Unified Approach to Image Distortion

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Abstract

We propose a new unified approach to deal with two formulations of image distortion and a method for estimating the distortion parameters by using the both formulation; So far either of the two formulations has been developed separately. The proposed method is based on image registration and consists of nonlinear optimization to estimate parameters including view change and radial distortion. Experimental results demonstrat that our approach works well for both formulations.

1 Introduction

Calibrating the intrinsic camera parameters and correcting image distortion are important processes for computer vision. Much research on computer vision formulate the problems without considering distortion because of simplicity. However, distortion is inevitable when we use an ordinary lens installed on an inexpensive camera; sometimes a point may be displaced more than ten pixels around the corner of the image. Pre-calibration of the intrinsic camera parameters and correction of distorted image are thus required for such research to produce a quality image.

Many studies on correcting distortion have been done by the image registration[3, 4] or the correspondence between corners[5], circles[6], curves [7] or feature points[8]. Although all of these researches is based on a distortion model proposed in an early study in photogrammetry[9], two different formulations have been used by different papers and this has caused a confusion for developing calibration methods.

In this paper, we show the relation between the two formulations to make the confusion clear, and propose a method for estimating the distortion parameters by using the both formulations; two formulations have not developed together. The proposed method is based on the image registration, and estimates the parameters of the transformations of view change and radial distortion by a nonlinear optimization that minimizes residuals between the distorted image and a calibration pattern.

In section 2, we explain the distortion model and its formulations. Then We describe how to estimate the distortion parameters and correct the image with both formulations in section 3. Finally, we present experimental results in section 4.

2 Distortion model and formulations

Usually the distortion of image is observed by the following two steps. At first, a point in a three-dimensional space is projected onto the image plane through a camera lens. Let $\boldsymbol{p}^u = (x^u, y^u)^T$ be the projected, undistorted coordinates on the image. Then $\boldsymbol{p}^u$ is moved by the distortion to the distorted point $\boldsymbol{p}^d = (x^d, y^d)^T$ (see Fig.2 and 3).

The relationship between $\boldsymbol{p}^u$ and $\boldsymbol{p}^d$ in an image is often modeled by five intrinsic camera parameters[10, 11] $\boldsymbol{\theta}^d = (\kappa_1, \kappa_2, c_x, c_y, s_x)^T$ [1]: the radial distortion parameters κ_1 and κ_2, the image center $(c_x, c_y)^T$, and the horizontal scale factor s_x. The radial distortion at a point $\boldsymbol{p} = (x, y)$ is represented by the following function with respect to the image center.

$$\boldsymbol{f}(\boldsymbol{p}, \boldsymbol{\theta}^d) = \begin{pmatrix} \dfrac{x-c_x}{s_x}(1+\kappa_1 R(\boldsymbol{p})^2 + \kappa_2 R(\boldsymbol{P})^4) + c_x \\ (y-c_y)(1+\kappa_1 R(\boldsymbol{p})^2 + \kappa_2 R(\boldsymbol{p})^4) + c_y \end{pmatrix} \tag{1}$$

$$R(\boldsymbol{p}) = \sqrt{\left(\frac{x-c_x}{s_x}\right)^2 + (y-c_y)^2} \tag{2}$$

Note that the inverse of $\boldsymbol{f}$ is not expressed in a closed-form.

The problem is that it is ambiguous to which coordinates the function is applied. There are two formulations. One is Distorted-to-Undistorted (**D-U**) formulation in which the undistorted coordinates is expressed as a function of the distorted coordinates.

$$\boldsymbol{p}^u = \boldsymbol{f}(\boldsymbol{p}^d, \boldsymbol{\theta}^d) \tag{3}$$

[1]Although we consider only the radial distortion, the following discussion can be applied to another model involving higher-order term or decentering distortion[12, 9]

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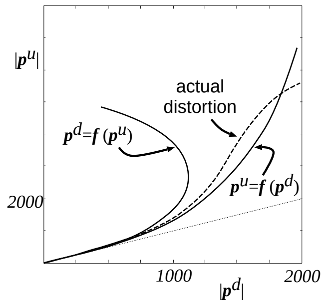


Figure 1. Example of the distortion approximated by different formulations.

