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Analysis of Thickness and Quality factor of a Double Paddle Oscillator at Room Temperature

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Abstract

In this paper, we evaluate the quality (Q) factor and the resonance frequency of a double paddle oscillator (DPO) with different thickness using analytical, computational and experimental methods. The study is carried out for the 2nd anti-symmetric resonance mode that provides extremely high experimental Q factors on the order of 10^5. The results show that both the Q factor and the resonance frequency of a DPO increase with the thickness at room temperature.

Keywords

double paddle oscillator; thermoelastic damping; simulation; quality factor; torsional resonance mode

I. Introduction

High quality (Q) factor micro-electro-mechanical systems (MEMS) based silicon resonators have been used for a wide variety of timing [1], inertial [2] and gravimetric sensing applications [3]. Single-crystal double-paddle oscillators (DPOs) have proved to be an invaluable tool to study the elastic properties of thin metal [4] and monolayer graphene films [5], characterize the thermodynamic properties of different atmospheric gases at cryogenic temperatures [6], detect magnetic forces [7] and investigate quantum effects [8]. Among different observable mechanical resonant modes of a DPO, the 2nd anti-symmetric (AS2) torsional mode exhibits an extremely high Q on the order of 10^5 at room temperature.

The ability to accurately model the Q and resonant modes of the oscillators is critical for design optimization. The quantitative modelling of Q and resonance frequency (f_o) of the flexure mode of a simple beam oscillator is widely studied [9, 10]. However, predictable quantitative and computational modelling of more complex geometries and resonant modes of oscillators like DPOs is limited. Thermoelastic dissipation (TED) is considered a

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dominant intrinsic energy loss mechanism in MEMS oscillators. The f_o of the AS2 mode of different DPOs is approximated by a formula (equation 3) [11]. We study a modified Zener approximation [9] model for TED to quantitatively compute the Q of DPOs with three different thicknesses. Finite element modelling (FEM) simulations are carried out using COMSOL Multiphysics and include the effect of crystalline anisotropy, to study TED of the AS2 mode. In the end, experimental results for Q factors and resonance frequencies of 300 μm, 400 μm and 500 μm thick DPOs are compared with both analytical and computational values. The results show that both f_o and Q increase with the thickness at room temperature. Moreover, both analytical and computational models underestimate the resonance frequencies and overestimate the Q values.

II. Experimental Section

A. Fabrication

The DPO shown in Fig. 1 is similar to the earlier reported design [6]. DPOs were produced from highly resistive (>10 k Ω cm), double side polished, float zone and <100> oriented single crystal silicon wafers with 300 μm, 400 μm and 500 μm thicknesses. DPOs were fabricated using well-established lithography and wet anisotropic etching using KOH [6]. One of the critical aspects of fabrication, to produce high quality oscillators, involves the alignment of the (110) wafer plane parallel to the axis of the DPO leg. Due to the preferential etching of different planes by KOH, one side of the DPO (top side) is smaller than the bottom side resulting in beveled edges along (111) planes [6]. The larger side of each DPO was then coated with T-shaped chromium (≈ 5 nm thick) and gold (≈ 50 nm thick) layers that act as electrodes.

B. Apparatus

The complete details of the experimental setup are explained elsewhere [12]. Briefly, DPOs were installed inside an ultra-high vacuum chamber with a typical base pressure of $\approx 6.5 \times 10^{-7}$ Pa. Each DPO is capacitively coupled to two round metal electrodes for actuation and detection purposes. After the identification of different eigenmodes between 0.2 kHz and 10 kHz, the DPO is excited in a self-tracking closed loop in the AS2 mode. The resonance frequency of DPO is continuously monitored using a high resolution frequency counter. The system is equipped with temperature sensors and a close-loop PID controller to control the temperature of the DPO.

III. Results and Discussion

A. Analytical Modelling

We used analytical expressions to predict the Q and the resonance frequency of the AS2 mode for different DPO thicknesses. In the absence of any fluidic damping (chamber pressure $\approx 6.5 \times 10^{-7}$ Pa) and assuming no anchor losses (a good approximation for the AS2 mode), TED is the dominant energy loss mechanism. A simplified analytical expression for a thin isotropic beam resonator operating in a flexure mode was derived by Zener [9]. This expression is inadequate to accurately predict the Q of complex micro structures like the DPO. Houston *et al.* [13] developed a simple model to predict the internal friction arising from TED for the torsional modes of DPO

$$\frac{1}{Q}=p_f\frac{E\alpha^2T}{C_v}\cdot\frac{\omega\tau}{1+(\omega\tau)^2} \quad (1)$$

Where p_f is the flexure mode participation factor (for the AS2 mode $p_f=0.0755$ [13]), E is the Young's modulus, α is the isotropic thermal expansion coefficient, T is the temperature, C_v is the specific heat per unit volume, $\omega$is the angular resonant frequency and τ is the thermal relaxation time given by

$$\tau=\frac{C_vt_s^2}{\pi^2K} \quad (2)$$

Where t_s is the thickness of oscillator and K is the thermal conductivity of silicon.

