A Robust Nonlinear Observation Strategy for the Control of Flexible Manipulators

Brian K. Post and Wayne J. Book

***Abstract*— Flexibility is often an unavoidable consequence of the desire for high speed and performance manipulators. This paper proposes a method that improves the performance of flexible manipulators through the employment of robust state estimation techniques. These techniques are based on discrete time Kalman filtering and sliding mode principles. A simple model for a single degree of freedom flexible manipulator is derived and a control scheme is chosen and implemented. The latter includes a robust non-linear estimator. Simulation and preliminary experimental results are presented that demonstrate the validity of the proposed control scheme.**

I. INTRODUCTION

Industrial manipulators often sacrifice performance and efficiency for accuracy. The limitations placed on current state of the art manipulators for precision come at the cost of high speed manipulation, payload capability, and safety. Demand for rigid positioning requires most industrial manipulators to operate with payloads equal to 3% to 5% of their total weight [1]. Increased manipulator weight also results in a need for larger, more powerful, and inefficient motors and drives to maintain the same performance characteristics as lighter weight arms.

The increased demand for link stiffness has also severely limited the workspaces of modern industrial robots. Current long-reach, lightweight robots like that of the space shuttle manipulator are operated far below their performance limits in order to avoid exciting undesired vibration. Recently, advanced control algorithms have improved the capabilities of these complex motion systems through feed-forward algorithms like inverse dynamics and command generation [2] [3]. These control algorithms effectively drive the manipulator in a way designed to avoid causing residual vibration. Fig. 1. shows an example of a robot to which these techniques have been applied greatly improving the utility of its long reach capabilities. However, while these techniques ideally minimize vibration along a trajectory, they offer no compensation for disturbances or unmodeled effects.

Feedback control can correct for external disturbances but generally requires precise knowledge of the current system state variables, which in practical applications, are rarely directly measurable [4] [5]. This estimation challenge is exacerbated by the nonlinearities and complexities of flexible motion systems. Thus the solution to the problem falls on

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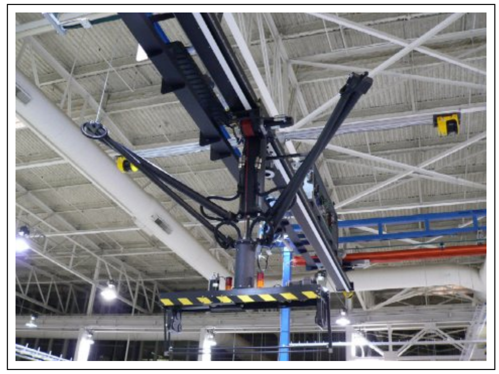


Fig. 1. CAMotion Depalletizing Robot

the development of state estimators to recreate an accurate portrayal of the current state of the dynamic system from available measurements.

Typically estimators of this type require an accurate model of the system, and in the case of flexible manipulators, models are either too complex for real time control or fail to capture the true dynamic behavior of the system, resulting in instability of the control system. Therefore, it is desirable for the estimator to exhibit robustness with respect to disturbances, parameter variation, and modeling inaccuracies [6].

In this study we propose an observation strategy built on the discrete time Kalman filter [7] and utilizing a sliding mode discrete switching algorithm developed by Walcott and Zak [8] to add robustness to the state estimates of a single link flexible manipulator. A simple model of a single axis of a gantry style packaging robot is derived and implemented in the developed estimation scheme. Simulation results are presented to examine the robustness and accuracy of the proposed algorithm and preliminary experimental results are included to verify practical applicability to industrial systems.

II. MODELING

Fig. 2. illustrates a simple model for a single degree of freedom of a belt driven gantry robot. Flexibility in the system comes from the elastic belt and the thin beam attached to the cart which is free to slide on a track with the assumption of viscous friction.

