Cooperative Localization of an UAV Fleet using Distributed MHE with EKF Pre-estimation and Nonlinear Measurements

Matthieu Borelle, Sylvain Bertrand, Cristina Stoica Maniu, Teodoro Alamo, Eduardo Fernandez Camacho

Cooperative Localization of an UAV Fleet using Distributed MHE with EKF Pre-estimation and Nonlinear Measurements

Matthieu Borelle, Sylvain Bertrand, Cristina Stoica, Teodoro Alamo, Eduardo F. Camacho

***Abstract*—This paper proposes a Distributed Moving Horizon Estimation (DMHE) with an Extended Kalman Filter (EKF)-based pre-estimation to solve the constrained cooperative localization problem for a Multi-Agent System (MAS) using nonlinear measurements. The proposed DMHE strategy uses a fused arrival cost obtained by a consensus among neighbors to efficiently spread the relevant estimation information across the communication network. The EKF pre-estimation enables to reduce the number of optimization variables and, thus, the computation time of the constrained nonlinear optimization problem over the horizon length, while preserving the accuracy of the estimation. A simulation case study of cooperative localization of a fleet of Unmanned Aerial Vehicles (UAVs) is proposed. Comparison with existing distributed estimation methods is carried out to confirm the effectiveness of the proposed DMHE algorithm in terms of estimation accuracy, computation time, and constraints handling.**

I. Introduction

Distributed State Estimation (DSE) is a fundamental problem in numerous engineering applications on multi-robot systems communicating via a wireless Sensor Network. In the context of civil or military applications for multi-vehicle localization [1] or tracking [2], the state of a dynamic Multi-Agent System (MAS) is observed by a network of sensors (external or embedded on the vehicles), each with limited sensing and communication capabilities. The goal of distributed state estimation is to estimate the state (e.g., position, velocity) of the MAS accurately and efficiently, despite the limitations of individual sensors and communication constraints.

Compared to a centralized state estimation scheme where a central unit processes data and shares its state estimation with the entire MAS, DSE has numerous benefits, in particular in the context of multi-vehicle localization. Indeed, it provides increased autonomy, fault-tolerance, scalability, computational efficiency, while it needs a smaller communication range (i.e. using only local information from neighbors) with respect to centralized approaches. The main challenge is to design distributed estimation algorithms that preserve as much as possible the stability and performance characteristics of their centralized equivalent. Several strategies such as distributed/decentralized Kalman filters and Extended Kalman Filter (EKF) techniques [3], [4], have been studied for DSE applied to multi-robot localization [5]. These approaches rely on probabilistic assumptions on disturbances and noises to minimize the error variance of the state estimate. In addition, a consensus step (e.g., consensus on information [6], on measurements [7], [8]) is usually considered to efficiently fuse the sensors information from different sensors[1].

However, methods based on (Extended) Kalman Filter can become suboptimal in presence of strong nonlinearities or even unstable in case of large initial estimation error. Moreover, they cannot deal with constraints (e.g., minimum and maximum altitude of drone, maximum speed of a robot). This encourages the development of Moving Horizon Estimation (MHE) approaches for linear [9] and nonlinear systems [10], as this method can account for constraints and nonlinearities in its formulation. Previous work already proposed distributed moving horizon observers using neighborhood measurements, with consensus steps on the a priori state at the beginning of the horizon [11] and on the arrival cost [12], [13]. In [12], the authors proposed a Distributed Moving Horizon Estimation (DMHE) algorithm with multiple-step consensus to ensure stability of the estimation error dynamics in all the nodes of the Sensor Network, under the assumptions of network connectivity and collective observability. The consensus steps enable to spread the information through the network in order to provide each agent the necessary information to estimate parts of the MAS state (which could be locally or even regionally non observable) from other agents [14]. To reduce the computation time and, thus, to empower a real-time implementation on low-cost processors, the work in [15] proposed a DMHE with a ”pre-estimation” strategy based on a Luenberger observer. However, this estimation method has only been designed for systems with linear dynamics and linear measurements. The increasingly recurrent use of low-cost embedded sensors (e.g., lidar, ultra-wideband mounted on mobile robots) that provide nonlinear measurements (e.g., angle and/or distance) justifies the interest of developing DMHE algorithms which can handle nonlinear measurements.

In this context, the main contribution of the paper consists of extending the estimation technique of [15] in order to handle nonlinear measurements. Therefore, the Luenberger observer used in [15] for linear prediction over the estimation window is replaced here by a nonlinear counterpart using an Extended Kalman Filter. Furthermore, an observability rank-based technique (see [16]) permitting to efficiently fuse the information from neighboring nodes is also extended to the

M. Borelle and C. Stoica are with Université Paris-Saclay, CNRS, CentraleSupélec, Laboratoire des signaux et systèmes, 91190, Gif-sur-Yvette, France (e-mail: {matthieu.borelle; cristina.stoica}@l2s.centralesupelec.fr).

M. Borelle and S. Bertrand are with Université Paris-Saclay, ONERA, Traitement de l'information et systèmes, 91123, Palaiseau, France (e-mail: {matthieu.borelle; sylvain.bertrand}@onera.fr).