The resonant frequency of the AS2 mode for each thickness is approximated by the following expression [11]

$$f_{AS2}=\frac{1}{\pi}\cdot\sqrt{3}\cdot\sqrt{\frac{1}{c^3d}\cdot Gt_s^2\cdot\frac{\beta}{\rho e}f} \quad (3)$$

Where c, d, e and f are different dimensions of the DPO (listed in Fig. 1), G is the shear modulus of high purity silicon along the (100) orientation at room temperature, $t_s$is the thickness and β is a unitless parameter equal to 0.25.

Using equations 1, 2 and 3 along with the room temperature material properties of single crystal silicon (Table I [14]), we calculated both the resonance frequency and the Q factors for three different thicknesses of DPO (Table II).

B. Computational Modelling

COMSOL Multiphysics v 5.2 is used to model the free vibration motions of the DPO. Model geometries are generated using Autodesk Inventor and imported into the COMSOL environment. Subsequently each geometry is rotated by a 45° angle to transform the imported model plane axis to the silicon crystal coordinate system. The transformation ensures that the stress axis of the model is properly aligned with reference to the fabricated DPO. The *thermoelasticity* (TE) module built into COMSOL Multiphysics is used for both modal shape and temperature deviation analysis. The built-in equations in the TE module couple the heat transfer model to the solid mechanics model. All the boundaries of the DPO are assumed to be thermally insulated. Similarly, all the boundaries are assumed mechanically free except for the three boundaries at the base of the DPO (Fig. 1). Moreover, the plane perpendicular (along the thickness) to boundary 2 (Fig. 1) is defined as a zero-temperature-deviation plane. A free triangular mesh with maximum element size of 0.5 mm (side length of 28.58 mm) and minimum element size of 6 μm is used to discretize the DPO with 7 elements through its thickness. The material properties of single crystal silicon listed in Table I are used for the FEM analysis. To account for the material anisotropy of silicon, an elasticity matrix with the elastic constants c_{11} = 165.6 GPa, c_{12} = 63.9 GPa and c_{44} = 79.5 GPa [15] is used rather than the Young's Modulus and the Shear Modulus.

The TE module solves for the resonant modes, temperature deviations and the corresponding Q factors. We solved for the first 10 resonant modes and the corresponding Q factors. Here we show only the modal shape of the low loss AS2 mode of a 400 μm DPO along with the surface plot of the temperature deviation (Fig.2). The computational modelling for the 300 μm and 500 μm thick DPOs was carried out with similar settings.

C. Experimental Results

In order to measure the Q, each DPO is first electrostatically excited by an external AC signal superimposed onto a 200 V DC signal at the AS2 resonance frequency (f_{AS2}). Subsequently, each DPO is put in a self-tracking closed loop and the frequency of each DPO is measured using repetitive measurements for a minimum of 1 hour. The quality factor is a measure of the time rate of decay (τ) of oscillation after the excitation voltage is turned off. The Q is given by the following relation

$$Q=\pi \cdot \tau \cdot f_{AS2} \quad (4)$$

The time decay is determined from a single ring-down measurement using an exponential decay function (Fig. 3). Table II shows the complete results of Q factors and resonant frequencies from analytical modelling, FEM modelling and experiments. The underestimation of the frequencies for FEM models is primarily due to the fact that the shape of the fabricated paddles is different due to convex corner under-cutting. The undercutting is more pronounced for the thicker DPOs. Table II clearly shows better agreement between the modelling and experiment for 300 μm thick DPO compared to the other thicknesses. Overall the trend shows that both resonant frequency and Q values increase with thickness at room temperature.

IV. Conclusion

In this paper, we study analytical and computational modelling techniques to evaluate the AS2 mode resonance frequencies and the corresponding Q factors of a DPO with three different thicknesses. The results are compared with experimental values. The FEM modelling could be further improved to include the effect of convex corner undercutting. Similarly, the effect of modal participation factor should be studied in detail to predict the analytical Q values more accurately.

Acknowledgments

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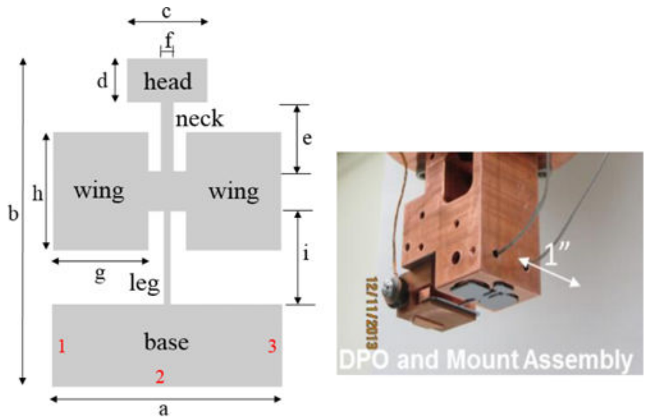

Name	Dimension	(mm)
Total Width	a	20.37
Total Height	b	28.58
Head Width	c	6.85
Head Height	d	3.65
Neck Length	e	1.10
Neck Width	f	5.95
Wing Width	g	8.11
Wing Height	h	10.06
Leg Length	i	7.98

Fig. 1.