The other is Undistorted-to-Distorted (**U-D**) formulation, the distorted coordinates is expressed by the undistorted coordinates.

$$\boldsymbol{p}^d = \boldsymbol{f}(\boldsymbol{p}^u, \boldsymbol{\theta}^d) \tag{4}$$

Historically, the D-U model was proposed to correct the plate coordinates of a photographed point on a film[9] and have been used for long time in papers of photogrammetry and computer vision [11, 10, 13, 7, 14, 8, 15]. Since the inverse of $\boldsymbol{f}$ is not a explicit function, it is inconvenient to make a combined transformation with projection and distortion. Therefore, sometimes the U-D model is used as an approximation of the former[6].

The confusion is that the U-D model is used as the exact formulation[4, 5, 16] and moreover the D-U model is regarded as the approximation of the other[12, 17]. Such researches have not disclosed the reason of this usage, however, performed as well as the former method.

Actually, almost vision applications require not the distortion parameters but just a corrected, distortion-free image. Therefore, any formulation or even the nonparametric approach[18] is employed if it can correct the distortion that an application takes into account.

Figure 1 illustrates an example that the barrel distortion is approximated by both formulations (here $|\boldsymbol{p}|$ denotes the distance between $\boldsymbol{p}$ and the image center because the distortion is usually represented with respect to the image center). In this case, both the formulations are close to the actual distortion for small $|\boldsymbol{p}^d|$. Since we assume that $|\boldsymbol{p}^d| \leq 300 \sim 400$ for an ordinary digital image, the difference between the two formulations is not significant and both can be used for correction.

Nevertheless, it is important to develop a method for both formulations and to choose an appropriate one.

3 Correction methods

The proposed method estimates the distortion parameters expressed in both formulations, while conventional methods deal with either of them.

The idea is that the image registration establishes the correspondence between an ideal (distortion-free) calibration pattern I_1 and a distorted image I_2 of the printed pattern observed by a camera[1, 2]. The observation is modeled by successive two transformations; view change and distortion. I_2 is regarded as an image generated from I_1 by applying these functions. The proposed method estimates the parameters of the functions by minimizing the difference between I_1 and I_2, that is, the sum of the squares of the intensity residuals of the two images.

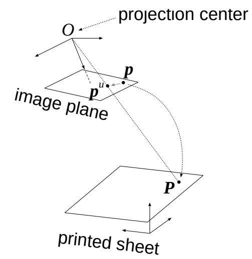


Figure 2. View change

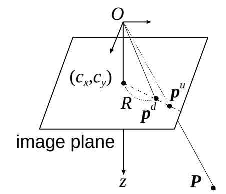


Figure 3. Distortion

3.1 Modeling view change

Given two images of a planar object from different viewpoints, the relationship between them is described by the planar perspective motion model with eight parameters[19, 20]. As shown in Fig.2, I_1 and I_2 can be considered as the different views of the same plane.

The model warps a point $\boldsymbol{p} = (x, y)^T$ on I_1 to the corresponding point $\boldsymbol{p}^u = (x^u, y^u)^T$ on I_2 by

$$\boldsymbol{p}^u = \boldsymbol{u}(\boldsymbol{p}, \boldsymbol{\theta}^u) = \frac{1}{\theta_1^u x + \theta_2^u y + 1} \begin{pmatrix} \theta_3^u x + \theta_4^u y + \theta_5^u \\ \theta_6^u x + \theta_7^u y + \theta_8^u \end{pmatrix} \tag{5}$$

where $\boldsymbol{\theta}^u = (\theta_1^u, \ldots, \theta_8^u)^T$.

