

Total Power Minimization: Joint Antenna Selection and Beamforming Design

Mostafa Medra* Andrew W. Eckford† Raviraj Adve*

* Department of Electrical and Computer Engineering, University of Toronto, ON, Canada.

† Department of Electrical Engineering and Computer Science, York University, Toronto, ON, Canada.

Abstract—In this paper, we consider the total power minimization problem when we have signal-to-interference-plus-noise ratio (SINR) constraints. The consumed power in the circuits depends on the number of active antennas, which can be modeled using zero-norm. Due to the difficulty of dealing with the non-convex zero-norm, we used the standard alternate weighted one-norm approach. We addressed the total power minimization for a narrowband system with and without per-antenna power constraints (PAPCs). We derived iterative closed-form expressions in both cases. Then we analysed the case when we have multiple bands operating at the same time. Analogous closed-form expressions are provided. Our simulation results show that significant gains can be obtained in terms of the total power required compared to standard methods that do not take into account the circuit power.

I. INTRODUCTION

When several antennas are available at a base station (BS), those antennas allow the BS to serve multiple users simultaneously; e.g., [1]–[7]. The BS serves multiple users through the use of beamforming techniques. Since the computational complexity is an important factor in design, typically linear beamforming techniques are used. Among conventional linear precoders are the maximum ratio transmission (MRT) [8], zero-forcing (ZF) [9] and regularized ZF (RZF) [10]. Those beamformers are of low-computational complexity and can be obtained in closed-form expressions. When the users have single antenna each, the beamformers that minimize the transmission power subject to certain signal-to-interference-plus-noise ratio (SINR) constraints can be optimally found [2], [7]. That problem can be formulated as a convex problem that can be efficiently solved. The KKT conditions of which also allow for an iterative closed-form expressions [7].

In practice, each antenna at the BS will be driven by its own power amplifier, and, hence, constraints on the transmitted power from each antenna should be included to the beamforming design. When per-antenna power constraints (PAPCs) are introduced to the power minimization problem under SINR constraints, the problem remains convex since the PAPCs are convex. Using the KKT conditions, iterative closed-form solutions can be obtained [11], [12]. The PAPCs can be directly applied when the system is a narrowband system. Systems operating simultaneously on many bands; e.g., OFDM

systems, would have an IFFT operation changing the power distribution across the antennas.

The problem of power minimization under certain SINR constraints is well-studied. However, most of the formulations of this problem ignore the power consumed in the RF circuits, and focus only on the transmitted power. Realistic power consumption models that also take the hardware-consumed power into account were studied; e.g., [13], [14]. While the transmit power required to meet fixed SINR constraints decays in an inverse fashion to the number of BS antennas [15], [16], the RF circuit power is linear in this number. There is, therefore, an optimal number of antennas that requires the least possible total power [13], [14]. In [17], the authors showed that the energy efficiency is a quasi-concave function of the number of BS antennas in the case of massive MIMO downlink system.

Introducing circuit power minimization into the power minimization problem is challenging because the consumed power depends on the number of active antennas. While the sparsity of a signal can be determined by using the zero-norm (ℓ_0 norm) of the power on each antenna, such a norm is not convex and the associated problems are NP hard; e.g., [18], [19]. One possible approach is to replace the zero-norm with a weighted one-norm. We will show that we can transform the total power minimization problem using weighted one-norm into a convex iterative problem that can be efficiently solved. The problem can be viewed as jointly solving the antenna selection (e.g., [20], [21]) and beamforming design, when the objective is power minimization. This problem was solved for fixed beamforming directions (ZF and MRT) when the number of antennas is large [22]. The analysis therein is based on asymptotic results and only provides the number of antennas, whereas our work is applicable to any number of available antennas and is capable of determining which antennas are on. The ℓ_0 norm was used for minimizing the number of antennas for a given rate constraints in [23]. In [24], for a given uplink sum rate, the authors investigated the optimal numbers of BS antennas and users for a single-cell system. The number of active antennas was also considered in a point-to-point large-scale MIMO channel in [25]. The minimum BS power consumption for a given sum rate in a large scale MIMO system is addressed in [26].

