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***Abstract*— One of the major challenges with the increase in wind power generation is the uncertain nature of wind speed. So far the uncertainty about wind speed has been presented through probability distributions. Also the existing models that consider the uncertainty of the wind speed primarily view the distributions of the wind speed over a wind farm as being homogeneous. However, the uncertainty about these wind speed models has not yet been considered. In this paper the Bayesian approach to taking into account the uncertainty inherent in the wind speed model has been presented. The proposed Bayesian predictive model of the wind speed aggregates the non-homogeneous distributions into a single continuous distribution. Therefore, the result is able to capture the variation among the probability distributions of the wind speeds at the turbines' locations in a wind farm. More specifically, instead of using a wind speed distribution whose parameters are known or estimated, the parameters are considered as random whose variations are according to probability distributions. The Bayesian predictive model for a Rayleigh which only has a single model scale parameter has been proposed. Also closed-form posterior and predictive inferences under different reasonable choices of prior distribution in sensitivity analysis have been presented.**

***Keywords*— *Prior distribution, Posterior distribution, Markov Chain Monte Carlo (MCMC), Gamma prior* (key words)**

I. INTRODUCTION

Environmental concerns have made wind power an appealing source of clean and renewable energy, and as this field continues to grow, calibrated and smart probabilistic forecasts can help to make wind power a more financially competitive alternative. However, this is challenging due to the uncertain nature of wind speed. Statistical methods have been applied to three different time scales: short-term, medium-term and long-term. Short-term wind speed forecasts play a central role in estimating various engineering parameters, such as power outputs, extreme wind loads, and fatigue loads [1]. The wind speed forecasts for this time scale are only a few hours ahead of target time [2]-[6]. Medium-term wind speed forecasts look several days ahead are generally based on weather prediction models, which can then be statistically post-processed [7]-[10]. Long-term wind speed forecasts require analysis of wind speed data over a number of years [11]. Probability distributions are used primarily to take into account the uncertainly of wind speed in all three time scales.

The quality of wind speed modeling depends on the suitability of the chosen probability models to describe the wind speed frequency distribution. An overview of the wind speed models used in recent literature is as follows. [1], [12] and [13] used a Weibull distribution to forecast the wind speed and assessed wind energy potential. [14] compared fit of a Rayleigh distribution and another Weibull distribution to wind speed data and showed that the Weibull model provided a better fit. In [15], a wind speed distribution was shown to be satisfactorily described by a Lognormal distribution. In [16] Weibull and Lognormal distributions were used to fit wind speed frequencies and concluded that the Weibull distribution better fit the data. [17] used Rayleigh, Weibull, and Gamma distributions to model wind speeds both on and offshore. In addition Gumbel and the Generalized Extreme Value distributions were used to model extreme wind speeds [18]-[21].

II. PROPOSED METHOD

This section will develop Bayesian stochastic models which incorporate the uncertainty about the wind speed distributional parameters in order to use a Bayesian predictive distribution of the wind speed in the context of wind-penetrated power systems. More specifically, instead of using a wind speed distribution $f(w|\theta)$, where θ is a vector of parameters whose values are known or estimated, we will view θ as a random vector whose variation is according to a prior probability distribution $p(\theta)$ which can be updated in light of data $\mathbf{D}$ into a posterior distribution $p(\theta|\mathbf{D})$ via the Bayes' rule as follows:

$$p(\theta\,|\,\mathbf{D}) \propto p(\mathbf{D}\,|\,\theta)\,p(\theta) \qquad (1)$$

Where " $\propto$ " stands for proportional" and $p(\theta|\mathbf{D})$ is the likelihood function obtained from the probability model of the data.

The Bayesian approach is prudent because of the inclusion of the uncertainty about the parameters in the model, which induces uncertainty about the wind speed distribution itself. The Bayesian prior predictive distribution of the wind speed is given by

$$f(w) = \int f(w|\theta)\,p(\theta)\,d\theta \qquad (2)$$

The posterior predictive distribution uses the same formula where $p(\theta)$ is replaced with $p(\theta|\mathbf{D})$.

$$f\left(w_{n+1}\right)|\boldsymbol{D})=\int f\left(w_{n+1}|\theta\right)p\left(\theta|\boldsymbol{D}\right)d\theta \tag{3}$$

where $f\left(w_{n+1}|\theta\right)$ is the conditional density function of w_{n+1}.