3.2 Distortion by U-D formulation

At first, we consider the U-D formulation; the undistorted point $\boldsymbol{p}^u$ is further moved to $\boldsymbol{p}^d$ by Eq.(4). The Jacobian of $\boldsymbol{p}^d$ is derived straightforward by using the chain rule of vector differentiation[21].

$$\frac{\partial \boldsymbol{p}^d}{\partial \boldsymbol{\theta}} = \begin{pmatrix} \frac{\partial \boldsymbol{p}^d}{\partial \boldsymbol{\theta}^d} & \frac{\partial \boldsymbol{p}^d}{\partial \boldsymbol{\theta}^u} \end{pmatrix} = \begin{pmatrix} \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\theta}^d} & \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^u} \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{\theta}^u} \end{pmatrix} \tag{6}$$

where $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{13})^T = (\boldsymbol{\theta}^u, \boldsymbol{\theta}^d)^T$. In this case, The Jacobian of the combined transformation with $\boldsymbol{u}$ and $\boldsymbol{f}$ is also derived straightforward.

3.3 Distortion by D-U formulation

Next, we consider the D-U formulation. Eq.(3) is rewritten as follows.

$$\boldsymbol{p}^d = \boldsymbol{f}^{-1}(\boldsymbol{p}^u, \boldsymbol{\theta}^d) \equiv \boldsymbol{d}(\boldsymbol{p}^u, \boldsymbol{\theta}^d) \tag{7}$$

where $\boldsymbol{d}$ is the inverse function of $\boldsymbol{f}$ and is implemented by an iterative procedure[10] because $\boldsymbol{d}$ is not expressed in a closed-form. Therefore, the Jacobian of $\boldsymbol{d}$ as well as that of the combined transformation with $\boldsymbol{u}$ and $\boldsymbol{d}$ is difficult to calculate, and most researchers have tried to avoid the difficulty.

Here we introduce the *implicit function theorem*[22] for systems[23]. This theorem can represent the Jacobian of

$\boldsymbol{d}$ as an explicit form through $\boldsymbol{f}$. Let $\boldsymbol{F}$ be a function of $\boldsymbol{q} = (\boldsymbol{p}^u, \boldsymbol{\theta}^d)$ and $\boldsymbol{p}^d$ represented by

$$\boldsymbol{F}(\boldsymbol{q}, \boldsymbol{p}^d) = \boldsymbol{p}^u - \boldsymbol{f}(\boldsymbol{p}^d, \boldsymbol{\theta}^d) \tag{8}$$

If $\boldsymbol{F}(\boldsymbol{q}, \boldsymbol{d}(\boldsymbol{q})) = 0$ is satisfied for $\forall \boldsymbol{q}$, $\boldsymbol{p}^d = \boldsymbol{d}(\boldsymbol{q})$ is called an implicit function determined by $\boldsymbol{F}(\boldsymbol{q}, \boldsymbol{p}^d) = 0$. [2]

According to the theorem, the Jacobian is given by the following equations.

$$\frac{\partial \boldsymbol{d}}{\partial \boldsymbol{q}} = -\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}^d}^{-1} \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{q}} = -\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}^d}^{-1} \begin{pmatrix} \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}^u} & \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{\theta}^d} \end{pmatrix} \tag{9}$$

unless $\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}^d}$ is singular. On the other hand, the Jacobian is also decomposed into two parts as follows.

$$\frac{\partial \boldsymbol{d}}{\partial \boldsymbol{q}} = \begin{pmatrix} \frac{\partial \boldsymbol{d}}{\partial \boldsymbol{p}^u} & \frac{\partial \boldsymbol{d}}{\partial \boldsymbol{\theta}^d} \end{pmatrix} \tag{10}$$

Therefore, the following is the desired gradient of $\boldsymbol{d}$.

$$\frac{\partial \boldsymbol{d}}{\partial \boldsymbol{\theta}^d} = -\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}^d}^{-1} \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{\theta}^d} = -\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^d}^{-1} \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\theta}^d} \tag{11}$$

$$\frac{\partial \boldsymbol{d}}{\partial \boldsymbol{p}^u} = -\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}^d}^{-1} \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}^u} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^d}^{-1} \tag{12}$$

3.4 Minimization

Image registration seeks to minimize the residuals r_i of intensities of I_1 and I_2. The function to be totally minimized is the sum of the squares of the residuals over the image I_1.