In this paper, we will first review the quality-of-service (QoS) problem for a given set of SINR targets and the closed-

form expressions for that problem. We will also review the required modifications for the PAPCs case. Then, we will show how the circuit power minimization can be introduced using the weighted one-norm approach. We will formulate the problem as a convex problem, and we will show that by analysing the KKT, we can obtain closed-form expressions for the optimal beamformers. We will then introduce the PAPCs, and show how to modify the closed-form expressions to accommodate for the extra constraints. For the multi-band case, we will show that convex formulations that allow for closed-form expressions are still possible. In the simulation section, we will show the trade-off between the number of antennas and the total consumed power. We show that significant gains can be obtained by turning many antennas off for the single narrow band case even when PAPCs are on. We also show that gains can be obtained even when many narrow bands are on.

II. SYSTEM MODEL

We consider a multiple-input single-output (MISO) downlink system where K users, each with a single antenna, are served by a base station (BS) with N_t antennas. We assume that the BS is provided with perfect channel state information of the users. We let $\mathbf{h}_k \in \mathbb{C}^{N_t}$ denote the channel between the BS and user k , and let s_k denote the intended normalized data symbol for that user. We design the precoding vector \mathbf{w}_k for transmission from the BS to user k . When the BS transmits $\sum_{k=1}^K \mathbf{w}_k s_k$, we can write the received signal at user k as:

$$y_k = \mathbf{h}_k^H \mathbf{w}_k s_k + \sum_{i \neq k} \mathbf{h}_k^H \mathbf{w}_i s_i + n_k, \quad (1)$$

in which n_k is zero-mean circular Gaussian noise of variance σ_k^2 . We will express each user's QoS constraint by an SINR constraint: $\text{SINR}_k \geq \gamma_k$. This SINR constraint can be written as

$$\text{SINR}_k = \frac{\mathbf{h}_k^H \mathbf{w}_k \mathbf{w}_k^H \mathbf{h}_k}{\mathbf{h}_k^H (\sum_{i \neq k} \mathbf{w}_i \mathbf{w}_i^H) \mathbf{h}_k + \sigma_k^2} \geq \gamma_k, \quad (2)$$

or equivalently $\mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2 \geq 0$, where

$$\mathbf{Q}_k = \mathbf{w}_k \mathbf{w}_k^H / \gamma_k - \sum_{i \neq k} \mathbf{w}_i \mathbf{w}_i^H. \quad (3)$$

When we deal with many narrow bands, we will use the superscript j to indicate the narrow band index, and N_b as the number of narrow bands. For single bands systems, the index j will be dropped to simplify the notations.

If we denote the signal transmitted from antenna i by x_i , then the average transmitted power from the BS can be written as $\sum_{i=1}^{N_t} E|x_i|^2$. In the case of zero-mean independent data symbols of normalized power, this becomes $\sum_{k=1}^K \mathbf{w}_k^H \mathbf{w}_k$, and the average transmitted power from the i th antenna is $\left[\sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i}$. The BS has to minimize the consumed power while satisfying the SINR constraints.

The conventional problem of minimizing the transmitted beamforming power under SINR constraints can, therefore, be written as

$$\min_{\mathbf{w}_k} \sum_k \mathbf{w}_k^H \mathbf{w}_k \quad (4a)$$

$$\text{s.t. } \mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2 \geq 0, \quad \forall k. \quad (4b)$$

This formulation is not convex. However, since the SINR expression does not change with the phase of \mathbf{w}_k , we can express the SINR constraint as

$$\mathbf{w}_k^H \mathbf{h}_k \sqrt{1 + 1/\gamma_k} \geq \sqrt{\sum_i |\mathbf{w}_i^H \mathbf{h}_k|^2 + \sigma_k^2}.$$

This equivalent formulation is a convex conic constraint that can be efficiently solved; e.g., using CVX tool [27] accessible from MATLAB. The KKT conditions of this problem allow for closed-form expressions for the optimal beamformers as well [2], [3].

When PAPCs are introduced, the power minimization problem can be written as

$$\min_{\mathbf{w}_k} \sum_k \mathbf{w}_k^H \mathbf{w}_k \quad (5a)$$

$$\text{s.t. } \mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2 \geq 0, \quad \forall k. \quad (5b)$$

$$p_a \geq \left[\sum_k \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i}, \quad \forall i, \quad (5c)$$

where p_a is the PAPC. Closed-form iterative solutions are available as well [11], [12].