Bayesian methods are available for situations where there is a complete absence of knowledge about the parameters as well as when some partial information about the distribution of the parameters. In the case of the complete absence of knowledge about the parameter, several methods for developing non-informative priors have been suggested in the statistics literature [22] and for the case of partial information developing prior distribution based on the maximum entropy approach are used [22] and [23].

Two important consequences of a Bayesian stochastic model are as follows.

a) Inclusion of uncertainty about the parameters of the wind speed distribution results in using a more prudent predictive distribution for the wind speed. That means, on average, the predictive distribution is more disperse than the probability distributions when the uncertainty about the parameters is ignored. Consequently, for example, for a range of the wind speed with a given probability, the range under the Bayesian predictive distribution is wider than that of ignoring the parameter uncertainty. Conversely, for a given probability, the range of the wind speed under the Bayesian predictive distribution is narrower than that of ignoring the parameter uncertainty.

b) The probability distributions of the parameters can be viewed in terms of the heterogeneity of the distributions of the wind speed over a wind farm. The wind speed distributions for various turbines in a farm may belong to the same family of models, such as the Weibull, and the model parameters of each turbine may vary randomly according to some probability distributions. The Bayesian predictive distribution aggregates the non-homogeneous distributions into a single distribution that captures the variation among the probability distributions of the wind speeds at the turbines' locations in a wind farm.

III. Example: Rayleigh Model for Data

The Rayleigh distribution is the simplest distribution commonly used to describe wind speeds [22], [24] and [25] because it only has a single model parameter c. Assume that the wind speed distribution is Rayleigh with the PDF,

$$f\left(w|c\right)=\left(\frac{2}{c}\right)\left(\frac{w}{c}\right)e^{-\left(\frac{w}{c}\right)^2}. \tag{4}$$

Let $D=(\mathrm{w}_1,\ldots,\mathrm{w}_\mathrm{n})$ denote the wind speed profile. Model (4) provides the following likelihood function under the independency assumption:

$$\begin{aligned}p\left(\mathbf{D}\,|\,c\right)&=f\left(w_1,\ldots,w_N\,|\,c\right)\\&=f\left(w_1\,|\,c\right)f\left(w_2\,|\,c\right)\ldots f\left(w_N\,|\,c\right)\\&=\left(\frac{2}{c}\right)^n e^{-\frac{\sum w_i^2}{c^2}}\prod\frac{w_i}{c}.\end{aligned} \tag{5}$$

For simplifying the illustration we re-parameterize $\theta=\frac{1}{c^2}$ and let $\sum w_i^{\,2}=T_n$.

Because w_i are not a function of θ then,

$$L\left(\theta\right)\propto\theta^n e^{-\theta T_n} \tag{6}$$

A. Prior information

Consider the general problem of inferring a distribution for a parameter θ given some datum or data w_i. From Bayes' theorem, the posterior distribution is proportional to the product of the likelihood function $\theta\rightarrow p\left(w_i\,|\,\theta\right)$ and prior $p(\theta)$, $p\left(\theta\,|\,w\right)\propto L\left(\theta\right)p\left(\theta\right)$.

