Optimal Switching Operation of PT-Symmetric Dimmers with Nonuniform Gain/Loss and Coupling Profiles

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ABSTRACT

We assess through a variational optimization approach the optimal gain-loss profile for a non-uniform PT-symmetric coupler allowing the realization of a binary transfer function and minimizing the deviation of the total traveling light intensity as compared to that holding in a conservative system. We bring evidence that the gain-loss profile fulfilling this requirement corresponds to a non-conventional situation when light intensity is conserved in every point along the propagation distance in the PT-symmetric system. Furthermore, the optimal profile thus found corresponds to a practically important case of optical switching operation achieved with a minimal amount of amplification level. We show that switching architectures using such type of gain-loss profiles are very substantially advantageous as compared to conventional uniform PT-symmetric couplers. Furthermore, this type of optimal profile turns out to be robust with respect to fabrication imperfections. This opens new prospects for functional applications of PT-symmetric devices in photonics.

Keywords: Parity-Time symmetry, Optical switches, Waveguides and couplers, Variational and optimization methods

1. INTRODUCTION

The progress of nanotechnologies has triggered the emergence of many photonic artificial structures: photonic crystals, metamaterials, plasmonic resonators. Recently the intriguing class of PT-symmetric devices – referring to Parity-Time symmetry [1],– has attracted much attention. The distinctive feature of PTSS is that the refractive index profile of the structures is complex-valued due to the gain and/or loss, which are spatially separated in the system. A pair of coupled waveguides with gain and loss represents the simplest configuration of PT-symmetric optical system, which was extensively studied in view of all-optical linear signal manipulation [2-5]. Beside academic motivations, the tremendous interest in these artificial systems is strongly driven by the practical outcomes from several unique properties of PT-symmetric systems (PTSS). In such a PTSS the effective detuning of the propagation constants between the odd and even supermodes, and thus the inverse of their beat length, is gradually reduced upon increasing the level of combined gain/loss in the system until the imaginary part of these constants reaches a critical point [Fig. 1(a)]. This can be advantageously exploited for implementing switches and modulators. This avenue would largely mitigate the lack of electro-optical tunability in integrated and fiber optics, metamaterials and plasmonics. A simple platform for these concepts, detailed below, is a pair of coupled waveguides, one with gain and the other with losses, as illustrated in Fig. 1(b). In the given example the design of the III-V waveguides is basically similar to that of twin semiconductor optical amplifiers (SOA) with separate electrodes.

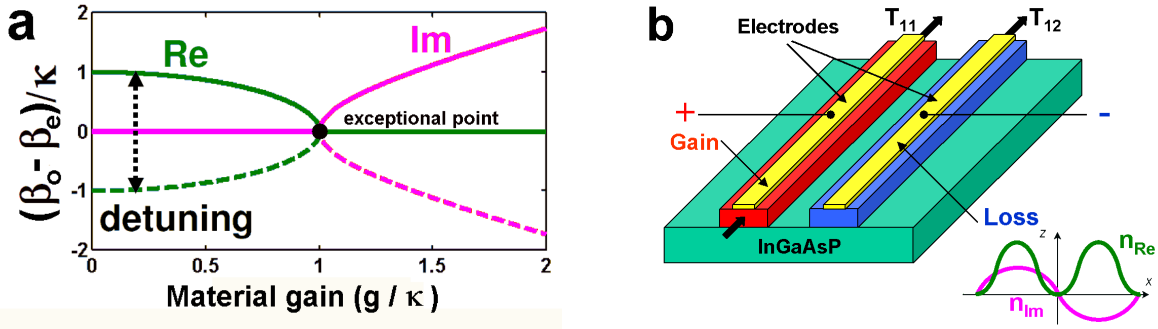

Figure 1. (a) Real (green) and Imaginary parts (magenta) of eigenvalues difference β_o-β_e normalized by the coupling coefficient κ; (b) Sketch of a PT-symmetric directional coupler and its complex index profile.

