support complex operations like range queries. To design our algorithms, we derive a theoretical framework which captures the communication characteristics of non cache-coherent architectures. This framework may be of independent interest. In this spirit, we further provide full theoretical proofs of correctness for some of the algorithms we present.

We have performed experiments on top of a non cache-coherent 512-core architecture, built using the Formic [4] hardware prototyping board. We distill the experimental observations into a metric expressing the scalability potential of such implementations. The experiments illustrate nice scalability characteristics for some of the proposed techniques and reveal the performance and scalability power of the hierarchical approach.

2 Related Work

Distributed transactional memory (DTM) [23, 24, 25, 26, 27, 28, 29, 30] is a generic approach for achieving synchronization, so data structures can be implemented on top of it. Transactional memory (TM) [31, 32] is a programming paradigm which provides the transaction's abstraction; a transaction executes a piece of code containing accesses to data items. TM ensures that each transaction seems to execute sequentially and in isolation. However, it introduces significant performance overheads whenever reads from or writes to data items take place, and requires the programmer to write code in a transactional-compatible way. (When the transactions dynamically allocate data, as well as when they synchronize operations on dynamic data structures, compilers cannot detect all possible data races without trading performance, by introducing many false positives.) Our work is on a different avenue: towards providing a customized library of highly-scalable data structures, specifically tailored for non cache-coherent machines.

 TM^2C [26] is a DTM proposed for non cache-coherent machines. The paper presents a simple distributed readers/writers lock service where nodes are responsible for controlling access to memory regions. It also proposes two contention management (CM) schemes (Wholly and FairCM) that could be used to achieve starvation-freedom. However, in Wholly, the number of transaction T may abort could be as large as the number of transactions the process executing T has committed in past, whereas in FairCM, progress is ensured under the assumption that there is no drift [33, 34] between the clocks of the different processors of the non cache-coherent machine. Read-only transactions in TM^2C can be slow since they have to synchronize with the lock service each time they read a data item, and in case of conflict, they must additionally synchronize with the appropriate CM module and may have to restart several times from scratch. Other existing DTMs [23, 24, 35, 30, 36], also impose common DTM overheads.

The data structure implementations we propose do not cause any space overhead, readonly requests are fast, since all nodes that store data of the implemented structure search for the requested key in parallel, and the number of steps executed to perform each request is bounded. We remark that, in our algorithms, information about active requests is submitted to the nodes where the data reside, and data are not statically assigned to nodes, so our algorithms follow neither the data-flow approach [37, 36] nor the control-flow approach [23, 35] from DTM research.

Distributed directory protocols [38, 39, 40, 41, 42] have been suggested for locating and moving objects on a distributed system. Most of the directory protocols follow the simple idea that each object is initially stored in one of the nodes, and as the object moves around, nodes store pointers to its new location. They are usually based either on a spanning tree [43, 42] or a hierarchical overlay structure [38, 40, 41]. Remarkably, among them, COMBINE [38] attempts to cope with systems in which communication is not uniform. Directory protocols could potentially serve for managing objects in DTM. However, to implement a DTM system using a directory protocol, a contention manager must be integrated with the distributed directory implementation. As pointed out in [39], this is not the case with the current contention managers and distributed directory protocols. It is unclear how to use these protocols to get efficient versions of the distributed data structures we present in this paper.

Previous research results [44, 45, 46, 47] propose how to support dynamic data structures on distributed memory machines. Some are restricted on tree-like data structures [46, 47], other focus on data-parallel programs [44], some favor code migration, whereas other focus on data replication. We optimize beyond simple distributed memory architectures by exploiting the communication characteristics of non cache-coherent multicore architectures. Some techniques from [44, 45, 46] could be of interest though to further enhance performance, fault-tolerance and scalability in our implementations.

Distributed data structures have also been proposed [48, 49, 50, 51, 52] in the context of peer-to-peer systems or cluster computing, where dynamicity and fault-tolerance are main issues. They tend to provide weak consistency guarantees. Our work is on a different avenue.

Hazelcast [19] is an in-memory data grid middleware which offers implementations for maps, queues, sets and lists from the Java concurrency utilities interface. These implementations are optimized for fault tolerance, so some form of replication is supported. Lists and sets are stored on a single node, so they do not scale beyond the capacity of this node. The queue stores all elements to the memory sequentially before flushing them to the datastore. Like Hazelcast, GridGain [53], an in-memory data fabric which connects applications with datastores, provides a distributed implementation of queue from the Java concurrency utilities interface. The queue can be either stored on a single grid node, or be distributed on different grid nodes using the datastore that exists below GridGain. In a similar vein, Grappa [54] is a software system that can provide distributed shared memory over a cluster of machines. So-called delegate operations are used in order to access shared memory, relieving the programmer of having to reason about remote or local memory. However, Grappa does not provide the programmer with a data structure library.

We extend some of the ideas from hierarchical lock implementations and other synchronization protocols for NUMA cache-coherent machines [14, 15, 55] to get hierarchical implementations for a non cache-coherent architecture. Tudor et al. [56] attempt to identify patterns on search data structures, which favor implementations that are portably scalable in cachecoherent machines. The patterns they came up with cannot be used to automatically generate a concurrent implementation from its sequential counterpart; they rather provide hints on how to apply optimizations when designing such implementations.

3 Abstract Description of Hardware

Inspired by the characteristics of non cache-coherent architectures [2] and prototypes [4], we consider an architecture which features m islands (or clusters), each comprised of c cores (located in one or more processors). The main memory is split into modules, with each module associated to a distinct island (or core). A fast cache memory is located close to each core. No hardware cache-coherence is provided among cores of different islands: this means that different copies of the same variable residing on caches of different islands may be inconsistent. The islands are interconnected with fast communication channels.

The architecture may provide cache-coherence for the memory modules of an island to processes executing on the cores of the island, i.e. the cores of the same island may see the memory modules of the island as cache-coherent shared memory. If this is so, we say that the architecture is *partially cache-coherent*; otherwise, it is *non cache-coherent*. A process can send messages to other processes by invoking **send** and it can receive messages from other processes by invoking **receive**. The following means of communication between cores (of the same or

different islands) are provided:

Send/Receive mechanism: Each core has its own mailbox which is a FIFO queue (implemented in hardware). (If more than one processes are executed on the same core, they share the same hardware mailbox; in this case, the functionality of **send** and **receive** can be provided to each process through the use of software mailboxes.) A process executing on the core can send messages to other processes by invoking **send**, and it can receive messages from other processes by invoking **receive**. Messages are not lost and are delivered in FIFO order. An invocation of **receive** blocks until the requested message arrives. The first parameter of an invocation to **send** determines the core identifier to which the message is sent.

Reads and Writes through DMA: A Direct Memory Access (DMA) engine allows certain hardware subsystems to access the system's memory without any interference with the CPU. We assume that each core can perform DMA(A, B, d) to copy a memory chunk of size d from memory address A to memory address B using a DMA (where A and B may be addresses in a local or a remote memory module). We remark that DMA is not executed atomically. It rather consists of a sequence of atomic reads of smaller parts, e.g. one or a few words, (known as *bursts*) of the memory chunk to be transferred, and atomic writes of these small parts to the other memory module. DMA can be used for performance optimization. Once the size of the memory chunk to be transferred becomes larger (by a small multiplicative factor) than the maximum message size supported by the architecture, it is more efficient to realize the transfer using DMA (in comparison to sending messages). Specifically, we denote by MMS the maximum size of a message supported by the architecture. (Usually, this size is equal to either a few words or a cache line). Consider that the chunk of data that a core wants to send has size equal to B. To send these data using messages, the core must send $\frac{B}{MMS}$ messages. Each message has a cost C_M to set it up. So, in total, to transfer the data using messages, the overhead paid is $\frac{B}{MMS} * C_M$. Setting up a DMA has a cost C_D , which in most architectures is by a small constant factor larger than C_M . However, C_D is paid only once for the entire transfer of the chunk of data since it is done with a single DMA. Additionally, sending and receiving $\frac{B}{MMS}$ messages requires that a core's CPU will be involved $\frac{B}{MMS}$ times to send each message and another core's CPU will be involved $\frac{B}{MMS}$ times to receive these messages. This cost will be greatly avoided when using a single DMA to transfer the entire chunk of data. Thus, it is beneficial in terms of performance, to use DMA for transferring data whenever the size of the data to be transferred is not too small.

4 Theoretical Framework

An implementation of a data structure (DS) stores its state in the memory modules and provides an algorithm, for each process, to implement each operation supported by the data structure. We model the submission and delivery of messages sent by processes by including incoming and outgoing message buffers in the state of each process (as described in standard books [33, 57] on distributed computing).

We model each process as a state machine. We model a DMA request as a code block which contains a sequence of interleaved burst reads from a memory module and burst writes to a memory module. A DMA engine executes sequences of DMA requests, so it can also be modeled as a simple state machine whose state includes a buffer storing DMA requests that are to be executed. A *configuration* is a vector describing the state of each process (including its message buffers), the state of each DMA engine, the state of the caches (or the shared variables in case shared memory is supported among the cores of each island), and the states of the memory modules. In an *initial configuration*, each process and DMA engine is in an initial state, the

shared variables and the memory modules are in initial states and all message and DMA buffers are empty. An *event* can be either a step by some process, a step by a DMA engine, or the delivery of a message; in one step, a process may either transmit exactly one message to some process and at least one message to every other process, or access (read or write) exactly one shared variable, or initiate a DMA transfer, or invoke an operation of the implemented data structure. In a *DMA step* a burst is read from or written to a memory module. All steps of a process should follow the process's algorithm. Similarly, all steps of a DMA engine should be steps of the code block that performs a DMA request submitted to this DMA engine.

An execution is an alternating sequence of configurations and events starting with an initial configuration. The execution interval of an instance of an operation op that completes in an execution α is the subsequence of α starting with the configuration preceding the invocation of this instance of op and ending with the configurations that follows its response. If an instance of an operation op does not complete in α , then the execution interval of op is the suffix of α starting with the configuration preceding the invocation of this instance of op and ending the invocation of this instance of op. An execution α starting with the configuration preceding the invocation of this instance of op. An execution α imposes a partial order \prec_{α} on the instances of its operations, such that for two instances op_1 and op_2 of some operations in α , $op_1 \prec_{\alpha} op_2$ if the response of op_1 precedes the invocation of op_2 in α . If it neither holds that $op_1 \prec_{\alpha} op_2$ not that $op_2 \prec_{\alpha} op_1$, then we say that op_1 and op_2 are concurrent. If no two operation instances in α are concurrent, then α is a sequential execution.

A step is *enabled* at a configuration C, if the process or DMA engine will execute this step next time it will be scheduled. A finite execution α is fair if, for each process p and each DMA engine e, no step by p or e is enabled at the final configuration C of α and all messages sent in α have been delivered by C. An infinite execution α is fair if the following hold:

- for each process p, either p takes infinitely many steps in α , or there are infinitely many configurations in α such that in each of them (1) no step by p is enabled, (2) for every prefix of α that ends at such a configuration, all messages sent by p have been delivered.
- for each DMA engine e, either e takes infinitely many steps in α , or there are infinitely many configurations in α such that in each of them no step by e is enabled (we remark that a DMA engine e always have an enabled step as long as its DMA buffer is not empty).

Correctness. For correctness, we consider *linearizability* [58]. This means that, for every execution α , there is a sequential execution σ , which contains all the completed operations in α (and some of the uncompleted ones) so that the response of each operation in σ is the same as in α and σ respects the partial order of α . In such a linearizable execution α , one can assign a *linearization point* to each completed operation instance (and to some of the uncompleted ones) so that the linearization point of each operation occurs after the invocation event and before the response event the operation, and so that the order of the linearization points is the same as the order of the operations in σ .

Progress. We aim at designing algorithms that always terminate, i.e. reach a state where all messages sent have been delivered and no step is enabled. We do not cope with message or process failures.

Communication Complexity. Communication between the cores of the same island is usually faster than that across islands. Thus, the communication complexity of an algorithm for a non cache-coherent architecture is measured in two different levels, namely the intra-island communication and the inter-island communication. The *inter-island communication complexity* of an instance *inst* of an operation *op* in an execution α is the total number of messages sent by every core *c* to cores residing on different islands from that of *c* for executing *inst* in α . The inter-island communication complexity of *op* in α is the maximum, over all instances *inst* of *op* in α , of the inter-island communication complexity of *inst* in α . The inter-island communication complexity of *inst* in α .

complexity of op for an implementation I is the maximum, over all executions α produced by I, of the inter-island communication complexity of op in α . We remark that communication can be measured in a more fine-grained way in terms of bytes transferred instead of messages sent, as described in [57]. For simplicity, we focus on the higher abstraction of measuring just the number of messages as described in [33].

If the architecture is non cache-coherent, then the *intra-island communication complexity* is defined as follows. The *intra-island communication complexity* of an instance *inst* of *op* in α is the maximum, over all islands, of the total number of messages sent by every core *c* of an island to cores residing on the same island as *c* for executing *inst* in α .

If the architecture is partially cache-coherent, then we measure the inter-island communication complexity in terms of cache misses following the cache-coherence (CC) shared-memory model (see e.g. [59, 60]). Specifically, in the (CC) shared memory model, accesses to shared variables are performed via cached copies of them; an access to a shared variable is a *cache miss* if the cached copy of this variable is invalid. In this case, a cache miss occurs and a valid copy of the variable should be fetched in the local cache first before it can be accessed. Once the cache miss is served and as long as the variable is not updated by processes that are being executed on other cores, future accesses to the variable by processes that are being executed on this core do not lead to further cache misses. In such a model, the *inter-island communication complexity* of an instance *inst* of an operation *op* is the maximum, over all islands, of the total number of cache-misses that the cores of the island experience to execute *inst*.

We remark that independently of whether the architecture is partially or fully non cachecoherent, the intra-island communication complexity of op in α is the maximum, over all instances *inst* of op in α , of the intra-island communication complexity of *inst* in α . Moreover, the intra-island communication complexity of op for an implementation I is the maximum, over all executions α produced by I, of the intra-island communication complexity of op in α .

The DMA communication complexity of an instance inst of an operation op, is the total number of DMA requests initiated by every process to execute inst; in a more fine-grained model, we could instead measure the total number of bursts performed by these DMA requests. The DMA communication complexity of op in α and of op in I are defined as for the other types of communication complexities.

Time complexity. Consider a fair execution α of an implementation I. A timed version of α is an enhanced version of α where each event has been associated to a non-negative real number, the time at which that event occurs. We define the delay of a message in a timed version of α to be the time that elapses between the computation event that sends the message and the event that delivers the message. We denote by \mathcal{T}_{α} those timed versions of α for which the following conditions hold: (1) the times must start at 0, (2) must be strictly increasing for each individual process and the same must hold for each individual DMA engine, (3) must increase without bound if the execution is infinite, (4) the timestamps of two subsequent events by the same process (or the same DMA engine) must differ by at most 1, and (4) the delay of each message sent must be no more than one time unit. Let $\mathcal{T} = \bigcup_{\forall \alpha \text{ produced by } I} \{\mathcal{T}_{\alpha}\}$.

The time until some event ρ is executed in an execution α is the supremum of the times that can be assigned to ρ in all timed versions of α in \mathcal{T}_{α} . The time between two events in α is the supremum of the differences between the times in all timed versions of α in \mathcal{T}_{α} . The time complexity of an instance inst of an operation op in α is the time between the events of its invocation and its response. The time complexity of op in α is the maximum, over all instances inst of op in α , of the time complexity of inst in α . The time complexity of an operation op for I is the maximum, over all executions α produced by I, of the time complexity of op in α .

Space Complexity. The *space complexity* of I is determined by the memory overhead introduced by I, and by the number and type of shared variables employed (in case of partial non cache-coherence).

5 Directory-based Stacks, Queues, and Deques

The state of the data structure is stored in a highly-scalable distributed *directory* whose data are spread over the local memory modules of the NS servers. The directory supports the operations **DirInsert**, **DirDelete**, **BlockDirDelete**, and **DirSearch**. **DirDelete** and **BlockDirDelete** both remove elements from the directory; however, **DirDelete** returns \perp if the requested element is not contained in the directory, while **BlockDirDelete** blocks until the element is found in the directory. To implement it, we employ a simple highly-efficient distributed hash table implementation (also met in [18, 19, 20]) where hash collisions are resolved by using hash chains (*buckets*). Each server stores a number of buckets. For simplicity, we consider a simple hash function which employs mod and works even if the key is a negative integer. It returns an index which is used to find the server where a request must be sent, and the appropriate bucket at this server, in which the element resides (or must be stored).

To perform an operation, each client must first access a fetch&add object to get a unique sequence number which it uses as the key for the requested data. This object can be implemented using a designated server, called the *synchronizer* and denoted by s_s . The client then communicates with the appropriate server to complete its operation. The server locally processes the request and responds to the process that initiated it. This approach suites better to workloads where the state of the data structure is large.

The hash table implementation we use as our directory is presented in Section 5.1, for completeness. Similar hash table designs have been presented (or discussed) in [18, 20, 19]. Section 5.2 presents the details of the directory-based distributed stack. The directory-based queue implementation is discussed in Section 5.3. Section 5.5 provides the directory-based deque. We remark that our directory-based data structures would work even when using a different directory implementation.

5.1 Distributed Hash Table

A hash table stores elements, each containing a key and a value (associated with the key). Each server stores hash table elements to a local data structure. This structure can be a smaller hash table or any other data structure (array, list, tree, etc.) and supports the operations INSERT, SEARCH and DELETE. To perform an operation on the DHT, a client c finds the appropriate server to submit its request by hashing the key value of interest. Then, it sends a message to this server, which performs the operation locally and sends back the result to c. A server s processes all incoming messages sequentially. After receiving a response message from the server, all client functions return a boolean value depending on whether the operation was successful or not.

\mathbf{A}	gorithm 2 Push operation for a	
cli	ent of the directory-based stack.	
8	<pre>void ClientPush(int cid, Data data) {</pre>	
9	$\operatorname{send}(\operatorname{sid}, \langle \operatorname{PUSH}, \operatorname{cid} \rangle);$	
10	key = receive(sid);	
11	11 $status = DirInsert(key, data);$	
12	return <i>status</i> ;	

}

Algorithm 3 Pop operation for a client of the directory-based stack.

Data ClientPop(int cid) { 13 $send(sid, \langle POP, cid \rangle);$ 14key = receive(sid);15if (key = -1) status = \perp 1617else { do { 18 status = DirDelete(key); 19} while $(status == \bot);$ 20} 21return status; }

5.2 Directory-Based Stack

Algorithm 1 Events triggered in the synchronizer of the directory-based stack.

```
int top\_key = -1;
1
     a message \langle op, cid \rangle is received:
1
\mathbf{2}
       if (op == PUSH)
         top\_key + +;
3
       send(cid, top_key);
4
       if (op == POP) {
\mathbf{5}
         if (top\_key \neq -1)
6
7
         top_key - -;
       }
```

To implement a stack, the synchronizer s_s maintains a variable *top* which stores an integer counting the number of elements that are currently in the stack.

To apply an operation op a client sends a message to the synchronizer s_s . If op is a push operation, s_s uses top_key variable to assign unique keys to the newly inserted data. Each time it receives a push request, s_s sends the value stored in top_key to the client after incrementing it by one. Once a client receives a key from s_s for the push operation it has initiated, it inserts the new element in the directory by invoking DirInsert. Similarly, if op is a pop operation, s_s sends the value stored in top_key to the client and decrements it by one. The client then invokes DirDelete repeatedly, until it successfully removes from the directory the element with the received key. Notice that keys of elements are greater than or equal to 0 and therefore, top_key has initial value -1, which indicates that the stack is empty.

Event-driven pseudocode for the synchronizer is described in Algorithm 1 and the code for the ClientPush() and ClientPop() operations, is presented in Algorithms 2 and 3.

The synchronizer receives, processes, and responds to clients' messages. The messages have an *op* field that represents the operation to be performed (PUSH or POP), and a *cid* field with the client's identification number, needed for identifying the appropriate client to communicate with.

The ClientPop() function, presented in Algorithm 3 is analogous to the push operation: it sends a POP message to s_s and waits for its response (line 15). Using the key that was received

as argument, DirDelete() is repeatedly called (line 19). This is necessary since another client responsible for inserting the key may not have finished yet its insertion. In this case DirDelete returns \perp (line 20). However, since the key was generated previously by s_s , it is certain that it will be eventually inserted into the directory service.

5.2.1 Proof of Correctness

Let α be an execution of the directory-based stack implementation. We assign linearization points to push and pop operations in α by placing the linearization point of an operation *op* in the configuration resulting from the execution of line 4 by the synchronizer for *op*.

Let op be a push or a pop operation invoked by a client c in α and assume that the synchronizer executes line 4 for it. By inspection of the pseudocode, we have that this line is executed after the synchronizer receives a message by c (line 1) and before c receives the synchronizer's response (line 10 for a push operation, line 15 for a pop operation). By the way the linearization points are assigned, we have the following lemma.

Lemma 1. The linearization point of a push (pop) operation op is placed within its execution interval.

Denote by L the sequence of operations (which have been assigned linearization points) in the order determined by their linearization points. Let C_i be the configuration in which the *i*-th operation op_i of L is linearized. Denote by α_i , the prefix of α which ends with C_i and let L_i be the prefix of L up until the operation that is linearized at C_i . We denote by top_i the value of the local variable top_key of s_s at configuration C_i ; Let C_0 be the initial configuration an let $top_0 = -1$.

Notice that since only s_s executes Algorithm 1, we have the following.

Observation 2. Instances of Algorithm 1 are executed sequentially, i.e. their execution does not overlap.

By the way linearization points are assigned, further inspection of the pseudocode in conjunction with Observation 2 leads to the following.

Observation 3. For each integer $i \ge 1$, the following hold at C_i : Let op_i and op_{i+1} be two operations in L.

- If op_i is a push operation and op_{i+1} is a push operation, then $top_{i+1} = top_i + 1$.
- If op_i is a push operation and op_{i+1} is a pop operation, then $top_{i+1} = top_i$.
- If op_i is a pop operation and op_{i+1} is a pop operation, then, if $top_i \neq -1$, $top_{i+1} = top_i 1$.
- If op_i is a pop operation and op_{i+1} is a push operation, then, if $top_i \neq -1$, $top_{i+1} = top_i$.

Let σ be the sequential execution that results by executing the operations in L sequentially, in the order of their linearization points, starting from an initial configuration in which the queue is empty. Let σ_i be the prefix of σ that contains the operations in L_i . Denote by S_i the state of the sequential stack that results if the operations of L_i are applied sequentially to an initially empty stack. Denote by d_i the number of elements in S_i . We associate a sequence number with each stack element such that the elements from the bottommost to the topmost are assigned $0, \ldots, d_i - 1$, respectively. Denote by sl_{d_i} the d_i -th element of S_i .

Let L'_i be the projection of L_i that contains all operations in L_i except those pop operations that return \perp . Denote those operations by op'_i . Denote by C'_i the configuration in which the *i*-th operation of L'_i is linearized. Let S'_i be the state of the sequential stack after the operations of L'_i have been applied to it, assuming an initially empty stack and denote by d'_i the number of elements in S'_i . Again we associate a sequence number with each stack element such that the elements from the bottommost to the topmost are assigned $0, \ldots, d'_i - 1$, respectively. Denote by $sl_{d'_i}$ the d'_i -th element of S'_i . We denote by top'_i the value of the local variable $top_k ey$ of s_s at configuration C'_i . Let k_i be the number of pop operations in L'_i .

By inspection of the pseudocode, it follows that:

Observation 4. If op_i is a push operation, it inserts a pair $\langle key, data \rangle$ in the directory, where data is the argument of ClientPush executed for op_i , and $key = top_i$. If op_i is a pop operation then, if $top_i \neq -1$, it removes a pair $\langle key, data \rangle$ with $key = top_{i-1}$ from the directory and returns data; if $top_i = -1$, it does not remove any pair from the directory and returns \bot .

Lemma 5. For each integer i > 0, it holds that:

- If op'_i is a push operation, then it inserts element d'_i into the stack and $top'_i = d'_i 1$.
- If op'_i is a pop operation, then it removes element d'_{i-1} from the stack, $top'_i = d'_{i-1} 1$ and top'_i is equal to the value of top_key of s_s for the $i 2 * k_i + 1$ -th push operation in L'_i .

Proof. We prove the claim by induction on i.

Base case. We prove the claim for i = 1. Recall that $S'_0 = \epsilon$ and $top'_0 = -1$. By definition of L'_i , op'_1 is a push operation. By observation of the pseudocode (line 3) we have that $top'_1 = -1 + 1 = 0$ and op'_1 inserts an element with sequence number $d'_1 - 1 = 0$ into the stack. Thus, the claim holds.

Hypothesis. Fix any i, i > 0, and assume that the claim holds for C'_i .

Induction step. We prove that the claim also holds at C'_{i+1} . First, consider the case where op'_{i+1} is a push operation. We distinguish two cases. Case (i): op'_i is a push operation, as well. For op'_i , the induction hypothesis holds. Therefore, op'_i inserts the d'_i -th element into the stack and $top'_i = d'_i - 1$. By Observation 3, $top'_{i+1} = top'_i + 1$. Since S'_i has d'_i elements, op'_{i+1} inserts element $d'_{i+1} = d'_i + 1$, with sequence number $d'_{i+1} - 1 = (d'_i + 1) - 1 = d'_i = top'_i + 1 = top'_{i+1}$ and the claim holds. Case (ii): op'_i is a pop operation, which removes an element from the stack. Since the induction hypothesis holds, $top'_i = d'_{i-1} - 1$. Conversely, op'_{i+1} inserts an element into the stack. By Observation 3, we have that $top'_{i+1} = top'_i$ and the claim holds.

From the above lemmas we have the following.

Theorem 6. The directory-based distributed stack implementation is linearizable.

5.3**Directory-Based** Queue

Algorithm 4 Events triggered in the synchronizer of the directory-based queue.

```
int head\_key = 0, tail\_key = 0;
\mathbf{2}
    a message \langle op, cid \rangle is received:
       if (op == ENQ) {
3
         tail_key++;
4
         send(cid, tail_key);
\mathbf{5}
\mathbf{6}
       } else if (op == DEQ) {
         if (head_key < tail_key) {
7
           head_key++;
8
           send(cid, head_key);
9
         } else
10
           send(cid, NACK);
11
         }
       }
```

1

The directory-based distributed queue is implemented in a way similar to the directorybased stack implementation. To implement a queue, the synchronizer s_s maintains two counters, head_key and tail_key, which store the key associated with the first and the last, respectively, element in the queue.