In the Bayesian analysis, if the posterior distributions $p(\theta|w_i)$ are in the same family as the prior distribution $p(\theta)$, the prior and posterior are then called conjugate distributions, and the prior is called a conjugate prior for the likelihood function. A conjugate prior is an algebraic convenience, giving a closed-form expression for the posterior; otherwise a difficult numerical integration may be necessary. Further, conjugate priors may give intuition by more transparently showing how a likelihood function updates a prior distribution. All members of the exponential family have conjugate priors. The family of conjugate priors for (6) is gamma prior $G\left(\alpha,\beta\right)$. The Gamma PDF, expected value and variance are as follows:

$$G\left(\alpha,\beta\right)=\frac{\beta^\alpha}{\Gamma\left(\alpha\right)}\theta^{\alpha-1}e^{-\beta\theta}, \tag{7}$$

$$E\left(\theta\right)=\frac{\alpha}{\beta}, \tag{8}$$

$$Var\left(\theta\right)=\frac{\alpha}{\beta^2}. \tag{9}$$

From the Bayes' rule (1) gives the posterior:

$$\begin{aligned}p\left(\theta|T_n\right)&\propto\theta^{\alpha-1}e^{-\beta\theta}\theta^n e^{-\theta T_n}\\&\propto\theta^{n+\alpha-1}e^{-(\beta+T_n)\theta}\end{aligned}. \tag{10}$$

That means the posterior distribution is also $G\left(n+\alpha,\beta+T_n\right)$ and under quadratic loss the Bayes' estimate is:

$$\tilde{\theta}=E\left(\theta|\boldsymbol{D}\right)=\frac{n+\alpha}{\beta+T_n}, \tag{11}$$

and the posterior variance is:

$$Var\left(\theta|\boldsymbol{D}\right)=\frac{n+\alpha}{\left(\beta+T_n\right)^2}, \tag{12}$$

The Bayesian predictive distribution of wind speed is:

$$
\begin{aligned}
f\left(w|D\right) &\propto \int \theta w e^{-\theta w^{2}} . \theta^{n+\alpha-1} e^{-(\beta+T_n)\theta} d\theta \\
&\propto w \int \theta^{n+\alpha+1-1} \; . e^{-(w^2+\beta+T_n)\theta} d\theta \\
&\propto w(w^2+\beta+T_n)^{-(\alpha+n)+1} \\
&\propto w\left(1+\frac{w^2}{\beta+T_n}\right)^{-(\alpha+n)+1}
\end{aligned} \tag{13}
$$

Expression (14) is the kernel of the Pareto Type IV distribution, P(IV), with scale parameter $(T_n + \beta)^{1/2}$, shape parameter 2, and the tail index $\alpha + n$. The complete PDF of predictive wind speed is:

$$
f\left(w|D\right) = 2w(\alpha+n)\left\{(T_n+\beta)\left(1+\frac{w^2}{T_n+\beta}\right)\right\}^{-(\alpha+n+1)}, w \geq 0. \tag{14}
$$

The survival function, $\overline{F}\left(w|D\right)$, derived from $f(w|\boldsymbol{D})$ is:

$$
\overline{F}\left(w|D\right) = \left(1+\frac{w^2}{\beta+T_n}\right)^{-(\alpha+n)}, \tag{15}
$$

The mean of (14) is:

$$
\begin{aligned}
E\left(W\right) &= \frac{\left(T_n+\beta\right)^{1/2} . \Gamma\left(\alpha+n-2\right) . \Gamma\left(3\right)}{\Gamma\left(\alpha+n\right)} \\
&= \frac{2\left(T_n+\beta\right)^{1/2}}{\left(\alpha+n-1\right)\left(\alpha+n-2\right)}
\end{aligned} \tag{16}
$$

The 2-moment of (14) is:

$$
\begin{aligned}
E\left(W^2\right) &= \frac{\left(T_n+\beta\right)^2 . \Gamma\left(\alpha+n-4\right) . \Gamma\left(5\right)}{\Gamma\left(\alpha+n\right)} \\
&= \frac{24\left(T_n+\beta\right)^2}{\left(\alpha+n-1\right)\left(\alpha+n-2\right)\left(\alpha+n-3\right)\left(\alpha+n-4\right)}
\end{aligned} \tag{17}
$$

The variance of (14) is:

$$
\begin{aligned}
Var\left(W\right) &= E\left(W^2\right) - E\left(W\right)^2 \\
&= \frac{24\left(T_n+\beta\right)^2}{\left(\alpha+n-1\right)\left(\alpha+n-2\right)\left(\alpha+n-3\right)\left(\alpha+n-4\right)} \\
&\quad - \frac{4\left(T_n+\beta\right)}{\left(\alpha+n-1\right)^2\left(\alpha+n-2\right)^2}
\end{aligned} \tag{18}
$$

The mean and variance of the distribution (14) exist because $2 < \alpha + n$.