III. NARROWBAND TOTAL POWER MINIMIZATION

As we can see, the problem in (4) does not take into account the power consumed in the circuits driving the antennas. Such a number depends on how many antennas are active. Accordingly, the number of active antennas is then $\sum_i \|P_i\|_0$, where P_i is the power emitted from the i th antenna. If we assume that the power consumed per antenna is denoted by c_1 and c_2 models the amplifier efficiency, then the more general problem of minimizing the *total consumed power* while satisfying the SINR constraints can be written as

$$\min_{\mathbf{w}_k, P_i} c_1 \sum_i \|P_i\|_0 + c_2 \sum_k \mathbf{w}_k^H \mathbf{w}_k \quad (6a)$$

$$\text{s.t. } \mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2 \geq 0, \quad \forall k. \quad (6b)$$

$$P_i \geq \left[\sum_k \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i}, \quad \forall i. \quad (6c)$$

Since ℓ_0 norm is not convex and hard to deal with, we will use the standard weighted one-norm approach. Using that approach, (6) can be written as

$$\min_{\mathbf{w}_k, P_i} c_1 \sum_i s_i P_i + c_2 \sum_k \mathbf{w}_k^H \mathbf{w}_k \quad (7a)$$

$$\text{s.t. } \mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2 \geq 0, \quad \forall k. \quad (7b)$$

$$P_i \geq \left[\sum_k \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i}, \quad \forall i. \quad (7c)$$

The weighted one-norm updates the weights s_i iteratively as $s_i = 1/(P_i + \delta)$, where δ is a regularization constant.

If we let ν_k and λ_i denote the dual variable of the k th constraint in (7b), and the i th constraint in (7c), respectively, then we can write the Lagrangian of (7) as

$$\mathcal{L}(\mathbf{w}_k, P_i, \nu_k, \lambda_i) = c_1 \sum_i s_i P_i + c_2 \sum_k \mathbf{w}_k^H \mathbf{w}_k - \sum_k \nu_k (\mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2) - \sum_i \lambda_i \left(P_i - \left[\sum_k \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i} \right).$$

From the KKT conditions, we have that $c_1 s_i = \lambda_i$. If we let Λ denote a diagonal matrix whose (i, i) th element is λ_i then the beamforming vector should satisfy the KKT condition

$$c_2 \mathbf{w}_k = \left(\frac{\nu_k}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H - \sum_{j \neq k} \nu_j \mathbf{h}_j \mathbf{h}_j^H - \Lambda \right) \mathbf{w}_k, \quad (8)$$

which can be rearranged, similar to [2], to show that the dual variables $\{\nu_k\}$ should satisfy the fixed-point equations

$$\nu_k^{-1} = \mathbf{h}_k^H \left(c_2 \mathbf{I} + \sum_j \nu_j \mathbf{h}_j \mathbf{h}_j^H + \Lambda \right)^{-1} \mathbf{h}_k \left(1 + \frac{1}{\gamma_k} \right). \quad (9)$$

Once we obtain ν_k , we can solve (8) as an eigen equation to obtain the directions. The power loading; $\|\mathbf{w}_k\|$, can be obtained from solving the K SINR constraints satisfied by equality at optimality. If this is not the case for constraint k , then we can decrease \mathbf{w}_k , which will still satisfy all the constraints, and provide a lower objective, which contradicts the presumed optimality. These steps are summarized in Alg. 1. Detailed complexity analysis and implementation issues are addressed for a similarly structured problem in [28].

Algorithm 1 Narrowband total power minimization

- 1: Initialize $s_k = 1$ and obtain the corresponding Λ .
 - 2: **for** A certain number of iterations **do**
 - 3: Solve the fixed-point equations in (9) to obtain ν_k .
 - 4: Solve the eigen equation in (8) to find the direction of \mathbf{w}_k .
 - 5: Solve the K linear equations arising from (7b) holding with equality at optimality to obtain the power loading; $\|\mathbf{w}_k\|$.
 - 6: Update the weights $s_i = 1/(P_i + \delta)$ and Λ .
 - 7: **end for**
-

IV. NARROWBAND TOTAL POWER MINIMIZATION UNDER PER-ANTENNA POWER CONSTRAINTS

The previous section focused on the case of minimizing the total power with no individual antenna power constraints. However, in practice, each antenna is driven by its own power amplifier, and the addition of PAPCs is inevitable. In this section, we will show how the previous analysis and algorithm can be tailored to address the PAPCs case as well.