In order to perform an enqueue or dequeue operation, a client calls ClientEnqueue() or ClientDequeue(), respectively (Algorithms 5 and 6, respectively). To apply an operation op, a client sends a request to s_s , in order to receive the key of the element to be inserted or deleted. The client then calls DirInsert to insert the new element in the directory (line 5). Each time s_s receives an enqueue request, it increments tail_key and then sends the value stored in tail_key to the client.

Each time s_s receives a dequeue request, if head key \neq tail key, then s_s sends the value of *head_key* to the client and increments *head_key* to store the key of the next element in the queue. The client then uses this value as the key of the element to remove from the directory (line 23). Otherwise, in case $head_key = tail_key$, s_s sends NACK to c without changing $head_key$.

A dequeue operation for which s_s sends a key value to the client by executing line 9, is referred to as a successful dequeue operation. On the other hand, a dequeue operation for which s_s sends NACK to the client by executing line 11, and for which in turn, the client returns \perp , is referred to as an unsuccessul dequeue operation.

5.3.1**Proof of Correctness**

Let α be an execution of the directory-based queue implementation. We assign linearization points to enqueue and dequeue operations in α as follows: The linearization point of an enqueue operation op is placed in the configuration resulting from the execution of line 5 for op by s_s . The linearization point of a dequeue operation op is placed in the configuration resulting from the execution of either line 9 or line 11 for op (whichever is executed) by s_s .

Lemma 7. The linearization point of an enqueue (dequeue) operation op is placed within its execution interval.

Proof. Assume that op is an enqueue operation and let c be the client that invokes it. The linearization point of op is placed at the configuration resulting from the execution of line 5

	_ (.		1.	$\overline{)}$
client of the d	lirectory-bas	sed queue.		
Algorithm	5 Enqueue	operation	for	a

12	<pre>void ClientEnqueue(int cid, Data data) {</pre>
13	sid = get the server id;
14	$\operatorname{send}(sid, \langle \operatorname{ENQ}, cid \rangle);$
15	$tail_key = receive(sid);$
16	<pre>DirInsert(tail_key, data);</pre>
	}

Algorithm 6 Dequeue operation for a client of the directory-based queue.

17	Data ClientDequeue(int cid) {
18	sid = get the server id;
19	$\operatorname{send}(\operatorname{sid}, \langle \mathtt{DEQ}, \operatorname{cid} \rangle);$
20	$head_key = receive(sid);$
21	$if(head_key == \texttt{NACK})$
22	$\texttt{return} \perp;$
23	$data = \texttt{BlockDirDelete}(head_key);$
24	return data;
	}

for op by s_s . This line is executed after the request by c is received, i.e. after c invokes ClientEnqueue. Furthermore, it is executed before c receives the response by the server and thus, before ClientEnqueue returns. Therefore, the linearization point is included in the execution interval of enqueue.

The argument regarding dequeue operations is similar.

Denote by L the sequence of operations which have been assigned linearization points in α in the order determined by their linearization points. Let C_i be the configuration in which the *i*-th operation op_i of L is linearized; denote by C_0 the initial configuration. Denote by α_i , the prefix of α which ends with C_i and let L_i be the prefix of L up until the operation that is linearized at C_i . We denote by *head_i* the value of the local variable *head_key* of s_s at configuration C_i , and by $tail_i$ the value of the local variable $tail_key$ of s_s at C_i . By the pseudocode, we have that the initial values of $tail_key$ and $head_key$ are 0; therefore, we consider that $head_0 = tail_0 = 0$.

Notice that since only s_s executes Algorithm 4, we have the following.

Observation 8. Instances of Algorithm 4 are executed sequentially by s_s , i.e. their execution does not overlap.

By inspection of Algorithm 4, we have that for some instance of it, either lines 3 - 4, or lines 7 - 9, or lines 10 - 11 are executed, where either *tail_key* or *head_key* is incremented. Then, by the way linearization points are assigned, and by Observation 8, we have the following.

Observation 9. Given two configurations C_i , C_{i+1} , $i \ge 0$, in α , there is at most one step in the execution interval between C_i and C_{i+1} that modifies either head_key or tail_key.

More specifically, regarding the values of $head_i$ and $tail_i$, we obtain the following lemma.

Lemma 10. For each integer $i \ge 1$, the following hold at C_i :

- 1. If op_i is an enqueue operation, then $tail_i = tail_{i-1} + 1$ and $head_i = head_{i-1}$.
- 2. If op_i is a dequeue operation and $head_{i-1} \neq tail_{i-1}$, then $head_i = head_{i-1} + 1$ and $tail_i = lead_{i-1} + l_i$. $tail_{i-1}$; otherwise $head_i = head_{i-1}$ and $tail_i = tail_{i-1}$.
- 3. $head_i \leq tail_i$.

Proof. We prove the claims by induction.

Base case. We prove the claims for i = 1. Assume first that op_1 is an enqueue operation. Then, the linearization point of op_1 is placed in the configuration resulting from the execution of line 5. By inspection of the pseudocode, we have that $tail_key$ is incremented by s_s between C_0 and C_1 , before the linearization point of op_1 . Notice also that because of Observation 8 no process other than s_s modifies neither *tail_key* nor *head_key* between C_0 and C_1 . Thus, $tail_1 = tail_0 + 1$. The value of *head_key* is not modified by enqueue operations (lines 3 - 4), therefore $head_1 = head_0$. Thus, claim 1 holds.

Next, assume that op_1 is a dequeue operation. Then, the linearization point of op_1 may be placed at the configuration resulting from the execution of line 11 or line 9, whichever is executed by s_s for it. By inspection of the pseudocode (line 7), line 11 is executed only in case $head_0 < tail_0$. Since $head_0 = tail_0 = 0$, line 11 is not executed and op_1 is linearized at the configuration resulting from the execution of line 9. By the pseudocode, line 8 and by Observation 8, $head_key$ is incremented by 1 in the execution step preceding the linearization point of op_1 , i.e. between configurations C_0 and C_1 . Thus, $head_1 = head_0 + 1$. The value of $tail_key$ is not modified by dequeue operations (lines 7 - 8), therefore $tail_1 = tail_0$. By the above, claim 2 also holds.

From the previous reasoning, we have that in case op_1 is an enqueue operation, then $tail_1 = 1$ and $head_1 = 0$, while if op_1 is a dequeue operation, $tail_1 = 0$ and $head_1 = 0$. In either case, $head_1 \leq tail_1$, thus claim 3 also holds.

Hypothesis. Fix any $i, i \ge 1$, and assume that the claims hold for i - 1.

Induction Step. We prove that the claims also hold for *i*. First, assume that op_i is an enqueue operation. Then, the linearization point of op_i is placed in the configuration resulting from the execution of line 5. By inspection of the pseudocode, we have that $tail_key$ is incremented by s_s between C_{i-1} and C_i , before the linearization point of op_i . Notice also that because of Observation 8 no process other than s_s modifies neither $tail_key$ nor $head_key$ between C_{i-1} and C_i . Thus, $tail_i = tail_{i-1} + 1$. The value of $head_key$ is not modified by enqueue operations (lines 3 - 4), therefore $head_i = head_{i-1}$. Thus, claim 1 holds.

Next, assume that op_i is a dequeue operation. Then, the linearization point of op_i may be placed at the configuration resulting from the execution of line 11 or line 9, whichever is executed by s_s for it. Let op_i be linearized at the execution of line 11. By the induction hypothesis, $head_{i-1} \leq tail_{i-1}$. By inspection of the pseudocode (line 7), line 11 is executed only in the case that $head_{i-1} = tail_{i-1}$. By inspection of the pseudocode (lines 7 - 11) and by Observation 8, it follows that in this case $head_key$ is not modified in the execution interval between C_{i-1} and C_i . Therefore, $head_i = head_{i-1}$. Since a dequeue operation does not modify $tail_key$, it also holds that $tail_i = tail_{i-1}$. Finally, let op_i be linearized at the execution of line 9. In this case, by the induction hypothesis and reasoning as previously, $head_{i-1} \neq tail_{i-1}$. By the pseudocode, line 8 and by Observation 8, $head_key$ is incremented by 1 in the computation step preceding the linearization point of op_i , i.e. between configurations C_{i-1} and C_i . Thus, $head_i = head_{i-1} + 1$. The value of $tail_key$ is not modified by dequeue operations (lines 7 - 8), therefore $tail_i = tail_{i-1}$. By the above, claim 2 also holds.

By the induction hypothesis, we have that $head_{i-1} \leq tail_{i-1}$. From the previous reasoning, we have that in case op_i is an enqueue operation, then $tail_i = tail_{i-1} + 1$ and $head_i = head_{i-1}$. It follows that $head_i \leq tail_i$. On the other hand, if op_i is a dequeue operation, $tail_i = tail_{i-1}$ and $head_i = head_{i-1}$ in case $head_{i-1} = tail_{i-1}$. Otherwise, in case $head_{i-1} < tail_{i-1}$, then $tail_i = tail_{i-1}$ and $head_i = head_{i-1} + 1$. Since $head_{i-1} < tail_{i-1}$, and since both $head_key$ and $tail_key$ are both incremented in steps of 1, it follows that $head_i \leq tail_i$. In either of the previous cases, claim 3 also holds.

Let σ be the sequential execution that results by executing the operations in L sequentially, in the order of their linearization points, starting from an initial configuration in which the queue is empty. Let σ_i be the prefix of σ that contains the operations in L_i . Let Q_i be the state of the queue after the operations of L_i have been applied to an empty queue sequentially. Let the size of Q_i (i.e. the number of elements contained in Q_i) be d_i . Denote by sl_i^j the *j*-th element of Q_i , $1 \leq j \leq d_i$. Consider a sequence of elements S. If e is the first element of S, we denote by $S \setminus e$ the suffix of S that results by removing only element e from the first position of S. We further denote by $S' = S \cdot e$ the sequence that results by appending some element e to the end of S.

By inspection of the pseudocode, it follows that:

Observation 11. If op_i is an enqueue operation, it inserts a pair $\langle data, key \rangle$ in the directory, where data is the argument of ClientEnqueue executed for op_i , and $key = tail_i$. If op_i is a dequeue operation then, if $head_i \neq tail_i$, it removes a pair $\langle data, key \rangle$ with $key = head_i$ from the directory and returns data; if $head_i = tail_i$, it does not remove any pair from the directory and returns \perp .

Lemma 12. For each *i*, let m_i be the number of enqueue operations in L_i and k_i be the number of successful dequeue operations in L_i . Then:

- 1. Q_i contains the elements inserted by the $m_i k_i$ last enqueue operations in L_i , in order,
- 2. $tail_i = m_i$,
- 3. $head_i = k_i$,
- if op_i is a dequeue operation, then it returns the same response in α and σ_i. If op_i is a successful dequeue operation that removes the pair (data, -), from the directory, then its response is the data inserted to the queue by the k_i-th enqueue operation in L_i, whereas if op_i is unsuccessful, then it returns ⊥.

Proof. We prove the claims by induction.

Base case. We prove the claim for i = 1. First, assume that op_1 is an enqueue operation. Then, $m_1 = 1$ and $k_1 = 0$, and Q_1 contains a single element, namely the element inserted by the $(m_1 - k_1)$ -th enqueue operation in L_1 , which is op_1 , thus claim 1 holds. By Lemma 10, we have that at C_1 , $tail_1 = tail_0 + 1 = 1 = m_1$, and $head_1 = head_0 = 0 = k_1$. Thus, claims 2 and 3 also hold. Claim 4 holds trivially.

Next, assume that op_1 is a dequeue operation. Thus, $m_1 = 0$. By inspection of the pseudocode, $head_0 = tail_0 = 0$. By the way linearization points are assigned and by Observation 9, op_1 is the operation for which the code of s_s is executed for the first time. Thus, line 11 is executed and op_1 is unsuccessful. Therefore, $k_1 = 0$. By Lemma 10, $head_1 = 0$ and $tail_1 = 0$. It follows that $head_1 = m_1$ and $tail_1 = k_1$. Thus, claims 2 and 3 hold.

Since op_1 is the first operation in σ and since it is a dequeue operation, it is an unsuccessful dequeue operation in σ . Since Q_0 is empty, it follows that $Q_1 = \emptyset$, so claim 1 follows. By inspection of the pseudocode, op_1 returns \perp in α . Since op_1 is an unsuccessful dequeue in σ , it also returns \perp in σ . Thus, claim 4 also holds.

Hypothesis. Fix any i, i > 0 and assume that the claims hold for i - 1.

Induction step. We prove that the claims also hold for *i*. First, assume that op_i is an enqueue operation and let the *data* argument of ClientEnqueue for op_i be *e*. By the induction hypothesis, Q_{i-1} contains the elements inserted by the $m_{i-1} - k_{i-1}$ last enqueue operations in L_{i-1} , in order. Since op_i is an enqueue operation, it follows that $Q_i = Q_{i-1} \cdot e$, $m_i = m_{i-1} + 1$, and claim 1 holds. Moreover, $k_i = k_{i-1}$. By Lemma 10, we have that $tail_i = tail_{i-1} + 1$ and that $head_i = head_{i-1}$. By the induction hypothesis, $tail_{i-1} = m_{i-1}$ and $head_i = k_{i-1}$. It follows that $tail_i = m_{i-1} + 1 = m_i$ and that $head_i = head_{i-1} = k_i$. Thus, claims 2 and 3 also hold. Claim 4 holds trivially.

Now let op_i be a dequeue operation. First, assume that op_i is a successful dequeue operation, i.e. s_s executes line 9 for op_i . By inspection of the pseudocode, by Observation 9 and the way linearization points are assigned, it follows that when line 7 is executed, *head_key* and *tail_key* have the values *head*_{i-1} and *tail*_{i-1} respectively. Since line 9 is executed, it follows that $head_{i-1} < tail_{i-1}$. By the induction hypothesis, $head_{i-1} = k_{i-1}$ and $tail_{i-1} = m_{i-1}$. Since op_i is a dequeue operation, $m_i = m_{i-1}$ and $k_i = k_{i-1} + 1$. By Lemma 10, we have that $tail_i = tail_{i-1}$ and that $head_i = head_{i-1} + 1$. So, $tail_i = m_i$ and $head_i = k_i$. Thus, claims 2 and 3 hold.

By the induction hypothesis, Q_{i-1} contains the elements inserted by the $m_{i-1} - k_{i-1}$ last enqueue operations in L_{i-1} . Since $m_{i-1} - k_{i-1} = m_{i-1} - (k_i - 1)$, Q_{i-1} contains $m_{i-1} - k_i + 1$ elements. Since op_i is a dequeue, it removes the first element of Q_{i-1} , so Q_i contains $m_i - k_i$ elements, and claim 1 follows.

Recall that $head_{i-1} < tail_{i-1}$. By the induction hypothesis (claims 2 and 3), it follows that $k_{i-1} < m_{i-1}$. Thus, Q_{i-1} is not empty and op_i is a successful dequeue operation also in σ .

By the induction hypothesis, Q_{i-1} contains the elements inserted by the $m_{i-1} - k_{i-1} = m_i - (k_i - 1) = m_i - k_i + 1$ last enqueue operations in L_{i-1} , in ascending order from head to tail. Since m_i is the total number of successful enqueue operations in L_i and k_i the total number of successful dequeue operations, this means that in σ , op_i removes the element inserted to the queue by the k_i -th enqueue operation. By inspection of the pseudocode, the client removes the key with value $head_i$ from the directory and returns the data that are associated with this key. Since $head_i = k_i$, op_i returns the data from a $\langle data, key \rangle$ pair it removes from the directory. By inspection of the pseudocode, the key used to remove the data has the value $head_i$. Therefore, also in α , op_i returns the data associated with the k_i -th enqueue operation in L_i , and claim 4 holds.

Assume now that op_i is an unsuccessful dequeue operation, i.e. s_s executes line 11 for op_i . By inspection of the pseudocode, by Observation 9 and the way linearization points are assigned, it follows that when line 7 is executed, $head_key$ and $tail_key$ have the values $head_{i-1}$ and $tail_{i-1}$ respectively. Since line 11 is executed, it follows that $head_{i-1} = tail_{i-1}$. By the induction hypothesis, $head_{i-1} = k_{i-1}$ and $tail_{i-1} = m_{i-1}$. Since op_i is an unsuccessful dequeue operation, $m_i = m_{i-1}$ and $k_i = k_{i-1}$. By Lemma 10, we have that in this case, $tail_i = tail_{i-1}$ and that $head_i = head_{i-1}$. So, $tail_i = m_i$ and $head_i = k_i$. Thus, claims 2 and 3 hold. Furthermore, since $head_{i-1} = tail_{i-1}$, it follows that $m_{i-1} = k_{i-1}$, which means that Q_{i-1} is empty, and thus, op_i is unsuccessful in σ . As $m_i = m_{i-1}$ and $k_i = k_{i-1}$ and op_i is unsuccessful, claim 1 holds. Since op_i is unsuccessful in α , it removes no $\langle data, key \rangle$ pair from the directory and returns \bot . Thus, claim 4 also holds.

From the above lemmas we have the following:

Theorem 13. The directory-based queue implementation is linearizable.

5.4 Queues with Special Functionality

Synchronous queue. A synchronous queue Q_S is an implementation of the queue data type. Instead of storing elements, a synchronous queue matches instances of Dequeue() with instances of Enqueue() operations. Thus, if op_e is an instance of an $Enqueue(x, Q_S)$ operation and op_d an instance of a $Dequeue(Q_S)$ such that op_d returns the element x enqueued by op_e , then a synchronous queue ensures that the execution intervals of op_e and op_d are overlapping.

In order to derive a distributed synchronous queue from the directory-based queue proposed here, s_s must respond to a dequeue request with the value of *head* and increment *head*, even if *head* = *tail*. Moreover, s_s must use a local queue to store active enqueue requests together with the keys it has assigned to them (notice that there can be no more such requests that the number of clients); s_s must send the key k for each such enqueue request to the client that initiated it, at the time that *head* becomes equal to k. In this way, the execution interval of an enqueue operation for element e overlaps that of the dequeue operation which gets e as a response, as specified by the semantics of a synchronous queue.

Delay queue. A delay queue Q_D implements the queue abstract data type. Each element e of a delay queue is associated with a delay value t_e that represents the time that e must remain in the queue before it can be removed from it. Thus, an $Enqueue(e, t_e, Q_D)$ inserts an element e with time-out value t_e to Q_S . $Dequeue(Q_D)$ returns the element e residing at the head of Q_D if t_e has expired and blocks (or performs spinning) if this is not the case. Notice that this implementation can easily be provided by associating each element inserted in the directory with a time-out value. We also have to change the way that the directory works so that it takes into consideration the delay of each element before removing it.

5.5 Directory-Based Double-Ended Queue (Deque)

The implementation of the directory-based deque follows similar principles as the stack and queue implementations. In order to implement a deque, s_s also maintains two counters, *head* and *tail*, which store the key associated with the first and the last, respectively, element in the deque. However, in this case, counters *head* and *tail* may store negative integers and are incremented or decremented based on the operation to be performed.

5.5.1 Algorithm Description

Event-driven pseudocode for the synchronizer s_s is presented in Algorithm 7; s_s now performs a combination of actions presented for the synchronizers of the stack and the queue implementations (Algorithms 1 and 4).

The synchronizer s_s has two counters, *head_key* and *tail_key* (line 1), that store the key associated with the first and the last, respectively, element in the deque. The *head_key* is modified when operations targeting the front are received by s_s and the *tail_key* is modified when operations targeting the back are received by s_s . Because each endpoint of a deque behaves as a stack, the actions for enqueuing and dequeuing are similar as in Algorithm 1.

Algorithm 7 Events triggered in the synchronizer of the directory-based deque.

```
int head\_key = 0, tail\_key = 0;
1
\mathbf{2}
    a message \langle op, cid \rangle is received:
      switch (op) {
3
4
        case ENQ_T:
5
          tail_key + +;
          send(cid, tail_key);
6
          break;
7
        case DEQ_T:
8
          if (tail_key == head_key) {
9
            send(cid, NACK);
10
          } else {
11
            do {
12
               status = DirDelete(tail_key);
13
             } while (status == \bot);
14
15
            tail_key - -;
            send(cid, status);
16
          }
17
          break;
        case ENQ_H:
18
19
          send(cid, head_key);
          head_key - -;
20
          break;
21
22
        case DEQ_H:
          if (tail_key == head_key) {
23
24
            send(cid, NACK);
          } else {
25
            head_key + +;
26
            do {
27
               status = DirDelete(head_key);
28
            } while (status == \bot);
29
30
            send(cid, status);
          }
31
32
          break;
      }
```

Upon a message receipt, if s_s receives a request ENQ_T (line 4) it increments *tail_key* by one (line 5), and then sends the current value of *tail_key* to the client (line 6). The client uses the value that s_s sends to it, as the key for the data to insert in the directory. Likewise, if s_s receives a request ENQ_H (line 18), it sends the current value of *head_key* to the client (line 19), and then decrements *head_key* by one (line 20).

When a message of type DEQ_T arrives (line 8), s_s first checks whether the deque is empty (line 9). If this is so, s_s sends a NACK to the client (line 10). Otherwise, the synchronizer repeatedly calls DirDelete(*tail_key*) to remove the element corresponding to a key equal to the value of *tail_key* from the directory (line 13), and then decrements *tail_key* (line 15). Finally, s_s sends the data to the client (line 16). The synchronizer performs similar actions for a DEQ_H message, but instead of decrementing the *tail_key*, it increments the *head_key* (line 26).

The code for the clients operations for enqueue, is presented in Algorithm 8. For enqueuing to the back of the deque, the client sends an ENQ_T message to s_s and blocks waiting for its response. When it receives the unique key from s_s , the client is free to insert the element lazily.

Algorithm 8 Enqueue operations for a	Algorithm 9 Dequeue operation for a
client of the directory-based deque.	client of the directory-based deque.
<pre>33 void EnqueueTail(int cid, Data data) { 34 sid = get the synchronizer id; 35 send(sid, ⟨ENQ_T, cid⟩); 36 key = receive(sid); 37 DirInsert(key, data); }</pre>	45 Data DequeueTail(int cid) { 46 $sid = get$ the synchronizer id; 47 $send(sid, \langle DEQ_T, cid \rangle);$ 48 $status = receive(sid);$ 49 $return status;$ }
<pre>38 39 void EnqueueHead(int cid, Data data) { 40 sid = get the synchronizer id; 41 send(sid, (ENQ_H, cid)); 42 key = receive(sid); 43 DirInsert(key, data); 44 }</pre>	50 Data DequeueHead(int cid) { 51 sid = get the synchronizer id; 52 send(sid, (DEQ_H, cid)); 53 status = receive(sid); 54 return status; }

For enqueuing to the front of the deque, the client sends an ENQ_H message and performs the same actions as for enqueuing to the back.

The client code for dequeue to the front and dequeue to the back, is presented in Algorithm 9. For dequeuing to the back of the deque, the client sends an DEQ_T message to s_s and blocks waiting for its response. The synchronizer performs the dequeue itself and sends back the response. For dequeuing to the front of the deque, the client sends an DEQ_H message and performs the same actions as for enqueue to the back.

5.5.2 **Proof of Correctness**

Let α be an execution of the directory-based deque implementation. We assign linearization points to enqueue and dequeue operations in α as follows:

The linearization point of an enqueue back operation op is placed in the configuration resulting from the execution of line 6 for op by s_s . The linearization point of a dequeue back operation op is placed in the configuration resulting from the execution of either line 10 or line 16 for op (whichever is executed) by s_s . The linearization point of an enqueue front operation op is placed in the configuration resulting from the execution of line 19 for op by s_s . The linearization point of a dequeue front operation op is placed in the configuration resulting from the execution of either line 24 or line 30 for op (whichever is executed) by s_s .

Lemma 14. The linearization point of an enqueue (dequeue) operation op executed by client c is placed within its execution interval.

Proof. Assume that op is an enqueue front (back) operation and let c be the client that invokes it. After the invocation of op, c sends a message to s_s (line 41) and awaits a response from it. Recall that routine **receive()** (line 42) blocks until a message is received. The linearization point of op is placed at the configuration resulting from the execution of line 19 for op by s_s . This line is executed after the request by c is received, i.e. after c invokes **EnqueueHead** (**EnqueueTail**). Furthermore, it is executed before c receives the response by the server and thus, before **EnqueueHead** (**EnqueueTail**) returns. Therefore, the linearization point is included in the execution interval of enqueue front (back).

The argumentation regarding dequeue front (back) operations is similar.

Denote by L the sequence of operations which have been assigned linearization points in α in the order determined by their linearization points. Let C_i be the configuration in which the *i*-th operation op_i of L is linearized; denote by C_0 the initial configuration. Denote by α_i , the prefix of α which ends with C_i and let L_i be the prefix of L up until the operation that is linearized at C_i . We denote by *head_i* the value of the local variable *head_key* of s_s at configuration C_i , and by *tail_i* the value of the local variable *tail_key* of s_s at C_i . By the pseudocode, we have that the initial values of *tail_key* and *head_key* are 0; therefore, we consider that $head_0 = tail_0 = 0$.

By analogous reasoning as the one followed in the case of the directory-based queue, inspection of the pseudocode leads to the following observations.

Observation 15. Instances of Algorithm 7 are executed sequentially, i.e. their execution does not overlap.

Observation 16. Given two configurations C_i , C_{i+1} , $i \ge 0$, in α , there is at most one step in the execution interval between C_i and C_{i+1} that modifies tail_key.

Denote by D_i the sequential deque that results if the operations of L_i are applied sequentially to an initially empty queue. Let the size of D_i (i.e. the number of elements contained in D_i) at C_i be d_i . Denote by sl_i^j the *j*-th element of D_i , $1 \le j \le d_i$. Each element of D_i is a pair of type $\langle key, data \rangle$ where the elements from the bottommost to the topmost are assigned integer keys as follows: Let f_i be the key of element sl_i^1 and l_i be the key of element $sl_i^{d_i}$ in some configuration C_i . We denote the key field of the $\langle data, key \rangle$ pair that comprises some element sl_i^j , $1 \le j \le d_i$, of D_i by sl_i^j .key. Then, if $d_i > 1$, sl_i^{j+1} .key = sl_i^j .key + 1, $1 \le j \le d_i$. We consider that if op_1 is an enqueue front operation, then $f_1 = l_1 = 0$, while if it is an enqueue back operation, then $f_1 = l_1 = 1$. Notice that $l_i - f_i + 1 = d_i$.

