Wideband Information in MOM Obtained from Narrowband Data

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***Abstract*— When solving radiation and/or scattering problems, the Method of Moments can give wideband information by using a Model Based Parameter Estimation technique for the expansion coefficients. The parameters are obtained by computing the values of expansion coefficients and their frequency derivatives at a fixed center frequency. This requires computing the moment matrix and its frequency derivatives. The technique is illustrated for a thin wire scatterer. Piecewise sinusoids are used as expansion functions and point matching is used for testing. The moment matrix and its derivatives can then be computed analytically. Computed results show that depending on the number of derivatives used, accurate results, including resonances, can be obtained over an octave of bandwidth.**

Keywords-component; frequency derivative; method of moments; parameter estimation; wideband

I. INTRODUCTION

It is well known that a Model Based Parameter Estimation (MBPE) technique in Method of Moments (MOM) can be used to obtain wideband information from narrowband data [1]-[6]. This technique has also been used for the scattering problems involving two-dimensional as well as Body of Revolution (BOR) scatterers [7]-[8]. Here, using a thin wire as a scatterer at a fixed center frequency (f_0), it is demonstrated that the current distribution induced on the wire can be obtained at frequencies away from f_0 using the value of the expansion coefficient and its frequency derivatives at f_0. This requires the knowledge of the moment matrix, its inverse, and the derivatives of the moment matrix and excitation vector. When piecewise sinusoids are chosen as expansion functions and point matching is used for testing, the moment matrix, excitation vector and their derivatives can be found analytically.

The expansion coefficients are expressed as a rational function of frequency and its model coefficients. The model coefficients of the rational function can be determined using both frequency and frequency derivatives (FD) samples at f_0.

Once the model coefficients of the rational function are found for each expansion coefficient, the expansion coefficients can be obtained at any frequency away from f_0 in the band. Using these expansion coefficients, the Backscattering Radar Cross Section (BSRCS) and the current distribution on a thin wire can be efficiently computed in the band.

The numerical data obtained using MBPE are compared with results calculated a MOM solution using a point-by-point approach.

II. BASIC PROCEDURE

A. Model-Based Parameter Estimation

Starting with the well known MOM equation

$$\sum_{j=1}^{N} Z_{ij} I_j = V_i, \quad \text{for } i = 1,2,\cdots,N \tag{1}$$

The i^{th} expansion coefficient in MOM is given by

$$I_i = \sum_{j=1}^{N} Y_{ij} V_j \tag{2}$$

where Y is the inverse of the moment matrix Z, and V and I are the excitation and expansion coefficient vector, respectively.

The i^{th} expansion coefficient (I_i) can be expressed using MBPE technique as a rational function of frequency.

$$\begin{aligned} I_i(s) &= N_i(s) \,/\, D_i(s) \\ &= \left[\sum_{j=0}^{n} N_{ij} s^j\right] / \left[\sum_{j=0}^{d} D_{ij} s^j\right] \\ &= \frac{N_{i0}+N_{i1}s+N_{i2}s^2+\cdots+N_{in}s^n}{D_{i0}+D_{i1}s+D_{i2}s^2+\cdots+D_{id}s^d} \end{aligned} \tag{3}$$

in which there are n+d+1 coefficients N_{ij}'s and D_{ij}'s to be determined $(D_{i0} = 1)$.

The order of polynomials in both numerator and denominator affects the accuracy of the representation. In general, more model coefficients can be used to cover a wider frequency band.