In order to design a suitable compensator for a flexible manipulator, the dynamic behavior of the flexible system must first be adequately modeled. Many approaches to the dynamic modeling of flexible manipulators have been

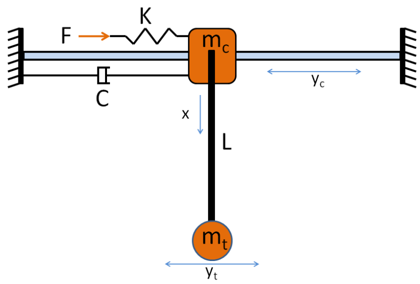


Fig. 2. System Schematic

proposed including finite element analysis [2], solutions to partial differential equations for continuous vibrating beams [9], transfer matrix methods [10] [11], and lumped parameter approximations [12] [9] [2]. For the purpose of this study we used a less accurate lumped parameter model based on the Ritz series expansion that permitted examination of the behavior of the proposed observation scheme.

Due to the continuous nature of flexible motion systems, a lumped parameter model will ultimately fail to characterize the true dynamic nature of the system. Inaccuracies arise from the fact that any continuous vibratory system has an infinite number of resonant modes. As a matter of practicality, this infinite series is truncated to consider only the first few modes. This approximation leads to residual mode spillover and aliasing, which can result in uncompensated high frequency modes of vibration and ultimately possible instability [6]. Therefore, model-based estimators like the Luenberger observer and Kalman filter built on inexact models will provide inaccurate state estimates, potentially destabilizing the system.

The Ritz series method for the description of vibrating beams, often called the "assumed modes method," is an energy based technique derived from the framework of Lagrange's equations. The basic procedure utilizes a series expansion of basis functions to represent the dependence of the displacement field of the vibrating beam. Thus the central issue in the Ritz series formulation becomes the selection of appropriate basis functions for the given type of vibration and constraints [9], in our case flexure with fixed free end conditions. Thus we can define the corresponding basis function (1).

$$\psi_j = cos\left(\frac{(j-1)\pi x}{2L}\right) \tag{1}$$

Where L is the beam length and x is the displacement along the beam. From the kinetic energy of the system the effective inertia matrix can be determined.

$$M_{jn} = \int_0^L \psi_j \psi_n \rho A dx + \sum m\psi_j(x_m)\psi_j(x_m) \tag{2}$$

Where the first term comes from the distributed mass ρA of the continuous beam and the second from the point masses at the base (cart mass) and tip (tip mass). Similarly the elasticity matrix can be determined from the system's total potential energy. Again the distributed elasticity EA of the beam is accounted for by the first term of (3), where E is Young's modulus and A is the cross-sectional area. The attachment of elastic elements at fixed points is enabled by the second term.

$$K_{jn} = \int_0^L EA\frac{d\psi_j}{dx}\frac{d\psi_n}{dx}dx + \sum k\psi_j(x_k)\psi_j(x_k) \tag{3}$$

Damping in the physical system stems from friction between the cart and track and internal damping in the flexible beam, which is characterized by a loss factor γ. The damping matrix itself is derived from the energy dissipated by the system.

$$C_{jn} = \int_0^L \gamma EA\frac{d\psi_j}{dx}\frac{d\psi_n}{dx}dx + \sum c\psi_j(x_c)\psi_j(x_c) \tag{4}$$

The generalized forces applied to the system are derived from the energy introduced to the system by external forces and consist of distributed forces f_x and point forces F.

$$Q_j = \int_0^L f_x\psi_j dx + \sum F\psi_j(x_f) \tag{5}$$

The system equation of motion is given by (6).

$$M\ddot{q} + C\dot{q} + Kq = Q \tag{6}$$

This equation of motion is a system of coupled differential equations, which can be decoupled by converting to modal coordinates η. This transformation is accomplished by solving the eigenvalue problem $(K - \omega_n^2 M)v = 0$ resulting in the natural frequencies w_n and mode shapes v. Transformation to modal coordinates is accomplished by (7).

$$q = \Phi\eta \tag{7}$$

Where $\Phi = [v_1, v_2, ..., v_n]$ resulting in the modal equation of motion (8).

$$\ddot{\eta} + \Phi^T C\Phi\dot{\eta} + \omega_n^2\eta = \Phi^T Q \tag{8}$$

An infinite number of modes must be considered for complete description of a continuous flexible motion system. However, the series is truncated to the first three terms: a rigid body mode and first and second flexible modes, for the purposes of this study.