Consider a sequence of elements S. If e is the first element of S, we denote by $S \setminus_f e$ the suffix of S that results by removing only element e from the first position of S. If e is the last element of S, we denote by $S \setminus_b e$ the prefix of S that results by removing only element e from the last position of S. If e is an element not included in S, we denote by $S' = S \cdot e$ the sequence that results by appending element e to the end of S, and by $S'' = e \cdot S$ the sequence that results by prefixing S with element e.

Lemma 17. For each integer $i \ge 1$, the following hold at C_i :

- 1. If op_i is an enqueue back operation, then $tail_i = tail_{i-1} + 1$.
- 2. If op_i is a dequeue back operation, then it holds that $tail_i = tail_{i-1} 1$ if $tail_{i-1} \neq head_{i-1}$; otherwise, $tail_i = tail_{i-1}$.

Proof. Fix any $i \ge 1$. If op_i is an enqueue back operation, the linearization point of op_i is placed at the configuration resulting from the execution of line 6. By inspection of the pseudocode (lines 5-6), we have that in the instance of Algorithm 7 executed for op_i , $tail_key$ is incremented before it is sent to the client c that invoked op_i . By Observations 15 and 16, this is the only increment that occurs on $tail_key$ between C_{i-1} and C_i . Thus, Claim 1 holds.

If op_i is a dequeue back operation, the linearization point of op_i is placed either at the configuration resulting from the execution of line 10 or at the configuration resulting from the execution of line 16. Let op_{i-1} be a dequeue back operation that is linearized at the execution of line 10. By inspection of the pseudocode (line 9), this occurs only in case $head_{i-1} = tail_{i-1}$. Since the execution of this line does not modify $tail_key$, $tail_i = tail_{i-1}$ and Claim 2 holds.

Now let op_i be a dequeue back operation that is linearized at the configuration resulting from the execution of line 16. By the pseudocode (lines 15-16) and by Observation 8, it follows that the execution of line 15 is the only step in which *tail_key* is modified in the execution interval between C_{i-1} and C_i . Since line 16 is executed, it holds that the condition of the if clause of line 9 evaluates to false, i.e. it holds that $head_{i-1} \neq tail_{i-1}$. Furthermore, because of the execution of line 15, $tail_i = tail_{i-1} - 1$. Thus, Claim 2 holds.

Lemma 18. For each integer $i \ge 1$, the following hold at C_i :

- 1. In case op_i is an enqueue front operation, then, if i > 1 and op_{i-1} is an enqueue front operation, it holds that $head_i = head_{i-1} 1$; otherwise $head_i = head_{i-1}$.
- 2. In case op_i is a dequeue front operation, then, if $head_{i-1} \neq tail_{i-1}$, i > 1, and op_{i-1} is not an enqueue front operation, it holds that $head_i = head_{i-1} + 1$; otherwise $head_i = head_{i-1}$.

Proof. Fix any $i \geq 1$. Let op_i be an enqueue front operation. If i = 1, then by inspection of the pseudocode, we have that $head_key$ is not modified before the execution of line 19. Since $head_0 = 0$ and the execution of line 19 does not modify $head_key$, it follows that $head_1 = 0 =$ $head_0$ and Claim 1 holds. Now let i > 1. By inspection of the pseudocode and by Observation 15 we have that $head_key$ is not modified by enqueue back and dequeue back operations. By the pseudocode, Observation 15 and the way linearization points are assigned, we have that although $head_key$ is modified by dequeue front operations only before the configuration in which the operation is linearized, it is modified by enqueue front operations in the step (line 20) right after the configuration in which an enqueue front operation is linearized. Therefore, if op_{i-1} is an enqueue front operation, then $head_key$ is decremented once (line 20) in the execution interval between C_{i-1} and C_i . Thus, if op_{i-1} is an enqueue front operation, then $head_i = head_{i-1} + 1$, while if op_{i-1} is any other type of operation, $head_i = head_{i-1}$. Thus, Claim 1 holds.

Now let op_i be a dequeue operation. If i = 1, then by inspection of the pseudocode, we have that $head_key$ is not modified before the execution of line 15. By the pseudocode and by Observation 15, $tail_key$ is not modified as well before the execution of line 15. Thus, $head_0 = 0 = tail_0$ and the if condition of line 9 evaluates to true. Then, op_1 is linearized in the configuration resulting from the execution of line 24. Notice that the execution of this line does not modify $head_key$. It follows that $head_1 = 0 = head_0$ and that Claim 2 holds.

Now let i > 1. The linearization point of op_i may be placed at the configuration resulting from the execution of line 24 or line 30, whichever is executed by s_s for it. Let the linearization point be placed in the configuration resulting from the execution of line 24. In that case, $head_{i-1} = tail_{i-1}$. Notice that the execution of that line does not modify $head_key$. Therefore, $head_i = head_{i-1}$, and Claim 2 holds. Now let the linearization point be placed in the configuration resulting from the execution of line 30. In case op_{i-1} is an enqueue back or dequeue tail operation, $head_key$ is not modified by it. Therefore, since line 26 is executed before line 30, $head_i = head_{i+1} + 1$ and Claim 2 holds. The same also holds if op_{i-1} is a dequeue front operation. If op_{i-1} is an enqueue front operation, then by inspection of the pseudocode (line 20), we have that $head_key$ is decremented in the step following the configuration in which op_{i-1} is linearized. Therefore, in this case and by Observation 15, $head_key$ is decremented and then incremented once in the execution interval between C_{i-1} and C_i . This in turn implies that $head_i = head_{i-1} - 1 + 1 = head_{i-1}$ and Claim 2 holds. \Box

Recall that $sl_i^j key = f_i + j - 1$ or $sl_i^j key = l_i - j + 1$. By inspection of the pseudocode (lines 19/6), we see that, when op_i is an enqueue front/back operation, $head_i/tail_i$ is sent by s_s to the client c that invoked op_i . By further inspection of the pseudocode (lines 42-43/37-37), we see that c uses $head_i/tail_i$ as the key field of the element it enqueues, i.e. uses it as argument for auxiliary function DirInsert()/DirDelete(). When op_i is a dequeue front/back operation, by inspection of the pseudocode (lines 24/10), we have that when $head_key = tail_key$, s_s sends NACK to c, and that when c receives NACK, it does not enqueue any element and instead, returns \perp (lines 53-54/48-49). When $head_key \neq tail_key$, s_s uses $head_i/(tail_i + 1)$ as the key field in

order to determine which element to dequeue (lines 28/13). Then, the following observation holds.

Observation 19. If op_i is an enqueue back operation, it inserts a pair with $key = tail_i$ into the directory. If op_i is a dequeue back operation, then, if $head_i \neq tail_i$, it removes a pair with $key = tail_i+1$ from the directory; if $head_i = tail_i$, it does not remove any pair from the directory. If op_i is an enqueue front operation, it inserts a pair with $key = head_i$ into the directory. If op_i is a dequeue front operation then, if $head_i \neq tail_i$, it removes a pair with $key = head_i$ from the directory; if $head_i = tail_i$, it does not remove any pair from the directory.

Lemma 20. At C_i , $i \ge 1$, the following hold:

- 1. If op_i is an enqueue back operation, then $tail_i = sl_i^{d_i}$.key.
- 2. If op_i is a dequeue back operation, then if $D_{i-1} \neq \epsilon$, $tail_i = sl_{i-1}^{d_{i-1}}$.key. If $D_{i-1} = \epsilon$, then $head_i = tail_i$.
- 3. If op_i is an enqueue front operation, then $head_i = sl_i^1$.key.
- 4. If op_i is a dequeue front operation, then if $D_{i-1} \neq \epsilon$, $head_i = sl_{i-1}^1$.key. If $D_{i-1} = \epsilon$, then $head_i = tail_i$.

Proof. We prove the claims by induction.

Base case. We prove the claim for i = 1.

Consider the case where op_1 is an enqueue back operation. Then, $d_1 = 1$ and by definition, D_1 contains only the pair $\langle 1, data \rangle$. By Observation 15, it is the first operation in α for which an instance of Algorithm 7 is executed by s_s . Therefore, by Lemma 17, $tail_1 = tail_0 + 1 = 1$. Thus, $tail_1 = sl_1^{d_1}$. key and Claim 1 holds.

Next, consider the case where op_1 is a dequeue back operation. By Observation 15, op_1 is the first operation in α for which an instance of Algorithm 7 is executed by s_s . Notice that then, $Q_1 = \epsilon$. Therefore, by Lemma 17, $tail_1 = tail_0 = 0$. Since $head_key$ is not modified by dequeue back operations, $head_1 = head_0 = 0$. Thus, $head_1 = tail_1$, so Claim 2 holds.

Next, consider the case where op_1 is an enqueue front operation. Again, by definition, $d_1 = 1$ and D_1 contains only the pair $\langle 0, data \rangle$. By Observation 15, it is the first operation in α for which an instance of Algorithm 7 is executed by s_s . Therefore, by Lemma 18, $head_1 = head_0 = 0$. Thus, $head_1 = sl_1^1$. key and Claim 3 holds.

Finally, consider the case where op_1 is a dequeue front operation. By Observation 15, op_1 is the first operation in α for which an instance of Algorithm 7 is executed by s_s . Notice that then, $Q_1 = \epsilon$. Therefore, by Lemma 18, $head_1 = head_0 = 0$. Since $tail_key$ is not modified by dequeue front operations, $tail_1 = tail_0 = 0$. Thus, $head_1 = tail_1$ and Claim 4 holds.

Hypothesis. Fix any i, i > 0 and assume that the lemma holds at C_i .

Induction step. We prove that the claims also hold at C_{i+1} . Assume that op_{i+1} is an enqueue back operation. By the induction hypothesis, if op_i is an enqueue back operation, then $sl_i^{d_i}.key = tail_i = l_i$. Similarly, if op_i is a dequeue back operation, then by the induction hypothesis, $sl_{i-1}^{d_{i-1}}.key = tail_i$. Since the dequeue back operation removes the last element in D_{i-1} , it follows that the last element $sl_i^{d_i}$ of D_i is $sl_{i-1}^{d_{i-1}}$. Thus, here also, $tail_i = sl_i^{d_i}.key = l_i$. Notice that enqueue front and dequeue front operations do not modify $tail_key$. Since these types of operation do not affect the back of the sequential dequeue, it still holds that $tail_i = sl_i^{d_i}.key = l_i$. Since op_{i+1} is an enqueue back operation, by Lemma 17, we have that $tail_{i+1} = tail_i + 1$. By Observation 19, we have that the client c that initiated op_{i+1} inserts a pair with $key = tail_{i+1} = tail_i + 1$ into the directory. By definition, $sl_{i+1}^{d_{i+1}}.key = sl_i^{d_i}.key + 1$. Thus, $sl_{i+1}^{d_{i+1}}.key = tail_i + 1$, and Claim 1 holds.

Now assume that op_{i+1} is a dequeue back operation. We examine two cases. First, let $D_i \neq \epsilon$. By Lemma 17, it then holds that $tail_{i+1} = tail_i - 1$. By Observation 19, we have that

a pair with $key = tail_{i+1} = tail_i - 1$ is removed from the directory. By definition, we have that $D_{i+1} = D_i \setminus_b sl_i^{d_i}$. Also by definition, we have that $sl_i^{d_i}.key = sl_i^{d_i-1}.key + 1$. Because of $op_{i+1}, sl_i^{d_i-1} = sl_{i+1}^{d_{i+1}}$. Since $tail_{i+1} = tail_i - 1$, Claim 2 holds. Now let $D_i = \epsilon$. In this case, op_{i+1} cannot have any effect on the state of the deque. By inspection of the pseudocode, this corresponds to the operation being linearized in the configuration resulting from the execution of line 10. Notice that in order for this to be the case, the **if** condition of line 9 must evaluate to **true**. This occurs if $head_i = tail_i$, thus Claim 2 holds.

Next assume that op_{i+1} is an enqueue front operation. By the induction hypothesis, if op_i is an enqueue front operation, then $sl_i^1 key = head_i = f_i$. By Lemma 18, it holds then that $head_{i+1} = head_i - 1$. Since op_{i+1} is an enqueue front operation, it prepends an element to D_i and therefore, $sl_i^1 = sl_{i+1}^2$. By definition of D_{i+1} , $sl_{i+1}^1 key = sl_{i+1}^2 key - 1$. Since $sl_{i+1}^2 key = head_i$, $sl_{i+1}^1 key = head_{i+1}$ and Claim 3 holds.

On the other hand, if op_i is a dequeue front operation, then by the induction hypothesis, $sl_{i-1}^1 key = head_i$. By Lemma 18, it also follows that in this case, $head_{i+1} = head_i$. Notice that by definition, op_i removes element sl_{i-1}^1 from D_{i-1} . Then, for element sl_i^1 of D_i , by definition, $sl_i^1 key = sl_{i-1}^1 key + 1$. This means that $sl_{i-1}^1 key = head_i = sl_i^1 key - 1$. Since $head_{i+1} = head_i$, Claim 3 holds.

Notice that enqueue back and dequeue back operations do not modify *head_key*.

Finally, assume that op_{i+1} is a dequeue front operation. We examine two cases. First, let $D_i \neq \epsilon$. By Lemma 18, it then holds that $head_{i+1} = head_i$. By Observation 19, we have that a pair with $key = head_{i+1} = head_i$ is removed from the directory. By definition, we have that $D_{i+1} = D_i \setminus_b sl_i^1$. Also by definition, we have that $sl_i^1.key = sl_i^2.key - 1$. Because of op_{i+1} , $sl_i^2 = sl_{i+1}^1$. Since $head_{i+1} = head_i$, Claim 4 holds. Now let $D_i = \epsilon$. In this case, op_{i+1} cannot have any effect on the state of the deque. By inspection of the pseudocode, this corresponds to the operation being linearized in the configuration resulting from the execution of line 24. Notice that in order for this to be the case, the **if** condition of line 23 must evaluate to **true**. This occurs if $head_i = tail_i$, thus Claim 4 holds.

From the above lemma, we have the following corollary.

Corollary 21. $D_i = \epsilon$ if and only if $head_i = tail_i$.

Lemma 22. If op_i is a dequeue back operation, then it returns the value of the field data of $sl_{i-1}^{d_{i-1}}$ or \perp if $D_{i-1} = \epsilon$.

Proof. Consider the case where $D_{i-1} \neq \epsilon$. By definition of D_i , we have that $D_i = D_{i-1} \setminus_b sl_{i-1}^{d_{i-1}}$. Let op_j be the enqueue operation that is linearized before op_i and inserts an element with key $tail_i + 1$ to the queue. Notice by the pseudocode (lines 12-16), that the parameter of DirDelete is $tail_i + 1$. By the semantics of DirDelete, if at the point that the instance of DirDelete is executed in the do - while loop of lines 12-13 for op_i , the instance of DirInsert of op_j has not yet returned, then DirDelete returns $\langle \perp, - \rangle$.

By Lemma 17, $tail_i + 1$ is the key of the last pair $sl_{i-1}^{d_{i-1}}$ in D_{i-1} . Therefore, when DirDelete returns a $status \neq \bot$, it holds that it returns the data field of $sl_{i-1}^{d_{i-1}}$, the last element in D_{i-1} . Notice that this value is sent to the client c that invoked op_i (line 16) and that c uses this value as the return value of op_i (lines 48-49). Thus, the claim holds.

Now consider the case where $D_{i-1} = \epsilon$. Since, by Corollary 21, when this is the case, $head_i = tail_i$, NACK is sent c and, by inspection of the pseudocode, op_i returns \perp , i.e. the claim holds.

In a similar fashion, we can prove the following.

Lemma 23. If op_i is a dequeue front operation, then it returns the value of the field data of sl_{i-1}^1 or \perp if $D_{i-1} = \epsilon$.

From the above lemmas we have the following:

Theorem 24. The directory-based deque implementation is linearizable.

5.6 Hierarchical approach, Elimination, and Combining.

In this section, we outline how the hierarchical approach, described in Section ??, is applied to the directory-based designs.

Each island master m_i performs the necessary communication between the clients of its island and s_s . In the stack implementation, each island master applies elimination before communicating with s_s . To further reduce communication with s_s , m_i applies a technique known as combining [61]. In the case of stack, once elimination has been applied, there is only one type of requests that must be sent to the synchronizer; for all these requests, m_i sends just one message containing their number f and their type to the synchronizer. In case of push operations, this method allows the synchronizer to directly increment top by f and respond to m_i with the value g that top had before the increment. Once m_i receives g, it informs the clients (which initiated these requests) that the keys for their requests are $g, g+1, \ldots, g+f-1$. In the case of queue, each message of m_i to s_s contains two counters counting the number of active enqueue and dequeue requests from clients of island i. When s_s receives such a message it responds with a message containing the current values of *tail* and *head*. It then increments *tail* and *head* by the value of the first and second counter, respectively. Server m_i assigns unique keys to active enqueue and dequeue operations, based on the value of *tail* and *head* it receives, in a way similar as in stacks. Combining can be used for deques (in addition to elimination) in ways similar to those described above.

6 Token-based Stacks, Queues, and Deques

We start with an informal description of the token-based technique that we present in this section. We assume that the servers are numbered from 0 to NS -1 and form a logical ring. Each server has allocated a chunk of memory (e.g. one or a few pages) of a predetermined size, where it stores elements of the implemented DS. A DS implementation employs (at least) one token which identifies the server s_t , called the *token server*, at the memory chunk of which newly inserted elements are stored. (A second token is needed in cases of queues and deques.) When the chunk of memory allocated by the token server becomes full, the token server gives up its role and appoints another (e.g. the next) server as the new token server. A client remembers the server that served its last request and submits the next request it initiates to that server; so, each response to a client contains the id of the server that served the client's request. Servers; this is done until the request reaches the appropriate token server. A server allocates a new (additional) chunk of memory every time the token reaches it (after having completed one more round of the ring) and gives up the token when this chunk becomes full.

Section 6.1 presents the details of the token-based distributed stack. The token-based queue implementation appears in Section 6.2. Section 6.3 provides the token-based deque. We start by presenting *static versions* of the implementations, i.e. versions in which the total memory allocated for the data structure is predetermined during an execution and once it is exhausted the data structure becomes full and no more insertions of elements can occur. We then describe in Section 6.5, how to take dynamic versions of the data structures from their static analogs.

6.1 Token-Based Stack

To implement a distributed stack, each server uses its allocated memory chunk to maintain a local stack, *lstack*. Initially, s_t is the server with id 0. To perform a push (or pop), a client c sends a push (or pop) request to the server that has served c's last request (or, initially, to server 0) and awaits for a response. If this server is not the current token server at the time that it receives the request, it forwards the request to its next or previous server, depending on whether its local stack is full or empty, respectively. This is repeated until the request reaches the server s_t that has the token which pushes the new element onto its local stack and sends an ACK to c. If s_t 's local stack does not have free space to accommodate the new element, it sends the push request of c, together with an indication that it gives up its token, to the next server. POP is treated by s_t in a similar way.

6.1.1 Algorithm Description

```
Algorithm 10 Events triggered in a server
of the token-based stack.
    LocalStack lstack = \emptyset:
1
\mathbf{2}
    int my_sid;
                          /* each server has a unique id */
    int token = 0;
3
    a message \langle op, data, id, tk \rangle is received:
4
       switch (op) {
\mathbf{5}
         case PUSH:
6
            if (tk == TOKEN) token = my_sid;
\overline{7}
            if (token \neq my\_sid) {
8
9
             send(token, \langle op, data, id, tk \rangle);
              break:
10
            }
            if (!IsFull(lstack)) {
11
              push(lstack, data);
12
             \operatorname{send}(id, \langle \operatorname{ACK}, my\_sid \rangle);
13
            } else if (my\_sid \neq NS-1) {
14
              token = find\_next\_server(my\_sid);
15
              send(token, \langle op, data, id, TOKEN \rangle);
16
17
            } else /* It's the last server in the order, thus the stack is full */
              send(id, \langle NACK, my\_sid \rangle);
18
           break;
19
         case POP:
20
            if (tk == TOKEN) token = my_sid;
21
            if (token \neq my\_sid) {
22
              send(token, \langle op, data, id, tk \rangle);
23
              break:
24
            }
            if (!IsEmpty(lstack)) {
25
              data = pop(lstack);
26
              send(id, \langle data, my\_sid \rangle);
27
            } else if (my_{-sid} \neq 0) {
28
              token = find_previous_server(my_sid);
29
              send(token, \langle op, data, id, TOKEN \rangle);
30
31
            } else /* It's the first server in the order, thus the stack is empty */
              send(id, \langle NACK, my\_sid \rangle);
32
33
           break;
       }
```

Initially the elements are stored in the memory space allocated by server s_0 , the first server in the ring. At this point, s_0 is the token server; the token server manages the top of the stack. Once the memory chunk of the token server becomes full, the token server notifies the next server (s_1) in the ring to become the new token server.

The pseudocode for the server is presented in Algorithm 10. Each server s, apart from a local stack (*lstack*), maintains also a local variable *token* which identifies whether s is the token server. The messages that are transmitted during the execution are of type PUSH and POP,

Algorithm 11 Push operation for a client of the token-based stack.

OI	of the token-based stack.		
34	sid = 0;		
35	Data ClientPush(int cid, Data data) {		
36	$\operatorname{send}(sid, \langle \operatorname{PUSH}, data, cid, \bot \rangle);$		
37	$\langle status, sid \rangle = receive();$		
38	return <i>status</i> ;		
	}		

Algorithm 12 Pop operation for a client of the token-based stack.

sid = 0;Data ClientPop(int cid) { send(sid, (POP, \perp , cid, \perp)); (status, sid) = receive(); return status; }

which are sent from clients that want to perform the mentioned operation to the servers, or are forwarded from any server towards the token server. Each message has four fields: (1) op with the operation to be performed, (2) data, containing data in case of ENQ and \perp otherwise, (3) id that contains the id of the sender and (4) a one-bit flag tk which is set to TOKEN only when a forwarded message denotes also a token transition.

39

40 41

42

43

If the message is of type PUSH (line 6), s first checks whether the message contains a token transition. If tk is marked with TOKEN, s changes the token variable to contain its id (line 7). If s is not a token server, it just forwards the message to the next server (line 9). Otherwise, it checks if there is free space in lstack to store the new request (line 11). If there is such space, the server pushes the data to the stack, and sends back an ACK to the client. In this implementation, the push() function (line 12) does not need to return any value, since the check for memory space has already been performed by the server on line 11, hence push() is always successful.

If s does not have any free space, it must notify the next server to become the new token server. More specifically, if s is not the server with id NS - 1 (line 14), it forwards to the next server the PUSH message it received from the client, after setting the tk field to TOKEN (line 16). On the other hand, if s is the server with id NS - 1, all previous servers have no memory space available to store a stack element. In this case, s sends back to the client a message NACK(line 18).

If the message is of type POP (line 20) similar actions take place: s checks whether the message contains a token transition and if its true, it changes its local variable *token* appropriately. Then s checks if it is the token server (line 22). If not, it just forwards the message towards the server it considers as the token server (line 23). If s is the token server, it checks if its local stack is empty (line 25). If it is not empty, the pop operation can be executed normally. At the end of the operation, s sends to the client the data of the previous top element (line 27). In case of an empty local stack, if s is not s_0 (line 28), it forwards to the previous server the client's POP message, after setting the tk field to TOKEN (line 30). On the other hand, if the server that received the POP request is s_0 (id == 0), then all the servers have empty stacks and the server sends back to the client a NACK message (line 32).

The clients execute the operations push and pop, by calling the functions ClientPush() and ClientPop(), respectively. Each of these functions sends a message to the server. Initially, the clients forward their requests to s_0 . Because the server that maintains the top element might change though, the clients update the *sid* variable through a lazy mechanism. When a client c wants to perform an operation, it sends a request to the server with id equal to the value of *sid* (lines 34 and 39). If the message was sent to an incorrect server, it is forwarded by the servers till it reaches the server that holds the token. That server is going to respond with the status value of the operation and with the its id. This way, c updates the variable *sid*.

During the execution of the ClientPush() function, described in Algorithm 11, the client

sends a PUSH message to the server with id *sid* (line 36). It then, waits for its response (line 37). When the client receives the response, it updates the *sid* variable (line 37) and returns the *status*. The *status* is either ACK for a successful push, or NACK for a full stack.

The ClientPop() function operates in a similar fashion. The client sends a POP message to the server with id *sid* (line 41). It then, waits for its response (line 42). The server responds with a NACK (for empty queue), or with the value of the top element (otherwise). The server also forwards its id, which is stored in client's variable *sid*. The client finally, returns the *status* value and terminates.

6.1.2 Proof of Correctness

Let α be an execution of the token-based stack algorithm presented in Algorithms 10, 11, and 12. Let *op* be any operation in α . We assign a linearization point to *op* by considering the following cases:

- op is a push operation. Let s_t be the token server that responds to the client that initiated op (i.e. the **receive** of line 37 in the execution of op receives a message from s_t). If op returns ACK, the linearization point is placed at the configuration resulting from the execution of line 13 by s_t for op. Otherwise, the linearization point of op is placed at the configuration resulting from the execution of line 18 by s_t for op.
- op is a pop operation. Let s_t be the token server that responds to the client that initiated op (line 42). If the operation returns NACK, the linearization point of op is placed at the configuration resulting from the execution of line 32 by s_t for op. Otherwise, the linearization point of op is placed at the configuration resulting from the execution of line 27 by s_t for op.

Denote by L the sequence of operations (which have been assigned linearization points) in the order determined by their linearization points.

Lemma 25. The linearization point of a push (pop) operation op is placed in its execution interval.

Proof Sketch. Assume that op is a push operation and let c be the client that invokes it. After the invocation of op, c sends a message to some server s and awaits a response. Recall that routine receive() (line 37) blocks until a message is received. The linearization point of op is placed either in the configuration resulting from the execution of line 13 by s_t for op, where s_t is the token server in this configuration, or in the configuration resulting from the execution of line 18 by s_t for op.

Either of these lines is executed after the request by c is received, i.e. after c invokes ClientPush. Furthermore, they are executed before c receives the response by s_t and thus, before ClientPush returns. Therefore, the linearization point is inside the execution interval of push.

The argumentation regarding pop operations is analogous.