For demonstration purpose, we assume three expansion coefficients which are expressed by rational functions of frequency as follows.

$$I(s) = \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} \frac{N_1(s)}{D_1(s)} \\ \frac{N_2(s)}{D_2(s)} \\ \frac{N_3(s)}{D_3(s)} \end{bmatrix} = \begin{bmatrix} \frac{\sum_{j=0}^{n} N_{1j} s^j}{\sum_{j=0}^{d} D_{1j} s^j} \\ \frac{\sum_{j=0}^{n} N_{2j} s^j}{\sum_{j=0}^{d} D_{2j} s^j} \\ \frac{\sum_{j=0}^{n} N_{3j} s^j}{\sum_{j=0}^{d} D_{3j} s^j} \end{bmatrix}$$

$$
= \begin{bmatrix} \frac{N_{10}+N_{11}s+N_{12}s^2+\cdots+N_{1n}s^n}{1+D_{11}s+D_{12}s^2+\cdots+D_{1d}s^d} \\ \frac{N_{20}+N_{21}s+N_{22}s^2+\cdots+N_{2n}s^n}{1+D_{21}s+D_{22}s^2+\cdots+D_{2d}s^d} \\ \frac{N_{30}+N_{31}s+N_{32}s^2+\cdots+N_{3n}s^n}{1+D_{31}s+D_{32}s^2+\cdots+D_{3d}s^d} \end{bmatrix} \tag{4}
$$

B. *Computing Model Coefficients Using Both Frequency and Frequency-Derivative Information*

The model coefficients N_{ij}'s and D_{ij}'s in (3) can be determined using both frequency and FD samples. Differentiating (3) k times with respect to s gives

$$
\begin{gathered}
I_i(s)D_i(s) = N_i(s) \\
I_i'(s)D_i(s) + I_i(s)D_i'(s) = N_i'(s) \\
I_i''(s)D_i(s) + 2I_i'(s)D_i'(s) + I_i(s)D_i''(s) = N_i''(s) \\
\vdots \\
I_i^{(k)}(s)D_i(s) + k\, I_i^{(k-1)}(s)D_i'(s) + \cdots + \binom{k}{k-m} I_i^{(m)}(s)D_i^{(k-m)}(s) \\
+ \cdots + k\, I_i'(s)D_i^{(k-1)}(s) + I_i(s)D_i^{(k)}(s) = N_i^{(k)}(s)
\end{gathered} \tag{5}
$$

where $\binom{k}{k-m}$ is the binomial coefficient. The system of $k+1$ equations in (5) can be used to determine the model coefficients.

If the frequency-derivatives are known at only a single frequency s_0, Equation (5) can be simplified by replacing s by $s - s_0$, where $s - s_0$ represents the frequency deviation from s_0. Then setting $D_{i0} = 1$ and $k = n + d$, we have the following linear equations for the unknown coefficients.

$$
\begin{gathered}
N_{i0} = I_{i0} \\
N_{i1} - I_{i0}D_{i1} = I_{i1} \\
N_{i2} - I_{i1}D_{i1} - I_{i0}D_{i2} = I_{i2} \\
\vdots \\
N_{in} - I_{i(n-1)}D_{i1} - I_{i(n-2)}D_{i2} - \cdots - I_{i(n-d)}D_{id} = I_{in} \\
-I_{in}D_{i1} - I_{i(n-1)}D_{i2} - \cdots - I_{i(n-d+1)}D_{id} = I_{i(n+1)} \\
\vdots \\
-I_{i(k-1)}D_{i1} - I_{i(k-2)}D_{i2} - \cdots - I_{i(k-d)}D_{id} = I_{ik}
\end{gathered} \tag{6}
$$

where

$$
I_{im} = \frac{1}{m!} I_i^{(m)}(0) \qquad m = 0,1,\ldots,k
$$

where I_i is regarded as a function of $s - s_0$. Subsequently, we obtain the following matrix equation for the unknown model coefficients.