The problem in (7) can be modified to include PAPCs as follows

$$\min_{\mathbf{w}_k, P_i} c_1 \sum_i s_i P_i + c_2 \sum_k \mathbf{w}_k^H \mathbf{w}_k \quad (10a)$$

$$\text{s.t. } \mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2 \geq 0, \quad \forall k. \quad (10b)$$

$$P_i \geq \left[\sum_k \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i}, \quad \forall i. \quad (10c)$$

$$p_a \geq \left[\sum_k \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i}, \quad \forall i. \quad (10d)$$

Following a similar KKT analysis, if we let q_i denote the dual variable for the PAPC, $\hat{\mathbf{Q}}$ denote a diagonal matrix whose (i, i) th element is q_i , then the Lagrangian of the problem in (10) can be written as

$$\mathcal{L}(\mathbf{w}_k, P_i, \nu_k, \lambda_i, q_i) = c_1 \sum_i s_i P_i + c_2 \sum_k \mathbf{w}_k^H \mathbf{w}_k - \sum_k \nu_k (\mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2) - \sum_i \lambda_i \left(P_i - \left[\sum_k \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i} \right) - \sum_i q_i \left(p_a - \left[\sum_k \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i} \right). \quad (11)$$

Similar to the previous case, we can observe from the KKT conditions that $c_1 s_i = \lambda_i$. Also the beamforming vector should satisfy the KKT condition

$$c_2 \mathbf{w}_k = \left(\frac{\nu_k}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H - \sum_{j \neq k} \nu_j \mathbf{h}_j \mathbf{h}_j^H - \Lambda - \hat{\mathbf{Q}} \right) \mathbf{w}_k, \quad (12)$$

which can be similarly rearranged to show that the dual variables $\{\nu_k\}$, in the PAPC case, should satisfy the fixed-point equations

$$\nu_k^{-1} = \mathbf{h}_k^H \left(c_2 \mathbf{I} + \sum_j \nu_j \mathbf{h}_j \mathbf{h}_j^H + \Lambda + \hat{\mathbf{Q}} \right)^{-1} \mathbf{h}_k \left(1 + \frac{1}{\gamma_k} \right). \quad (13)$$

And once the beamforming directions are obtained, the power loading can be calculated assuming that the SINR constraints hold with equality at optimality.

The above derivations are based on the values of the dual variable q_i , which are not known in advance. However, we can use efficient subgradient algorithms to obtain these values in an iterative way. Subgradient algorithms and their convergence analysis were proposed in [11], and parameter selection and convergence speed were enhanced in [12]. The basic idea of such algorithm is to solve the KKT conditions for an initial value of q_i , then check the PAPCs. If they are met, the algorithm terminates, other wise, the values of q_i are updated. We update q_i (or $\hat{\mathbf{Q}}$) such that [12]

$$q_i^{n+1} = \max \left(q_i^n + t_n \left(\left[\sum_k \mathbf{w}_k \mathbf{w}_k^H \right]_{i,i} - p_a \right), 0 \right), \quad (14)$$

where t_n is the step size, and n is the iteration index. The maximum operator guarantees that when the PAPC is not

active, then the corresponding q_i is zero. We can summarize the proposed algorithm as shown in Alg. 2. The complexity analysis in each iteration is the same as that of Alg. 1.

Algorithm 2 Power minimization under PAPCs

- 1: Initialize $q_i = 0$, $s_k = 1$ and obtain the corresponding Λ .
 - 2: **for** A certain number of iterations **do**
 - 3: **while** $p_a < [\sum_k \mathbf{w}_k \mathbf{w}_k^H]_{i,i}$ for any i **do**
 - 4: Solve (13) to obtain ν_k .
 - 5: Solve (12) to find the direction of \mathbf{w}_k .
 - 6: Solve the K linear equations arising from (10b) holding with equality at optimality to obtain the power loading; $\|\mathbf{w}_k\|$.
 - 7: Update $\hat{\mathbf{Q}}^{n+1}$ using (14).
 - 8: **end while**
 - 9: Update the weights $s_i = 1/(P_i + \delta)$ and Λ .
 - 10: **end for**
-