Each server maintains a local variable *token* with initial value 0 (initially, the server with id equal to 0 is the token server). Whenever some server s_i receives a TOKEN message, i.e. a message with its *tk* field equal to TOKEN (line 7), the value of *token* is set to *i*. By inspection of the pseudocode, it follows that the value of *token* is set to the id of the next server if the local stack of s_i is full (line 15); then, a TOKEN message is sent to the next server (line 16). Moreover, the value of *token* is set to the id of the previous server if the local stack of s_i is empty (line 28); then, a TOKEN message is sent to the previous server (lines 29-30). (Unless the server is s_0 in which case a NACK is sent to the client (line 32 but no TOKEN message to any server.) Thus, the following observation holds.

Observation 26. At each configuration in α , there is at most one server s_i for which the local variable token has the value *i*.

At each configuration C, the server s_i whose token variable is equal to i is referred to as the token server at C.

Observation 27. A TOKEN message is sent from a server with $id i, 0 \le i < NS - 1$, to a server with id i + 1 only if the local stack of server i is full. A TOKEN message is sent from a server with $id i, 0 < i \le NS - 1$, to a server with id i - 1 only when the local stack of server i is empty.

By the pseudocode, namely the if clause of line 8 and the if clause of line 22, the following observation holds.

Observation 28. Whenever a server s_i performs push and pop operations on its local stack (lines 12 and 26), it holds that its local variable token is equal to i.

Let C_i be the configuration at which the *i*-th operation op_i of L is linearized. Denote by α_i , the prefix of α which ends with C_i and let L_i be the prefix of L up until the operation that is linearized at C_i . Denote by S_i the sequence of values that a sequential stack contains after applying the sequence of operations in L_i , in order, starting from an empty stack; let $S_0 = \epsilon$, i.e. S_0 is the empty sequence.

Lemma 29. For each $i, i \ge 0$, if s_{k_i} is the token server at C_i and ls_i^j are the contents of the local stack of server $j, 0 \le j \le k_i$, at C_i , then it holds that $S_i = ls_i^0 \cdot ls_i^1 \cdot \ldots \cdot ls_i^{k_i}$ at C_i .

Proof. We prove the claim by induction on *i*. The claim holds trivially for i = 0. Fix any $i \ge 0$ and assume that at C_i , it holds that $S_i = ls_i^0 \cdot ls_i^1 \cdot \ldots \cdot ls_i^{k_i}$. We show that the claim holds for i+1.

We first assume that op_{i+1} is a push operation initiated by some client c. Assume first that $s_{k_i} = s_{k_{i+1}}$. Then, by induction hypothesis, $S_i = ls_i^0 \cdot \ldots \cdot ls_i^{k_i}$. In case the local stack of s_{k_i} is not full, s_{k_i} pushes the value v_{i+1} of field *data* of the request onto its local stack and responds to c. Since no other change occurs to the local stacks of s_0, \ldots, s_{k_i} from C_i to C_{i+1} , at C_{i+1} , it holds that $S_{i+1} = ls_i^0 \cdot \ldots \cdot ls_i^k \cdot \{v_{i+1}\} = ls_i^0 \cdot \ldots \cdot ls_{i+1}^{k_i}$. In case that the local stack of s_{k_i} is full, since $s_{k_i} = s_{k_{i+1}}$ and it is the token server, it follows that $s_{k_i} = s_{NS-1}$. In this case, s_{k_i} responds with a NACK to c and the local stack remains unchanged. Thus, it holds that $S_{i+1} = ls_i^0 \cdot \ldots \cdot ls_i^k = S_i$.

Assume now that $s_{k_i} \neq s_{k_{i+1}}$. This implies that the local stack of s_{k_i} is full just after C_i . Observation 27 implies that s_{k_i} forwarded the token to s_{k_i+1} in some configuration between C_i and C_{i+1} . Notice that then, $s_{k_i+1} = s_{k_{i+1}}$. Observation 28 implies that the local stack of s_{k_i+1} is empty. Thus, the **if** condition of line 11 evaluates to **true** for server s_{k_i+1} and therefore, it pushes the value v_{i+1} of op_{i+1} onto its local stack. Thus, at C_{i+1} , $ls_{i+1}^{k_i+1} = \{v_{i+1}\}$. By definition, $S_{i+1} = S_i \cdot \{v_{i+1}\}$. Therefore, $S_{i+1} = ls_i^0 \cdot \ldots \cdot ls_{i+1}^{k_i+1}$. And since by Observations 26 and 28, the contents of the local stacks of servers other than $k_i + 1$ do not change, it holds that $S_{i+1} = ls_{i+1}^0 \cdot \ldots \cdot ls_{i+1}^{k_i+1} = ls_i^0 \cdot \ldots \cdot ls_{i+1}^{k_i+1}$.

The reasoning for the case where op_{i+1} is an instance of a pop operation is symmetrical. \Box

From the above lemmas and observations, we have the following.

Theorem 30. The token-based distributed stack implementation is linearizable. The time complexity and the communication complexity of each operation op is O(NS).

6.2 Token-Based Queue

To implement a queue, two tokens are employed: at each point in time, there is a head token server s_h and a tail token server s_t . Initially, server 0 plays the role of both s_h and s_t . Each server s_r , other than s_t (s_h), that receives a request (directly) from a client c, it forwards the request to the next server to ensure that it will either reach the appropriate token server or return back to s_r (after traversing all servers). Servers s_t and s_h work in a way similar as server s_t in stacks.

To prevent a request from being forwarded forever due to the completion of concurrent requests which may cause the token(s) to keep advancing, each server keeps track of the request that each client c (directly) sends to it, in a *client table* (there can be only one such request per client). Server s_t (and/or s_h) now reports the response to s_r which forwards it to c. If s_r receives a response for a request recorded in its client table, it deletes the request from the client table. If s_r receives the token (stack, tail, or head), it serves each request (push and pop, enqueue, or dequeue, respectively) in its client array and records its response. If a request, from those included in s_r 's client array, reaches s_r again, s_r sends the response it has calculated for it to the client and removes it from its client array. Since the communication channels are FIFO, the implementations ensures that all requests, their responses, and the appropriate tokens, move from one server to the next, based on the servers' ring order, until they reach their destination. This is necessary to argue that the technique ensures termination for each request.

6.2.1 Algorithm Description

The queue implementation follows similar ideas to those of the token-based distributed stack, presented in Section 6.1. However, the queue implementation employs two tokens, one for the queue's tail and one for the queue's head, called *head token* and *tail token*, respectively. The tokens for the global head and tail are initially held by s_0 . However, they can be reassigned to other servers during the execution. If the local queue of the server that has the tail token becomes full, the token is forwarded to the next server. Similarly, if the local queue of the server that has the head token becomes empty, the head token is forwarded to the next server. If the appropriate token server receives the request and serves it, it sends an ACK message back to the server that initiated the forwarding. Then, the initial server responds to the client with an ACK message, which also includes the id of the server that currently holds the token.

The clients in their initial state store the id of s_0 , which is the first server to hold the head and tail tokens. The clients keep track of the servers that hold either token in a lazy way. Specifically, a client updates its local variable (either enq_sid or deq_sid depending on whether its current active operation is an enqueue or a dequeue, respectively) with the id of the token server when it receives a server response.

In this scheme, a client request may be transmitted indefinitely from a server to the next without ever reaching the appropriate token server. This occurs if both the head and the tail tokens are forwarded indefinitely along the ring. Then, a continuous, never-ending race between a forwarded message and the appropriate token server may occur. To avoid this scenario, we do the following actions. When a server s receives a client's request r, if it does not have the appropriate token to serve it, it stores information about r in a local array before it forwards it. Next time that the server receives the tail (head) token, it will serve all enqueue (dequeue) requests. Notice that since channels preserve the FIFO order and servers process messages in the order they arrive, the appropriate token will reach s earlier than the r. When s receives r, it has already processed it; however, it is then that s sends the response for r to the client.

In Algorithm 13, we present the local variables of a server. Each server s holds its unique id my_sid and a local queue lqueue that stores its part of the queue. Also keeps two boolean flag

\mathbf{A}	Algorithm 13 Token-based queue server's local variables.		
1	int <i>my_sid</i> ;		
2	LocalQueue $lqueue = \emptyset;$		
3	LocalArray $clients = \emptyset;$	/* Array of three values: <op, data,="" is<br="">Served >$\ */$</op,>	
4	boolean $fullQueue = \texttt{false};$	/* True when tail and head are in the same	
		server and tail is before head $*/$	
5	boolean $hasHead;$	/* Initially has Head and has Tail are true in server 0, and false in the rest */	
6	boolean $hasTail;$		

variables, (hasHead and hasTail), indicating whether s has the head token or the tail token, and one more bit flag (fullQueue) indicating whether the queue is full. Finally, s has a local

\mathbf{A}	Algorithm 14 Events triggered in a server of the token-based queue.		
7	a message $\langle op, data, cid, sid, tk \rangle$ is received:		
8	if (!clients[cid] AND clients[cid].isServed) {	/* If message has been served earlier. */	
9	$send(cid, \langle ACK, clients[cid].data, my_sid \rangle);$		
10	$clients[cid] = \bot;$		
11	} else {		
12	switch (op) {		
13	case ENQ:		
14	if $(tk == \texttt{TAIL_TOKEN})$ {		
15	hasTail = true;		
16	if(hasHead) fullQueue = true;		
17	ServeOldEnqueues();		
	}		
18	$if (!hasTail) \{$	/* Server does not have token */	
19	$nsid = find_next_server((my_sid);$		
20	$\texttt{if} (sid == -1) \{$	/* From client. $*/$	
21	$clients[cid] = \langle ENQ, data, false \rangle;$		
22	$\operatorname{send}(nsid, \langle \texttt{ENQ}, data, cid, my_sid, \bot \rangle);$		
23	} else {	/* From server. $*/$	
24	$\operatorname{send}(nsid, \langle \texttt{ENQ}, data, cid, sid, \bot \rangle);$		
	}		
25	$\}$ else if $(!IsFull(lqueue))$ {	/* Server can enqueue. $*/$	
26	enqueue(lqueue, data);		
27	if(sid == -1)	/* From client. */	
28	$\operatorname{send}(cid, \langle \texttt{ACK}, \bot, my_sid \rangle);$		
29	else	/* From server. $*/$	
30	$\operatorname{send}(\operatorname{sid}, \langle \operatorname{ACK}, \bot, \operatorname{cid}, my_\operatorname{sid}, \bot \rangle);$		
31	$\}$ else if $(fullQueue)$ {	/* Global Queue full */	
32	if(sid == -1)	/* From client. */	
33	$\operatorname{send}(\operatorname{cid}, \langle \texttt{NACK}, \bot, my_\operatorname{sid} \rangle);$		
34	else	/* From server */	
35	$\operatorname{send}(sid, \langle \texttt{NACK}, \bot, cid, my_sid, \bot \rangle);$		
36	} else {	/* Server moves the tail token to the next server */	
37	$nsid = \text{find_next_server}(my_sid);$		
38	fullQueue = false;		
39	hasTail = false;		
40	$send(nsid, \langle op, data, cid, my_sid, \texttt{TAIL_TOKEN} \rangle);$		
	}		
41	break;		

42	case DEQ:	
43	if $(tk == \text{HEAD_TOKEN})$ {	
44	hasHead = true;	
45	ServeOldDequeues();	
40	}	
46	if (!hasHead) {	
47	$nsid = \text{find_next_server}(my_sid);$	
48	if $(sid = = -1)$ {	/* From client */
49	$clients[cid] = \langle DEQ, \bot, false \rangle;$, , ,
50	$send(nsid, \langle DEQ, \bot, cid, my_sid \rangle);$	
51	} else {	/* From server */
52	$\operatorname{send}(nsid, \langle DEQ, \bot, cid, sid, \bot \rangle);$	
	}	
53	} else if (!IsEmpty(lqueue)) {	/* Server can dequeue. */
54	data = dequeue(lqueue);	
55	if(sid == -1)	/* From client */
56	$\operatorname{send}(\operatorname{cid}, \langle \operatorname{ACK}, \operatorname{data}, \operatorname{my_sid} \rangle);$	
57	else	/* From server */
58	$\operatorname{send}(sid, \langle \texttt{ACK}, data, cid, my_sid, \bot \rangle);$	
59	$\}$ else if $(hasTail AND ! fullQueue)$ {	/* Queue is empty $*/$
60	if(sid == -1)	/* From client */
61	$\operatorname{send}(\operatorname{cid}, \langle \operatorname{NACK}, \bot, my_sid \rangle);$	
62	else	/* From server */
63	$\operatorname{send}(sid, \langle \operatorname{NACK}, \bot, cid, my_sid, \bot \rangle);$	
64	$\}$ else $\{$	/* Server moves the head token to the next server */
65	$nsid = \text{find_next_server}(my_sid);$	
66	hasHead = false;	
67	$send(nsid, \langle op, \bot, cid, my_sid, \texttt{HEAD_TOKEN} \rangle);$	
	}	
68	break;	
69	case ACK:	
70	$clients[cid] = \bot;$	
71	$send(cid, \langle \texttt{ACK}, data, sid \rangle);$	
72	break;	
73	case NACK:	
74	$clients[cid] = \bot;$	
75	$send(cid, \langle NACK, \bot, sid \rangle);$	
76	break;	
	}	
	}	

array of size n, where n is the maximum number of clients, used for storing all direct requests from clients (called s's *clients* array). In their initial state, all servers have *fullQueue* set to **false** and their *clients* array and local queue empty. Also, all servers apart from server 0, have both their flags *hasHead* and *hasTail* set to **false**, whereas in server 0, they are set to **true**, as described above.

The messages sent to a server s_i are of type ENQ or DEQ, describing requests for enqueue or dequeue operations, respectively, sent by either a server or a client, and ACK or NACK sent by another server s_j which executed a forwarded request, whose forwarding was initiated by s_i . The token transition is encapsulated in a message of type ENQ or DEQ. The messages have five fields: (1) op, which describes the type of the request (ENQ, DEQ, ACK or NACK), (2) data, which stores an element in case of ENQ, and \perp otherwise, (3) cid, which stores the id of the client

that issued the request, (4) *sid* which contains the id of the server if the message was sent by a server, and -1 otherwise, and (5) tk, which contains TAIL_TOKEN or HEAD_TOKEN in forwarded messages of type ENQ or DEQ, respectively, to indicate if an additional tail (or head, respectively) token transition occurs, and it is equal to \perp otherwise.

Event-driven pseudocode for the server is presented in Algorithm 14. When a server s receives a message of type ENQ (line 13), it first checks if it contains a token transfer from another server (line 14). If it does, the server sets its token *hasTail* to **true** (line 15) and if it also had the head token from a previous round, it changes *fullQueue* flag to **true** as well (line 16). Then, s serves all pending ENQ messages stored in its *clients* array (line 17).

Then, s continues to execute the ENQ request. It checks first whether it has the tail token. If it does not (line 18), it finds the next server s_{next} (line 19), to whom s is going to forward the request. Afterwards, s sends the received request to s_{next} (lines 22 and 24) and if that request came directly from a client (line 20), s updates its *clients* array storing in it information about this message (line 21).

If s has the token, then it attempts to serve the request. If s has remaining space in its local queue (*lqueue*), it enqueues the given data and informs the appropriate server with an ACK message (lines 25-30). If the implemented queue is full, s sends a NACK message to the client (line 31-35). In any remaining case, the server s must give the tail token to the next server (line 36). So, s forwards the ENQ message to s_{next} , after encapsulating in the message the tail token (line 40). After releasing the tail token, s changes the values of its local variables (*hasTail* and fullQueue) to false.

In case a DEQ message is received (line 42), the actions performed by s are similar to those for ENQ. Server s checks whether the request message contains a token transition (line 43). If it does, the server sets its token *hasHead* to **true** (line 44). Then, s serves all pending DEQ messages stored in its *clients* array (line 45), and then attempts to serve the request. If s does not hold the head token (line 46), it finds the next server s_{next} in the ring (line 47), to whom s is going to forward the request. Afterwards, s sends the received request to s_{next} (lines 50, 52) and if that request came directly from a client, s updates its *clients* array storing in it information about this message (line 49).

If s has the head token, it does the following actions. If its local queue (*lqueue*) is not empty (line 53), s performs a dequeue on its local queue and sends an ACK along with the dequeued data to the appropriate server. If s holds both head and tail tokens, but no other server has a queue element stored, and s's *lqueue* is empty, it means that the global queue is empty (line 59). Thus, s sends a NACK message to the appropriate server. In the remaining cases, s must forward its head token (line 64). Server s finds the next server s_{next} (line 65), which is going to receive the forwarded message and the head token transition. Server s sets the message field tk to HEAD_TOKEN and sends the message (line 67). After releasing the head token, s sets the value of its local variable *hasHead* to false (line 66).

Algorithm 15 Auxiliary functions for a server

```
of the token-based queue.
77
   void ServeOldEnqueues(void) {
      if (!fullQueue) {
78
       for each cid such that clients[cid].op == ENQ 
79
         if (!IsFull(lqueue)) {
80
81
           enqueue(lqueue, clients[cid].data);
82
           clients[cid].isServed = true;
   } } }
83
   void ServeOldDequeues(void) {
84
     for each cid such that clients[cid].op == DEQ {
85
86
       if (!IsEmpty(lqueue)) {
         clients[cid].data = dequeue(lqueue);
87
         clients[cid].isServed = true;
88
   } } }
```

If s received a message of type ACK (line 69) or NACK (line 73), then s sets the entry *cid* of its *clients* array to \perp (lines 70, 74) and sends an ACK (line 71) or a NACK (line 75) to that client. The ACK and NACK messages a server s receives, are only sent by other servers and signify the result of the execution of a forwarded message sent by s.

On lines 21 and 49, s stores the client request in its *clients* array when it does not hold the appropriate token. A request recorded in the *clients* array is removed from the array either when an ACK or NACK message is received for it (lines 70 and 74) or when the server receives again the request (after a round-trip on the ring) (lines 9-10). Thus, the server, upon any message receipt, first checks whether the message exists in its *clients* array and has already been served. In case of ENQ s answers with an ACK message, whereas in case of DEQ s answers with ACK and the dequeued data. Then, server s proceeds with the deletion of the entry in its clients array (lines 9, 10).

Functions ServeOldEnqueues() and ServeOldDequeues() are described in more detail in Algorithm 15. ServeOldEnqueues() (line 77) processes all ENQ requests stored in the *clients* array, if the local queue has space (line 80). Similarly, ServeOldDequeues() (line 84) processes all DEQ requests stored in the *clients* array, if the local queue is not empty (line 86).

The clients call the functions ClientEnqueue() and ClientDequeue(), presented in Algorithm 16, in order to perform one of these operations. In more detail, during enqueue, the client sends an ENQ message to the *enq_sid* server, and waits for a response. When the client receives the response, it returns it. Likewise, in ClientDequeue() the client sends a DEQ message to server *deq_sid* and blocks waiting for a response. When it receives the response, it returns it.

```
Algorithm 16 Enqueue and Dequeue op-
erations for a client of the token-based
queue.
    int enq\_sid = 0;
89
90
    int deq\_sid = 0;
    Data ClientEnqueue(int cid, Data data) {
91
      send(enq\_sid, \langle ENQ, data, cid, -1 \rangle);
92
93
       \langle status, \bot, enq\_sid \rangle = receive(enq\_sid);
94
      return status;
    }
   Data ClientDequeue(int cid) {
95
      send(deq\_sid, \langle DEQ, \bot, cid \rangle);
96
       \langle status, data, deq\_sid \rangle = receive(deq\_sid);
97
98
      return data;
    }
```

6.2.2 Proof of Correctness

Let α be an execution of the token-based queue algorithm presented in Algorithms 14, 15, and 16. Each server maintains local boolean variables *hasHead* and *hasTail*, with initial values **false**. Whenever some server s_i receives a **TAIL_TOKEN** message, i.e. a message with its tkfield equal to **TAIL_TOKEN** (line 14), the value of *hasTail* is set to **true** (line 15). By inspection of the pseudocode, it follows that the value of *hasTail* is set to **false** if the local queue of s_i is full (line 25, 36- 39); then, a **TAIL_TOKEN** message is sent to the next server (line 40). The same holds for *hasHead* and HEAD_TOKEN messages, i.e. messages with their tk field equal to HEAD_TOKEN. Thus, the following observations holds.

Observation 31. At each configuration in α , there is at most one server for which the local variable has Head (has Tail) has the value true.

Observation 32. In some configuration C of α , TAIL_TOKEN message is sent from a server s_j , $0 \leq j < NS - 1$, to a server s_k , where $k = (j + 1) \mod NS$ only if the local queue of s_j is full in C. Similary, a HEAD_TOKEN message is sent from s_j to s_k only if the local queue of s_j is empty in C.

By inspection of the pseudocode, we see that a server performs an enqueue (dequeue) operation on its local queue *lqueue* either when executing line 26 (line 45) or when executing ServeOldEnqueues (ServeOldDequeues). Further inspection of the pseudocode (lines 14-17, lines 25-31, as well as lines 46-52, lines 53-59), shows that these lines are executed when hasTail = true. Then, the following observation holds.

Observation 33. Whenever a server s_j performs an enqueue (dequeue) operation on its local queue, it holds that its local variable hasTail (hasHead) is equal to true.

By a straight-forward induction, the following lemma can be shown.

Lemma 34. The mailbox of a client in any configuration of α contains at most one incoming message.

If hasTail = true (hasHead = true) for some server s in some configuration C, then we say that s has the tail (head) token. The server that has the tail token is referred to as *tail token server*. The server that has the head token is referred to as *head token server*.

Let op be any operation in α . We assign a linearization point to op by considering the following cases:

- If *op* is an enqueue operation for which a tail token server executes an instance of Algorithm 14, then it is linearized in the configuration resulting from the execution of either line 26, or line 81, or line 33, whichever is executed for *op* in that instance of Algorithm 14 by the tail token server.
- If *op* is a dequeue operation for which a head token server executes an instance of Algorithm 14, then it is linearized in the configuration resulting from the execution of either line 54, or line 87, or line 56, whichever is executed for *op* in that instance of Algorithm 14 by the head token server.

Lemma 35. The linearization point of an enqueue (dequeue) operation op is placed in its execution interval.

Proof. Assume that op is an enqueue operation and let c be the client that invokes it. After the invocation of op, c sends a message to some server s (line 92) and awaits a response. Recall that routine receive() (line 93) blocks until a message is received. The linearization point of op is placed either in the configuration resulting from the execution of line 26 by s_t for op, in the configuration resulting from the execution of line 33 by s_t for op, or in the configuration resulting from the execution of line 81 by s_t for op. Notice that either of these lines is executed after the request by c is received, i.e. after c invokes ClientEnqueue, and thus, after the execution interval of op starts.

By definition, the execution interval of op terminates in the configuration resulting from the execution of line 94. By inspection of the pseudocode, this line is executed after line 93, i.e. after c receives a response by some server. In the following, we show that the linearization point of op occurs before this response is sent to c.

Let s_j be the server that c initially sends the request for op to. By observation of the pseudocode, we see that c may either receive a response from s_j if s_j executes lines 28 or 33, or if s_j executes lines 70-71 or lines 74-75, or if s_j executes line 9. To arrive at a contradiction, assume that either of these lines is executed in α before the configuration in which the linearization point of op is placed. Thus, a tail token server s_t executes lines 26, 81, or 33 in a configuration following the execution of lines 28, or 33, or 70-71 or 74-75, or line 9 by s_j . Since the algorithm is event-driven, inspection of the pseudocode shows that in order for a tail token server to execute these lines, it must receive a message containing he request for op either from a client or from another server.

Assume first that a tail token server executes the algorithm after receiving a message containing a request for op from a client. This is a contradiction, since, on one hand, c blocks until receiving a response, and thus, does not sent further messages requesting op or any other operation, and since op terminates after c receives the response by s_j , and on the other hand, any other request from any other client concerns a different operation op'.

Assume next that a tail token server executes the algorithm after receiving a message containing the request for op from some other server. This is also a contradiction since inspection of the pseudocode shows that after s_j executes either of the lines that sends a response to c, it sends no further message to some other server and instead, terminates the execution of that instance of the algorithm.

The argumentation regarding dequeue operations is analogous.

Denote by L the sequence of operations which have been assigned linearization points in α in the order determined by their linearization points. Let C_i be the configuration at which the *i*-th operation op_i of L is linearized. Denote by α_i , the prefix of α which ends with C_i and let L_i be the prefix of L up until the operation that is linearized at C_i . Denote by Q_i the sequence of values that a sequential queue contains after applying the sequence of operations in L_i , in order, starting from an empty queue; let $Q_0 = \epsilon$, i.e. Q_0 is the empty sequence. In the following, we denote by s_{t_i} the tail token server at C_i and by s_{h_i} the head token server at C_i .

Lemma 36. For each $i, i \geq 0$, if lq_i^j are the contents of the local queue of server s_j at C_i , $h_i \leq j \leq t_i$, at C_i , then it holds that $Q_i = lq_i^{h_i} \cdot lq_i^{h_i+1} \cdot \ldots \cdot lq_i^{t_i}$ at C_i .

Proof. We prove the claim by induction on i. The claim holds trivially at i = 0. Fix any $i \ge 0$ and assume that at C_i , it holds that $Q_i = lq_i^{h_i} \cdot lq_i^{h_i+1} \cdot \ldots \cdot lq_i^{t_i}$. We show that the claim holds for i + 1.

First, assume that op_{i+1} is an enqueue operation by client c. Furthermore, distinguish the following two cases:

• Assume that $t_i = t_{i+1}$. Then, by the induction hypothesis, $Q_i = lq_i^{h_i} \cdot lq_i^{h_i+1} \cdot \ldots \cdot lq_i^{t_i}$. In case the local queue of s_{t_i} is not full, s_{t_i} enqueues the value v_{i+1} of the data field of the request for op_{i+1} in the local queue (line 26 or line 81). Notice that, by Observation 33 changes on the local queues of servers occur only on token servers. Notice also that those changes occur only in a step that immediately precedes a configuration in which an operation is linearized. Thus, no further change occurs on the local queues of $s_{h_i}, s_{h_i+1}, \ldots, s_{t_i}$ between C_i and C_{i+1} , other than the enqueue on lq_i^t . Then, it holds that $Q_{i+1} = Q_i \cdot v_{i+1} = lq_i^{h_i} \cdot lq_i^{h_i+1} \cdot \ldots \cdot lq_i^{t_i} \cdot v_{i+1} = lq_i^{h_i} \cdot lq_i^{h_i+1} \cdot \ldots \cdot lq_{i+1}^{t_i} = lq_{i+1}^{h_i} \cdot lq_{i+1}^{h_i+1} \cdot \ldots \cdot lq_{i+1}^{t_i}$, and if the head token server does not change between C_i and C_{i+1} , then $h_{i+1} = h_i$ and $Q_{i+1} = lq_{i+1}^{h_{i+1}} \cdot lq_{i+1}^{h_{i+1}+1} \cdot \ldots \cdot lq_{i+1}^{t_{i+1}}$ and the claim holds. If the head token server changes, i.e., if $h_{i+1} \neq h_i$, then by Observation 32, $lq_{i+1}^{h_i} = \emptyset$ and the claim holds again.