$$
\begin{bmatrix}
1 & 0 & \cdots & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 1 & \cdots & \cdots & 0 & -I_{i0} & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 & -I_{i1} & -I_{i0} & \cdots & 0 \\
\vdots & \cdots & \cdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \cdots & 1 & -I_{i(n-1)} & -I_{i(n-2)} & \cdots & -I_{i(n-d)} \\
0 & 0 & \cdots & \cdots & 0 & -I_{in} & -I_{i(n-1)} & \cdots & -I_{i(n-d+1)} \\
\vdots & \cdots & \cdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \cdots & 0 & -I_{i(k-1)} & -I_{i(k-2)} & \cdots & -I_{i(k-d)}
\end{bmatrix}
\begin{bmatrix} N_{i0} \\ N_{i1} \\ N_{i2} \\ \vdots \\ N_{in} \\ D_{i1} \\ \vdots \\ D_{id} \end{bmatrix}
=
\begin{bmatrix} I_{i0} \\ I_{i1} \\ I_{i2} \\ \vdots \\ I_{in} \\ I_{i(n+1)} \\ \vdots \\ I_{ik} \end{bmatrix} \tag{7}
$$

C. *Computing Frequency Derivatives in a Moment-Method Model*

The expansion coefficient $I_i(s)$ and its derivatives can be determined using the main MOM equation (1). Differentiating (1) with respect to frequency, we obtain

$$
\sum_{j=1}^{N} \left(Z_{ij} I_j' + Z_{ij}' I_j \right) = V_j' \tag{8}
$$

Then the derivative of the i^{th} expansion coefficient is given by

$$
I_i' = \sum_{j=1}^{N} Y_{ij} \left[V_j' - \sum_{r=1}^{N} Z_{jr}' I_r \right] \tag{9}
$$

In general, the k^{th} frequency derivative is given by

$$
I_i^{(k)} = \sum_{j=1}^{N} Y_{ij} \left[V_j^{(k)} - \sum_{m=1}^{k} \binom{k}{m} \sum_{r=1}^{N} Z_{jr}^{(m)} I_r^{(k-m)} \right] \tag{10}
$$

where $\binom{k}{m}$ is the binomial coefficient, the superscript in parenthesis indicates the order of differentiation with respect to frequency, and N is the order of the moment matrix Z. Note that, each successive derivative involves only the inverse of the moment matrix and the derivative of the inverse matrix is not needed.

Substituting the expansion coefficient I_i and its k derivatives at center frequency f_0 given in (10) into (7) one can readily obtain the N_{ij}'s and D_{ij}'s of (3). Replacing the variable s by f-f_0 in (3), I_i can be written as

$$
I_i(f - f_0) = \frac{N_{i0} + N_{i1}(f-f_0) + N_{i2}(f-f_0)^2 + \cdots + N_{in}(f-f_0)^n}{1 + D_{i1}(f-f_0) + D_{i2}(f-f_0)^2 + \cdots + D_{id}(f-f_0)^d} \tag{11}
$$

III. Sample Numerical Results

The method described above is applied to a thin wire scatterer of length L and radius a=L/148.4 which is illuminated by a plane wave incident on its broadside. Piecewise sinusoidal expansion functions and 65 expansion coefficients are used for the results. The tangential component of the electric field produced by such functions is analytically given in [9].

The current distribution and BSRCS at any frequency away from f_0 are computed. Specifically, the numerical data obtained using MBPE using both frequency and 9$^{\text{th}}$ order FD samples at f_0=300 MHz are compared with the results calculated the MOM solution using a point-by-point approach. The computer program is implemented by using MATLAB.

In Fig.1 the solid curve shows the current distribution on the thin wire computed using MOM at f=1.65f_0. The dashed curve shows the current distribution using MBPE with 9 derivatives (n=3, d=6). Similarly, Fig.2 shows the current distribution at f=0.5f_0. The physical length of the wire was L=1m. Since the solid and dashed curves agree very well, it is clear from these figures that MBPE technique can produce very accurate results over a 3:1 wide band of frequencies.