T. Alamo and E.F. Camacho are with Department of Ingeniería de Sistemas y Automática, Universidad de Sevilla, 41092 Sevilla, Spain (e-mail: talamo@us.es, eduardo@esi.us.es).

[1] Heterogeneous sensors (e.g., camera, radar, lidar) can be considered.

nonlinear case in order to define the weights used in the consensus step. Another contribution of this paper consists of applying the developed DMHE technique on the constrained cooperative localization problem of a fleet of Unmanned Aerial Vehicles (UAVs). Nonlinear measurements including the relative distance between the UAVs neighbors are further considered.

II. Distributed State Estimation for Cooperative Localization

This section describes the problem of Distributed State Estimation (DSE) of a Multi-Agent System (MAS) for cooperative localization.

A. Problem description

The main problem addressed in this paper is the constrained cooperative distributed self-localization of a fleet of n_a drones. Thus, each drone performs the following operations: 1. Communicates its own measurements (provided by its own embedded sensors) to its neighbors; 2. Receives information sent through communication links[2] by its one-step neighbors (measurements from embedded sensors of each neighbor and prior estimation of the state of the fleet sent by each neighbor); 3. Computes an estimate of the entire MAS state.

B. Considered model

The dynamics of each UAV i of the MAS are described by the discrete-time Linear Time-Invariant (LTI) model:

$$x_{t+1}^i = A^i x_t^i + B^i u_t^i + B^i w_t^i, \quad i \in \{1, \ldots, n_a\} \tag{1}$$

with the state vector x^i, the input vector u^i, the input perturbation vector w^i, the evolution matrix A^i and the input matrix B^i of corresponding dimensions.

Then the dynamics of the global MAS can be defined by:

$$\boldsymbol{x}_{t+1} = \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{B}\boldsymbol{u}_t + \boldsymbol{B}\boldsymbol{w}_t \in \mathbb{R}^{n_x} \tag{2}$$

with the collective (global) state $\boldsymbol{x} = \mathrm{col}(x^1, x^2, \ldots, x^{n_a}) = [(x^1)^\top, (x^2)^\top, \ldots, (x^{n_a})^\top]^\top$, the collective input $\boldsymbol{u} = \mathrm{col}(u^1, \ldots, u^{n_a})$, the collective perturbation $\boldsymbol{w} = \mathrm{col}(w^1, \ldots, w^{n_a})$, and the collective block-diagonal matrices $\boldsymbol{A} = \mathrm{diag}(A^1, \ldots, A^{n_a})$ and $\boldsymbol{B} = \mathrm{diag}(B^1, \ldots, B^{n_a})$.

Remark 1: Each agent i is assumed to have access to a noisy measurement of its own input vector only $u_t^i + w_t^i$ (e.g., acceleration measurement from its IMU for the drone case). Inputs of other agents $j \neq i$ are assumed to be unavailable to agent i. Notice that they could be transmitted by neighbouring agents, but this would require exchanging much more information (that could to be difficult, e.g., considering high frequency data such as measured accelerations). Therefore, it has been chosen to consider the inputs of other agents as unknown inputs by agent i and to denote by:

$$\hat{\boldsymbol{u}}^i = \mathrm{col}(0, \ldots, u^i + w^i, \ldots, 0) \in \mathbb{R}^{n_u} \tag{3}$$

the knowledge of agent i on the collective input of the MAS.

Measurements are locally performed by each agent i:

$$y_t^i = h^i(\boldsymbol{x}_t) + \nu_t^i, \quad i \in \{1, \ldots, n_a\} \tag{4}$$

with the output vector y^i and the measurement perturbation $\nu^i \in \mathcal{V}^i$. A nonlinear dependence on the MAS system state is considered via h^i. The nonlinear time-invariant measurement function h^i can be different for each agent $i \in \{1, \ldots, n_a\}$, depending on the type of onboard sensors or on the proximity with other agents (enabling proximity distance measurements).

C. Communication network topology

In the considered Distributed State Estimation scheme, only nearby agents can share data among each other. Hence, the communication network can be described by an undirected connected graph $\mathcal{G} = (\mathcal{N}_a, \mathcal{E})$, where $\mathcal{N}_a = \{1, 2, \ldots, n_a\}$ is the set of all nodes (agents) and $\mathcal{E} \subseteq \mathcal{N}_a \times \mathcal{N}_a$ is the set of all edges (communication links between agents). Therefore, the pair $(i, j) \in \mathcal{E}$ exists if and only if the agent j can receive information from the agent i.

Remark 2: This paper assumes a time-invariant communication network and no communication failure.

The notions of local, regional and global information are further described. Indeed, *local* information refers only to the local agent i (e.g., x^i indicates its local state vector), the *regional* information refers to the one step neighborhood of agent i denoted by $\mathcal{N}_a^i$ (e.g., $\bar{y}^i$ indicates the regional measurement of agent i), while the *global* information considers the entire MAS (e.g., $\boldsymbol{x}$ as defined in (2)).