Understanding how changes in the model inputs influence the outputs is a concern. In this chapter, we do the sensitivity analyses via changing the number of wind samples, parameters of prior distributions and class of prior distributions.

IV. Simulations and Results

In this section we simulate small wind speed samples, say n = 3, 5 and 20, from a known Rayleigh model without uncertainty with c = 9.24. Then we do posterior and predictive inferences under two informative priors on the parameter $\theta = 1/c^2$. We begin with presenting priors for θ as shown in Figure 1.

Prior 1, Gamma with α = 1 and β = 1, G(1,1), which represents an exponential distribution

Prior 2, Gamma with α = 10 and β = 10, G(10,10),

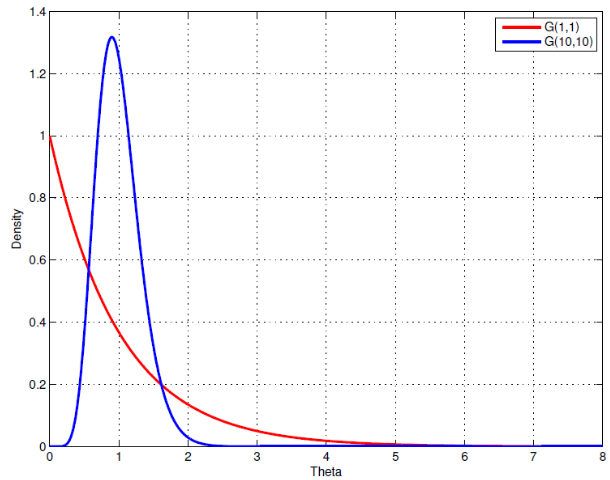


Fig. 1. PDF of Gamma prior with parameters α = 1, β = 1 and α = 1 0, β = 10.

Table 1 illustrates the summary statistics of closed-form and simulated posteriors under different prior distributions. Table 1 gives the Mean, Standard Deviation, Quantiles, Median and Estimates of posteriors' distributional parameters for θ and c. The first column of the table shows the wind speed sample size, n, and the sum of the squared randomly selected wind speed samples, T_n. It is seen that T_n increases while n increases. This always happens when the sample size is large. But this statement is not necessarily true for the small sample sizes like n = 3 and 5. We have randomly selected n samples multiple times, say one thousand times. The mean of the distribution of T_n for n = 5 is always greater than that of n = 3. We have chosen those wind speed samples which have the value of T_n nearest the mean. Table 1 illustrates that the mean values of the posterior distributions of the c parameter get closer to the scale parameter value of the Rayleigh distribution without uncertainty while n and T_n increase. Also the variance of the posterior distribution of the c parameter decreases when n and T_n increase. Table 2 gives the summary statistics for the closed-form and simulated predictive wind speed under different choices of prior while changing n and T_n. This table illustrates that the mean value of predictive wind speed distributions are getting closer to the mean value of the Rayleigh distribution, 8.23, since increasing n and T_n under same choices of prior. From the engineering point of view, it interprets that if the sample size is small or available data provide only indirect information about the parameters of interest, the prior distribution becomes more important. Figure 2 shows the distributions of the Rayleigh as well as the closed-

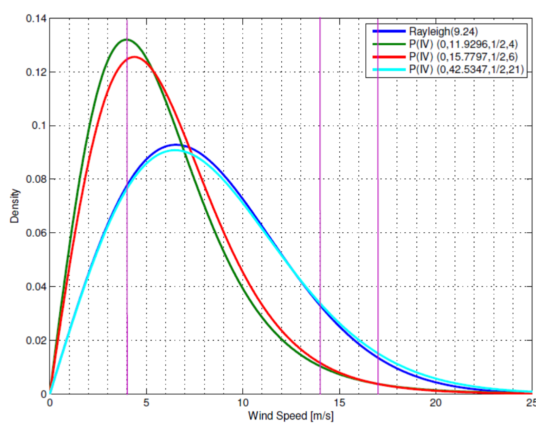
Table 1 Summary statistics of closed-form and simulated posteriors under different choices of prior distribution while changing n and T_n.