In case the local queue of s_{t_i} is full and since by assumption, $s_{t_i} = s_{t_{i+1}}$, it follows by inspection of the pseudocode (line 31) and the definition of linearization points, that $s_{t_{i+1}} = s_{h_{i+1}}$. In this case, $s_{t_{i+1}}$ responds with a NACK to c and the local queue remains unchanged. Since no token server changes between C_i and C_{i+1} , $Q_{i+1} = Q_i = lq_i^{h_i} \cdot lq_i^{h_{i+1}} \cdot \dots \cdot lq_i^{t_i} = lq_{i+1}^{h_{i+1}} \cdot lq_{i+1}^{h_{i+1}+1} \cdot \dots \cdot lq_{i+1}^{t_{i+1}}$ and the claim holds.

• Next, assume that $t_i \neq t_{i+1}$. This implies that the local queue of s_{t_i} is full just after C_i . Observation 32 implies that s_{t_i} forwarded the token to s_{t_i+1} in some configuration between C_i and C_{i+1} . Notice that then, $s_{t_i+1} = s_{t_{i+1}}$. If the local queue of $s_{t_{i+1}}$ is not full, then the condition of line 25 evaluates to true and therefore, line 26 is executed, enqueueing value v_{i+1} to it. Then at C_{i+1} , $lq_{i+1}^{t_{i+1}} = v_{i+1}$. By definition, $Q_{i+1} = Q_i \cdot v_{i+1}$, and therefore, $Q_{i+1} = lq_i^{h_i} \cdot lq_i^{h_i+1} \cdot \ldots \cdot lq_i^{t_i} \cdot v_{i+1} = lq_{i+1}^{h_{i+1}} \cdot lq_{i+1}^{h_{i+1}+1} \cdot \ldots \cdot lq_{i+1}^{t_i} \cdot v_{i+1} =$ $lq_{i+1}^{h_{i+1}} \cdot lq_{i+1}^{h_{i+1}+1} \cdot \ldots \cdot lq_{i+1}^{t_i} \cdot lq_{i+1}^{t_{i+1}}$ and the claim holds. If the local queue of $s_{t_{i+1}}$ is full, then the condition of line 25 evaluates to false and therefore, line 35 is executed. The operation is linearized in the resulting configuration and NACK is sent to c. Notice that in that case, the local queue of the server is not updated. Then, $Q_{i+1} = Q_i = lq_i^{h_i} \cdot lq_i^{h_i+1} \cdot \ldots \cdot lq_i^{t_i} \cdot lq_{i+1}^{t_{i+1}} =$ $lq_{i+1}^{h_{i+1}} \cdot lq_{i+1}^{h_{i+1}+1} \cdot \ldots \cdot lq_{i+1}^{t_i} \cdot lq_{i+1}^{t_{i+1}}$, and the claim holds.

The reasoning for the case where op_{i+1} is an instance of a dequeue operation is symmetrical.

From the above lemmas and observations we have the following theorem.

Theorem 37. The token-based distributed queue implementation is linearizable. The time complexity and the communication complexity of each operation op is O(NS).

6.3 Token-Based Double Ended Queue (Deque)

The dequeue implementation is a natural generalization of the stack and queue implementations described previously. The deque implementation is analogous to the queue implementation described in section 6.2. To provide a deque, we add actions to the queue's design to support the additional operations supported by a deque. We retain the static ordering of the servers and the head and tail tokens. Again, each server uses a local data structure, this time a deque, on which the server is allowed to execute enqueues or dequeues to the appropriate end, only if it has either the *tail token* or the *head token*. The head and tail tokens are initially held by s_0 , but can be reassigned to other servers during the execution.

6.3.1 Algorithm Description

Algorithm 17 presents the events triggered in a server s and s's actions for each event. Each server, in addition to its id (my_sid) , maintains a local deque (ldeque) to store elements of the implemented deque. For the token management, each server has two **boolean** flags (hasHead and hasTail, which are initialized **true** for the server s_0 , and **false** for the rest. Furthermore, the servers maintain a fullDeque flag, similar to the fullQueue flag in Algorithm 13 of Section 6.2. Finally, the servers maintain a local array (called s's clients array) for storing the requests they receive directly from clients, which is used in a similar way as in Section 6.2.

The types of messages a server s can receive are ENQ_T, DEQ_T for enqueuing at and dequeuing from the tail, ENQ_H, DEQ_H for enqueuing at and dequeuing from the head, and ACK or NACK sent by other servers as responses to s's forwarded messages. Every message m a server receives, contains five fields: (1) the op field that represents the type of the request, (2) a data field, that contains either the data to be enqueued or \perp , (3) a cid field that contains the client id that requested the operation op, (4) a sid field that contains the id of the server which started forwarding the request, and -1 otherwise, and (5) a tk field, a flag used to pass tokens from one server to another. The values that tk can take are either HEAD_TOKEN for the head token transition, TAIL_TOKEN for the tail token transition, or \perp for no token transition.

When server s receives the head token, that means that s can serve all operations in its client array regarding the head endpoint (EnqueueHead(), DequeueHead()). In analogous way, when server s receives the token for the tail, that means that s can serve all operations in its client array for the tail (EnqueueTail(), DequeueTail()). For this purpose, we use two functions, ServeOldHeadOps() and ServeOldTailOps(). The clients whose requests are served, are not informed until server s receives their requests completes a round-trip on the ring and returns back to s.

When a message is received (line 7), server s checks if the message is stored in its *client* array (line 8). If it is, s sends an ACK message to client with *cid* (line 9) and removes the entry in the *clients* array (line 10). If it is not, s checks the message's operation code and acts accordingly (line 12). Messages with operation code ENQ_T, DEQ_T, ENQ_H, DEQ_H (lines 13, 16, 19 and 22, respectively) are handled by functions ServerEnqueueTail() (line 14), ServerDequeueTail() (line 17), ServerEnqueueHead() (line 20) and ServerDequeueHead() (line 23), respectively. These functions are presented in Algorithms 18-21. If the message is of type ACK (line 25) or NACK (line 29), s sets the *cid* entry of the clients array to \perp (lines 26, 30), and sends an ACK (line 27) or NACK (line 31) to the client.

Algorithm 18 presents pseudocode for function ServerEnqueueTail(), which is called by a server s when an ENQ_T message is received. First, s checks whether the tk field of the message contains TAIL_TOKEN (line 34). In such a case, the message received by s denotes a tail token transition. So, s sets its *hasTail* flag to true (line 35) and if it also had the head token from a previous round, it sets its *fullDeque* flag to true (line 36). At this point, s has just received

Algorithm 17	Events	triggered	in	\mathbf{a}	server
--------------	--------	-----------	----	--------------	--------

1	$int my_sid;$		
2	LocalDeque $ldeque = \emptyset;$		
3	LocalArray $clients = \emptyset;$		/* Array of three values ; op, data, is Served; *,
4	boolean $fullDeque = \texttt{false};$		
		/* True when tail and h	ead are in the same server and tail is before head $*/$
5	boolean $hasHead;$	/* Initially hasHead and	l has Tail are true in server 0, and false in the rest $^{\ast}/$
6	boolean $hasTail;$		
7	a message $\langle op, data, cid, sid, tk \rangle$ is rece		
8	if $(\text{clients}[cid] \neq \bot \text{ AND clients}[cid]$	d]. $isServed$) {	/* If request was served earlier. */
9	$send(cid, \langle ACK, clients[cid].data, m)$	y_sid);	
10	$clients[cid] = \bot;$		
11	} else {		
12	$\texttt{switch} \ (op) \ \{$		
13	case ENQ_T:		
14	ServerEnqueueTail(op, data, calculate)	id, sid, tk);	
15	break;		
16	case DEQ_T:		
17	ServerDequeueTail(op, cid, sid)	(t,tk);	
18	break;		
19	case ENQ_H:		
20	ServerEnqueueHead(op, data, data)	cid, sid, tk);	
21	break;		
22	case DEQ_H:		
23	ServerDequeueHead(op, cid, si	(id, tk);	
24	break;		
25	case ACK:		
26	$clients[cid] = \bot;$		
27	$\operatorname{send}(\operatorname{cid}, \langle \texttt{ACK}, \operatorname{data}, \operatorname{sid} \rangle);$		
28	break;		
29	case NACK:		
30	clients $[cid] = \bot;$		
31	$\operatorname{send}(\operatorname{cid}, \langle \operatorname{NACK}, \bot, \operatorname{sid} \rangle);$		
32	break;		
	}		
	}		

the global deque's tail, so it must serve all old operations that clients have requested directly from this server to be performed in the deque's tail. For that purpose, the server calls the ServeOldTailOps() function (line 37). Then, the server checks whether the global deque is full and also the *ldeque* is full and if it is full, means that there is no free space left for the operation, thus s responds to the request with with a NACK (lines 40, 42). Otherwise, if the server can serve the request, it enqueues the received data to the tail of *ldeque* (line 44) and responds with an ACK (lines 46, 48). In any other case, s cannot serve the request received, but some other server might be capable to do so, so the message must be forwarded. Server s finds the next server (line 50) and if s handles the global deque's tail, turns its flags (*hasTail* and *fullDeque*) to false (line 52) and marks the token as TAIL_TOKEN for the token transition (line 54). If the message to the next server (lines 59, 61).

Algorithm 19 presents pseudocode for function ServerDequeueTail(), which is called by a

tail void ServerEnqueueTail(int op, Data data, int cid, int sid, enum tk) { 33 if $(tk == TAIL_TOKEN)$ { 34 hasTail = true;35 if (hasHead) fullDeque = true; 36 ServeOldTailOps(clients, fullDeque); 37ł if (fullDeque AND IsFull(ldeque)) { 38 /* Deque is full, can't enqueue */ if (sid = = -1)39 /* From client */ 40 $\operatorname{send}(id, \langle \operatorname{NACK}, \bot, my_sid \rangle);$ else 41 /* From server */ send(*sid*, $\langle NACK, \bot, cid, my_s id, \bot \rangle$); 42 } else if (*hasTail* AND !IsFull(*ldeque*)) { 43/* Server can enqueue */ enqueue_tail(*deque*, *data*); 44 45if (sid = = -1)/* From client */ send(*cid*, $\langle ACK, \bot, my_sid \rangle$); 46 else 47/* From server */ $\operatorname{send}(sid, \langle \operatorname{ACK}, \bot, cid, my_sid, \bot \rangle);$ 48} else { 49/* Server can't enqueue and global deque is not full */ $nsid = find_next_server(my_sid);$ 5051if (hasTail) { hasTail = false;52fullDeque = false;53 $tk = \text{TAIL}_{\text{TOKEN}};$ 5455} else { 56 $tk = \bot;$ if (sid = = -1) { 57/* From client */ clients[*cid*] = $\langle ENQ_T, data, false \rangle$; 58 $send(nsid, \langle ENQ_T, data, cid, my_sid, tk \rangle);$ 5960 } else { /* From server */ $send(nsid, \langle ENQ_T, data, cid, sid, tk \rangle);$ 61 ł }

Algorithm 18 Server helping function for handling an enqueue request to the global deque's

server s when a DEQ_T message is received. First, s checks whether the tk field of the message contains TAIL_TOKEN (line 63). In such a case, the message received by s, denotes a tail token transition. So, s sets its has Tail flag to true (line 64). then, s serves all old operations that clients have requested directly from this server to be performed in the deque's tail. For that purpose, the server calls ServeOldTailOps() (line 65). Afterwards, s checks if the "global" deque is empty and if it is empty, s responds with a NACK (lines 68, 70). Otherwise, if s can serve the request, it dequeues the data from the tail of its local deque (line 71), and sends them to the appropriate server or client with an ACK (lines 74, 76). In any other case, s cannot serve the request received, but some other server might be capable to do so, thus the message must be forwarded. Server s finds the previous server s_{prev} (line 78) and if s handles the global deque's tail, turns its flag for the tail to false (line 80) and marks tk as TAIL_TOKEN for the token transition (line 81). If the message received was send by a client, s stores the request to its *clients* array (line 85). Finally, s forwards the message to the previous server (lines 58, 59).

Algorithm 20 presents pseudocode for function ServerEnqueueHead(), which is called by a

Algorithm 19 Server helping function for handling an dequeue request to the global deque's tail void ServerDequeueTail(int op, int cid, int sid, enum tk) { 62if $(tk == TAIL_TOKEN)$ { 63 64hasTail = true:65ServeOldTailOps(clients, fullDeque); } if (hasHead AND hasTail AND !fullDeque AND IsEmpty(ldeque)) { 66/* Deque is empty, can't dequeue */ 67 if (sid = = -1)/* From client */ $send(cid, \langle NACK, \bot, my_sid \rangle);$ 68 69 else /* from server */ send(*sid*, $\langle NACK, \bot, cid, my_sid, \bot \rangle$); 70} else if (*hasTail* AND !IsFull(*ldeque*)) { 71/* Server can dequeue */ $data = dequeue_tail(ldeque);$ 7273if (sid = = -1)/* From client */ 74 $send(cid, \langle ACK, data, my_sid \rangle);$ else 75/* from server */ $send(sid, \langle ACK, data, cid, my_sid, \bot \rangle);$ 76} else { 77 /* Server can't dequeue and global deque is not empty. */ $psid = \text{find_previous_server}(my_sid);$ 7879if (hasTail) { hasTail = false;80 $tk = \text{TAIL}_{\text{TOKEN}};$ 81 } else { 82 $tk = \bot;$ 83if (sid = -1) { 84 /* From client */ clients[*cid*] = $\langle \text{DEQ}_T, \bot, \texttt{false} \rangle$; 85 $send(psid, \langle DEQ_T, \bot, cid, my_sid, tk \rangle);$ 8687 } else { /* from server */ $\operatorname{send}(psid, \langle \mathsf{DEQ}_{-}\mathsf{T}, \bot, cid, sid, tk \rangle);$ 88 } }

server s when an ENQ_H message is received. First, s checks if it has the token for global deque's head (line 90) and if this is the case, the server sets its hasHead flag to true (line 91). If s already has the token for the global deque's tail, it sets its fullDeque flag to true (line 92). At this point, s has just received the global deque's head, so it must serve all old operations that clients have requested directly from this server to be performed in the deque's head. For that purpose, server s calls ServeOldHeadOps() (line 93), which iterates s's clients array and serves all operations for the deque's head. Then, the server checks whether the global deque is full and also the *ldeque* is full and if it is full, means that there is no free space left for the operation, thus s responds to the request with with a NACK (lines 96, 98). Otherwise, if the server can serve the request, it enqueues the received data to the deque's head (line 100) and responds with an ACK (lines 102, 104). In any other case, s cannot serve the request received, but some other server might be capable to do so, so the message must be forwarded. Server s finds the previous server (line 106) and if s handles the global deque's head, turns its flags (*hasHead* and *fullDeque*) to false (line 108) and marks the tk as HEAD_TOKEN for the token transition (line 110). If the message received was send by a client, s stores the request to its

head void ServerEnqueueHead(int op, Data data, int cid, int sid, enum tk) { 89 if $(tk == \text{HEAD_TOKEN})$ { 90 hasHead = true;91 if (hasTail) fullDeque = true; 92 ServeOldHeadOps(clients); 93ł if (fullDeque AND IsFull(ldeque)){ 94/* Deque is full, can't enqueue */ if (sid = = -1)95/* From client */ $send(cid, \langle NACK, \bot, my_sid \rangle);$ 96 97 else /* from server */ send(*sid*, $\langle NACK, \bot, cid, my_sid, \bot \rangle$); 98 } else if (*hasHead* AND !IsFull(*ldeque*)) { 99/* Server can Server can enqueue. */ enqueue_head(*ldeque*, *data*); 100 101 if (sid = = -1)/* From client */ send(*cid*, $\langle ACK, \bot, my_sid \rangle$); 102else 103/* from server */ $\operatorname{send}(sid, \langle \operatorname{ACK}, \bot, cid, my_sid, \bot \rangle);$ 104} else { 105/* Server can't dequeue and global deque is not full. */ $psid = \text{find_previous_server}(my_sid);$ 106107if (hasHead) { hasHead = false;108 fullDeque = false;109 $tk = \text{HEAD}_{\text{TOKEN}};$ 110 111} else { 112 $tk = \bot;$ if (sid = = -1) { 113/* From client */ clients[*cid*] = $\langle ENQ_H, data, false \rangle$; 114 $send(psid, \langle ENQ_H, data, cid, my_sid, tk \rangle);$ 115116} else { /* from server */ $send(psid, \langle ENQ_H, data, cid, sid, tk \rangle);$ 117} }

Algorithm 20 Server helping function for handling an enqueue request to the global deque's

clients array (line 114). Finally, s forwards the message to the previous server (lines 115, 117).

Algorithm 21 presents pseudocode for function ServerDequeueHead(), which is called by a server when a DEQ_H message is received by some server s. First, s checks if it has the token for global deque's head (line 119) and if this is the case, the server sets its hasHead flag to true (line 120). At this point, server s has just received the global deque's head, so it must serve all old operations that clients have requested directly from this server to be performed in the deque's head. For that purpose, the server calls the ServeOldHeadOps() routine (line 121), which iterates s's *clients* array and serves all operations for the deque's head. Then, the server checks if the global deque is empty and if it is, s responds to the request with a NACK (lines 124, 126). Otherwise, if the server can serve the request, it dequeues the data from the head (line 128) and sends them to the sender or client together with an ACK (lines 130, 132). In any other case, s cannot serve the request received, but some other server might be capable to do so, so the message must be forwarded. Server s finds the next server (line 134) and if s has the token for the global head, turns its flag for the head to false (line 135) and and marks the tk

```
Algorithm 21 Server helping function for handling an dequeue request to the global deque's
head
118 void ServerDequeueHead(int op, int cid, int sid, enum tk) {
       if (tk == \text{HEAD_TOKEN}) {
119
          hasHead = true;
120
         ServeOldHeadOps(clients);
121
       }
       if (hasHead AND hasTail AND !fullDeque AND IsEmpty(ldeque)){
122
                                                                                                 /* Deque is empty, can't dequeue */
         if (sid = = -1)
123
                                                                                                                    /* From client */
            send(cid, \langle NACK, \bot, my\_sid \rangle);
124
125
          else
                                                                                                                    /* from server */
            \operatorname{send}(\operatorname{sid}, \langle \operatorname{NACK}, \bot, \operatorname{cid}, my \operatorname{sid}, \bot \rangle);
126
       } else if (hasHead AND !IsFull(ldeque)) {
127
                                                                                                 /* Server can Server can enqueue. */
         data = dequeue\_head(ldeque);
128
129
          if (sid = = -1)
                                                                                                                    /* From client */
            send(cid, \langle ACK, data, my\_sid \rangle);
130
          else
131
                                                                                                                    /* from server */
            send(sid, \langle ACK, data, cid, my\_sid, \bot \rangle);
132
       } else {
133
                                                                             /* Server can't dequeue and global deque is not empty. */
         nsid = find\_next\_server(my\_sid);
134
135
            hasHead = false;
            tk = \text{HEAD}_{\text{-}}\text{TOKEN};
136
          } else {
137
            tk = \bot;
138
139
          if (sid = -1) {
                                                                                                                    /* From client */
            clients[cid] = \langle DEQ_H, \bot, false \rangle;
140
            send(nsid, \langle DEQ_H, \bot, cid, my\_sid, tk \rangle);
141
          } else {
142
                                                                                                                    /* from server */
            send(nsid, \langle \text{DEQ_H}, \bot, cid, sid, tk \rangle);
143
          }
       }
     }
```

as HEAD_TOKEN for the token transition (line 136). If the message received was send by a client, s stores the request to its *clients* array (line 140). Finally, s forwards the message to the next server (lines 141, 143).

Algorithm 22 presents pseudocode for the client functions. These are EnqueueTail(), DequeueTail(), EnqueueHead(), DequeueHead(). All clients have two local variables, *head_sid* and *tail_sid*, to store the last known server to have the head token and the tail token, respectively. These variables initially store the id of the server zero, but may change values during runtime. The messages received by clients contain two fields: *status* which contains either the data from a dequeue operation, or \perp in case of an enqueue, and *tail_sid* or *head_sid*, depending on the function, which contains the id of the server that currently holds the token.

```
Algorithm 22 Enqueue and dequeue op-
erations for a client of the token-based
deque.
144 int tail\_sid = 0, head\_sid = 0;
145 Data EnqueueTail(int cid, Data data) {
       send(tail_sid, \langle ENQ_T, data, cid, -1, \bot \rangle);
146
       \langle status, tail\_sid \rangle = receive();
147
       return status;
148
    }
149 Data DequeueTail(int cid) {
       send(tail_sid, \langle \mathsf{DEQ}_{-}\mathsf{T}, \bot, cid, -1, \bot \rangle);
150
       \langle status, tail\_sid \rangle = receive();
151
      return status;
152
    }
153 Data EnqueueHead(int cid, Data data) {
       send(head_sid, \langle ENQ_H, data, cid, -1, \bot \rangle);
154
155
       \langle status, head\_sid \rangle = receive();
156
       return status;
    }
157 Data DequeueHead(int cid) {
       send(head_sid, \langle \mathsf{DEQ\_H}, \bot, cid, -1, \bot \rangle);
158
159
       \langle status, head\_sid \rangle = receive();
      return status;
160
    }
```

For enqueuing at the deque's tail, clients calls EnqueueTail() (line 145). This function sends an ENQ_T message to the last known server, which has the token for the global tail (line 146). The server with the tail token may have changed, but the client is still unaware of the change. In that case, server s, which received the message, stores this message in its *clients* array and then forwards the message to the next server in the order, as described in Algorithm 18. During this time, the client blocks waiting for a server's response. Once the client receives the response (line 147), it returns the contents of the status variable (line 148).

Algorithm 23 Auxiliary functions for a server

```
of the token-based deque.
161 void ServeOldTailOps(void) {
      LocalSet eliminated = \emptyset
162
      for each cid1 \notin eliminated such that clients[cid1].op == ENQ_T {
163
        if there is cid2 \notin eliminated such that clients[cid2].op == DEQ_T {
164
          clients[cid2].data = clients[cid1].data;
165
          clients[cid1].isServed = true;
166
          clients[cid2].isServed = true;
167
          eliminated = eliminated \cup \{cid1, cid2\};
168
        }
      }
      if (!fullDeque) {
169
        for each cid such that clients[cid].op == ENQ_H {
170
171
          if (!IsFull(ldeque)) {
            enqueue_tail(ldeque, clients[cid].data);
172
            clients[cid].isServed = true;
173
          }
        }
      }
      for each cid such that clients [cid].op == DEQ_T \{
174
175
        if (!IsEmpty(ldeque)) {
176
          clients[cid].data = dequeue\_tail(ldeque);
          clients[cid].isServed = true;
177
    } } }
178 void ServeOldHeadOps(void) {
      LocalSet eliminated = \emptyset
179
      for each cid1 \notin eliminated such that clients[cid1].op == ENQ_H \{
180
        if there is cid2 \notin eliminated such that clients[cid2].op == DEQ_H {
181
182
          clients[cid2].data = clients[cid1].data;
          clients[cid1].isServed = true;
183
          clients[cid2].isServed = true;
184
          eliminated = eliminated \cup \{cid1, cid2\};
185
        }
      }
186
      if (!fullDeque) {
        for each cid such that clients[cid].op == ENQ_H \{
187
          if (!IsFull(ldeque)) {
188
            enqueue_head(ldeque, clients[cid].data);
189
            clients[cid].isServed = true;
190
          }
        }
      }
      for each cid such that clients[cid].op == DEQ_H {
191
        if (!IsEmpty(ldeque)) {
192
193
          clients[cid].data = dequeue\_head(ldeque);
194
          clients[cid].isServed = true;
    } } }
```

The enqueuing at the deque's head is symmetrical to this approach and is achieved with the function EnqueueHead() (line 153). The only difference is that the server which is send the message to, is the server with id *head_sid* (line 154) and the message's operation code is ENQ_H

instead of ENQ_T.

For dequeuing at the deque's tail, clients call DequeueTail() (line 149). This function sends a DEQ_T message to the last known server which has the token for the global tail (line 150). Then, the client waits for server to respond. Once the client receives the response (line 151), it returns the contents of the *status* variable (line 152).

The dequeuing at the deque's head is symmetrical to this approach and is achieved with the function DequeueHead() (line 157). The only difference is that the server, to whom the message was sent, is the server with id *head_sid* (line 158) and the message's operation code is DEQ_H instead of DEQ_T.

6.3.2 Proof of Correctness

Let α be an execution of the token-based deque algorithm presented in Algorithms 17, 18, 19, 20, 21, 22, and 23.

Each server maintains local boolean variables hasHead and hasTail, with initial values false. Whenever some server s_i receives a TAIL_TOKEN message, i.e. a message with its tk field equal to TAIL_TOKEN (line 34, line 63), the value of hasTail is set to true (line 35, line 64). By inspection of the pseudocode, it follows that the value of hasTail is set to false if the local deque of s_i is full (line 52, line 80); then, a TAIL_TOKEN message is sent to the next or previous server (line 61, line 88). The same holds for hasHead and HEAD_TOKEN messages, i.e. messages with their tk field equal to HEAD_TOKEN. Thus, the following observations holds.

Observation 38. At each configuration in α , there is at most one server for which the local variable has Head (hasTail) has the value true.

Observation 39. In some configuration C of α , a TAIL_TOKEN message is sent from a server s_j , $0 \le j < NS - 1$, to a server s_k , where $k = (j + 1) \mod NS$, only if the local deque of s_j is full in C. A TAIL_TOKEN message is sent from a server s_j , $0 \le j < NS - 1$, to a server s_k , where $k = (j - 1) \mod NS$, only if the local deque of s_j is empty in C.

Similary, a HEAD_TOKEN message is sent from s_j to s_k , where $k = (j + 1) \mod NS$, only if the local deque of s_j is empty in C. HEAD_TOKEN message is sent from s_j to s_k , where $k = (j-1) \mod NS$, only if the local deque of s_j is full in C.