$$\min_{\boldsymbol{\theta}} \sum_i {r_i}^2 , \quad \boldsymbol{p}_i \in I_1 \tag{13}$$

$$r_i = I_1(\boldsymbol{p}_i) - I_2(\boldsymbol{p}_i^d) \tag{14}$$

$$\boldsymbol{p}_i^d = \boldsymbol{f}(\boldsymbol{p}_i^u, \boldsymbol{\theta}^d) \quad \text{for U-D model} \tag{15}$$

$$\boldsymbol{p}_i^d = \boldsymbol{d}(\boldsymbol{p}_i^u, \boldsymbol{\theta}^d) \quad \text{for D-U model} \tag{16}$$

$$\boldsymbol{p}_i^u = \boldsymbol{u}(\boldsymbol{p}_i, \boldsymbol{\theta}^u) \tag{17}$$

Estimating the parameters $\boldsymbol{\theta}$, the objective function is minimized by the Gauss-Newton method[21]. To calculate the decent direction of the cost function, the following Jacobian of r with respect to $\boldsymbol{\theta}$ is required.

$$\frac{\partial r}{\partial \boldsymbol{\theta}} = \begin{pmatrix} \frac{\partial r}{\partial \boldsymbol{\theta}^u} & \frac{\partial r}{\partial \boldsymbol{\theta}^d} \end{pmatrix} \tag{18}$$

We show the derivations for both formulations based on the discussions above.

For D-U model:

$$\begin{aligned} \frac{\partial r}{\partial \boldsymbol{\theta}^u} &= \frac{\partial r}{\partial I_2} \frac{\partial I_2}{\partial \boldsymbol{p}^d} \frac{\partial \boldsymbol{p}^d}{\partial \boldsymbol{p}^u} \frac{\partial \boldsymbol{p}^u}{\partial \boldsymbol{\theta}^u} = -\frac{\partial I_2}{\partial \boldsymbol{p}^d} \frac{\partial \boldsymbol{d}}{\partial \boldsymbol{p}^u} \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{\theta}^u} \\ &= -\nabla I_2(\boldsymbol{p}^d) \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^d}^{-1} \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{\theta}^u} \end{aligned} \tag{19}$$

$$\frac{\partial r}{\partial \boldsymbol{\theta}^d} = \frac{\partial r}{\partial I_2} \frac{\partial I_2}{\partial \boldsymbol{p}^d} \frac{\partial \boldsymbol{p}^d}{\partial \boldsymbol{\theta}^d} = \nabla I_2(\boldsymbol{p}^d) \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^d}^{-1} \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\theta}^d} \tag{20}$$

[2] In our case, the condition is theoretically always satisfied because we defined $\boldsymbol{d}$ as the inverse of $\boldsymbol{f}$, and numerically Eq.(8) is almost 0 (it can be less than 10^{-10}).

For U-D model:

$$\begin{aligned} \frac{\partial r}{\partial \boldsymbol{\theta}^u} &= \frac{\partial r}{\partial I_2} \frac{\partial I_2}{\partial \boldsymbol{p}^d} \frac{\partial \boldsymbol{p}^d}{\partial \boldsymbol{p}^u} \frac{\partial \boldsymbol{p}^u}{\partial \boldsymbol{\theta}^u} = -\frac{\partial I_2}{\partial \boldsymbol{p}^d} \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^u} \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{\theta}^u} \\ &= -\nabla I_2(\boldsymbol{p}^d) \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^u} \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{\theta}^u} \end{aligned} \tag{21}$$

$$\frac{\partial r}{\partial \boldsymbol{\theta}^d} = \frac{\partial r}{\partial I_2} \frac{\partial I_2}{\partial \boldsymbol{p}^d} \frac{\partial \boldsymbol{p}^d}{\partial \boldsymbol{\theta}^d} = -\nabla I_2(\boldsymbol{p}^d) \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\theta}^d} \tag{22}$$

Every iteration of the oprimization, the decent direction is calculated by the equations above until the estimation converges.