n, Tn	Priors	Par.		Mean	Std.	2.5%	Med.	97.5%	Est. of $G(\alpha_0, \beta_0)$
n=3 T_n=141.3158	G(1,1)	θ	Tru.	0.0285	0.0144	0.0080	0.0264	0.0655	(4.0000,142.3158)
			Sim.	0.0284	0.0165	0.0072	0.0252	0.0674	(3.0289,106.8099)
		c	Sim.	6.8288	2.6271	3.8519	6.2944	11.7590	
	G(10,10)	θ	Tru.	0.0268	0.0136	0.0075	0.0248	0.0616	(13.0000,151.3158)
			Sim.	0.1037	0.0306	0.0517	0.1000	0.1729	(11.5370,111.2580)
		c	Sim.	3.2108	0.4941	2.4049	3.1623	4.3988	
n=5 T_n=247.9977	G(1,1)	θ	Tru.	0.0240	0.0101	0.0085	0.0227	0.0485	(6.0000,248.9977)
			Sim.	0.0236	0.0106	0.0078	0.0222	0.0493	(4.7752,197.5440)
		c	Sim.	7.0756	1.8970	4.5408	6.7116	11.3228	
	G(10,10)	θ	Tru.	0.0231	0.0095	0.0081	0.0217	0.0451	(15.0000,257.9977)
			Sim.	0.0639	0.0168	0.0327	0.0628	0.1008	(13.9899,218.8347)
		c	Sim.	4.0661	0.5792	3.1497	3.9903	5.5249	
n=20 T_n=1808.2000	G(1,1)	θ	Tru.	0.0116	0.0025	0.0072	0.0114	0.0171	(21.0000,1809.2000)
			Sim.	0.0122	0.0028	0.0075	0.0120	0.0180	(19.4000,1585.2000)
		c	Sim.	9.2166	1.0710	7.4514	9.1211	11.5380	
	G(10,10)	θ	Tru.	0.0165	0.0030	0.0112	0.0163	0.0229	(30.0000,1818.2000)
			Sim.	0.0176	0.0032	0.0119	0.0174	0.0245	(29.6000,1878.8000)
		c	Sim.	7.6301	0.7115	6.3641	7.5853	9.1478	

Table 2 Summary statistics of closed-form and simulated predictive wind speed under different choices of prior distribution while changing n and T_n.

n, Tn	Priors	Dist.	Mean	Std.	2.5%	Med.	97.5%	Est. of P(IV) (σ, γ, a)
		Ray.	8.2300	4.2205	1.4740	7.7068	17.7540	
n=3 T_n=141.3158	G(1,1)	P(IV)	5.8559	3.9765	0.9680	5.1929	14.6960	(11.9296,1/2,4)
		Pred.	5.9290	3.8250	0.9653	5.1590	15.7900	
	G(10,10)	P(IV)	3.1144	1.0251	0.6600	3.4980	7.5680	(12.3010,1/2,13)
		Pred.	2.8740	1.5930	0.4919	2.6820	6.5170	
n=5 T_n=247.9977	G(1,1)	P(IV)	6.0997	4.1362	0.9688	5.1045	13.5100	(15.7797,1/2,6)
		Pred.	6.3700	3.8950	1.0510	5.6800	15.7000	
	G(10,10)	P(IV)	3.7706	1.1473	0.9470	3.4970	11.5720	(16.0623,1/2,15)
		Pred.	3.5900	2.0220	0.6576	3.2330	8.4760	
n=20 T_n=1808.2000	G(1,1)	P(IV)	8.3764	2.1267	1.4520	7.6120	18.1940	(42.5347,1/2,21)
		Pred.	8.1780	4.4080	1.8120	7.3760	18.4900	
	G(10,10)	P(IV)	6.9871	1.4704	1.2410	7.7008	13.8160	(42.6404,1/2,30)
		Pred.	6.7240	3.6370	1.2180	6.2360	15.0800	