By inspection of the pseudocode, we see that a server performs an enqueue (dequeue) back operation on its local deque *ldeque* either when executing line 44 (line 72) or when it executes ServeOldTailOps. Further inspection of the pseudocode (lines 34-36, line 43, as well as lines 63-65, line 71), shows that these lines are executed when hasTail = true. By inspection of the pseudocode, the same can be shown for hasHead. Then, the following observation holds.

Observation 40. Whenever a server s_j performs an enqueue or dequeue back (front) operation on its local deque, it holds that its local variable hasTail (hasHead) is equal to true.

If hasTail = true (hasHead = true) for some server s in some configuration C, then we say that s has the tail (head) token. The server that has the tail token is referred to as *tail token server*. The server that has the head token is referred to as *head token server*.

By a straight-forward induction, the following lemma can be shown.

Lemma 41. The mailbox of a client in any configuration of α contains at most one incoming message.

Let op be any operation in α . We assign a linearization point to op by considering the following cases:

- If *op* is an enqueue back operation for which a tail token server executes an instance of Algorithm 17, then it is linearized in the configuration resulting from the execution of either line 40, or line 44, or line 165, or line 172, whichever is executed for *op* in that instance of Algorithm 17 by the tail token server.
- If *op* is a dequeue back operation for which a head token server executes an instance of Algorithm 17, then it is linearized in the configuration resulting from the execution of either line 68, or line 72, or line 165, or line 176, whichever is executed for *op* in that instance of Algorithm 17 by the tail token server.
- If *op* is an enqueue front operation for which a tail token server executes an instance of Algorithm 17, then it is linearized in the configuration resulting from the execution of either line 96, or line 100, or line 182, or line 189, whichever is executed for *op* in that instance of Algorithm 17 by the head token server.
- If *op* is a dequeue front operation for which a head token server executes an instance of Algorithm 17, then it is linearized in the configuration resulting from the execution of either line 124, or line 128, or line 182, or line 193, whichever is executed for *op* in that instance of Algorithm 17 by the head token server.

Lemma 42. The linearization point of an enqueue (dequeue) operation op is placed in its execution interval.

Proof. Assume that op is an enqueue back operation and let c be the client that invokes it. After the invocation of op, c sends a message to some server s (line 146) and awaits a response. Recall that routine **receive()** (line 147) blocks until a message is received. The linearization point of op is placed in the configuration resulting from the execution of either line 40, or line 44, or line 165, or line 172 by s_t for op. Notice that since the execution of Algorithm 17 by s_t is triggered by a message that contains the request for op, either of these lines is executed after the request by c is received, i.e. after c invokes EnqueueTail, and thus, after the execution interval of op starts.

By definition, the execution interval of op terminates in the configuration resulting from the execution of line 148. By inspection of the pseudocode, this line is executed after line 147, i.e. after c receives a response by some server. In the following, we show that the linearization point of op occurs before this response is sent to c.

Let s_j be the server that c initially sends the request for op to. By observation of the pseudocode, we see that c may either receive a response from s_j if s_j executes lines 9, or 40, or 46.

To arrive at a contradiction, assume that either of these lines is executed in α before the configuration in which the linearization point of op is placed. Thus, a tail token server s_t executes lines line 40, or line 44, or line 165, or line 172, in a configuration following the execution of lines 9, or 40, or 46 by s_j . Since the algorithm is event-driven, inspection of the pseudocode shows that in order for a tail token server to execute these lines, it must receive a message containing the request for op either from a client or from another server.

Assume first that a tail token server executes the algorithm after receiving a message containing a request for op from a client. This is a contradiction, since, on one hand, c blocks until receiving a response, and thus, does not sent further messages requesting op or any other operation, and since op terminates after c receives the response by s_j , and on the other hand, any other request from any other client concerns a different operation op'.

Assume next that a tail token server executes the algorithm after receiving a message containing the request for op from some other server. This is also a contradiction since inspection of the pseudocode shows that after s_j executes either of the lines that sends a response to c, it sends no further message to some other server and instead, terminates the execution of that instance of the algorithm.

The argumentation regarding dequeue back, enqueue front, and dequeue front operations is analogous. $\hfill \square$

Denote by L the sequence of operations which have been assigned linearization points in α in the order determined by their linearization points. Let C_i be the configuration at which the *i*-th operation op_i of L is linearized. Denote by α_i , the prefix of α which ends with C_i and let L_i be the prefix of L up until the operation that is linearized at C_i . Denote by D_i the sequence of values that a sequential deque contains after applying the sequence of operations in L_i , in order, starting from an empty deque; let $D_0 = \epsilon$, i.e. D_0 is the empty sequence. In the following, we denote by s_{t_i} the tail token server at C_i and by s_{h_i} the head token server at C_i .

Lemma 43. For each $i, i \ge 0$, if ld_i^j are the contents of the local deque of server s_j at C_i , $h_i \le j \le t_i$, at C_i , then it holds that $D_i = ld_i^{h_i} \cdot ld_i^{h_i+1} \cdot \ldots \cdot ld_i^{t_i}$ at C_i .

Proof. We prove the claim by induction on i. The claim holds trivially at i = 0.

Fix any $i \ge 0$ and assume that at C_i , it holds that $D_i = ld_i^{h_i} \cdot ld_i^{h_i + 1} \cdot \ldots \cdot ld_i^{t_i}$. We show that the claim holds for i + 1.

Assume that op_{i+1} is an enqueue back operation by client c. Furthermore, distinguish the following two cases:

• Assume that $t_i = t_{i+1}$. Then, by the induction hypothesis, $D_i = ld_i^{h_i} \cdot ld_i^{h_i+1} \cdot \ldots \cdot ld_i^{t_i}$. In case the local queue of s_{t_i} is not full, s_{t_i} enqueues the value v_{i+1} of the data field of the request for op_{i+1} in the local deque (line 44 or line 172). Notice that, by Observation 40 changes on the local deques of servers occur only on token servers. Notice also that those changes occur only in a step that immediately precedes a configuration in which an operation is linearized. Thus, no further change occurs on the local deques of $s_{h_i}, s_{h_i+1}, \ldots, s_{t_i}$ between C_i and C_{i+1} , other than the enqueue on ld_i^t . Then, it holds that $D_{i+1} = D_i \cdot v_{i+1} = ld_i^{h_i} \cdot ld_i^{t_i+1} \cdot \ldots \cdot ld_i^{t_i} \cdot v_{i+1} = ld_i^{h_i} \cdot ld_{i+1}^{h_i+1} \cdot \ldots \cdot ld_{i+1}^{t_i}$ and if the head token server does not change between C_i and C_{i+1} , then $h_{i+1} = h_i$ and $D_{i+1} = ld_{i+1}^{h_{i+1}} \cdot ld_{i+1}^{h_{i+1}+1} \cdot \ldots \cdot ld_{i+1}^{t_{i+1}}$ and the claim holds. If the head token server changes, i.e., if $h_{i+1} \neq h_i$, then by Observation 39, $ld_{i+1}^{h_i} = \emptyset$ and the claim holds again.

In case the local deque of s_{t_i} is full and since by assumption, $s_{t_i} = s_{t_{i+1}}$, it follows by inspection of the pseudocode (line 31) and the definition of linearization points, that $s_{t_{i+1}} = s_{h_{i+1}}$. In this case, $s_{t_{i+1}}$ responds with a NACK to c and the local deque remains unchanged. Since no token server changes between C_i and C_{i+1} , $D_{i+1} = D_i = ld_i^{h_i} \cdot ld_i^{h_{i+1}} \cdot \ldots \cdot ld_i^{t_i} = ld_{i+1}^{h_{i+1}} \cdot ld_{i+1}^{h_{i+1}} \cdot \ldots \cdot ld_{i+1}^{t_{i+1}}$ and the claim holds.

• Next, assume that $t_i \neq t_{i+1}$. This implies that the local deque of s_{t_i} is full just after C_i . Observation 39 implies that s_{t_i} forwarded the token to s_{t_i+1} in some configuration between C_i and C_{i+1} . Notice that then, $s_{t_i+1} = s_{t_{i+1}}$. If the local deque of $s_{t_{i+1}}$ is not full, then the condition of line 43 evaluates to **true** and therefore, line 44 is executed, enqueueing value v_{i+1} to it. Then at C_{i+1} , $ld_{i+1}^{t_{i+1}} = v_{i+1}$. By definition, $D_{i+1} = D_i \cdot v_{i+1}$, and therefore, $D_{i+1} = ld_i^{h_i} \cdot ld_i^{h_i+1} \cdot \ldots \cdot ld_i^{t_i} \cdot v_{i+1} = ld_{i+1}^{h_{i+1}} \cdot ld_{i+1}^{h_{i+1}+1} \cdot \ldots \cdot ld_{i+1}^{t_i} \cdot v_{i+1} = ld_{i+1}^{h_{i+1}+1} \cdot ld_{i+1}^{h_{i+1}+1} \cdot \ldots \cdot ld_{i+1}^{t_i}$ and the claim holds. If the local deque of $s_{t_{i+1}}$ is full, then the condition of line 43 evaluates to **false** and therefore, line 40 is executed. The operation is linearized in the resulting configuration and NACK is sent to c. Notice that in that case, the local deque of the server is not updated. Then, $D_{i+1} = D_i = ld_i^{h_i} \cdot ld_i^{h_i+1} \cdot \ldots \cdot ld_i^{t_i+1} = ld_{i+1}^{h_{i+1}+1} \cdot \ldots \cdot ld_{i+1}^{t_i+1}$ and the claim holds.

The reasoning for the case where op_{i+1} is an instance of a dequeue back, enqueue front, or enqueue back operation is symmetrical.

From the above lemmas and observations we have the following theorem.

Theorem 44. The token-based distributed deque implementation is linearizable. The time complexity and the communication complexity of each operation op is O(NS).

6.4 Hierarchical approach.

In this section, we outline how the hierarchical approach, described in Section ??, is applied to the token-based designs.

Only the island masters play the role of clients to the algorithms described in this section. So, it is each island master m_i that keeps track of the last server(s), which responded to its batches of requests. In the stack and deque implementations, m_i performs elimination before contacting a server. In the queue implementation, batching is done by having each batch containing requests of the same type. In the deque implementation, each batch contains requests of the same type that are to be applied to the same endpoint. A batch can be sent to a server using DMA; the same could be done for getting back the responses. A server that does not hold the appropriate token to serve a batch of requests, forwards the entire batch to the next (or previous) server. Since token-based algorithms exploit locality, a batch of requests will be processed by at most two servers.

6.5 Dynamic Versions of the Implementations

The implementations presented above (in Section 6) are static. Their dynamic versions retain the placement of servers in a logical ring, and the token that renders the server able to execute operations in its local partition. In the static versions of the algorithms, when the servers consume all their predefined space for the data structure, the global (implemented) data structure is considered full, and the token server was sending NACK to clients to notify them of this event.

In the dynamic version, though, there is no upper bound to the number of elements that can be stored in the data structure. In order to modify the static version of the structures of this section, we remove the mechanism that sends NACK messages to clients. Instead, every time a server *s* receives the token (regarding inserts), it allocates an additional chunk of memory for its local partition. Because of this circular movement of the token, the elements are stored along a spiral path, that spans over all servers. Each chunk is marked with a sequence number, associated with the coil of the spiral, to distinguish the order of allocation.

An example of the transformation of a static algorithm to a dynamic is the dynamic version of the queue algorithm, presented in Algorithm 24. In this design a server *s* uses two tokens, the head and tail token. In analogy, *s* deploys two variables (*tail_round* and *head_round*) to count the times the tokens have come to its possession. When a server *s* receives an ENQ message (line 7) but has no space left to store the element (line 24), it forwards the request along with the token to the next server in the ring. Afterwards, *s* increases by one the variable *tail_round* and allocates a new memory chunk, by calling allocate_new_space(), to be used during the next time the token comes to its possession.

For the DEQ operation, the server performs additional actions concerning the empty queue state (line 48), where after responding with a NACK, it re-initializes *tail_round* and *head_round* to be equal to zero (line 49). An empty queue implies that the allocated chunks for lqueue are also empty, hence they can be recycled and be used again anew. During the head token transition, *s* increases *head_round* by one chunk (line 57), so that when the head token comes to its possession to dequeue from the next memory.

The double ended queue (deque) algorithm is going to work verbatim after these modifications. For the stack implementation the modifications are analogous. In this design there is one token, hence each server associates one counter with the token rounds. Each time a server s moves the token to another server because the local stack is full, it increases the counter and allocates a new chunk for future use, and s moves the token due to an empty stack, the counter is decreased by one. However, the dynamic design for the stack would introduce the termination problem described for queues. Nevertheless, the problem can be solved by applying the same technique of using client arrays as we did to solve the problem in the queue implementation.

Alg	Algorithm 24 Events triggered in a server of a dynamic token-based deque.			
1	a message $\langle op, data, cid, sid, tk \rangle$ is received:			
2	if (!clients[cid] AND clients[cid].isServed) {			
		/* If message has been served earlier. */		
3	$send(cid, \langle ACK, clients[cid].data, my_sid \rangle);$			
4	$clients[cid] = \bot;$			
5	} else {			
6	$\texttt{switch} (op) \{$			
7	case ENQ:			
8	if $(tk == \texttt{TAIL_TOKEN})$ {			
9	hasTail = true;			
10	ServeOldEnqueues();			
	}			
11	if $(!hasTail)$ {	/* Server does not have token */		
12	$nsid = \text{find_next_server}(my_sid);$			
13	$if (sid == -1) \{$	/* From client. $*/$		
14	$clients[cid] = \langle ENQ, data, false \rangle;$			
15	$\operatorname{send}(nsid, \langle \texttt{ENQ}, data, cid, my_sid, \bot \rangle);$			
16	} else {	/* From server. */		
17	$\operatorname{send}(nsid, \langle \texttt{ENQ}, data, cid, sid, \bot \rangle);$			
	}			
18	$\}$ else if $(!IsFull(lqueue))$ {			
19	$enqueue(lqueue, data, tail_round);$			
20	if(sid == -1)	/* From client. $*/$		
21	$\operatorname{send}(\operatorname{cid}, \langle \texttt{ACK}, \bot, my_sid \rangle);$			
22	else	/* From server. */		
23	$\operatorname{send}(\operatorname{sid}, \langle \operatorname{ACK}, \bot, \operatorname{cid}, my_\operatorname{sid}, \bot \rangle);$			
24	$\}$ else $\{$	/* Server moves the tail token */		
25	$nsid = \text{find_next_server}(my_sid);$			
26	$send(nsid, \langle op, data, cid, my_sid, \texttt{TAIL}_\texttt{TOKEN} \rangle);$			
27	$tail_round + +;$			
28	$allocate_new_space(lqueue, tail_round);$			
29	hasTail = false;			
	}			
30	break;			

31	case DEQ:	
32	$if(tk == HEAD_TOKEN)$ {	
33	hasHead = true;	
34	ServeOldDequeues();	
	}	
35	if (!hasHead) {	
36	$nsid = \text{find_next_server}(my_sid);$	
37	$\texttt{if} (sid == -1) \{$	/* From client $*/$
38	$clients[cid] = \langle DEQ, \bot, false \rangle;$	
39	$\operatorname{send}(nsid, \langle \mathtt{DEQ}, \bot, cid, my_sid \rangle);$	
40	} else {	/* From server */
41	$\operatorname{send}(nsid, \langle \mathtt{DEQ}, \bot, cid, sid, \bot \rangle);$	
	}	
42	<pre>} else if (!IsEmpty(lqueue)) {</pre>	/* can dequeue. $*/$
43	$data = dequeue(lqueue, head_round);$	
44	if(sid == -1)	/* From client */
45	$\operatorname{send}(\operatorname{cid}, \langle \operatorname{ACK}, \operatorname{data}, \operatorname{my_sid} \rangle);$	
46	else	/* From server */
47	$\operatorname{send}(\operatorname{sid}, \langle \operatorname{ACK}, \operatorname{data}, \operatorname{cid}, \operatorname{my_sid}, \bot \rangle);$	
48	} else if $(tail_round == head_round)$ {	
49	$tail_round = head_round = 0;$	
50	if(sid == -1)	/* empty to client $*/$
51	$\mathrm{send}(cid, \langle \mathtt{NACK}, \bot, my_sid \rangle);$	
52	else	/* empty to server $*/$
53	$\operatorname{send}(sid, \langle \texttt{NACK}, \bot, cid, my_sid, \bot \rangle$);	
54	$\}$ else $\{$	/* Move the head token to next */
55	$nsid = \text{find_next_server}(my_sid);$	
56	$ ext{send}(nsid, \langle op, \bot, cid, my_sid, \texttt{HEAD_TOKEN} angle);$	
57	$head_round + +;$	
58	hasHead = false;	
	}	
59	break;	
60	case ACK:	
61	$clients[cid] = \bot;$	
62	$\operatorname{send}(\operatorname{cid}, \langle \operatorname{ACK}, \operatorname{data}, \operatorname{sid} \rangle);$	
63	break;	
64	case NACK:	
65	$clients[cid] = \bot;$	
66	$\operatorname{send}(\operatorname{cid}, \langle \texttt{NACK}, \bot, \operatorname{sid} \rangle);$	
67	break;	
	}	
	}	

7 Distributed Lists

A *list* is an ordered collection of elements. It can either be *sorted*, in which case the elements appear in the list in increasing (or decreasing) order of their keys, or *unsorted*, in which case

the elements appear in the list in some arbitrary order (e.g. in the order of their insertion). A list L supports the operations *Insert*, *Delete*, and *Search*. Operation Insert(L, k, I) inserts an element with key k and associated info I to L. Operation Delete(L, k) removes the element with key k from L (if it exists), while operation Search(L, k) detects whether an element with key k is present in L and returns the information I that is associated with k.

In this section, we first provide an implementation of an unsorted distributed list in which we follow a token-based approach for implementing *Insert*. In this implementation, Search and Delete are highly parallel. We then build on this approach in order to get a distributed implementation of a sorted list.

7.1 Unsorted List

The list state is stored distributedly in the local memories of several of the available servers, potentially spreading among all of them, if its size is large enough. The proposed implementation follows a token-based approach for implementing insert. Thus, we assume that the servers are arranged on a logical ring, based on their ids.

At each point in time, there is a server (not necessarily always the same), denoted by s_t , which holds the insert token, and serves insert operations. Initially, server s_0 has the token, thus the first element to be inserted in the list is stored on server s_0 . Further element insertions are also performed on it, as long as the space it has allocated for the list does not exceed a threshold. In case server s_0 has to service an insertion but its space is filled up, it forwards the token by sending a message to the next server, i.e. server s_1 . Thus, if server s_i , $0 \le i < NS$, has the token, but cannot service an insertion request without exceeding the threshold, it forwards the token to server $s_{(i+1) \mod NS}$. When the next server receives the token, it allocates a memory chunk of size equal to threshold, to store list elements. When the token reaches s_{NS-1} , if s_{NS-1} has filled all the local space up to a threshold, it sends the token again to s_0 . Then, s_0 allocates more memory (in addition to the memory chunk it had initially allocated for storing list elements) for storing more list elements. The token might go through the server sequence again without having any upper-bound restrictions concerning the number of round-trips. In order for a server to know whether the token has performed a round-trip on the ring, and hence all servers have stored list elements, it deploys a variable to count the number of ring round-trips it knows that the token has performed.

Algorithm 25 Events triggered in a server of the distributed unsorted list.

```
List llist = \emptyset;
1
\mathbf{2}
    int my_{id}, next_{id}, token = 0, round = 0;
    a message \langle op, cid, key, data, mloop, tk \rangle is received:
3
       switch(op)
4
         case INSERT:
\mathbf{5}
           if (tk == TOKEN) {
6
             token = my_id;
7
             allocate_new_memory_chunk(llist, round);
8
           ł
9
           status_1 = search(llist, key);
           if (status_1) send(cid, NACK);
10
11
           else {
              if (token \neq my_id) {
12
                next_id = get_next(my_id);
13
                if (my_{-id} \neq NS - 1) {
14
15
                  send(next_id, \langle op, cid, key, data, mloop, tk \rangle);
                } else
                          send(next_id, \langle op, cid, key, data, true, tk \rangle);
16
              } else {
17
                if ((my_id \neq NS - 1) \text{ AND } (round > 0) \text{ AND } !(mloop)) {
18
                  next_id = get_next(my_id);
19
20
                  send(next\_id, \langle op, cid, key, data, mloop, tk \rangle);
21
                } else {
22
                  status_2 = insert(llist, round, key, data);
                  if (status_2 == false) {
23
                    round + +;
24
                    token = get_next(my_id);
25
26
                    send(token, (op, cid, key, data, mloop, TOKEN));
                  } else send(cid, ACK);
27
                }
             }
           }
28
           break;
         case SEARCH:
29
           status_1 = \operatorname{search}(llist, key);
30
           if (status_1) send(cid, \langle ACK, my_id \rangle);
31
           else send(cid, \langle NACK, my_i d \rangle);
32
           break:
33
         case DELETE:
34
           status_1 = delete(llist, key);
35
           if (status_1) send(cid, ACK);
36
           else send(cid, NACK);
37
38
           break;
       }
```

Event-driven code for the server is presented in Algorithm 25. Each server s maintains a local list (*llist* variable) allocated for storing list elements, a *token* variable which indicates whether s currently holds the token, and a variable *round* to mark the ring round-trips the token has performed; *round* is initially 0, and is incremented after every transmission of the token to the next server.

Each message a server receives has five fields: (1) op that denotes the operation to be

executed, (2) *cid* that holds the id of the client that initiated a request, (3) *key* that holds the value to be inserted, (4) *mloop* stands for "message loop", a **boolean** value that denotes if the message has traversed the whole server sequence and (5) tk that is set when a forwarded message also denotes a token transition from one server to the other.

Algorithm 26 Insert, Search and Delete operation for a client of the distributed list.

```
boolean ClientInsert(int cid, int key, data data) {
39
40
      boolean status;
      send(0, (INSERT, cid, key, data, false, -1));
41
42
      status = receive();
      return status;
43
    ł
    boolean ClientSearch(int cid, int key) {
44
      int sid;
45
      int c = 0;
46
      boolean status;
47
      boolean found = false;
48
      send_to_all_servers(\langleSEARCH, cid, key, \bot, false, -1\rangle);
49
50
      do {
         \langle status, sid \rangle = receive();
51
         if (status == ACK) found = true;
52
        c + +;
53
      } while (c < NS);
54
      return found;
55
    ł
    boolean ClientDelete(int cid, int key) {
56
      int sid;
57
      int c = 0;
58
      boolean status;
59
      boolean deleted = false;
60
      send_to_all_servers(\langle \text{DELETE}, cid, key, \bot, \texttt{false}, -1 \rangle);
61
62
      do {
         \langle status, sid \rangle = receive();
63
        if (status == ACK) deleted = true;
64
65
         c + +;
      } while (c < NS);
66
      return deleted;
67
```

When a message is received, the server s first checks its type. If the message is of type INSERT (line 5), s first checks whether the message has the tk field marked. If it is marked (line 6), s sets a local variable token equal to its own id (line 7) and allocates additional space for its local part of the list (line 8).

Afterwards, s searches the part of the list that it stores locally, for an element with the same key (key variable inthe algorithm) as the one to be inserted (line 9). Searching *llist* for the element has to be performed independently of whether the server holds the token or not. Since this design does not permit duplicate entries, if such an element is found, the server responds with NACK to the client (line 10). Otherwise (line 11), s checks whether the new element can be stored in *llist*.

In case s does not hold the token (line 12), it is not allowed to perform an insertion, therefore it must forward the message to the next server in the ring. If s is not s_{NS-1} (line 14), it forwards to the next server the request (15). In case s is s_{NS-1} , it means that all servers have been searched for the ele-

ment and the element was not found. Server s sends the message to the next server (in order to eventually reach the token server), after marking the *mloop* field of the message as **true**, to indicate that the message has completed a full round-trip on the ring (line 16).

On the other hand, if s holds the token (line 17), it must first check whether there is room in *llist* to insert the element in it. If there is room in *llist* and the local variable *round* of s equals to **false** (which means that the list does not expand to the next servers) or the message has already performed a round-trip on the ring, then s inserts the element and returns ACK. If however, *round* > 0 and the message has not performed a round trip on the ring (mloop == false), s continues forwarding the message.

If the token server's local memory is out of sufficient space (line 23) (i.e. the insert() function was unsuccessful), s forwards the message to the next server the tk field with TOKEN (line 26) to indicate that this server will become the new token server after s. Also, s increments round by one to count the number of times the token has passed from it. The round variable is also used by function allocate_new_memory_chunk() that allocates additional space for the list (line 8).

Notice that, contrary to other token-based implementations presented in previous sections, the token server of the unsorted list does not need to rely on client tables in order to stop a message from being incessantly forwarded from one server to another, without ever being served. By virtue of having clients always sending their insert requests to s_0 , an insert request r_j that arrives at s_0 before some other insert request r_k , is necessarily served before r_k . The scenario where insert requests constantly arrive at the token server before r_j , making the token travel to the next server before r_j can be served, is thus avoided.

Upon receiving a SEARCH request from a client (line 29), a server searches for the requested element in its local list (line 30) and sends ACK to the server if the element is found (line 31) and NACK otherwise (line 32).

Upon receiving a DELETE request from a client (line 34), a server attempts to delete the requested element from its local list (line 35) and sends ACK to the server if the deletion was successful (line 36). Otherwise it sends NACK (line 37).

The pseudocode of the client is presented in Algorithm 26. Notice that insert operations in the proposed implementation are executed in sequence and must necessarily pass through server 0 and be forwarded through the server ring, if necessary due to space constraints. Search and Delete operations, on the contrary, are executed in parallel.

In order to execute an insertion, a client calls ClientInsert() (line 39) which sends an INSERT message (line 41) to server 0, regardless of which server holds the token in any given configuration, and then blocks waiting for a response (line 42). If the client receives ACK from a server, then the element was inserted correctly. If the client receives NACK, then the insertion failed, due to either limited space, or the existence of another element with the same key value.

For a search operation the client calls ClientSearch() (line 44). The client sends a SEARCH request to all servers (line 49) and waits to receive a response message (line 51) from each server (do while loop of lines 50-54). The requested element is in the list if the client receives ACK from some server (line 52). A delete operation proceeds similarly to ClientSearch(). It is initiated by a client by sending a DELETE request to all servers (line 61). The client then waits to receive a response message (line 63) from each server (do while loop of lines 62-66). The requested element has been found in the list of some client and deleted from there, if the client receives ACK from some server s.