D. Distributed state estimation approach

At each sample time t, each agent $i \in \mathcal{N}_a$ gets measurements from its neighbors (nodes in $\mathcal{N}_a^i$) and from its own sensors, then it exchanges information with its neighbors to conduct a consensus on a priori state, and finally, it locally solves a constrained optimization problem to determine a local estimate $\hat{\boldsymbol{x}}_t^i$ of the real global state $\boldsymbol{x}_t$ of the MAS.

Notice that the UAVs do not communicate their input vector with each other, as indicated in Remark 1.

III. DMHE for cooperative localization

This section formulates the Distributed Moving Horizon Estimation approach with EKF pre-estimation and observability rank-based weights to address the cooperative localization of a MAS with nonlinear measurements.

A. Local optimization problem

At each sample time t, given an estimation horizon length $N \geqslant 1$, each agent $i \in \mathcal{N}_a$ determines (based on the information received from its neighbors in $\mathcal{N}_a^i$) its estimate

[2]The communication range is supposed to be limited, and thus only one-step (also known as "one-hop") neighbors communication is allowed.

$\hat{\boldsymbol{x}}^i_{t-N|t}$ of the global state $\boldsymbol{x}_{t-N}$ of the MAS by solving the following constrained optimization problem:

$$\hat{\boldsymbol{x}}^i_{t-N|t} = \arg\min_{\hat{\boldsymbol{x}}^i_{t-N}} J^i_N(\cdot) \tag{5}$$

$$\text{s.t. } \hat{\boldsymbol{x}}^i_{k+1} = \hat{\boldsymbol{x}}^i_{k+1|k} + K^i_k \left(\bar{y}^i_{k+1} - \bar{h}^i(\hat{\boldsymbol{x}}^i_{k+1|k})\right), \tag{6}$$

$$\hat{\boldsymbol{x}}^i_{k+1|k} = \boldsymbol{A}\,\hat{\boldsymbol{x}}^i_k + \boldsymbol{B}\,\hat{\boldsymbol{u}}^i_k \tag{7}$$

$$\hat{\boldsymbol{x}}^i_k \in \mathcal{X}, \quad \bar{y}^i_{k+1} - \bar{h}^i(\hat{\boldsymbol{x}}^i_{k+1|k}) \in \bar{\mathcal{V}}^i, \tag{8}$$

$$\forall k = t-N, \ldots, t-1.$$

where the matrix gain K^i_k is computed as the Kalman gain of an EKF, as detailed in Section III-C.

The $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices in (6) refer to the global Multi-Agent System dynamics (2). The sequence of state estimates $\hat{\boldsymbol{x}}^i_{t-N+1|t}, \ldots, \hat{\boldsymbol{x}}^i_{t|t}$ is obtained from the optimal solution $\hat{\boldsymbol{x}}^i_{t-N|t}$, using the propagation by the dynamical equation (6).

Compared to the classical DMHE of [11], [12], the proposed DMHE with EKF pre-estimation (5) decreases the number of decision variables leading to a reduced computation time [17]. Note that the state and measurement noise constraints (8) are incorporated within the optimization problem.

The objective function in (5) is defined as:

$$J^i_N(\cdot) = \sum_{k=t-N}^{t} \left\| \bar{y}^i_k - \bar{h}^i(\hat{\boldsymbol{x}}^i_k) \right\|^2_{(\bar{R}^i)^{-1}} + \Gamma^i_t(\cdot). \tag{9}$$

In (9), the first term represents the sum over the horizon length N of a weighted square norm of the error between the regional measurements $\bar{y}^i_k$ and the regional predicted measurements $\bar{h}^i(\hat{\boldsymbol{x}}^i_k)$, obtained from the sequence of estimated state. The definite positive matrix $\bar{R}^i$ gathers the weights that can be related to regional measurement noises covariances, if available.

The second term $\Gamma^i_t(\cdot)$ is called the *initial penalty function* (related to the *arrival cost*[3]). Its update is crucial to ensure the stability of the estimation error dynamics. The initial penalty function defined by:

$$\Gamma^i_t(.) = \|\hat{\boldsymbol{x}}^i_{t-N} - \bar{\boldsymbol{x}}^i_{t-N}\|^2_{(\tilde{\Pi}^i_{t-N})^{-1}} \tag{10}$$

involves two weighted consensus terms (arrival-cost consensus as in [12]) detailed in Section III-B: $\tilde{\Pi}^i_{t-N}$ (definite positive weight matrix) and $\bar{\boldsymbol{x}}^i_{t-N}$ (initial state of the receding horizon window, averaged over the neighborhood). To obtain the weighted average state estimate $\bar{\boldsymbol{x}}^i_{t-N}$, a consensus on information is set up in section III-B. To make this consensus state more relevant in the case of regional non observability, the information must be broadcast among the MAS in a broader way. For this purpose and considering communication range limits, the possibility to carry out L loops of this consensus among neighborhood is retained, corresponding to the L-step consensus approach in [12]. At the next instant $t+1$, for the formulation of the new optimization problem, $\hat{\boldsymbol{x}}^i_{t-N+1}$ is assigned from the previously computed solution $\hat{\boldsymbol{x}}^i_{t-N+1|t}$.