forms of predictive wind speeds considering G(1,1) prior while changing n and T_n.The reference lines show the V_{CI} = 4, V_r = 14 and V_{CO} = 17 [m/s]. In the upper panel of the figure 2, it can be found that the PDFs are taking the form of the Rayleigh as n and T_n increase. From the lower panel of the figure 2, it can be seen that the probabilities of the wind speed availabilities are increasing while n increases. Figure 3 shows the distributions of the Rayleigh as well as the closed-forms of predictive wind speeds considering G(10,10) prior while changing n and T_n. From the upper panel of the figure 3, it is also seen that the PDFs are taking the form of the Rayleigh as n and T_n increase. From the lower panel of the figure, it can be found that the probabilities of the wind power production are increasing while n increases.



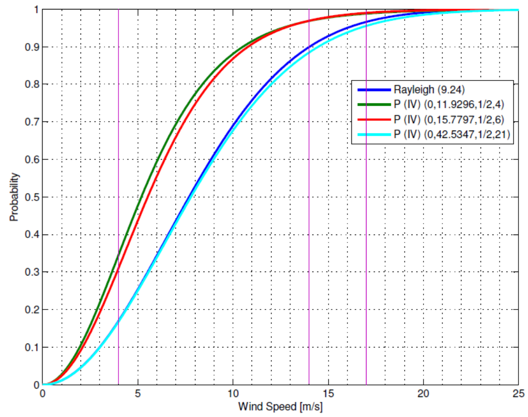


Fig. 2. Distributions of Rayleigh and closed-from P(IV) of predictive wind speed considering G(1,1) prior.

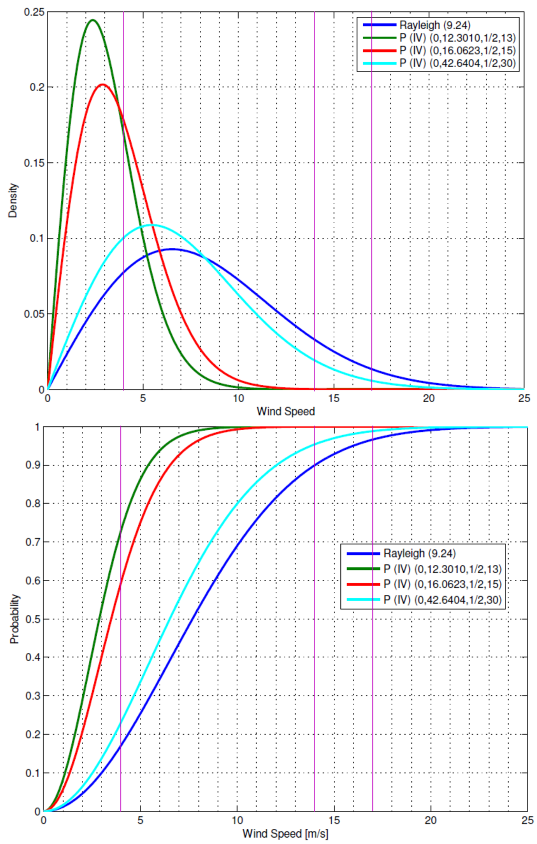


Fig. 3. Distributions of Rayleigh and closed-from P(IV) of predictive wind speed considering G(10,10) prior.