V. MULTI-BAND CASE

The previous sections dealt with the single-band case. However, to turn an antenna off in the multi-band case, that antenna should have no power over all the beamformers of the different bands. Accordingly, if we let P_i denote the power over all the N_b narrow bands, instead of only one band, then we can use the weighted one-norm approach to solving (6) as follows

$$\min_{\mathbf{w}_k^j, P_i} c_1 \sum_i s_i P_i + c_2 \sum_{k,j} (\mathbf{w}_k^j)^H \mathbf{w}_k^j \quad (15a)$$

$$\text{s.t.} \quad (\mathbf{h}_k^j)^H \mathbf{Q}_k^j \mathbf{h}_k^j - (\sigma_k^j)^2 \geq 0, \quad \forall k, j. \quad (15b)$$

$$P_i \geq \left[\sum_{k,j} \mathbf{w}_k^j (\mathbf{w}_k^j)^H \right]_{i,i}, \quad \forall i. \quad (15c)$$

Similar to the narrowband case, from the KKT conditions, we have that $c_1 s_i = \lambda_i$ and the beamforming vector satisfies

$$c_2 \mathbf{w}_k^j = \left(\frac{\nu_k^j}{\gamma_k^j} \mathbf{h}_k^j (\mathbf{h}_k^j)^H - \sum_{i \neq k} \nu_i^j \mathbf{h}_i^j (\mathbf{h}_i^j)^H - \Lambda \right) \mathbf{w}_k^j, \quad (16)$$

where the dual variables $\{\nu_k^j\}$ satisfy

$$(\nu_k^j)^{-1} = (\mathbf{h}_k^j)^H \left(c_2 \mathbf{I} + \sum_i \nu_i^j \mathbf{h}_i^j (\mathbf{h}_i^j)^H + \Lambda \right)^{-1} \mathbf{h}_k^j \left(1 + \frac{1}{\gamma_k^j} \right).$$

The power loading for the beamformers of the j th narrowband is then obtained from solving the K SINR constraints of that band satisfied by equality at optimality.

VI. SIMULATION RESULTS

In this section, we illustrate the performance of the proposed approaches in solving the total power minimization problem. We consider a system consisting of a BS with N_t antennas, serving $K = 4$ users per band. Fading is modelled using the standard Rayleigh model. We assume an SINR target of $\gamma = 3\text{dB}$ for all users and that each user has normalized noise power; $\sigma_k^2 = 1$. We set $c_1 = 0.3$ W/antenna and the amplifier

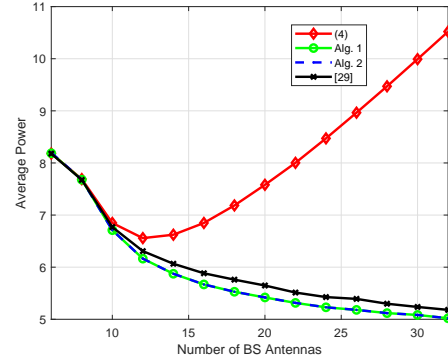


Fig. 1. The average total power versus the number of BS antennas for $K = 4$ users, $\gamma=3\text{dB}$, and $\sigma^2=1$.

TABLE 1
AVERAGE NUMBER OF ACTIVE ANTENNAS

N_t	8	12	16	20	24	28	32
Alg. 1	7.41	8.82	9.22	9.32	9.33	9.27	9.30
Alg. 2	7.47	8.83	9.22	9.32	9.33	9.27	9.30
[29]	7.80	9.91	9.87	9.71	9.36	9.12	9.01

efficiency to 30%; i.e., $c_2 = 1/0.3$. For the PAPCs, $P_a = 0.4$. For the weighted one-norm, we used a regularization factor of $\delta = 10^{-4}$, and 6 iterations.