2. OPTIMIZATION OF NON-UNIFORM COUPLER WITH LOCALLY PT-SYMMETRIC PROFILE

Though the physics of such non-Hermitian optical structures has been widely explored, most of the attention focused on uniform, (i.e., propagation-distance independent) complex refractive index profiles. The situation becomes however totally different when we move from a “flat landscape” to a “hill-and-valley landscape” formed by a non-uniform PT-symmetric complex index profile. In our study we are particularly interested in the relation between the shape of the complex index profile and the propagation-distance-dependent transmission properties of these structures. To this end we consider we consider the simplest model of directional coupler described by the equations:

$$\frac{dq_1}{dz} = \kappa q_2 + i\gamma(z) q_1, \qquad \frac{dq_2}{dz} = \kappa q_1 - i\gamma(z) q_2, \tag{1}$$

where κ is the coupling coefficient, γ(z) is the distance-varying gain-loss and the column vector $q(z) = \begin{pmatrix} q_1(z) \\ q_2(z) \end{pmatrix}$ stands for the field amplitudes in the two coupled waveguides. The length of the coupler is assumed to be $L = 2L_c$, where the coupling length is $L_c = \pi/\kappa$. In the absence of gain (loss), i.e. when $\gamma(z) \equiv 0$, light sloshes from guide 1 to guide 2 and back, and the coupler satisfies the obvious condition:

$$q(-L_c) = \begin{pmatrix} e^{-i\varphi} \\ 0 \end{pmatrix} \to \begin{pmatrix} e^{-i\varphi} \\ 0 \end{pmatrix} = q(L_c) \tag{2}$$

The introduction of gain (loss) changes the propagation constants of coupled waveguides such that they must now cause the realization of the binary switching operation:

$$q(-L_c) = \begin{pmatrix} e^{-i\varphi} \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ e^{i\varphi} \end{pmatrix} = q(L_c) \tag{3}$$

As established in ref.[6], in the case of constant gain-loss profile $\gamma(z) = \text{const}$ the condition for binary switching is $\gamma L_c = 0.6765\pi = 2.125$. It corresponds to an amplification level of 18.6 dB [related to $\exp(2\gamma L_c)$] in the waveguide with gain in absence of coupling. The light intensity profile along the waveguide corresponding to this distribution is shown in Fig. (2a). It is intuitively clear that, energetically, such switching is not optimal since we need first amplifying the signal and then attenuating its amplitude. As a gauge, the mean sum of intensities $\langle q_1^2 + q_2^2 \rangle$, which is the mean Stokes component $\langle S_0 \rangle$, reaches $\langle S_0 \rangle = 2.24$, a value significantly larger than that of the energy conservative passive system where $\langle S_0 \rangle = S \equiv 1$.