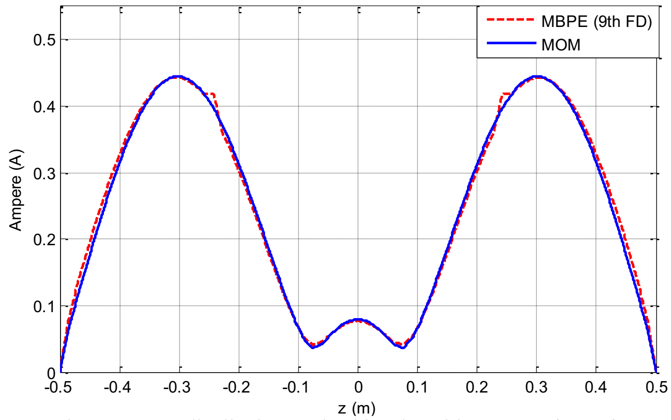

Fig. 1. Current distribution on the thin wire with L=1m at f=1.65f_o

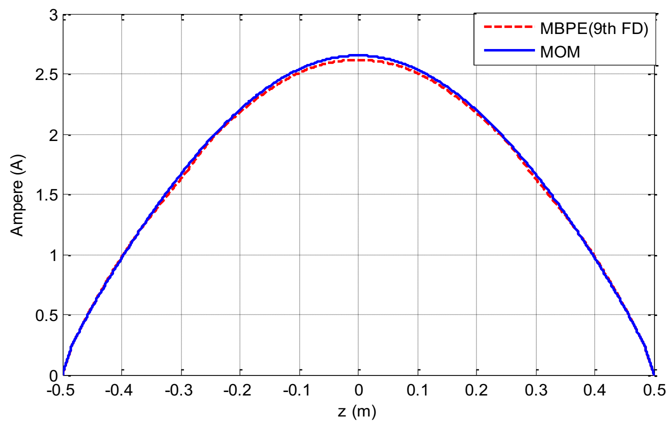

Fig. 2. Current distribution on the thin wire with L=1m at f=0.5f_o

In Fig.3 and Fig.4 the physical length L is taken to be 2m. The good agreement between the solid and the dashed curves suggests that MBPE technique with 9 derivatives (n=4, d=5) gives accurate results over an octave bandwidth.

The solid curve in Fig.5 shows the normalized BSRCS of the thin wire as a function of L/λ using a logarithmic scale for the y-axis. It is computed by using regular MOM at 200 equally spaced points from L/λ=0.25 to L/λ=2.25. The other three curves show the normalized BSRCS using MBPE. The red curve has 9 derivatives (n=4, d=5), the blue curve has 5 derivatives (n=2, d=3), and the green curve is with 3 derivatives (n=1, d=2). The center frequency of MBPE technique is corresponded to $L/\lambda = 1$. It is clear from these curves that increasing the number of derivatives gives more accurate results. It is also clear that with only 5 derivatives, one can obtain reasonably accurate results over a 2.0:0.4 band of frequencies. Note also that the resonances are captured accurately by the MBPE method.