[3]The non negative term $\Gamma^i_t(.)$ is used to summarize the effect of the past measurements, before time $t-N$.

B. L-step information consensus

This subsection details the process of the L-step information consensus [12]. First, the information matrix P^i_{t-N} and the information vector ξ^i_{t-N} are initialized at the beginning of the receding horizon window (i.e., at time $t-N$):

$$P^i_{t-N,0} = (\Pi^i_{t-N})^{-1} \tag{11}$$

$$\xi^i_{t-N,0} = P^i_{t-N,0}\bar{\boldsymbol{x}}^i_{t-N} \tag{12}$$

with Π^i_{t-N} the covariance of the estimation error of the initial state of the receding horizon (see [11]). Each step $l \in \{0, \ldots, L-1\}$ of the information consensus consists in computing:

$$P^i_{t-N,l+1} = k_{i,i}P^i_{t-N,l} + \sum_{j\in\mathcal{N}^i_a} k_{i,j}P^j_{t-N,l} \tag{13}$$

$$\xi^i_{t-N,l+1} = k_{i,i}\xi^i_{t-N,l} + \sum_{j\in\mathcal{N}^i_a} k_{i,j}\xi^j_{t-N,l} \tag{14}$$

with the consensus weights $k_{i,j}$ defined in Section III-D.

After L-steps of consensus, the following quantities:

$$\tilde{\Pi}^i_{t-N} = (P^i_{t-N,L})^{-1} \tag{15}$$

$$\bar{\boldsymbol{x}}^i_{t-N} = (P^i_{t-N,L})^{-1}\xi^i_{t-N,L} \tag{16}$$

are finally used to define the arrival cost (10).

Remark 3: Notice that $\tilde{\Pi}^i_{t-N}$ corresponds to the weight matrix Π^i_{t-N} after proceeding L-step information consensus.

The consensus phase is necessary to guarantee convergence of the state estimates to the state of the observed system even in lack of regional observability [11].

The positive definite matrix Π^i_{t-N+1} is then obtained from the matrix $\tilde{\Pi}^i_{t-N}$ using the discrete-time Riccati equation associated to an Extended Kalman filter (as in [10] for the centralized case):

$$\Pi^i_{t-N+1|t-N} = \boldsymbol{A}\tilde{\Pi}^i_{t-N}\boldsymbol{A}^\top + \boldsymbol{B}\boldsymbol{Q}^i\boldsymbol{B}^\top \tag{17}$$

$$\begin{aligned}\Pi^i_{t-N+1} = \Pi^i_{t-N+1|t-N} - \Pi^i_{t-N+1|t-N}(\bar{C}^i_{t-N+1})^\top \cdot \\ \left(\bar{C}^i_{t-N+1}\Pi^i_{t-N+1|t-N}(\bar{C}^i_{t-N+1})^\top + \bar{R}^i\right)^{-1} \cdot \\ \bar{C}^i_{t-N+1}\Pi^i_{t-N+1|t-N}\end{aligned} \tag{18}$$

with $\boldsymbol{Q}^i = \operatorname{diag}(Q^1, \ldots, Q^{n_a})$ and Q^i the covariance of the input noise vector w^i of agent $i \in \{1, \ldots, n_a\}$. The term $\bar{C}^i_{t-N+1}$ is obtained via the linearization of the regional measurement function around $\hat{\boldsymbol{x}}^i_{t-N+1}$:

$$\bar{C}^i_{t-N+1} = \left.\frac{\partial\,\bar{h}^i}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}^i_{t-N+1}} \tag{19}$$

C. EKF pre-estimation observer

For $k \in \{t-N, \ldots, t\}$, the matrix gain K^i_k is computed using an EKF observer update. The step $t-N$ consists of initializing the pre-estimation error covariance matrix $\Pi^i_{pre,t-N|t-N} = \tilde{\Pi}^i_{t-N}$. Then, for $k \in \{t-N+1, \ldots, t\}$, the prediction of the covariance evolution is performed as follows:

$$\Pi^i_{pre,k|k-1} = \boldsymbol{A}\Pi^i_{pre,k-1|k-1}\boldsymbol{A}^\top + \boldsymbol{B}\boldsymbol{Q}^i\boldsymbol{B}^\top \tag{20}$$

avec $\Pi^i_{pre,k|k-1}$ the a priori estimation matrix of covariance of the estimation error at time k.