III. STATE OBSERVATION

In order to build an observation strategy for a flexible motion system as described in section II. the modal model must first be converted to state-space form. State derivatives $\dot{x}$ and system outputs y are a linear function of the system states x and the inputs to the system u.

$$\dot{x} = Ax + Bu \tag{9a}$$
$$y = Cx + Du \tag{9b}$$

One convenient way to transform the modal equation of motion to state space is to define the modal coordinates and their derivatives as the states of the system.

$$x = \left\{ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \right\} = \left\{ \begin{array}{c} \eta_1 \\ \eta_2 \\ \eta_3 \\ \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \end{array} \right\} \tag{10}$$

For simplicity, let $\Phi^T C \Phi = \hat{C}$ and $\Phi^T Q = \hat{Q}$. Then by solving the modal equation of motion in (8) for the second derivative of the modal coordinates, the following state space realization can be defined.

$$A = \left[\begin{array}{cc} 0 & I_3 \\ -w_n^2 & \hat{C} \end{array} \right] \tag{11a}$$

$$B = \left\{ \begin{array}{c} 0 \\ \Phi^T \hat{Q} \end{array} \right\} \tag{11b}$$

$$C = \left[\begin{array}{cc} T(0)\Phi & 0 \\ T(L)\Phi\omega_n^2 & T(L)\Phi\hat{C} \end{array} \right] \tag{11c}$$

$$D = \left\{ \begin{array}{c} 0 \\ T(L)\Phi\hat{Q} \end{array} \right\} \tag{11d}$$

Where $T(x) = [\psi_1(x), \psi_2(x), \psi_3(x)]$ and cart position and tip acceleration are chosen as outputs.

A. Kalman Filter

Originally proposed by R.E. Kalman in 1960 [13], the discrete time Kalman filter has seen ubiquitous use in numerous fields. The Kalman filter is a recursive algorithm for the estimation of a dynamic system. Given a stochastic system model, the filter produces estimates that are optimal in the least squares sense [7]. It is this optimality in the presence of system noise that makes the Kalman filter a good choice for the first layer of our proposed estimation routine.

To implement the Kalman filter we must first define the stochastic system model (12a) and (12b).

$$\dot{x} = Ax + Bu + Gw \tag{12a}$$

$$y = Cx + Du + Hw + v \tag{12b}$$

Where w and v are the system process noise and measurement noise, G is a matrix relating the process noise to the system states, and H relates the process noise to the system outputs. For the discrete time Kalman filter formulation the system must be discretized.

$$x_{k+1} = A_d x_k + B_d u_k + G_d w_k \tag{13a}$$

$$y_k = C_d x_k + D_d u_k + H_d w_k + v_k \tag{13b}$$

We must also determine the process noise covariance matrix Q_p, measurement noise covariance matrix R and the cross covariance matrix N. Then the Kalman gain equation becomes (14).