3.5 Correcting distortion

After the distortion parameters $\boldsymbol{\theta}^d$ are estimated, we can use them for correction. For every point $\boldsymbol{p}^u$ in the corrected image I_2', the intensity is decided by that of the correspoinding point in the distorted image I_2 as follows.

$$I_2'(\boldsymbol{p}^u) = I_2(\boldsymbol{f}(\boldsymbol{p}^u, \boldsymbol{\theta}^d)) \quad \text{for U-D model} \tag{23}$$

$$I_2'(\boldsymbol{p}^u) = I_2(\boldsymbol{d}(\boldsymbol{p}^u, \boldsymbol{\theta}^d)) \quad \text{for D-U model} \tag{24}$$

Actually, U-D is faster than D-U because Eq.(24) involves the computation of the iterative procedure $\boldsymbol{d}$. Note that the Jacobian also involves $\boldsymbol{d}$, so the computation time for D-U is much longer than U-D.

4 Experimental Results

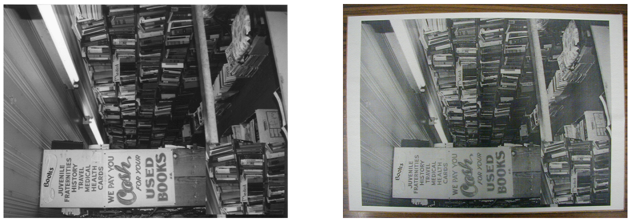
We conducted experiments with the proposed method using real distorted image. We used a scanned photograph (Fig.4(a)) as the calibration pattern I_1, then printed it with a laser printer and captured a distorted image I_2 (Fig.4(b)) of the printed sheet by a digital camera.

Table 1 shows the estimated parameters of both formulations. The image centers are almost identical, however, the horizontal scales differ greatly. The reason is that s_x is theoretically absorbed into $\boldsymbol{\theta}^u$ for U-D formulation; the view change stretches the image horizontally while $s_x(<1)$ makes the stretched image shrink. Therefore, it is difficult to estimate s_x accurately by U-D formulation.

Although the signs of the distortion parameters are inverted, in Fig.6 we can see that κ_1 and κ_2 have the same effect on the distortion curves of both formulations which are quite similar to each other for $|\boldsymbol{p}^d| < 400$ (the distance between a point in I_2 and the center is less than about 400) Note that we used $s_x = 1$ for U-D because of the reason above. The distorted images are corrected well (Fig.5) by both formulations (s_x=1 for U-D). Therefore, both of them are comparable with each other except the estimation of s_x and the computation time for correction (as shown in 3.5).

5 Conclusions

We have proposed a new unified approach to deal with two formulations of image distortion and a method for estimating the distortion parameters by using both formulation. The proposed method is based on image registration



(a) (b)

Figure 4. (a) Calibration pattern. (b) Captured image with distortion. (640×480)

Table 1. Estimated parameters

	κ_1	κ_2	c_x	c_y	s_x
U-D	-4.96e-07	7.49e-13	298.7	241.2	0.762
D-U	5.07e-07	-4.22e-13	297.7	241.2	0.978

and consists of nonlinear optimization to estimate parameters including view change and distortion. Experimental results demonstrated that our approach works well for both formulations of distortion.

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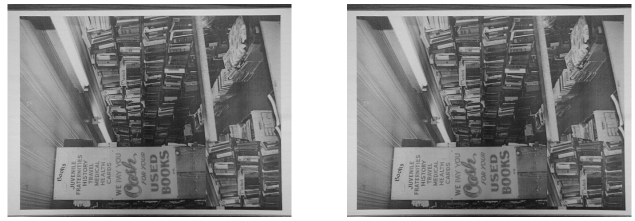

(a) (b)

Figure 5. Corrected images by (a) U-D ($s_x = 1$) and (b) D-U.

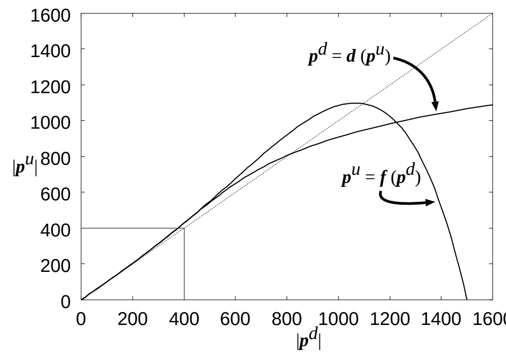

Figure 6. Distortion curves of both formulations

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