7.1.1 Proof of Correctness

We sketch the correctness argument for the proposed implementation by providing linearization points. Let α be an execution of the distributed unsorted list algorithm presented in Algorithms 25 and 26. We assign linearization points to insert, delete and search operations in α as follows:

• Insert. Let op be any instance of ClienInsert for which an ACK or a NACK message is sent by a token server. Then, if ACK is sent by a token server for op (line 27), the linearization point is placed in the configuration resulting from the execution of line 22 that successfully inserted the required element into the server's local list. If NACK is sent for op (line 10), then the linearization point is placed in the configuration resulting from the execution of line 9, where the search operation on the local list of the server returned true.

- Let op be any instance of ClientDelete for which an ACK or a NACK message is sent by a server. Then, if ACK is sent by a server s for op, the linearization point is placed in the configuration resulting from the execution of line 35 by the server that sent the ACK. Otherwise, if the key k that op had to delete was not present in any of the local lists of the servers in the beginning of the execution interval of op, then the linearization point of op is placed at the beginning of its execution interval. Otherwise, if k was present but was deleted by a concurrent instance op' of ClientDelete, then the linearization point is placed right after the linearization point of op'.
- Let op be any instance of ClientSearch for which an ACK or a NACK message is sent by a server. Then, if ACK is sent by a server s for op, the linearization point is placed in the configuration resulting from the execution of line 30 by the server that sent the ACK. Otherwise, if the key k that op had to find was not present in the list in the beginning of its execution interval, then the linearization point is placed there. Otherwise, if k was present but was deleted by a concurrent instance op' of ClientDelete, then the linearization point is placed right after the linearization point of op'.

Lemma 45. Let op be any instance of an insert, delete, or a search operation executed by some client c in α . Then, the linearization point of op is placed in its execution interval.

Proof. Let op be an instance of an insert operation invoked by client c. A message with the insert request is sent on line 41, after the invocation of the operation. Recall that routine receive() blocks until a message is received. Notice that both line 22 as well as line 9 are executed by a server before it sends a message to the client. Therefore, whether op is linearized at the point some server sends it a message on line 27 or on line 10, it terminates only after receiving it. Notice also that the operation terminates only after the client receives it. Thus, the linearization point is included in its execution interval.

By similar reasoning, if op is an instance of a delete operation that is linearized in the configuration resulting from the execution of line 35 or a search operation that is linearized in the configuration resulting from the execution of line 30, then the linearization point is included in the execution interval of op.

Let op be an instance of a delete operation that deletes key k and that terminates after receiving only NACK messages on line 63. If k is not present in the list in the beginning of the execution interval of op, then op is linearized at that point and the claim holds.

Consider the case where k is included in the list when op is invoked. By observation of the pseudocode (lines 34-38), we have that when a server receives a delete request by a client, it traverses its local part of the list and deletes the element with key equal to k (line 35), if it is included in it. By further observation of the pseudocode (lines 61-67), we have that after c invokes op, it sends a delete request to all servers (line 61) and then awaits for a response from all of them (do while loop of lines 62-66). By assumption, all servers responds with NACK. Notice that this implies that between the execution of line 63 and 65 the element with key k is removed from the local list of s because of some other concurrent delete operation op' invoked by some client c'. By scrutiny of the pseudocode, we have that a server that deletes an element from its local list, does so on line 35, which occurs before the server sends a response to the delete request. By definition, then, op' is linearized at the point s executes line 35, before it sends an ACK message to c'. Since op' causes the element with key k to be removed from the local list of s between the execution of lines 63 and 65 by c, its linearization point is included in the execution interval of op. Since we place the linearization point of op right after the linearization point of op', the claim holds.

The argument is similar for when op is an instance of a search operation for key k that terminates after receiving a NACK message from all the servers on lines 50-54.

Each server maintains a local variable *token* with initial value 0. Let some server s receive a message m in some configuration C. If the field tk of m is equal to TOKEN, we say that *receives a token message*. Observe that when s receives a token message (line 7), the value of *token* is set to s. Furthermore, when s executes line 25, where the value of *token* changes from s to s + 1, s also sends a token message to s + 1 (line 26). Notice that s can only reach and execute this line if the condition of the **if** clause of line 12 evaluates to **false**, i.e. if *token* = s. Then, the following holds:

Observation 46. At each configuration in α , there is at most one server s for which the local variable token has the value s.

This server is referred to as *token server*. By the pseudocode, namely the **if else** clause of lines 12, 17, and by line 22, the following observations holds.

Observation 47. A server s performs insert operations on its local list in α only during those subsequences of α in which it is the token server.

Each server maintains a local list collection, llist. By observation of the pseudocode, lines 9 and 10, we have that if an insert operation attempts to insert key k in either of the lists of a server s, but an element with that key already exists, then no second element for k is inserted and the operation terminates. Thus, the following holds:

Observation 48. The keys contained in the list collection of s in any configuration C of α form a set.

We denote this set by ll^s . By scrutiny of the pseudocode, we see that a new list object is allocated in *llist* each time a server receives a token message (lines 6-8). The new object is identified by the value of local variable *round*. By observation of the pseudocode, we further have that each time a server inserts a key into ll^s , it does so on the list object identified by *round* (line 22). We refer to this object as *current list object*. Then, based on lines 23-26 we have the following:

Observation 49. A token message is sent from a server s to a server $((s + 1) \mod NS)$ in some configuration C only if the current local list object of server s is full at C.

Further inspection of the pseudocode shows that the local list object of a server is only accessed by the execution of line 9, 22, 30, or 35. From this, we have the following observation.

Observation 50. If an operation op modifies the local list object of some server, then this occurs in the configuration in which op is linearized.

Let C_i be the configuration in which the *i*-th linearization point in α is placed. Denote by α_i , the prefix of α which ends just after C_i and let L_i be the sequence of linearization points that is defined by α_i . Denote by S_i the set of keys that a sequential list contains after applying the sequence of operations that L_i imposes. Denote by $S_i = \epsilon$ the empty sequence (the list is empty).

Lemma 51. Let k be the token server in some configuration C in which it receives a message m for an insert operation op with key k invoked by client c. Then at C, no element with key k is contained in the local list set of any other server $s \neq k$.

Proof. By inspection of the pseudocode, when a client c sends a message m to some server either on line 41, line 49, line 61, or line 65, the *mloop* field of m is equal to **false**. This field is set to **true** when server s_{NS-1} executes line 16. Notice that in the configuration in which this

line is executed by s_{NS-1} , it is not the token server (otherwise the condition of line 12 would not evaluate to **true** and the line would not be executed).

Consider the case where m reaches a server s at some configuration C and let ll^s contain an element with key k in C. By inspection of the pseudocode (lines 9-10) we have that in that case, m is not forwarded to a subsequent server.

Furthermore, by lines 12-16, we have that if s is not the token server and not s_{NS-1} , and provided that ll^s does not contain an element with key k, then s forwards m without modifying the *mloop* field. This implies that the *mloop* field of m is changed at most once in α from false to true, and that by server NS - 1, in a configuration C' in which k is not contained in ll^{NS-1} .

Lemma 52. Let C_i , $i \ge 0$, be a configuration in α in which server s_{t_i} is the token server. Let ll_i^j be the local list set of server s_j , $0 \le j < NS$, in C_i . Then it holds that $S_i = \bigcup_{i=0}^{NS-1} ll_i^j$.

Proof. We prove the claim by induction on i.

Base case (i = 0). The claim holds trivially at C_0 .

Hypothesis. Fix any i > 0 and assume that at C_i , it holds that $S_i = \bigcup_{j=0}^{NS-1} ll_i^j$. We show that the claim holds for i + 1.

Induction step. Let op_{i+1} be the operation that corresponds to the linearization point placed in C_{i+1} . We proceed by case study.

Let op_{i+1} be an insert operation for key k. Assume first that the linearization point of op_{i+1} is placed at the execution of line 9 by $s_{t_{i+1}}$ for it. Notice that when this line is executed, k is searched for in the local list of $s_{t_{i+1}}$. Recall that, by the way linearization points are assigned, the client c that invoked op_{i+1} receives NACK as response. Notice also that $s_{t_{i+1}}$ sends NACK as a response to c if k is present in the local list of $s_{t_{i+1}}$, and thus $status_1 = true$. In that case, lines 12 to 27 are not executed, and therefore, no new element is inserted into the local list of $s_{t_{i+1}}$ (line 22). Thus $ll_{i+1}^{s_{t_{i+1}}} = ll_i^{s_{t_{i+1}}}$. By the induction hypothesis, $S_i = \bigcup_{j=0}^{NS-1} ll_i^j$. By Observation 50 it follows that for any other server s_j , where $j \neq t_{i+1}$, $ll_{i+1}^{s_j} = ll_i^{s_j}$ as well. Then, $\bigcup_{j=0}^{NS-1} ll_{i+1}^j = \bigcup_{j=0}^{NS-1} ll_i^j$. Notice that since the server responds with NACK, $S_{i+1} = S_i$ by definition. Thus, $S_{i+1} = \bigcup_{j=0}^{NS-1} ll_{i+1}^j$ and the claim holds.

Now, assume that op_{i+1} is linearized at the execution of line 22 by the token server for it. By the way linearization points are assigned, this implies that when this line is executed, $status_2 = true$, and the insertion of an element with key k into the local list of s_t was successful. This in turn implies that at C_{i+1} , $ll_{i+1}^{s_t} = ll_i^{s_t} \cup \{k\}$. By Observation 50 it follows that for any other server s_j , where $j \neq t_{i+1}$, $ll_{i+1}^{s_j} = ll_i^{s_j}$ as well. Notice that since the server responds with ACK, by definition the insertion is successful and thus $S_{i+1} = S_i \cup \{k\}$. Since by the induction hypothesis, $S_i = \bigcup_{j=0}^{NS-1} ll_i^j$, it holds that $S_{i+1} = \bigcup_{j=0}^{NS-1} ll_i^j \cup \{k\} = \bigcup_{j=0}^{NS-1} ll_{i+1}^j$, thus, the claim holds.

Now consider that op_{i+1} is a delete operation for key k. Assume first that some server s_d responds with ACK, by executing line 36, to the client c that invoked op_{i+1} . Then op_{i+1} is linearized at the execution of this line by s_d . Notice that this line is executed by a server if $status_1 = true$, i.e. if the server was successful in locating and deleting an element with key k from its local list. Thus, $ll_{i+1}^{s_t} = ll_i^{s_t} \setminus \{k\}$. Furthermore, by definition, $S_{i+1} = S_i \setminus \{k\}$. By the induction hypothesis, $S_i = \bigcup_{j=0}^{NS-1} ll_i^j$ and since by Observation 50 no other modification occurred on the local list of some other server between C_i and C_{i+1} , it follows that $S_{i+1} = S_i \setminus \{k\} = \bigcup_{j=0}^{NS-1} ll_i^j \setminus \{k\} = \bigcup_{j=0}^{NS-1} ll_{i+1}^j$.

Assume now that op_{i+1} is a delete operation for which no server responds with ACK to the invoking client. Recall that in this case, by definition, $S_{i+1} = S_i$. By inspection of the pseudocode, it follows that no server finds an element with key k in its local list when it is executing line 35 for op_{i+1} . We examine two cases: (i) either no element with key k is contained in any local list of any server in the beginning of the execution interval of op_{i+1} , or (ii) an element with key k is contained in the local list of some server s_d in the beginning of op_{i+1} 's execution interval, but s_d deletes it while serving a different delete operation op', before it executes line 35 for op_{i+1} .

Assume that case (i) holds. Then, the linearization point is placed in the beginning of the execution interval of op_{i+1} . Notice that in this case, the invocation (nor in fact the further execution) of op_{i+1} has no effect on the local list of any server. Thus, between C_i and C_{i+1} no server local list is modified and, by the induction hypothesis, the claim holds.

Assume now that case (ii) holds. By Lemma 45, we have that a concurrent delete operation op' removes the element with key k from the local list of s_d during the execution interval of op_{i+1} . By the assignment of linearization points, Observation 50 and Lemma 45, it further follows that $op' = op_i$. Notice that in this case (ii) also, op_{i+1} has no effect on the local list of any server. Thus, since by the induction hypothesis it holds that $S_i = \bigcup_{j=0}^{NS-1} ll_i^j$, it also holds that $S_i = \bigcup_{j=0}^{NS-1} ll_{i+1}^j$, and since $S_i = S_{i+1}$, the claim holds.

Since a search operation does not modify the local list of any server, the argument is analogous as for the case of the delete operation. \Box

From the above lemmas and observations, we have the following.

Theorem 53. The distributed unsorted list is linearizable. The insert operation has time and communication complexity O(NS). The search and delete operations have communication complexity O(1).

7.1.2 Alternative Implementation

At each point in time, there is a server (not necessarily always the same), denoted by s_t , which holds the insert token, and serves insert operations. Initially, server s_0 has the token, thus the first element to be inserted in the list is stored on server s_0 . Further element insertions are also performed on it, as long as the space it has allocated for the list does not exceed a threshold. In case server s_0 has to service an insertion but its space is filled up, it forwards the token by sending a message to the next server, i.e. server s_1 . Thus, if server s_i , $0 \le i < NS$, has the token, but cannot service an insertion request without exceeding the threshold, it forwards the token to server $s_{(i+1) \mod NS}$. When the next server receives the token, it allocates a memory chunk of size equal to threshold, to store list elements. When the token reaches s_{NS-1} , if s_{NS-1} has filled all the local space up to a threshold, it sends the token again to s_0 . Then, s_0 allocates more memory (in addition to the memory chunk it had initially allocated for storing list elements) for storing more list elements. The token might go through the server sequence again without having any upper-bound restrictions concerning the number of round-trips.

Event-driven code for the server is presented in Algorithm 27. Each server s maintains a local list (*llist* variable) allocated for storing list elements, a *token* variable which indicates whether s currently holds the token, and a variable *round* to mark the ring round-trips the token has performed; *round* is initially 0, and is incremented after every transmission of the token to the next server. The pseudocode of the client is presented in Algorithm 28.

A client c sends an insert request for an element with key k to all servers in parallel and awaits a response. If any of the servers contains k in its local list, it sends ACK to c and the insert operation terminates. If no server finds k, then all reply NACK to c. In addition, the token server s_t encapsulates its id in the NACK reply. After that, c sends an insert request for k to s_t only. If s_t can insert it, it replies ACK to c. If k has in the meanwhile been inserted, s_t replies NACK to c. If s_t is no longer the token server, it forwards the request along the server ring until it reaches the current token server. Servers along the ring should check whether they contain k or not, and if some server does, then it replies NACK to c. Let s'_t be a token server that receives such a request. It also checks whether it contains k or not. If not, it attempts to insert k into its local list. Otherwise it replies NACK. When attempting to insert the element in the local list, it may occur that the allocated space does not suffice. In this case, the server forwards the request as well as the token to the next server in the ring, and increments the value of *round* variable. If the insertion at a token server is successful, the server then replies ACK to c.

To perform a search for an element e, a client c sends a search request to all servers and awaits their responses. A server s that receives a search request, checks whether e is present in its local part of the list and if so, it responds with ACK to c. Otherwise, the response is NACK. If all responses that c receives are NACK, e is not present in the list. Notice that if e is contained in the list, exactly one server responds with ACK. Delete works similarly; if a server s responds with ACK, then s has found and deleted e from its local list. Given that communication is fast and the number of servers is much less than the total number of cores, forwarding a request to all servers does not flood the network.

Algorithm 27 Events triggered in a server of the distributed unsorted list.

```
List llist = \emptyset;
1
\mathbf{2}
    int my_{id}, next_{id}, token = 0, round = 0;
    a message \langle op, cid, key, data, tk \rangle is received:
3
4
       switch (op) {
         case INSERT:
\mathbf{5}
           if (tk == TOKEN) {
6
              token = my_{id};
7
              allocate_new_memory_chunk(llist, round);
8
            }
           status_1 = search(llist, key);
9
           if (tk = -2) {
10
11
              if (status_1) {
                if (token == my_id) send(cid, \langle ACK, true \rangle);
12
                else send(cid, \langle ACK, false \rangle);
13
              } else {
14
                if (token == my_id) send(cid, \langle NACK, true \rangle);
15
16
                else send(cid, \langle NACK, false \rangle);
              }
            } else {
17
              if (status_1) send(cid, NACK);
18
19
              else {
                if (token \neq my_{-id}) {
20
21
                  next_id = get_next(my_id);
                  send(next_id, \langle op, cid, key, data, tk \rangle);
22
                } else {
23
                  status_2 = insert(llist, round, key, data);
24
25
                  if (status_2 == false) {
26
                    round + +;
27
                     token = get_next(my_id);
                     send(token, \langle op, cid, key, data, \mathsf{TOKEN} \rangle);
28
29
                  } else send(cid, ACK);
                }
              }
            }
30
           break;
         case SEARCH:
31
           status_1 = search(llist, key);
32
            if (status_1) send(cid, \langle ACK, my_i d \rangle);
33
34
           else send(cid, \langle NACK, my_id \rangle);
35
           break:
         case DELETE:
36
37
           status_1 = delete(llist, key);
           if (status_1) send(cid, ACK);
38
           else send(cid, NACK);
39
           break;
40
         }
```

7.2 Sorted List

Algorithm 28 Insert, Search and Delete oper-

ation for a client of the distributed list.

```
boolean ClientInsert(int cid, int key, data data) {
41
        boolean status;
42
43
        boolean found = false;
44
        int tid;
        send_to_all_servers(\langle INSERT, cid, key, \bot, -2 \rangle);
45
        do {
46
47
           \langle status, sid, is\_token \rangle = receive();
           if (status == ACK) found = true;
48
           if (is\_token) tid = sid;
49
          c + +;
50
51
         } while (c < NS);
52
         if (found == true) return false;
        send(tid, \langleINSERT, cid, key, data, -1\rangle);
53
54
        status = receive();
        if (status == NACK) return false;
55
        else return true;
56
      }
57
      boolean ClientSearch(int cid, int key) {
        int sid;
58
        int c = 0;
59
60
        boolean status;
        boolean found = false;
61
        send_to_all_servers(\langleSEARCH, cid, key, \bot, -1\rangle);
62
63
        do {
64
           \langle status, sid \rangle = receive();
           if (status == ACK) found = true;
65
66
          c + +;
67
         } while (c < NS);
        return found;
68
      }
      boolean ClientDelete(int cid, int key) {
69
70
        int sid;
        int c = 0;
71
        boolean status;
72
        boolean deleted = false;
73
        send_to_all_servers(\langle \text{DELETE}, cid, key, \bot, -1 \rangle);
74
75
        do {
76
           \langle status, sid \rangle = receive();
           if (status == ACK) deleted = true;
77
          c + +;
78
         } while (c < NS);
79
        return deleted;
80
      }
```

sort	ed list.
81	List $llist = \emptyset;$
82	int my_id , $next_id$, k_{max} , $cv[MC]$, $nbr_cv[MC]$;
83	data[0CHUNKSIZE] chunk1, chunk2;
84	boolean $status = false, served = false;$
85	a message $\langle op, cid, key, data \rangle$ is received:
86	switch (op) {
87	case REQC:
88	send(cid, cv);
89 00	<pre>chunk2 = receive(cid); if (not enough free space in local list to fit elements of chunk2) {</pre>
$90 \\ 91$	if $(my_sid == NS - 1)$ status = false;
92	else {
93	chunk1 = getChunkOfElementsFromLocalList(llist);
94	$status = ServerMove(next_id, chunk1);$
~	}
95 96	} else <i>status</i> = true;
96 07	if (status == true) { incort Chunk OfFlomentaInL coelList(llist_chunk2);
$\frac{97}{98}$	insertChunkOfElementsInLocalList(<i>llist</i> , <i>chunk2</i>); send(<i>cid</i> , ACK);
99	} else send(cid, NACK);
100	break;
101	case INSERT:
102	while $(served \neq \texttt{true})$ {
103	$k_{max} = \text{find}_{\max}(llist);$
104	if $(k_{max} > key \text{ and isFull}(llist) \neq \texttt{true})$ {
105	status = insert(llist, key, data);
$\frac{106}{107}$	send(cid, status); served = true;
107	} else if $(k_{max} > key)$ {
109	chunk1 = getChunkOfElementsFromLocalList(llist);
110	$status = $ ServerMove($next_id$, $chunk1$);
111	if $(status == true)$ {
112	removeChunkOfElementsFromLocalList(<i>llist</i> , <i>chunk</i> 1);
113	} else {
$\frac{114}{115}$	send(cid, NACK); served = true;
110	}
116	} else {
117	if $(my_id \neq NS - 1)$ send $(next_id, \langle INSERT, cid, key, data \rangle);$
118	else send(cid, NACK);
119	served = true;
	}
120	} break;
$120 \\ 121$	case SEARCH:
122	cv[cid] + +;
123	status = search(llist, key);
124	if $(status == false)$ send $(cid, NACK));$
125	else send(cid, ACK);
126	break;
$127 \\ 128$	case $DELETE$: cv[cid] + +;
$120 \\ 129$	co[cia] + +, status = search(<i>llist</i> , <i>key</i>);
130	$if (status == true) \{$
131	delete(llist, key);
132	send(cid, ACK);
133	} else send(cid, NACK);
134	break;
	}

Algorithm 29 Events triggered in a server of the distributed sorted list

The proposed implementation is based on the distributed unsorted list, presented in Section 7.1. Each server s has a memory chunk of predetermined size where it maintains a part of the implemented list so that all elements stored on server s_i have smaller keys than those stored on server $s_{i+1}, 0 \le i < NS - 1$. Because of this sorting property, an element with key k is not appended to the end of the list, so a token server is useless in this case. This is an essential difference with the unsorted list implementation.

Similarly to the unsorted case, a client sends an insert request for key kto server s_0 . The server searches its local part of the list for a key that is greater than or equal to In case that it finds k. such an element that is not equal to k, it can try to insert k to its local list, *llist*. More specifically, if the server has sufficient storage space for a new element, it simply creates a new node with key k and inserts it to the list. However, in case that the server does not have enough storage space, it tries to free it by forwarding a chunk of elements of *llist* to the next server. If this is possible, it serves the request. In case s_0

does not find a key that is greater than or equal to k in its *llist*, if forwards the message with the insert request to the next server, which in turn tries to serve the request accordingly.

Notice that this way, a request may be forwarded from one server to the next, as in the case of the unsorted list. However, for ease of presentation, in the following we present a static algorithm where this forwarding stops at s_{NS-1} . In case that an element with k is already present in the *llist* of some server s of the resulting sequence, then s sends an NACK message to the client that requested the insert.

As in the case of the unsorted list, a client performs a search or delete operation for key k by sending the request to all servers. If not handled correctly, then the interleaving of the arrival of requests to servers may cause a search operation to "miss" the key k that it is searching, because the corresponding element may be in the process to be moved from one server to a neighboring one. In order to avoid this, servers maintain a sequence number for each client that is incremented at every search and delete operation. Neighboring servers that have to move a chunk of elements among them, first verify that the latest (search or delete) requests that they have served for each client have compatible sequence numbers and perform the move only in this case.

Event-driven code for the server is presented in Algorithms 29 and 30. The clients access the sorted list using the same routines as they do in the case of the unsorted list (see Algorithm 26).

When an insert request for key k reaches a server s, s compares the maximal key stored in its local list to k. If k is greater than the maximal key and s is not s_{NS-1} , the request must be forwarded to the next server (line 117). Otherwise, if k is to be stored on s, s checks if *llist* has enough space to serve the insert. If it does, s inserts the element and sends an ACK to the client (line 105-106). If s does not have space for inserts, the operation cannot be executed, hence s must check whether a chunk of its elements can be forwarded to the next server to make room for further inserts. To move a chunk, s calls ServerMove() (presented in Algorithm 30) (line 110). If ServerMove() succeeds in making room in s's *llist*, the insert can be accommodated (line 111). In any other case, s responds to the client with NACK (line 114).

A server process a search request as described for the unsorted list, but it now pairs each such request with a sequence number (line 122). Delete is processed by a server in a way analogous to search.

In order to move a chunk of *llist* to the next server, a server s_i invokes the auxiliary routine **ServerMove()** (line 110). **ServerMove()** sends a **REQC** message to server s_{i+1} (line 138). When s_{i+1} receives this request, it sends its client vector to s_i (line 88). Upon reception (line 139), s_i compares its own client vector to that of s_{i+1} and as long as it lags behind s_{i+1} for any client, it services search and delete requests until it catches up to s_{i+1} (lines 140-142). Notice that during this time, s_{i+1} does not serve further client request, in order allow s_i to catch up with it. As soon as s_i and s_{i+1} are compatible in the client delete and search requests that they have served, s_i sends to s_{i+1} a chunk of the elements in its local list (lines 143-144) and awaits the response of s_{i+1} . We remark that in order to perform this kind of bulk transfer, as the one carried out between a server executing line 145 and another server executing line 89, we consider that remote DMA transfers are employed. This is omitted from the pseudocode for ease of presentation.

If s_{i+1} can store the chunk of elements, then it does so and sends ACK to s_i . Upon reception, s_i may now remove this chunk from its local list (line 111) and attempt to serve the insert request. Notice that if s_{i+1} cannot store the chunk of elements of s_i , then it itself initiates the same chunk moving procedure with its next neighbor (lines 92-94), and if it is successful in moving a chunk of its own, then it can accommodate the chunk received by s_i . Notice that in the static sorted list that is presented here, this protocol may potentially spread up to server s_{NS-1} (line 91). If s_{NS-1} does not have available space, then the moving of the chunk fails (line 113). The client then receives a NACK response, corresponding to a full list.

We remark that this implementation can become dynamic by appropriately exploiting the placement of the servers on the logical ring, in a way similar to what we do in the unsorted version.

8 Details on Hierarchical Approach

Algorithm 30 Auxiliary routine ServerMove for the servers of the distributed sorted list.

135 boolean ServerMove(int cid, data chunk1) {
136 boolean <i>status</i> ;
137 data $chunk2;$
138 $\operatorname{send}(next_id, \langle REQC, cid, 0, \bot \rangle);$
139 $nbr_cv = receive(next_id);$
140 while (for any element i , $cv[i] < nbr_cv[i]$) {
141 receiveMessageOfType($SEARCH$ or $DELETE$);
142 service request
}
143 $chunk2 = getChunkOfElementsFromLocalList(llist);$
144 $\operatorname{send}(next_id, chunk2);$
145 $status = receive(next_id);$
146 if $(status == true)$ {
147 removeChunkOfElementsFromLocalList($llist, chunk2$);
148 $insertChunkOfElementsInLocalList(llist, chunk1);$
149 return true;
} else return false;
}

We interpolate one or more communication layers by using intermediate servers between the servers that maintain parts of the data structure and the clients. The number of intermediate servers and the number of layers of intermediate servers between clients and servers can be tuned for achieving better performance.