The optimal Kalman gain (6) is computed as follows:

$$K^i_k = \Pi^i_{pre,k|k-1}(\bar{C}^i_k)^\top (S^i_k)^{-1} \tag{21}$$

with the pre-estimation error covariance matrix:

$$\Pi^i_{pre,k|k} = (I_{n_x} - K^i_k\,\bar{C}^i_k)\Pi^i_{pre,k|k-1} \tag{22}$$

and S^i_k the innovation covariance:

$$S^i_k = \bar{C}^i_k \Pi^i_{pre,k|k-1}(\bar{C}^i_k)^\top + \bar{R}^i \tag{23}$$

D. Observability rank-based weights technique

The weights $k_{i,j}$ used in the consensus steps (14) and (13) can be defined as the components of a stochastic matrix K associated to the graph $\mathcal{G}$. The weights tuning technique of [1] and [17] is extended here to the nonlinear cas by making use of the linearization (19). This method relies only on *regional* information available by each agent to compute its own component $k_{i,j}$ of K. Thus, it is suitable for a distributed scheme. The interest of this approach is that it enhances the accuracy of the estimates by means of exploiting observability properties of the neighborhoods.

Consider an agent i. Its regional observability matrix over the discrete-time window $[t-N,t]$ given by:

$$\bar{\mathcal{O}}^i_{N,t} = \begin{bmatrix}(\bar{C}^i_{t-N})^\top & (\bar{C}^i_{t-N+1}\boldsymbol{A})^\top & \cdots & (\bar{C}^i_t \boldsymbol{A}^N)^\top\end{bmatrix}^\top \tag{24}$$

is of full rank if and only if the pair $(\boldsymbol{A}, \bar{C}^i_k)$ is completely observable for any $k \in \{t-N,\dots,t\}$, i.e. $\text{rank}(\bar{\mathcal{O}}^i_{N,t}) = n_x$. For the sake of simplicity, the following notion is further used:

$$\rho^i_{\mathcal{O}} = \text{rank}(\bar{\mathcal{O}}^i_{N,t}). \tag{25}$$

This information could be used as the reliability of sensor i when choosing the weights $k_{i,j}$, which must be averaged among the neighbors $j \in \mathcal{N}^i_a$:

$$k_{i,j} = \frac{\rho^j_{\mathcal{O}}}{\sum_{j\in\mathcal{N}^i_a}\rho^j_{\mathcal{O}}}. \tag{26}$$

E. Proposed DMHE with EKF pre-estimation algorithm

The procedure of the proposed DMHE scheme with EKF pre-estimation is described in Algorithm 1.

In steps 6, new local measurements, together with the knowledge on the collective input are collected (e.g., acceleration measurements). Then, in step 7, the local measurements y^i_t are shared with neighbors $j \in \mathcal{N}^i_a$. Steps 8 to 11 are related to the implementation of the receding horizon strategy. As the regional measurement matrix $\bar{C}^i_t$ and thus the observability properties can change over time, it is necessary to update the observability-rank based weight $k_{i,j}$ accordingly (steps 12 to 14). At step 15, the L-step information consensus is conducted. At step 16, the local DMHE with pre-estimation optimization problem is solved. From step 18 to 20, the horizon window is still increasing in size thus not sliding, so the a priori state $\bar{x}^i_0$ is set to the newest state estimated at time $t=0$ and the weight matrix Π^i_0 is set to the L-step consensus weight matrix $\tilde{\Pi}^i_0$ determined at step 15. From step 21 to 23, to take into account the receding horizon window at time $t+1$, the weight matrix Π^i_{t+1-N} and the a priori state $\bar{x}^i_{t+1-N}$ are adequately updated.

Algorithm 1 DMHE with pre-estimation procedure

1: **Initialization:** $\forall i \in \mathcal{N}_a$, at the first time step $t=0$
2: **initialize** Π^i_0, $\hat{\boldsymbol{x}}^i_0$
3: **collect** a first local measurement y^i_0 and the knowledge on the initial collective input $\hat{\boldsymbol{u}}^i_0$
4: **receive** from the neighborhood $j \in \mathcal{N}^i_a$ their measurements y^j_0
5: **Online:** $\forall i \in \mathcal{N}_a$, $\forall t > 0$
6: **collect** the local measurement y^i_t and the knowledge on the collective input $\hat{\boldsymbol{u}}^i_t$ using (4) and (3)
7: **receive** from the neighbors $j \in \mathcal{N}^i_a$ the collected measurements in the step 6, form and store $\bar{y}^i_t$
8: **if** $1 \leqslant t \leqslant N$ **then**
9: **set** the horizon length $N_w = t$
10: **else**
11: **set** the horizon length $N_w = N$
12: **compute** $\mathcal{O}^i_{N_w,t}$ and $\rho^i_{\mathcal{O}}$ according to (24) and (25)
13: **exchange** $\rho^j_{\mathcal{O}}$ with $j \in \mathcal{N}^i_a$
14: **compute** the k_{ij} components according to (26)
15: **perform** L steps of the consensus algorithms (14)-(13) with the initialization (11)-(12) to get $\tilde{\Pi}^i_{t-N_w}$ and $\bar{\boldsymbol{x}}^i_{t-N_w}$ (15)-(16)
16: **solve** the local optimization problem of DMHE with EKF pre-estimation, minimizing $J^i_{N_w}$ as in (9) and (10) subject to the constraints (6)-(8)
17: **store** the solution $\hat{\boldsymbol{x}}^i_{t-N_w|t}$, $\hat{\boldsymbol{x}}^i_{t-N_w+1|t}$ and the corresponding estimate $\hat{\boldsymbol{x}}^i_{t|t}$
18: **if** $1 \leqslant t \leqslant N$ **then**
19: **set** $\bar{\boldsymbol{x}}^i_{0|t+1} = \hat{\boldsymbol{x}}^i_{0|t}$
20: **set** $\Pi^i_{0|t+1} = \tilde{\Pi}^i_{0|t}$
21: **else**
22: **compute** Π^i_{t+1-N} according to (17)-(18)
23: **compute** prediction $\bar{\boldsymbol{x}}^i_{t+1-N} = \hat{\boldsymbol{x}}^i_{t+1-N|t}$