A. *Wind power implications*

Table 3 shows the probabilities of wind power availability for the Rayleigh model without uncertainty as well as the predictive wind speed model under different choices of priors while changing n. More specifically the probability of available wind speed is the difference of the probabilities of the wind speed being less than or equal to V_{CO} and that less than or equal to V_{CI}. Table 3 also gives the probability of the maximum available wind power as the difference between the probabilities of the wind speed being less than or equal to V_{CO} and that less than or equal to V_r, in the last column. It can be seen that the availability of the wind power is 79.52% for the Rayleigh without uncertainty. The availability of the wind power are 64.11%, 67.84% and 78.67%, for the P(IV) predictive model with G(1,1) prior for n = 3, 5 and 20 respectively. Likewise these probabilities are 27.07%, 40.55% and 75.69% for the P(IV) predictive model with G(10,10) prior on the scale parameter. This table reveals that the available wind power and maximum available wind power probabilities increase to get closer to the true value of the Rayleigh model without uncertainty while n increases. Furthermore this table depicts that if the sample size is small or available data provide only indirect information about the parameters of interest, the prior distribution becomes more important. Figure 4 shows the effect of changes in T_n on the CDF of the closed-form P(IV) of predictive wind speed while keeping n and prior constant, e.g. n = 3 and G(10,10). It is obvious that the probability of the available wind power increases when T_n increases.

Table 3 Probabilities of available wind power for Rayleigh and predictive wind speed models under different choices of prior distribution on scale parameter while changing n.

Prior	Dist.	P($W \leq V_{CI}$)	P($W \leq V_{CO}$)	P(WPA)	P($W \leq V_r$)	P(WPA)-max
	Ray.	17.08	96.61	79.52	89.93	6.68
G(1,1)	P(IV) (n=3)	34.70	98.81	64.11	96.86	1.94
	P(IV) (n=5)	31.18	99.02	67.84	96.93	2.09
	P(IV) (n=20)	16.88	95.54	78.67	88.47	7.08
G(10,10)	P(IV) (n=3)	72.93	100.00	27.07	100.00	00.00
	P(IV) (n=5)	59.45	100.00	40.55	100.00	00.00
	P(IV) (n=20)	23.11	98.80	75.69	95.36	3.44

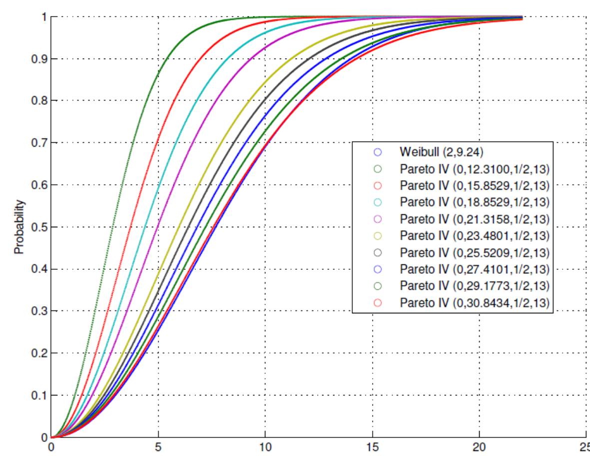


Fig. 4. CDFs of P(IV) of predictive wind speed while changing the T_n considering n=3 and G(10,10) prior.

V. CONCLUSION

The quality of wind speed modeling depends on the suitability of the chosen probability models to describe the wind speed frequency distribution. The models most commonly used in recent literature to describe wind speed distribution are Weibull and Gamma, both of which belong to the Generalized Gamma (GG) family. These models view the distributions of the wind speed over a wind farm as being homogenous. However, a wind farm has multiple turbines installed in different locations, each of which may have its own distribution model. The main aim of this paper was to develop a wind speed model that can aggregate the non-homogenous distributions into a single continuous distribution. For this purpose the authors considered a Rayleigh model which is an especial case of Weibull to describe the wind speed. A Bayesian predictive model has been developed to capture the uncertainty about the wind speed parameter. The closed-forms of posteriors and predictive wind speeds consider the conjugate priors for Rayleigh have been derived. Comparing posterior and predictive inferences under different reasonable choices of prior distribution in sensitivity analysis, (and, for that matter, different reasonable choices of probability models for data) showed that if the sample size is small, or if the available data provide only indirect information about the parameters of interest, the prior distribution becomes more important.

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