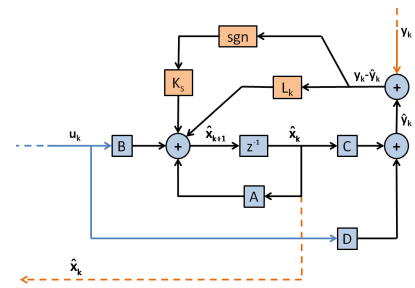


Fig. 3. State Estimator

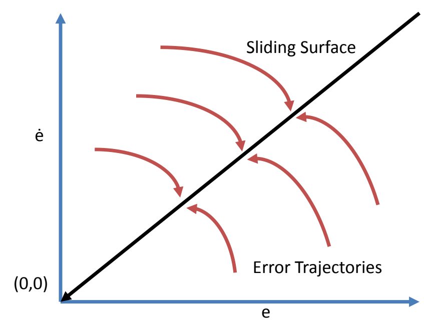


Fig. 4. Sliding Surface in Error Space

$$\begin{aligned} L_k = &\left[A_d P_k C_d^T + G_d Q_p H_d^T + G_d N\right] \\ &\left[C_d P_k C_d^T + H_d Q_p H_d^T + H_d N + N^T H_d^T + R\right]^{-1} \end{aligned} \tag{14}$$

Where P_k is the update covariance matrix and is defined in (15).

$$\begin{aligned} P_{k+1} = &\left[A_d P_k A_d^T + G_d Q_p G_d^T\right] \\ &- L_k \left[A_d P_k C_d^T + G_d Q_p H_d^T + G_d N\right]^T \end{aligned} \tag{15}$$

Thus it becomes a matter of measuring the covariance of the measurement signals and tuning the process noise covariance to determine the appropriate weights to give the model predictions and the noisy system measurements in the estimated state.

A significant drawback to the Kalman filter however, is its limited robustness. Thus for our application we seek a method to improve the robustness of the Kalman filter to unmodeled nonlinearities and parameter variation.

B. Sliding Mode Observer

To improve robustness we borrow a technique from variable structure systems theory and use a sliding mode term to compensate for bounded nonlinearities and variations in system parameters (Fig. 3.). The general function of this sliding term is to force the error trajectories of the compensated system to a sliding surface [4] [14] on which the error between estimated and true state is driven to zero according to the error dynamics of the Kalman filter (Fig. 4.). Our estimator then becomes (16) as described in [8].

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) + K_s\left[sgn(y - \hat{y})\right] \quad (16)$$

letting $e = \hat{x} - x$ the error dynamics become (17).

$$\dot{e} = (A - LC)e + K_s\left[sgn(y - \hat{y})\right] - \Delta Ax \quad (17)$$

Where ΔAx represents parametric variation due to modeling errors, simplifications in system dynamics, and nonlinearities in the plant.

It then becomes a matter of overpowering this parametric variation to achieve stable error dynamics. Walcott and Zak [8] suggest the following form of K_s to satisfy this requirement.

$$K_s = \rho P^{-1} C^T \quad (18)$$

Where P satisfies the Lyapunov equation (19)

$$(A - LC)P + P(A - LC)^T = -Q \quad (19)$$

We can show that the error dynamics are stable by choosing a Lyapunov candidate (20) and showing its derivative is negative definite. For a formal stability proof see [4], [8] or [6].

$$V = e^T P e \quad (20)$$

Defining $A_o = A - LC$, $\xi = (P^{-1}C^T)^{-1}(\Delta Ax)$ and substituting the relation $sgn(y - \hat{y}) = \frac{-C(\hat{x}-x)}{\|C(\hat{x}-x)\|}$ gives

$$\dot{V} = e^T(A_o^T P + PA_o)e - 2\rho\frac{e^T P(P^{-1}C^T Ce)}{\|Ce\|} - 2e^T C^T \xi \quad (21)$$

Assuming a worst case for the parameter variation by taking the euclidean norm of the last term in (21)

$$\dot{V} = -e^T Q e - 2\rho\|Ce\| + 2\|Ce\|\|\xi\| \quad (22)$$

Therefore to ensure stability of the estimate one must choose $\rho \geq \|\xi\|$ [15].

Assuming an adequately fast sampling rate, this criteria is sufficient to ensure that the estimate error is asymptotically stable. As the estimate error converges to the sliding surface, the sliding mode gain in (18) will switch signs infinitely fast. This effectively negates its impact on the overall system. In practice however this high gain switching algorithm leads to undesirable chatter at lower sampling rates.