IV. Simulation Results

A. Agents dynamics and measurements

Inspired from [18], a MAS composed of n_a UAVs is considered. The state vector of each UAV $i \in \{1,\dots,n_a\}$ is composed of its 3D position and velocity components in a common inertial reference frame $x^i = \text{col}\left(p^i_x, p^i_y, p^i_z, v^i_x, v^i_y, v^i_z\right)$. The translational dynamics of each UAV i is considered to be modelled as in (1) with:

$$A^i = \begin{bmatrix}1&0&0&\Delta t&0&0\\0&1&0&0&\Delta t&0\\0&0&1&0&0&\Delta t\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}, B^i = \begin{bmatrix}\frac{\Delta t^2}{2}&0&0\\0&\frac{\Delta t^2}{2}&0\\0&0&\frac{\Delta t^2}{2}\\\Delta t&0&0\\0&\Delta t&0\\0&0&\Delta t\end{bmatrix} \tag{27}$$

with the sampling period Δt. The input vector corresponds to the acceleration of the drone $u^i = \left[a_x^i \; a_y^i \; a_z^i\right]^\top$. Each UAV i has access to the acceleration measurement $u^i + w^i$ deduced from raw data provided by its IMU[4]. Three different types of measurements (linear and nonlinear) are considered. Each UAV i has access to measurements of its own position and speed norm as provided by a GPS:

$$\begin{bmatrix} p^i_{GPS} \\ v^i_{GPS} \end{bmatrix} = \begin{bmatrix} p_x^i \\ p_y^i \\ p_z^i \\ \sqrt{(v_x^i)^2 + (v_y^i)^2 + (v_z^i)^2} \end{bmatrix} + \nu^i_{GPS}. \tag{28}$$

Finally, each UAV i has also access to measurements of the distance to its close neighbors (e.g., deduced from the received communication signal strength or UWB distance sensors):

$$d^i_{radio} = \mathrm{col}\left(d_{i,j_1}, \ldots, d_{j_{n_a^i}}\right) + \nu^i_{radio} \tag{29}$$

with $\left\{j_1, \ldots, j_{n_a^i}\right\} \in \mathcal{N}_a^i$ and the distances $d_{i,j} = \|p^i - p^j\|_2$ between the position vectors $p^i = [p_x^i \; p_y^i \; p_z^i]^\top$ and $p^j = [p_x^j \; p_y^j \; p_z^j]^\top$ of the UAVS i and j, respectively.

The local measurement vector in (4) corresponds to $y^i = \mathrm{col}\left(p^i_{GPS}, v^i_{GPS}, d^i_{radio}\right)$ with the measurement noise vector:

$$\nu^i = \mathrm{col}\left(\nu^i_{GPS}, \nu^i_{radio}\right) \tag{30}$$

Remark 4: Notice that in practice, measurements from different sensors are available at different frequencies. Although not addressed here for simplicity reasons, this can be handled by the algorithm by considering measurements available in y at the appropriate time instants over the estimation horizon, and performing EKF update steps accordingly at these instants.

B. *Simulated scenarios*

Simulation results for a fleet of $n_a = 3$ UAVs are further described. The communication graph is represented in Fig. 1. In this communication topology, UAVs 1 and 3 can exchange information and get distance measurements only with UAV 2. Thus, agents 1 and 3 dispose of regional measurements which do not allow the regional observability[5] property to be respected.

The flight trajectory of each UAV is generated in simulation and a Linear Quadratic Regulator (LQR) control[6] stabilizes the vehicle to a constant reference position. Each coordinate of the acceleration input u^i is imposed to be inside $[-2, 2]$ m/s^2.

The measurement noises (e.g., on the acceleration) are considered uniformly distributed and centred on zero. At each time instant, the acceleration measurement noise w^i has been generated by drawing a 3-dimensional vector from a uniform distribution in $[-0.1, 0.1]^3$. For the measurement noise vector ν^i in (30), the GPS noise ν^i_{GPS} follows a uniform distribution in $[-0.15, 0.15]^3 \times [-0.08, 0.08]$, and ν^i_{radio} follows a uniform distribution in $[-0.1, 0.1]$. The covariance matrix Q^i of the acceleration measurement noise w^i and the covariance matrix R^i of the measurement noise vector ν^i derive directly from the uniform distribution bounds.