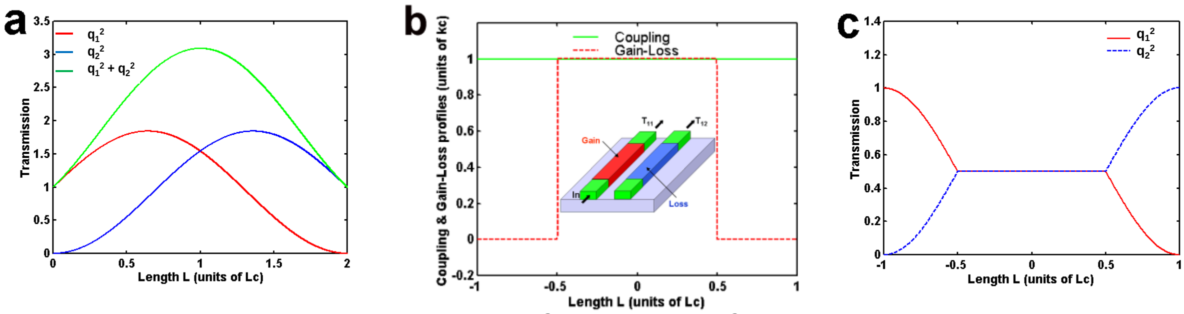

Figure 2: (a) Intensity distributions of Bar – q_1^2 – and cross – q_2^2 – sides ($\equiv T_{11}$ & T_{12}) and their sum $S_0 = q_1^2+q_2^2$ along the propagation direction for a PT-symmetric system of uniformly coupled waveguides with gain-loss level $\gamma = 0.6765\kappa$; (b) Optimal $\gamma(z)$ gain-loss distribution for PT-symmetric coupler of length $L = 2L_c$ with constant coupling profile; (c) Light intensity distribution along the bar and cross waveguides q_2^2 and q_2^2.

Our cost is logically the total gain needed to achieve switching, i.e. the integral $\Gamma = \int_{-L_c}^{L_c} \gamma(z)dz$ of the local gain $\gamma(z) \geq 0$, to be specified below, between device ends $\pm L_c$. The mathematical approach used for solving the optimization problem, thus finding the optimal $\gamma(z)$ profile of minimal Γ is detailed in [7]. Here we provide only the final result on the optimal non-uniform gain-loss profile $\gamma(z)$, which is plotted in Fig. 2(b). As can be seen, the optimal profile found through the optimization procedure is not trivial since $\gamma(z)$ is discontinuous along the propagation direction. Its particular feature is that the active region bearing gain and loss is located only in the middle of the coupler. Its length $L_a = L_c$ is equal to one coupling length. The gain-loss level is constant through the active region and equals the critical-point gain level: $\gamma(z) = \kappa_c = \pi/(2L_c)$. Outside the active region the coupler is passive and $\gamma(z)=0$. The gain-loss profile shown in Fig. 2(b) corresponds the lowest amount of amplification level that equals 13.65 dB for an isolated waveguide. The remarkable point is that in contrast to the case of uniform PT-symmetric coupler, the switching operation is obtained for a ≈ 5 dB lower amplification level.

The second very important result is that the coupling profile found from optimization problem turns out to be optimal with respect to the total intensity propagated in the coupler. The intensity distribution along the bar and cross channel waveguides q_1^2 and q_2^2 is represented Fig.2(c). It corresponds to the initial condition when the light is injected in the "gainy" waveguide. After the propagation through the passive region with length L_p=0.5L_c the light intensities become equal in both waveguides. The intensity distribution does not change when light propagates through the active region with critical gain level. The power exchange between waveguides resumes again in the passive region and eventually leads to the switching operation. The total amount of intensity is conserved in every point along the propagation direction:

$$S_0(z) = q_1^2(z) + q_2^2(z) \equiv T_{11}(z) + T_{12}(z) \equiv 1 \qquad (4)$$

Note that the situation is totally different – and no binary switching operation is obtained – for the initial condition when coupler length L = 2L_c and light is injected in the "lossy" waveguide. The binary switching operation with initial condition corresponding to light injection in the "lossy" waveguide becomes however possible when L ≥ 3L_c and considering more complex switching architectures involving transitions from higher order coupler states [6].

3. OPTIMAL PT-SYMMETRIC ARCHITECTURES FOR BINARY SWITCHING OPERATION

An important teaching from the above study is that the critical point is more than a simple transition between PT-symmetric and PT-broken phase. It provides a particular operation regime allowing conservative propagation of the properly conditioned supermode. The transfer matrix of the active PT-symmetric region operated at critical point is formally equivalent to that of a system formed by two *uncoupled* and *passive* waveguides:

$$q\left(z_{j+L_a}\right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} e^{i\beta_c L_a} q\left(z_j\right) \qquad (5)$$

Such kind of operation without energy exchange between waveguides provides a highly valuable functionality for binary switching architectures using PT-symmetric couplers. To illustrate this we consider first the example of a uniform PT-symmetric coupler with length L=3L_c. For a binary input signal, the output of a uniform passive coupler is now in a cross state (3 is odd). The introduction of the gain and in the system changes the supermodes beating length. It then becomes possible to perform two consecutive switching operations for this coupler [6]. Since the variation of eigenmodes propagations constants becomes notable only when γ is approaching the critical point, the first binary switching from cross to bar state occurs when γ = 0.7455 in this uniform γ context. This corresponds to an amplification (attenuation) level of 30.5 dB in the waveguide with gain in absence of coupling. The second binary switching from bar to cross state occurs when γ = 0.8388, i.e. for 34.3 dB of amplification (attenuation) level.