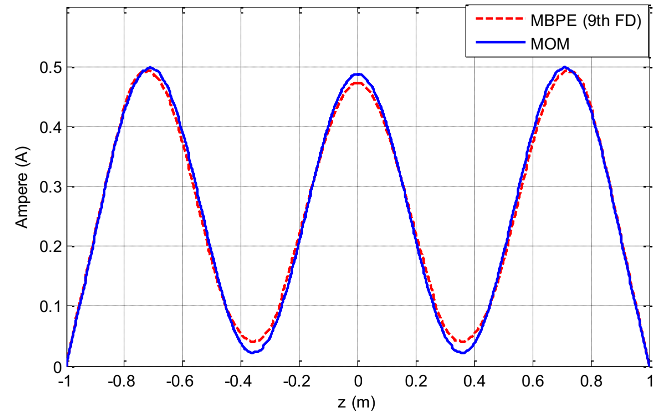

Fig. 3. Current distribution on the thin wire with L=2m at f=1.4f_o

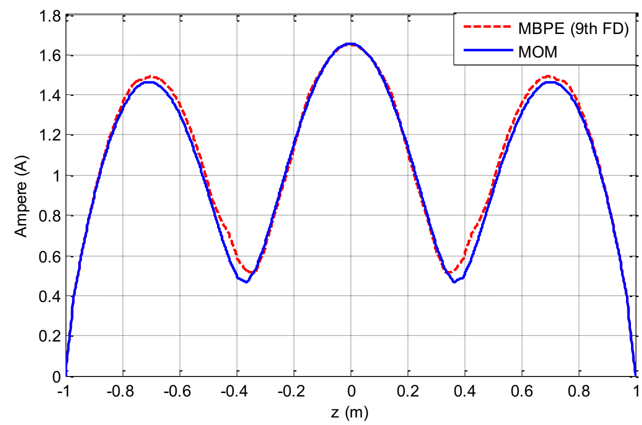

Fig. 4. Current distribution on the thin wire with L=2m at f=0.7f_o

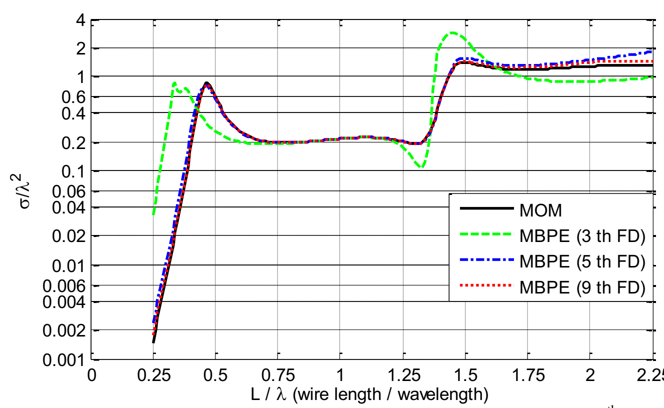

Fig. 5. Normalized BSRCS of the thin wire for different 3, 5, and 9th FD

In Fig.6, the solid curve shows the normalized BSRCS using MOM at 200 equally spaced points from L/λ=0.25 to L/λ=2.25. The other two curves show the normalized BSRCS using MBPE with the same number of derivatives being equal to 9. The red curve has the numerator of order 3, while the blue curve has the numerator of order 4. The blue curve seems to be slightly better.

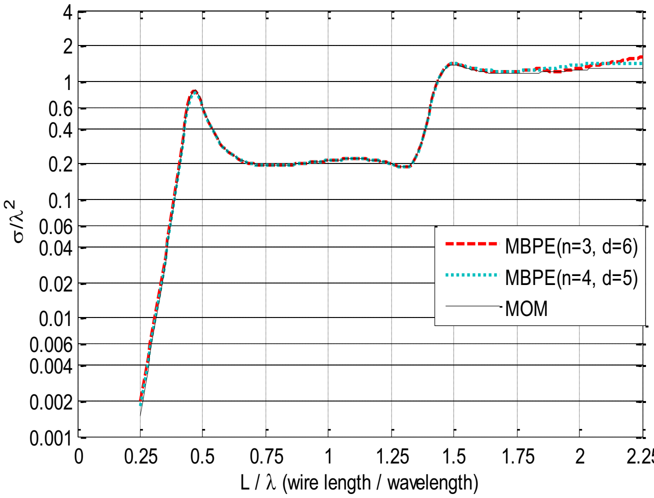

Fig. 6. Normalized BSRCS of the thin wire for different number of model coefficients in numerator and denominator with 9th FD

IV. Summary and Conclusion

It is well known that MOM can provide wideband information based on narrowband data when it is used with the MBPE scheme [1]-[8]. Here, this fact was once again demonstrated by using the example of the thin wire scatterer. Because of the special expansion functions chosen [9], the moment matrix and its frequency derivatives were obtained analytically.

The current distribution and the backscattering radar cross-section (BRSCS) at any frequency away from the center frequency f_0 are computed. They are in very good agreement with the results obtained by repeated use of regular MOM.

In general, it was observed that the large number of derivatives used the more accurate results were obtained over a wider frequency range. One can also obtain more accurate results by choosing different orders for the numerator and the denominator polynomials representing the expansion coefficients.

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