C. Boundary Layer Sliding Mode Observer (BLSMO)

A boundary layer is proposed to minimize this chatter [8] [6]. This yields a linear transition region below the boundary layer instead of a sharp discontinuity. Thus with small errors in the state estimate, a proportionally small effort will be applied to force the state error to the sliding surface.

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) + S \quad (23)$$

where

$$S = \begin{cases} \rho P^{-1} C^T sgn(y - \hat{y}) & \|y - \hat{y}\| > \lambda \\ \rho P^{-1} C^T \frac{(y - \hat{y})}{\lambda} & \|y - \hat{y}\| \leq \lambda \end{cases} \quad (24)$$

Tuning of the SMO and BLSMO requires the selection of a positive definite Q, a scalar ρ and in the case of BLSMO a boundary layer thickness λ. While intuition and iteration were used in this study, several approaches have been described in [16] and [17].

IV. SIMULATION

We performed a simulation to evaluate the effectiveness of the proposed state estimation algorithms. A model of a single axis of a CAMotion three axis gantry style packaging robot was developed using the assumed mode modeling technique described in section II. and the observer structures outlined in section III. were implemented using the LabView 2009 Control Design and Simulation Module.

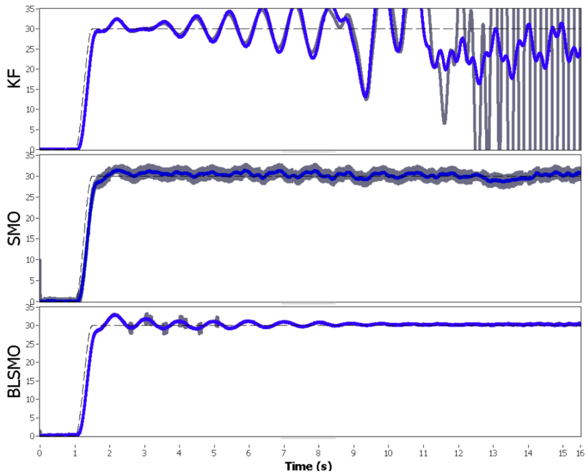
A linear quadratic regulator with penalties on cart position error, tip acceleration, and command effort was used as the controller. And for consistency all observers were evaluated using the same controller gains and trapezoidal velocity profiles.

Nonlinearities present in the physical system were implemented in the simulation, including saturation of the motor drives and dead-band. Likewise, noise characteristics from the system sensors, an angular encoder for cart position and a piezoelectric accelerometer for tip acceleration were measured and included. Possible sources of parameter variation including beam length, tip mass, cart mass, cart friction, and loss factor were identified and considered to examine the robustness of each estimator.

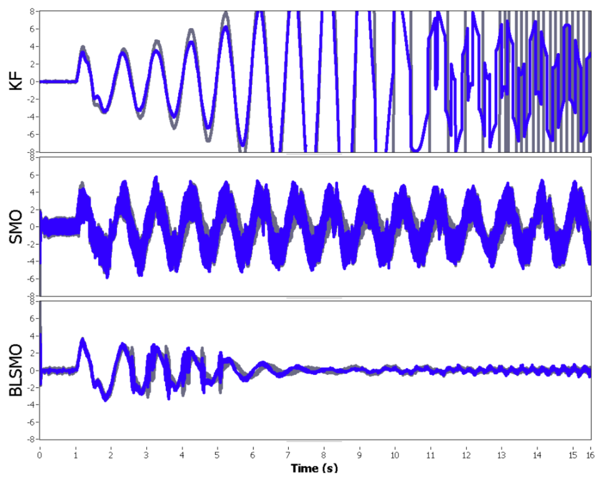
Fig. 5(a). and 5(b). represent a sample simulation result from a trapezoidal move of $30\,\text{cm}$ in $0.5\,\text{s}$ with a $+75\%$ variation in tip mass. Akin to the added mass of an object picked up by the gripper of the robot, the results illustrate the inherent robustness problem with the Kalman filter, as well as the benefits and behavior of the nonlinear observers and their effects on the controlled system. The actual output in each case is represented by the dark blue line and the observer estimates of the output by the lighter grey line.