[4]Raw measurements provided by the accelerometers are transformed from the body frame of the UAV to the inertial reference frame, and corrected from gravity.

[5]To guarantee regional observability, the rank of the regional observability matrix $\bar{\mathcal{O}}^i_{N,t}$ has to be less than the dimension of the MAS state, i.e., n_x.

[6]The tuning of this LQR control is beyond the scope of the paper and it is omitted here.

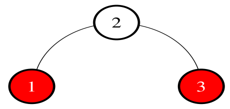

Fig. 1: Communication graph and distance measurement capabilities between the three drones.

The MHEs have been used with a horizon length $N = 2$, consensus step size $L = 1$, and the estimators initial values were set to $\hat{\boldsymbol{x}}_0 = \mathbf{0} \in \mathbb{R}^{18}$, $\Pi_0^i = \mathrm{diag}(\Pi^1_{0,local}, \ldots, \Pi^3_{0,local})$, with $\Pi^i_{0,local} = \mathrm{diag}(6, 6, 6, 0.01, 0.01, 0.01)$. The initial condition of each agent x_0^i has been generated by sampling each of its position components in $[-15, 15]$ (m) and each of its velocity components in $[-2, 2]$ (m/s) independently, using uniform distributions.

The optimization problem has been solved on Matlab using the `fmincon` solver with the interior-point algorithm.

C. *Results and analysis*

The performance of the proposed DMHE approach with EKF pre-estimation (denoted by DMHE-pre-EKF) is assessed by comparing its estimation accuracy and computation time with:

- a similar nonlinear extension of the DMHE algorithm of [12] (without pre-estimation), denoted by DMHE-1;
- the consensus-based on information Distributed Extended Kalman Filter (EKF) of [7] denoted by DEKF-CI.

The interest of the L-step information consensus (described in Section III-B) is highlighted by implementing a DMHE-pre-EKF observer with 2-step consensus (i.e., $L = 2$), denoted by DMHE-pre-EKF-2-step.

A Monte Carlo simulation of 20 runs with different initial conditions for the UAVs, measurements noise and input noise realizations has been conducted. As comparison metrics, the

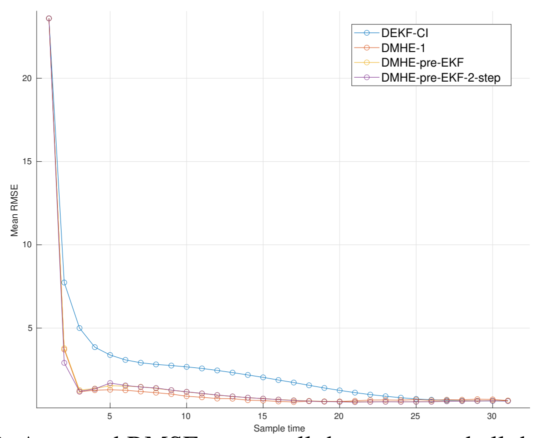

Fig. 2: Averaged RMSE among all the agents and all the trials.

averaged RMSE among agents and samples, the averaged RMSE over final values (denoted by RMSE final values) and the average computation time τ among agents and samples are exposed in Table I.

	RMSE	RMSE final values	τ (s)
DEKF-CI	2.7651	0.7059	0.0004
DMHE-1	1.6371	0.6906	0.4946
DMHE-pre-EKF	1.7009	0.6104	0.0413
DMHE-pre-EKF-2-step	1.6709	0.5980	0.0433

TABLE I: Comparative results of several estimation techniques

The simulations were carried out by a PC Linux Ubuntu 20.04.1 equipped with an Intel Core i7-10875H CPU. According to Fig. 2, the RMSE is reduced faster using DMHE-1 and DMHE-pre-EKF compared to DEKF-CI. However, starting from the sample time $t = 25$, the DEKF-CI error estimation is sensibly close to the one obtained using DMHE-1 and DMHE-pre-EKF. According to Fig. 3 and Table I, compared to the DMHE-1, the DMHE-pre-EKF allows to significantly reduce the computation time, due to the reduction of the number of optimization variables, while keeping an accurate estimate. Moreover, as expected, the DMHE-pre-EKF-2-step consensus results in a better RMSE reduction (see Table I).

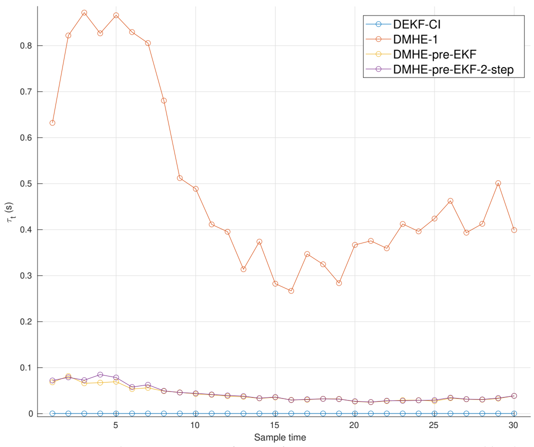

Fig. 3: Averaged computation time τ (s) among all the agents and all the trials.