For simplicity of presentation, we focus on the case where there is a single layer of intermediate servers. We present first the details for a fully non cache-coherent architecture.

For each island i, we appoint one process executing on a core of this island, called the island master (and denoted by m_i), as the intermediate server. Process m_i is responsible to gather messages from all the other cores of the island, and batch them together before forwarding them to the appropriate server. This way, we exploit the fast communication between the cores of the same island and we minimize the number of messages sent to the servers by putting many small messages to one batch. We remark that each batch can be sent to a server by performing DMA. In this case, m_i initiates the DMA to the server's memory, and once the DMA is completed, it sends a small message to the server to notify it about the new data that it has to process.

Algorithm 31 presents the events triggered in an island master m_i and its actions in order to handle them. m_i receives messages from clients that have type OUT (outgoing messages) and from servers that have type IN (incoming messages). The outbatch messages are stored in the *outbuf* array. Each time a client from island *i* wants to execute an operation, it sends a message to m_i ; m_i checks the destination server id (*sid*), recorded in the message, and packs this message together with other messages directed to *sid* (lines 4-6). Server m_i has set a timer, and it will submit this batch of messages to server *sid* (as well as other batches of messages to other servers), when the timer expires. When m_i receives an incoming message from a server, it unpacks it, and sends each message to the appropriate client on its island. When the timer is triggered, m_i places each batch of messages in an *outbuf* array that m_i maintains for the appropriate server. The transfer of all these message to add a message to an *outbuf* buffer of m_i , and split to split a batch of messages msg that have arrived to the particular messages of the batch which are then placed in the *inbuf* buffer of m_i .

Alg	orithm 31 Events triggered in an island master	r - Case of fully non cache-conerent arcm-
tect	ures.	
1]	$LocalArray \ outbuf = \varnothing;$	/* stores outgoing messages $*/$
2	LocalArray $inbuf = \emptyset;$	/* stores incoming messages $*/$
3	a message $\langle type, msg \rangle$ is received:	
4	$if(type == OUT) \{$	
5	sid = read field sid from msg;	
6	$add_message(outbuf, sid, msg);$	
7	} else if $(op == IN)$ {	/* m_i received a batch of messages from a server */
8	inbuf = split(msg);	/* unbundle the message to many small ones $\ */$
9	for each message m in $inbuf$ {	/* the batch is for all cores in the island $\ \ */$
10	cid = read field client from msg;	
11	send(cid, msg);	
	}	
	}	
12	timer is triggered:	/* Every timeout, m_i sends outgoing messages */
13	for each batch of messages in <i>outbuf</i> {	/* send a bach to a server $*/$
14	send(sid, batch);	/* this send can be done using DMA */
15	delete(outbuf, sid, batch);	
	}	

Algorithm 31 Events triggered in an island master - Case of fully non cache-coherent archi-

The code of the client does not change much. Instead of sending messages directly to the server, it sends them to the board master.

We remark that in order to improve the scalability of a directory-based algorithm, a localitysensitive hash functions could be a preferable choice. For instance, the simple currently employed mod hash function, can be replaced by a hash function that divides by some integer k. Then, elements of up to k subsequent insert (i.e. push or enqueue) operations may be sent to the same directory server. This approach suites better to bulk transfers since it allows for exploiting locality. Specifically, consider that m_i sends a batch of elements to be inserted to the synchronizer s_s of a directory-based data structure. Server s_s unpacks the batch and processes each of the requests contained therein separately. Thus, if mod is used, each of these elements will be stored in different buckets (of different directory servers). As a sample alternative, if the div hash function is used, more than one elements may end up to be stored in the same bucket. When later on a batch of remove (i.e. pops or dequeues) operations arrives to s_s , it can request from the directory server that stores the first of the elements to be removed, to additionally remove and send back further elements with subsequent keys that are located in the same bucket. Notice that in this case, the use of DMA can optimize these transfers.

We now turn attention to partially non cache-coherent architectures where the cores of an island communicate via cache-coherent shared memory. Algorithm 32 presents code for the hierarchical approach in this case. For each island, we use an instance of the CC-Synch combining synchronization algorithm, presented in [15]. All clients of island i participate to the instance of CC-Synch for island *i*, i.e. each such client calls CC-Synch (see Algorithm 32) to execute an operation.

CC-Synch employs a list which contains one node for each client that has initiated an operation; the last node of the list is a dummy node. After announcing its request by by recording

	itectures.	
st	truct Node {	
	Request req;	
	RetVal ret;	
	int id;	
	boolean wait;	
	boolean completed;	
	int sid;	
	Node *next;	
	};	
	shared Node *Tail;	
	private Node $*node_i$;	
	$LocalArray \ outbuf = \varnothing;$	/* stores outgoing messages */
	RetVal CC-Synch(Request req) {	/* Pseudocode for thread p_i */
	Node *nextNode, *tmpNode, *tmpNodeNext;	γ i beddeedd for unredd p_i γ
	int counter = 0;	
	Int counter $= 0$;	
16	$node_i \rightarrow wait = true;$	
17	$node_i \rightarrow next = $ null;	
18	$node_i \rightarrow completed = false;$	
19	$nextNode = node_i;$	
20	$node_i = Swap(Tail, node_i);$	
-0 21	$node_i \rightarrow req = req;$	/* p_i announces its request */
22	$node_i \rightarrow sid = \text{destination server};$	$j = p_i$ announces its request j
22	$node_i \rightarrow next = nextNode;$	
	,	
24	while $(node_i \rightarrow wait == true)$	/* p_i spins until it is unlocked */
~ ~	nop;	
25	if $(node_i \rightarrow completed == true)$	/* if p_i 's req is already applied */
26	$\texttt{return} \ node_i \to ret;$	/* p_i returns its return value */
27	$tmpNode = node_i;$	/* otherwise p_i is the combiner */
28	while $(\text{tmpNode} \rightarrow \text{next} \neq null \text{ AND counter} < h)$	
29	counter = counter + 1;	
30	$tmpNodeNext=tmpNode \rightarrow next;$	
31	$add_message(outbuf, tmpNode \rightarrow sid, tmpNode \rightarrow req);$	
32	tmpNode = tmpNodeNext;	/* proceed to the next node $*/$
	}	
33	for each batch of messages in <i>outbuf</i> {	/* send a batch of messages to servers $*/$
34	send(<i>sid</i> , batch of message);	/* where <i>sid</i> is the destination server for this batch */
35	delete(<i>outbuf</i> , <i>sid</i> , <i>fatm</i>);	/ where sta is the destination server for this batch /
00		
36	f = split(receive());	
37	$tmpNode = node_i;$	
38	while $(counter \ge 0)$ {	
39	counter = counter - 1;	
40	$tmpNodeNext=tmpNode \rightarrow next;$	
41	$tmpNode \rightarrow ret = find in inbuf$ the response for tmpN	lode→id;
42	$tmpNode \rightarrow completed = true;$	
43	$tmpNode \rightarrow wait = false;$	/* unlock the spinning thread $*/$
44	tmpNode = tmpNodeNext;	
	}	
45	$tmpNode \rightarrow wait = false;$	/* unlock next node's owner $*/$
46	return $node_i \rightarrow ret;$	
	}	

Algorithm 32 Pseudocode of hierarchical approach - Case of partially non cache-coherent architectures.

it in the last node of the list (i.e., in the dummy node) and by inserting a new node as the last node of the list (which will comprise the new dummy node), a client tries to acquire a global

lock (line 20) which is implemented as a queue lock. The client that manages to acquire the lock, called the *combiner*, batches those active requests, recorded in the list, that target the same server (lines 28-32) and forwards them to this server (line 33). Thus, at each point in time, the combiner plays the role of the island master. When the island master receives (a batch of) responses from a server, it records each of them in the appropriate element of the request list to inform active clients of the island about the completion of their requests (lines38-44). In the meantime, each such client performs spinning (on the element in which it recorded its request) until either the response for its request has been fulled by the island master or the global lock has been released (line 24).

We use the list of requests to implement the global lock as a *queue lock* [62, 63]. The process that has recorded its request in the head node of the list plays the role of the combiner.

9 Experimental Evaluation

We run our experiments on the Formic-Cube [4], which is a hardware prototype of a 512 core, non-cache-coherent machine. It consists of 64 boards with 8 cores each (for a total of 512 cores). Each core owns 8 KB of private L1 cache, and 256 KB of private L2 cache. None of these caches is hardware coherent. The boards are connected with a fast, lossless packet-based network forming a 3D-mesh with a diameter of 6 hops. Each core is equipped with its own local *hardware mailbox*, an incoming hardware FIFO queue, whose size is 4 KB. It can be written by any core and read by the core that owns it. One core per board plays the role of the island master (and could be one of the algorithm's servers), whereas the remaining 7 cores of the board serve as clients.

Our experiments are similar to those presented in [64, 15, 10]. More specifically, 10^7 pairs of requests (PUSH and POP or ENQUEUE and DEQUEUE) are executed in total, as the number of cores increases. To make the experiment more realistic, a random local work (up to 512 dummy loop iterations) is simulated between the execution of two consecutive requests by the same thread as in [64, 15, 10]. To reduce the overheads for the memory allocation of the stack nodes, we allocate a pool of nodes (instead of allocating one node each time).

In Figure 1.a, we experimentally compare the performance of the centralized queue (CQueue) to the performance of its hierarchical version (HQueue), and to those of the hierarchical versions of the directory-based queue (DQueue) and the token-based queue (TQueue). We measure the average throughput achieved by each algorithm. As expected, CQueue does not scale well. Specifically, the experiment shows that for more than 16 cores in the system, the throughput of the algorithm remains almost the same. We remark that the clients running on these 16 cores do not send enough messages to fill up the mailbox of the server. This allows us to conclude that when the server receives about 16 messages or more, for reading these messages, processing the requests they contain, and sending back the responses to clients, the server ends up to be always busy. We remark that reading each message from the mailbox causes a cache miss to the server. So, the dominant factor at the server side in this case is to perform the reading of these messages from the mailbox.

These remarks are further supported by studying the experiment for the HQueue implementation. Specifically, the throughput of HQueue does not further increase when the number of cores becomes 64 or more. Remarkably, the 64 active cores are located in 8 boards, so there exist 8 island masters in the system. Each island master sends two messages to the server – one for batched enqueue and one for batched dequeue requests. So, again, the server becomes saturated when it receives about 16 messages. These messages are read from the mailbox in about the same time as in the setting of CQueue with 16 running cores. However, in the case of HQueue, each message contains more requests to be processed by the server. Therefore, CQueue — HQueue — DQueue — TQueue — CQueue — CQueue — TQueue — CQueue — CQU

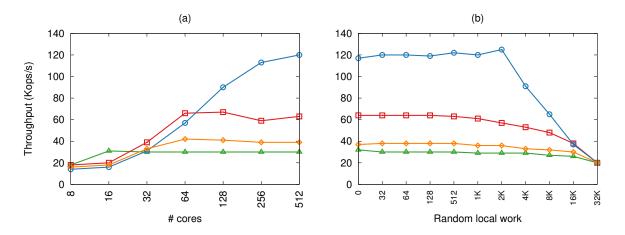


Figure 1: Performance evaluation of (a) distributed queue implementations, (b) distributed queue implementations while executing different amounts of local work (512 cores).

the average time needed to process a request is now smaller since the overhead of reading and processing a message is divided over the number of requests it contains. Notice that now, the server has more work to do in terms of processing requests. The experiment shows that the time required for this is evened out by the time saved for processing each request.

Similarly to CQueue and HQueue, the DQueue implementation uses a centralized component, namely the synchronizer. However, contrary to the HQueue case, in the DQueue, each island master sends one message instead of two, which contains the number of both the enqueues and dequeues requests. Since we do not see the throughput stop increasing at 128 cores, we conclude that now the dominant factor is not the time that the server requires to read the messages from its mailbox. The DQueue graph of Figure 1.a shows that DQueue scales well for up to 512 cores. Therefore, in the DQueue approach, the synchronizer does not pose a scalability problem. The reason for this is, not only that the synchronizer receives a smaller number of messages, but also that it has to do a simple arithmetic addition or subtraction for each batch of requests that it receives. This computational effort is significantly smaller than those carried out by the centralized component in the CQueue and HQueue implementations. Therefore, it is important that the local computation done by a server be small.

Notice that in the DQueue implementation, the actual request processing takes place on the hash table servers. So, clients do not initiate requests as frequently as in the previous algorithms, since they also have to communicate with the hash table servers. Moreover, the processing in this case is shared among the hash-table servers and therefore, this processing does not cause scalability problems. It follows that load balancing is also an important factor affecting scalability. In the case of the DQueue the local work on the synchronizer is a linear function of the amount of island masters. On the contrary, in the other two implementations, it is a function of the amount of clients. It follows that this is another reason for the good scalability observed on the DQueue implementation.

We remark that when the amount of clients, and therefore, of island masters, is so large as to cause saturation on the synchronizer, a tree-like hierarchy of island masters would solve the scalability problem. There, the DQueue algorithm can offer a trade-off between overloaded activity of the centralized component and the latency that is caused by the height of the tree

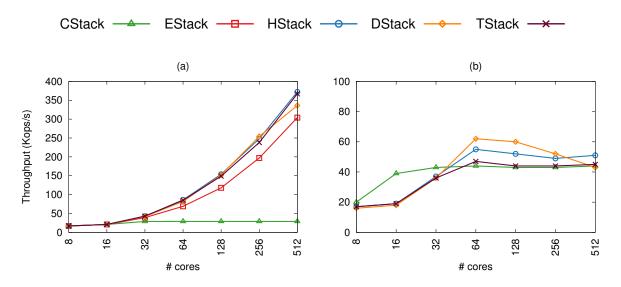


Figure 2: Performance evaluation of (a) distributed stack implementations with elimination, (b) distributed stack implementations without elimination.

hierarchy.

Figure 1.a further shows the observed throughput of TQueue. The behavior of TQueue in terms of scalability follows that of HQueue. Notice that TQueue works in a similar way as HQueue, with the difference that the identity of the centralized component may change, as the token moves from server to server. This makes TQueue more complex than HQueue. As a result, its throughput is lower than that of HQueue. However, TQueue is a nice generalization of HQueue, which can be used in cases where the expected size of the required queue is not known in advance and where a moderate number of cores are active.

In Figure 1.b, we fix the number of cores to 512 and perform the experiment for several different random work values (0 - 32K). It is shown that, for a wide range of values (0-512), we see no big difference on the performance of each algorithm. This is so because, for this range of values, the cost to perform the requests dominates the cost introduced by the random work. When the random work becomes too high (greater than 32K dummy loop iterations), the throughput of all algorithms degrades and the performance differences among them become minimal, since the amount of random work becomes then the dominant performance factor.

In Figure 2.a, we experimentally compare the performance of the centralized stack (CStack) with its hierarchical version where the island master performs elimination (EStack), an improved version of EStack where the island master performs batching (HStack), and the hierarchical versions of the directory-based stack (DStack) and the token based-stack (TStack). As expected, the centralized implementation does not scale for more than 16 cores. All other algorithms scale well for up to 512 cores. This shows that the elimination technique is highly-scalable. It is so efficient that it results in no significant performance differences between the algorithms that apply it.

In order to get a better estimation of the effect that the elimination technique has on the algorithms, we experimentally compared the performance that CStack, EStack, HStack, TStack, and DStack can achieve, when they are not performing elimination. Figure 2.b shows the obtained throughput. Notice that the scalability characteristics of CStack, HStack, and TStack are similar to those of CQueue, HQueue, and TQueue. This is not the case for DStack. The reason for this is the following. In our experiment, each client performs pairs of push and pops.

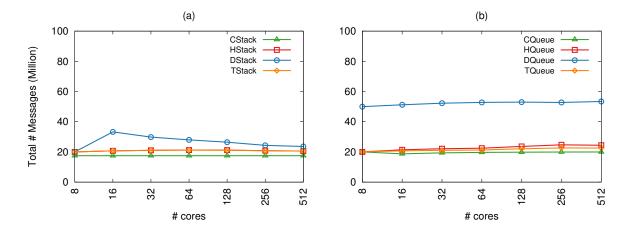


Figure 3: Total number of messages received in the proposed implementations by all servers in the system for 10^7 operations.

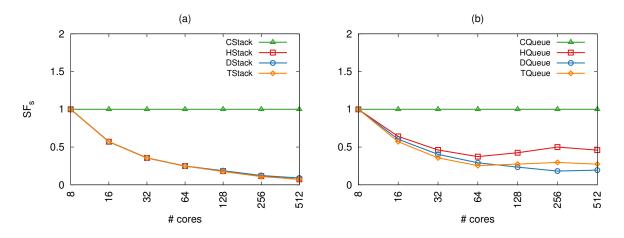


Figure 4: Scalability factors of presented algorithms.

By the way the synchronizer works, the push request and the pop request of each pair are often assigned the same key. This results in contention at the hash table. In an experiment where the number of pushes is not equal to the number of pops, the observed performance would be much better than that of other algorithms for all numbers of cores.

Figure 3.b shows the total number of messages sent in each experiment presented in Figure 1.a. Remarkably, there is an inverse relationship between these results and those of Figure 1.a. For instance, DQueue circulates the largest number of messages in the system. However, DQueue is the implementation that scales best. Thus, the total number of messages is not necessarily an indicative factor of the actual performance. This is so because a server may easily become saturated even by a moderate amount of messages, if the required processing is important. This is for example the case in the CQueue implementation. Therefore, in order to be scalable, an implementation should avoid overloading the servers in this manner. Backed up by Figure 3.b, becomes aparent that good scalability is offered when the implementation ensures good load balancing between the messages that the servers have to process.

We distill the empirical observations of this section into a metric. As observed, achieving load balancing in terms of evenly distributing both messages and the processing of requests to servers is important for ensuring scalability. If we fix an algorithm \mathcal{A} , this is reflected in the number of messages received by each server s for performing m requests when executing \mathcal{A} . By denoting this number as msg_s , we can define the scalability factor sf_s of a server s as follows:

$$sf_s = \lim_{m \to \infty} \frac{msg_s}{m} \tag{1}$$

where we assume that a maximum number of clients repeatedly initiate requests and these clients are scheduled fairly. We define the scalability factor sf of \mathcal{A} as the maximum of the scalability factors of all servers.

$$sf = \max\{sf_s\}\tag{2}$$

We remark that a low scalability factor indicates good scalability behavior. The intuition behind this metric is that the lower the sf_s fraction is, the more requests are batched in a message that reaches a server. Computational overhead for the reading, handling and decoding of a single message is then spread over multiple requests. As a scalability indicator, the scalability factor shows that it is of more interest to attempt not to minimize the total number of messages that are trafficked in a system, but to design implementations that minimize the total number of messages that a server has to process. Figure 4 shows graphs of the scalability factors of the the proposed implementations. The results agree with this theoretical perception.

10 Implementation of Shared-Memory Primitives

In Java, synchronization primitives are provided as methods of the library sun.misc.unsafe. In order to support most of the methods sun.misc.unsafe uses, we need to implement the atomic updates, the compare-and-swap primitives, and the pair of park()/unpark() methods used for thread synchronization. Once these primitives have been supported, the implementations of several data structures that are provided in java.util.concurrent will come for free.

In addition to these primitives, we have implemented the primitives fetch-and-increment and swap, to implement numeric operations used by the methods of package java.util.concurrent.atomic, described in Section 10, e.g. getAndSet(). These primitives need to be implemented on a lower level than the Java Virtual Machine, since sun.misc.unsafe mostly calls primitives that need either to access the system or hardware resource.

We have to point out that Formic does support reads and stores to remote memory locations but does not provide coherence. Thus, this remote store is different from the atomic write primitive we want to implement, due to the atomicity that the second provides. By adding this primitive, we aim to give safety when simple writes occur concurrently with other atomic primitives, such as CAS, etc..

Hence, we implemented the following atomic primitives:

- Read, which taks as arguments a memory address and returns the value that is stored in this address.
- Write, which takes as arguments a memory address and a value, and stores the new value to this address.
- Fetch-And-Increment, which takes as arguments a memory address and a value, and adds the new value to the existing one stored in the address. It then returns the new value.
- Swap, which takes as arguments a memory address, and a new value, and replaces the stored value with the new one. It then returns the previous value.
- Compare-And-Swap, which takes as arguments a memory address, an old value and a new value. If the value stored in the memory address equals to the old value, then it is replaced by the new value. Then, the function returns the old value.

11 Atomic Accesses Support

The java.util.concurrent.atomic package provides a set of atomic types (as classes). Instances of these classes must be atomically accessed. In shared-memory architectures these classes are implemented by delegating the complexity of synchronization handling to instructions provided by the underlying architecture, e.g., memory barriers, compare and swap etc. In the case of Formic, however, such instructions are not available. As a result the atomic types need to be implemented in software.

A naive implementation is to delegate the handling of such operations to a manager similar to the monitor manager, presented in Section 2.3 of Deliverable 1.1. This manager would be responsible for holding the values associated to instances of atomic types, as well as, for performing atomic operations on them. Such a manager is capable of reducing the memory traffic regarding synchronization in some cases.

\mathbf{A}	Algorithm 33 Example of getAndSet implementation.			
1	public final int getAndSet(int newValue) {			
2	for (;;) {			
3	int current $= get();$	/* Synchronization point $*/$		
4	<pre>if (compareAndSet(current, newValue))</pre>	/* Synchronization point */		
5	return current;			
6	}			
7	}			

For instance, in the implementation depicted in Algorithm 33, the JVM constantly tries to set the value of the object to newValue before its value changes between the get() and the compareAndSet invocations. This may result in many failures and unnecessary synchronizations in case of high contention. When using a manager there is no need for such a loop. Since the accesses are only possible by the manager, the manager can assume that the value will never change between the get() and a consequent set() invocation. As a result we could implement the logic of the above function in the manager and avoid the extra communication from the loop iterations. The equivalent algorithm in the manager is shown in Algorithm 34.

Algorithm 34 Implementation of getAndSet through the monitor manager.

```
8 public final int getAndSet(int newValue) {
9     int current = get();
10     set(newValue);
11     return current;
12   }
```

A similar effect can be achieved by using the synchronized classifier. Since the code inside a synchronized block or method can be seen as atomic we can implement getAndSet as above but without the need of a centralized manager by just adding synchronized to the method classifiers, as shown in Algorithm 35.

Note that, in shared-memory processors, this approach is probably less efficient than the first one when there is no contention on the object. In the lack of contention the first implementation would just perform a read instruction and a compare and swap, while in the third implementation it would also need to go through the monitor implementation of the virtual machine (at minimum an extra compare and swap, and a write). On the Formic, however, where

Algorithm 35 Implementation of getAndSet using synchronized.

```
13 public final synchronized int getAndSet(int newValue) {
14     int current = get();
15     set(newValue);
16     return current;
17   }
```

there is no compare and swap instruction, the first implementation would require at least two request messages (get and compareAndSet) to the manager and the corresponding two replies per iteration. The third implementation would also need two requests (a monitor acquire and a monitor release) and the corresponding two replies but with the difference that it never needs to repeat this process, resulting in reduced memory traffic.

To further improve this approach we avoid the use of regular monitors and replace them with readers-writers monitors. This way in the case of contented reads we do not restrict access to a single thread, allowing for increased parallelism. As a further optimization we slightly change the semantics of compareAndSet and make it *lazely* return False in case that the object is owned by another writer. This behavior is based on the heuristic that if an object is owned by a writer it is going to be written and the value will probably be different than the one the programmer provided as expected to the compareAndSet.

11.1 Readers-Writers Implementation

The readers-writers monitors are implemented using the same monitor manager, presented in Section 2.3 of Deliverable 1.1, but with different operation codes. We introduce four new operations code, READ_LOCK, WRITE_LOCK, READ_UNLOCK, and WRITE_UNLOCK. For each readers-writers monitor we keep two different queues, the readers queue and the writers queue. The readers queue holds the thread IDs of the threads waiting to acquire a read-lock on the monitor. Correspondingly, the writers queue holds the thread IDs of the thread IDs of the threads waiting to acquire a write-lock on the monitor.

When a monitor is read-locked we hold the count of threads sharing that read monitor, while for write-locked monitors we hold the thread ID of the thread owning that write monitor. As long as the writers lock is empty the monitor manager services read-lock requests by increasing the counter and sending an acknowledgement message to the requester. When a write-lock request arrives, it gets queued to the writers queue and read-lock requests start being queued in the readers-queue instead of being served, essentially giving priority to the writer-lock requests. Eventually, when all the readers release the read-locks they hold the write-lock requests will start being served, and read-lock requests will be served again only after the writers-queue gets empty again.

We chose to give priority to the write locks since in most algorithms writing a variable is less common than reading it (e.g., polling). In order to implement a more fair mechanism we could set a threshold on the number of read-lock and write-lock requests being served at each phase.

Finally, to support the *lazy fail* for compareAndSet we introduce another operation code, the TRY_LOCK. This is a special write-lock request that if the monitor is not free, it does not get queued in the write-queue, but a negative acknowledgement is send back to the requester, notifying him that the monitor is not free.

12 Conclusions

We have presented a comprehensive collection of data structures for future many-core architectures. The collection could be utilized by runtimes of high-productivity languages ported to such architectures. Our collection provides all types of concurrent data structures supported in Java's concurrency utilities. Other high-level productivity languages that provide shared memory for thread communication could also benefit from our library. Specifically, we provide several different kinds of queues, including static, dynamic and synchronous; our queue (or deque) implementations can be trivially adjusted to provide the functionality of *delay queues* (or *delay deques*) [8]. We do not provide a priority queue implementation, since it is easy to adopt a simplified version of the priority queue presented in [65] in our setting. Our list implementations provide the functionality of sets, whereas the simple hash table that we utilize to design some of our data structures can serve as a hash-based map.

We have outlined hierarchical versions of the data structures that we have implemented. These implementations take into consideration challenges that are raised in realistic scalable multicore architectures where communication is implicit only between the cores of an island whereas explicit communication is employed among islands.

We have performed experimental evaluation of the implemented data structures in order to examine both their throughput and energy efficiency. Our experiments show the performance and scalability characteristics of some of the techniques on top of a non cache-coherent hardware prototype. They also illustrate the scalability power of the hierarchical approach in such machines. We believe that the proposed implementations will exhibit the same performance characteristics, if programmed appropriately, in prototypes with similar characteristics as FORMIC, like Tilera or SCC. We expect this also to be true for future, commercially available, such machines.

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