It should be noted that with 0% parameter variation and ignoring the system nonlinearities, all three observer systems provide nearly identical performance. Aside from the observed chatter in the case of the SMO estimator, and the responses in each case remain stable. However, as the system nonlinearity and parameter variation increase, the result becomes immediately apparent.

It is clear from Fig. 5(a). that the controlled system is unstable when using the Kalman filter observer for the given amount of parameter variation and nonlinearity. This is due to the divergence of the estimated from the true system state. Using the sliding mode observer, the system remains stable but with significant residual tip vibration evident in Fig. 5(b). from the chattering effect. The boundary layer sliding mode observer, however, stabilizes the system and reduces residual tip vibration by decreasing the chatter present in the SMO



(a) Cart Position (cm)



(b) Tip Acceleration ($\frac{m}{s^2}$)

Fig. 5. 75% Tip Mass Variation

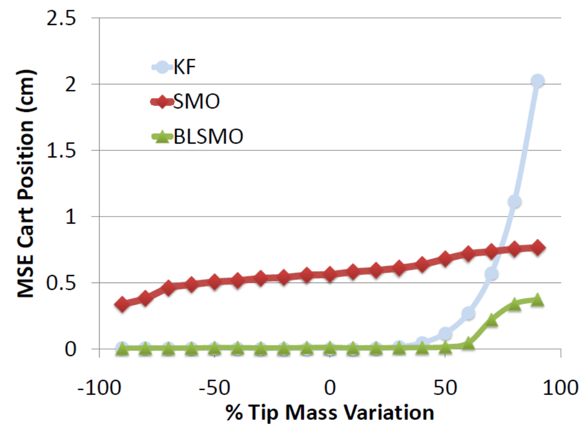


Fig. 6. Robustness Comparison

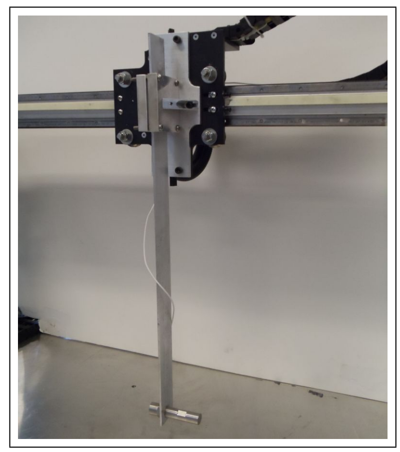


Fig. 7. Experimental Apparatus

system. Of particular interest are the portions of the response where the estimated and true outputs reach the edge of the boundary layer and the switching behavior dominates. This forces the errors back in as illustrated by small portions of the trajectory in Fig. 5(a). and 5(b).

A direct comparison of the robustness of the three observer types is shown in Fig. 6. Note that when the model is exact replica of the physical system, the Kalman filter provides the best estimates and the errors in the estimates of the sliding mode observer are significantly larger. As the amount of variation in the system model increases, the Kalman filter estimates worsen dramatically and both the sliding mode and boundary layer sliding mode observers offer substantially more robustness.

V. EXPERIMENTAL RESULTS

Preliminary experiments were carried out on a modified CAMotion packaging robot. An aluminum armature was affixed in place of the vertical axis as pictured in Fig. 7. to accentuate the flexibility of the system and bring the resonant natural frequencies of $8.13\,\mathrm{Hz}$ and $78.31\,\mathrm{Hz}$ into the controller bandwidth. Cart position was measured using a encoder mounted to the drive shaft of the belt drive API Turboservo AC servo motor, and tip acceleration was measured using a piezoelectric accelerometer from PCB Piezotronics Inc.