In contrast to the uniform coupler the switching operation of PT-symmetric coupler with optimized energy conservative profile is substantially different. The step-like function γ(z) requiring minimal amount of gain is represented Fig.3(a). In agreement with section II results following from Eqs. (11) and Theorems 5 and 6 Ref. [8], the gain-loss profile must be odd, γ(z) = – γ(–z). The optimal gain-loss profile corresponding to this case is represented Fig.3(a). The two opposite polarity active regions bearing gain and loss of length L_a = L_c/2 are disposed symmetrically with respect to the middle of the coupler. The gain-loss level is constant through the active regions and equals critical-point gain level. Outside the active regions the coupler is passive and γ(z) = 0. The active regions are separated by a passive section of length L_p = L_c located in the middle of the coupler. The amount of amplification (attenuation) level of 13.65 dB necessary for switching is identical to the previously considered case of a coupler with length L = 2L_c. The intensity distribution along the bar and cross channel waveguides q_1^2 and q_2^2 is represented Fig.3(b). It corresponds to the initial condition of light injected in the "gainy" waveguide. The power exchange between waveguides occurs in the passive region. The total amount of intensity is conserved in every point along the propagation direction. The introduction of the gain-loss in the parts corresponding to the active regions has the effect equivalent to remove one coupling length L_c, leading thus to the switching operation.

The second switching point can be attained in a quite similar manner. The optimal gain-loss profile corresponding to this case is represented Fig.3(c). This time the symmetric profile is symmetric with respect to the middle of the coupler and the length of the active region is L_a = 2L_c. The amount of amplification (attenuation) level is 27.3 dB, i.e. exactly twice that for the first switching point. The intensity distribution along the bar and cross channel waveguides q_1^2 and q_2^2 is represented Fig. 3(d).

As follows from the examples provided, the implementation of optimal gain-loss profile opens the possibility for much more efficient switching operation in terms of required amplification (attenuation) level as compared to uniform PT-symmetric couplers. Furthermore, this kind of switching operation can be further extended to couplers with arbitrary length L≥2L_c. The extra coupler length with respect to the lower integer multiple of 2L_c

can always be compensated by the insertion of additional active region “annihilating” this extra-length. This is a highly valuable property allowing a post-process compensation of the coupling length deviation from the nominal value due to fabrication imperfections (in length or in coupling strength). Finally, the deviation of the gain-loss profile from the optimal one is not critical. It can be shown that, in the limit of small deviations, the first additional term in a Taylor expansion comes with a power of as much as four of the deviating parameter.