Critical dimensions of the double paddle oscillator (table). 1, 2, and 3 are fixed boundaries in the FEM analysis (top left image). A 400 μm DPO mounted on a copper pedestal is shown (top right image).

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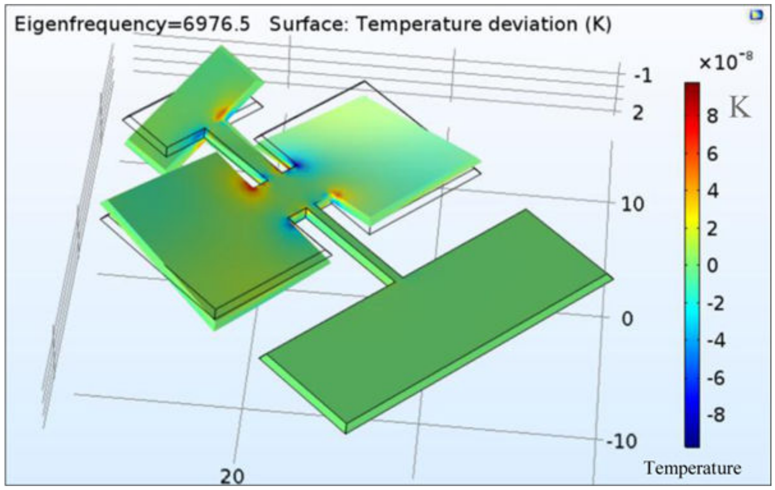

Fig. 2.

FEM simulation of the temperature deviation for the AS2 resonant mode of a 400 μm thick DPO at 298 K. Spatial dimensions of the DPO are listed in Fig. 1

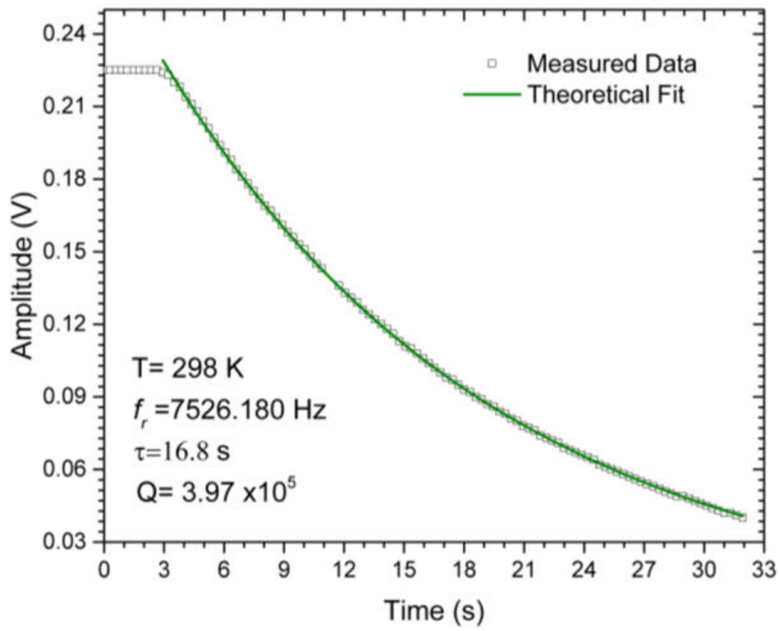

Fig. 3.

Ring down measurement of a 400 μm thick DPO at room temperature and $\approx 6.5 \times 10^{-7}$ Pa chamber pressure.

Table I

Room Temperature Material Properties of A High Purity Silicon [14]

Constant	**Value**
Density (ρ)	2330 kg/m^3
Thermal Expansion Coefficient (α)	2.55 ×10^{-6} 1/K
Thermal Conductivity (κ)	159 W/(m•K)
Specific Heat (C)	711.7 J/(kg•K)
Young's Modulus (E)	169 GPa
Shear Modulus (G)	61.7 GPa

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Table II

Comparison of analytical, FEM and Experimental Values of AS2 Frequency and Q Of DPOs With Different Thicknesses (Uncertainty in the experimental values is less than 1×10^{-7} for the resonance frequency and less than 0.2 % for the Q factor.)

DPO Thickness (μm)	Calculated Frequency (Hz)	FEM Frequency (Hz)	Experimental Frequency (Hz)	Modified Zener Q	FEM Quality Factor	Experimental Quality Factor
300	5349	5307	5568	2.53×10^5	2.25×10^5	1.91×10^5
400	7132	6976	7526	5.57×10^5	4.29×10^5	3.97×10^5
500	8915	8611	9385	1.06×10^6	7.09×10^5	5.71×10^5