Identical control structures and plant models to those used in the simulation were used for the experiments. The control system was implemented using Labview Real-time on a real time quad-core pc and utilizing a NI X-Series DAQ card for data acquisition and command generation. Controller implementation again was executed in LabView CD-SIM at a sampling rate of $1\,\mathrm{kHz}$.

For uniformity, each observer design was evaluated using identical control gains and the same trapezoidal velocity profiles. Similarly each commanded move was carried out on identical portions of the track to standardize the nonlinear effects apparent in the system including belt stiffness and track friction.

Fig. 8. shows an example result from one trial using the three observer systems. The respective cart positions have been omitted for space considerations and because they elicit no new information. This is because we are ultimately

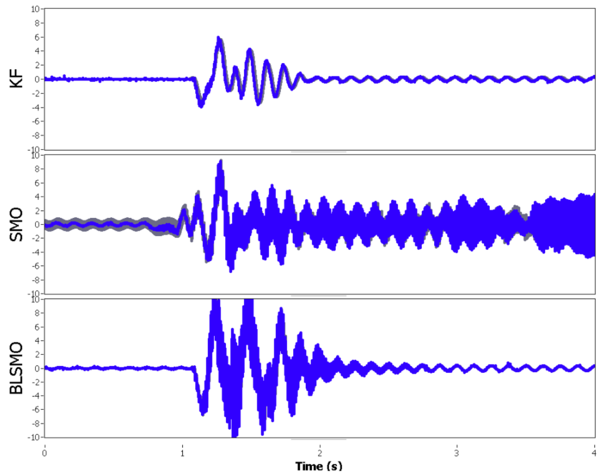


Fig. 8. Tip Acceleration $\mathrm{m/s^2}$

concerned with tip vibration reduction and each system tracks the desired trajectory equally well. As illustrated by the results, all three observer systems are stable. Note that both the Kalman filter and boundary layer sliding mode observers significantly reduce the residual tip acceleration.

Although on direct observation all three observer systems visibly reduce the magnitude of residual tip vibration relative to an uncompensated system, the sliding mode observer results in high residual acceleration of the beam tip. It is important to note that this residual vibration does not occur at the dominant natural frequency of the system at approximately 8 Hz. Instead, it occurs at a much higher frequency of approximately 71 Hz, indicating that the chattering effect is exciting the second system mode resulting in the extraneous residual acceleration. In fact, when operating with the SMO, an audible tone at a frequency of roughly 1 kHz is emitted from the servo drive.

VI. CONCLUSIONS AND FUTURE WORK

A. Conclusions

A single degree of freedom model for a flexible link manipulator was developed using the assumed modes lumped parameter method. A robust non-linear observer was discussed based on the boundary layer sliding mode principle, and built on top of a Kalman filter to provide optimal state estimates. A simulation was performed to test the robustness of the developed observer with respect to the Kalman filter and a sliding mode observer for a flexible motion system. Finally preliminary physical experiments were performed on a lightweight, high speed industrial robot to assess the behavior of the proposed observation strategy on a realistic system.

We find that the boundary layer sliding mode observer, built on the Kalman filter, offers a combination of both the noise filtering characteristics of the Kalman filter and the robustness of the sliding mode observer, while greatly reducing chatter.

B. Future Work

Experimental evaluation of the robustness of the observers is ongoing and extension, application, and evaluation of the algorithms to multiple degrees of freedom remains an important next step in the development of a general solution for the control of flexible motion systems. Feedback control of flexible systems is, at its best, imperfect because of the reliance on the existence of trajectory errors before any control effort is applied. Therefore, in order to be successful, any feedback control system must also be combined with a method for the generation of vibration reducing trajectories as well.

VII. ACKNOWLEDGEMENTS

This work would not have been possible without the prior work of Moshin Waqar, the generous equipment donations of National Instruments and CAMotion Inc. or the skill and expertise of James Huggins.

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