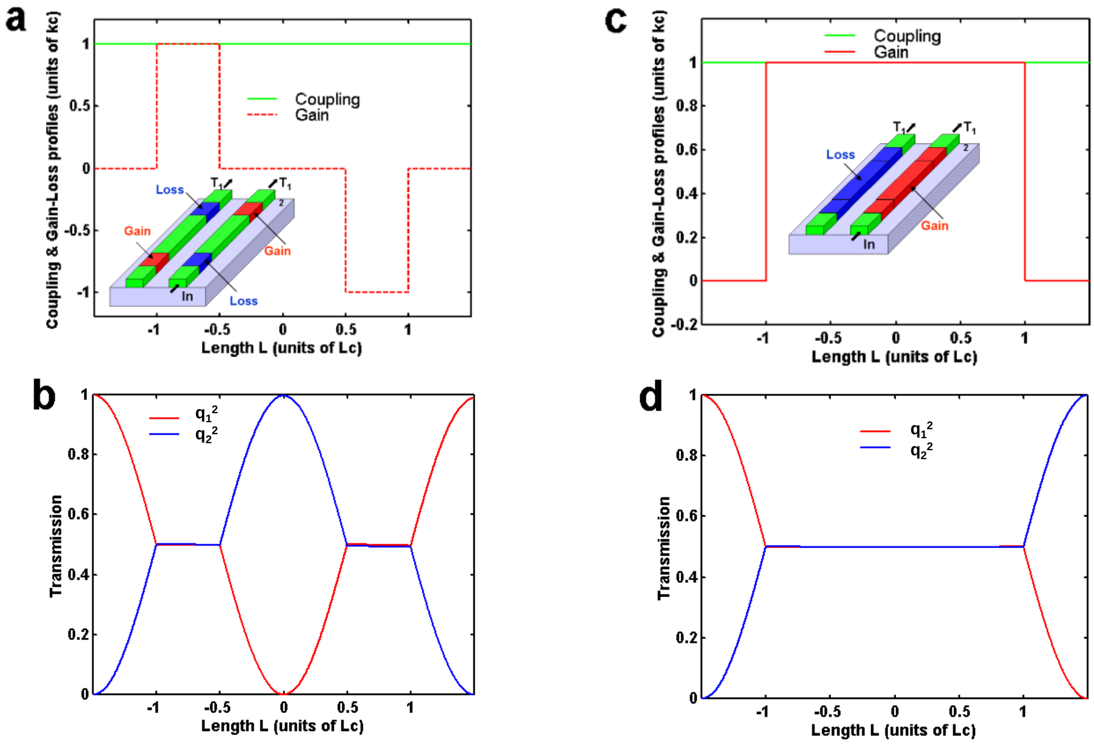

Figure 3: (a) First switching point optimal γ(z) profile for PT-symmetric coupler of length $L = 3L_c$ with constant coupling; (b) Light intensity distribution along the bar and cross channel waveguides q_1^2 and q_2^2 corresponding to the first switching point optimal γ(z) profile; (c) Second switching point optimal γ(z) gain-loss profile; (d) Light intensity distribution q_1^2 and q_2^2 corresponding to the second switching point optimal γ(z) profile.

4. SUMMARY AND CONCLUSIONS

The results discussed here show that a variational optimization approach can fruitfully be used to find non-conventional solutions for non-Hermitian PT-symmetric type systems. We expect that many of the techniques based on the optimization approach that were previously developed for passive type devices can be readily transposed and adapted to PT-symmetric cases, fostering thus a new generation of active photonic devices.

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REFERENCES

[1] C. M. Bender and S. Boettcher: “Real spectra in non-Hermitian Hamiltonians having PT-symmetry,” *Phys. Rev. Lett.* 80, 5243 (1998).
[2] C. E. Rüter, *et al*: “Observation of parity–time symmetry in optics,” *Nature Phys.* 6, 192–195 (2010).
[3] P. L. Chu, *et al*: “Time domain soliton filter based on a semidissipative dual-core coupler,” *Opt. Lett.* 20, 1092-1094 (1995).
[4] J. Čtyroký, *et al*: “Waveguide structures with antisymmetric gain/loss profile,” Opt. Express 18, 21585–21593 (2010).
[5] D. A. Zezyulin and V.V. Konotop: “Nonlinear Modes in finite-dimensional PT-symmetric systems,” *Phys. Rev. Lett.* 108, 213906 (2012).
[6] A. Lupu, H. Benisty, and A. Degiron: “Switching using PT-symmetry in plasmonic systems: positive role of the losses,” *Opt. Express* 21, 21651-21668 (2013).
[7] A. Lupu, V. V. Konotop, H. Benisty: “Optimal PT -symmetric switch features exceptional point,” *Scientific reports* 7, 13299 (2017).
[8] G. H. Song: “Theory of symmetry in optical filter responses,” *J. Opt. Soc. Am. A* 11, 2027-2037 (1994).