V. Conclusion and perspectives

This paper has proposed a DMHE algorithm with EKF-based pre-estimation for constrained cooperative localization of a Multi-Agent System. The algorithm accounts for nonlinear measurements performed by the agents and exchanges of information between them via a communication network. The proposed solution is compared with several existing distributed observers. The simulation results confirm the interest of the proposed method in handling constraints, and keeping a reduced computation load compared to standard DMHE approaches while preserving a good estimation accuracy.

Current work focuses on the practical implementation of the proposed estimation method on a fleet of several drones.

VI. Acknowledgment

The first author thanks the STIC doctoral school, University Paris-Saclay, for the co-funding of a 3-months research stay at the University of Seville, during his PhD thesis co-funded by ONERA and Agence de l'innovation de défense (AID) under the Grant 2022-65-0011-AID_ONERA. This work was supported by the Agence Nationale de Recherche (ANR)-France (Grant ANR-21-CE48-0003) and the Agencia Estatal de Investigación (AEI)-Spain (Grant PID2019-106212RB-C41/AEI/10.13039/501100011033) and the European Research Council under the advanced grant OCONTSOLAR Grant agreement ID: 789051.

References

[1] A. Venturino, S. Bertrand, C. Stoica Maniu, T. Alamo, and E. F. Camacho, "Multi-vehicle system localization by distributed moving horizon estimation over a sensor network with sporadic measurements," in *IEEE Conference on Control Technology and Applications*, 2022.

[2] Z. R. Yousefi and M. B. Menhaj, "Constrained distributed estimation based on consensus algorithm for mobile robots tracking," in *IEEE Int. Conf. on Control System, Computing and Engineering*, 2014.

[3] R. Olfati-Saber, "Distributed Kalman filtering for sensor networks," in *46th IEEE Conference on Decision and Control*, 2007, pp. 5492–5498.

[4] P. Alriksson and A. Rantzer, "Distributed Kalman filtering using weighted averaging," in *17th International Symposium on Mathematical Theory of Networks and Systems*, 2006.

[5] R. Madhavan, K. Fregene, and L. Parker, "Distributed heterogeneous outdoor multi-robot localization," in *IEEE International Conference on Robotics and Automation*, 2002.

[6] G. Battistelli, L. Chisci, S. Morrocchi, and F. Papi, "An information-theoretic approach to distributed state estimation," *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 12 477–12 482, 2011.

[7] G. Battistelli, L. Chisci, G. Mugnai, A. Farina, and A. Graziano, "Consensus-based linear and nonlinear filtering," *IEEE Transactions on Automatic Control*, vol. 60, no. 5, pp. 1410–1415, 2015.

[8] M. S. Mahmoud and H. M. Khalid, "Distributed Kalman filtering: a bibliographic review," *IET Control Theory & Applications*, vol. 7, no. 4, pp. 483–501, 2013.

[9] C. V. Rao, J. B. Rawlings, and J. H. Lee, "Constrained linear state estimation — a moving horizon approach," *Automatica*, vol. 37, 2001.

[10] C. Rao, J. Rawlings, and D. Mayne, "Constrained state estimation for nonlinear discrete-time systems: stability and moving horizon approximations," *IEEE Transaction on Automatic Control*, vol. 48, no. 2, 2003.

[11] M. Farina, G. Ferrari-Trecate, and R. Scattolini, "Distributed moving horizon estimation for linear constrained systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 11, pp. 2462–2475, 2010.

[12] G. Battistelli, "Distributed moving-horizon estimation with arrival-cost consensus," *IEEE Transactions on Automatic Control*, vol. 64, no. 8, pp. 3316–3323, 2018.

[13] A. Simonetto, D. Balzaretti, and T. Keviczky, "A distributed moving horizon estimator for mobile robot localization problems," *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 8902–8907, 2011.

[14] M. Farina, G. Ferrari-Trecate, and R. Scattolini, "Distributed moving horizon estimation for nonlinear constrained systems," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 2, 2012.

[15] A. Venturino, S. Bertrand, C. Stoica Maniu, T. Alamo, and E. F. Camacho, "Distributed moving horizon estimation with pre-estimating observer," in *24th International Conference on System Theory, Control and Computing*, 2020, pp. 174–179.

[16] A. Venturino, S. Bertrand, C. Stoica Maniu, T. Alamo, and E. F. Camacho, "A new ℓ-step neighbourhood distributed moving horizon estimator," in *60th IEEE Conference on Decision and Control*, 2021.

[17] A. Venturino, C. Stoica Maniu, S. Bertrand, T. Alamo, and E. F. Camacho, "Distributed moving horizon state estimation for sensor networks with low computation capabilities," *System Theory, Control and Computing Journal*, vol. 1, no. 1, pp. 81–87, 2021.

[18] E. D'Amato, I. Notaro, B. Iodice, G. Panico, and L. Blasi, "Decentralized moving horizon estimation for a fleet of UAVs," in *International Conference on Unmanned Aircraft Systems*, 2022, pp. 998–1005.