In Fig. 1, we plot the total power versus the number of BS antennas. As we can see for (4), the reduction in the transmitted power is significant when the number of antennas is low, then the circuit power dominates which causes the total power to grow almost linearly. We observe that Alg. 1 and Alg. 2 are not able to reduce the power by turning off antennas when the number of antennas is low, however, as the number of antennas increases, they provide significant gains compared to the naive approach in (4). As a benchmark, we also compare with the algorithm that uses antenna selection as described in [29], then solves the beamforming problem using (4). The antenna selection in [29] is based on finding the most correlated rows of the channel matrix then deleting the row of lower power. We keep deleting rows and calculating the power required using (4) till the minimum power is obtained. Since this method deletes one row at a time, it takes more iterations than our proposed methods that work with a fixed number of iterations. In addition, our methods provide lower average power and is easily extendable to the case of PAPCs or multi-band. In table 1, we list the average number of active antennas versus the total number of antennas. Note that our closed-form expressions provide the same results obtained by solving (7) or (10) using CVX tool [27]. We note that the problem with PAPCs can be infeasible. Using higher number of antennas can lower the transmitted power to avoid infeasibility.

In Fig. 2, we plot the total power versus the number of operating bands for $K = 4$ users per band, and an $N_t = 32$ antennas. We can see that up to a few bands, the algorithm is still able to obtain gains by turning antennas off. In table 2, we list the average number of active antennas versus the total number of active bands. While the power gains are smaller for

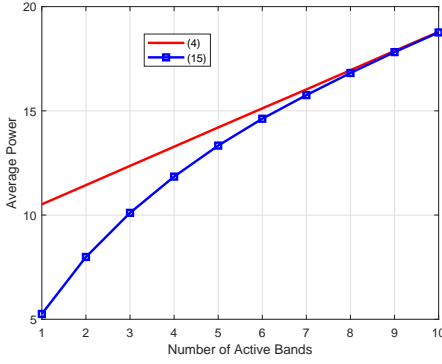


Fig. 2. The average power versus the number of active bands for $K = 4$ users per band, and an $N_t = 32$ antennas BS.

TABLE 2
AVERAGE NUMBER OF ACTIVE ANTENNAS

N_b	1	2	3	4	5	6	7	8	9	10
(15)	11	17	21	25	27	29	30	31	31	32

higher number of active bands, the smaller number of active antennas means that the computations would be easier and less power consuming.

VII. CONCLUSION

In this paper, we formulated the power minimization SINR-constrained problem such that it includes the power dissipated in the RF circuits powering the antennas. The number of active antennas can be modeled using the zero-norm of the antenna's power. We used the standard weighted one-norm approach to replace the zero-norm. We provided iterative closed-form expressions for our proposed algorithm. We then extended the algorithm to the case where we have per-antenna power constraints (PAPCs) and derived iterative closed-form expressions as well. Then we examined the case where many bands are on, and provided analogous closed-form expressions. Our simulations show that we can obtain significant power gains compared to the conventional naive approach that only minimizes the transmit power.

REFERENCES

- [1] D. Gesbert, M. Kountouris, R. W. Heath, C.-B. Chae, and T. Salzer, "Shifting the MIMO paradigm," *IEEE Signal Proc. Mag.*, vol. 24, no. 5, pp. 36–46, Sept. 2007.
- [2] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*, L. C. Godara, Ed. CRC Press, 2001, ch. 18.
- [3] F. Rashid-Farrokhi, L. Tassiulas, and K. J. R. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Trans. Commun.*, vol. 46, no. 10, pp. 1313–1324, Oct. 1998.
- [4] A. Wiesel, Y. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, Jan. 2006.
- [5] M. Alodeh, D. Spano, A. Kalantari, C. G. Tsinos, D. Christopoulos, S. Chatzinotas, and B. Ottersten, "Symbol-level and multicast precoding for multiuser multi-antenna downlink: A state-of-the-art, classification, and challenges," *IEEE Communications Surveys Tutorials*, vol. 20, no. 3, pp. 1733–1757, May 2018.

- [6] M. Schubert and H. Boche, "Iterative multiuser uplink and downlink beamforming under SINR constraints," *IEEE Trans. Signal Process.*, vol. 53, no. 7, pp. 2324–2334, July 2005.
- [7] E. Björnson, M. Bengtsson, and B. Ottersten, "Optimal multiuser transmit beamforming: A difficult problem with a simple solution structure," *IEEE Signal Process. Mag.*, vol. 31, no. 4, pp. 142–148, July 2014.
- [8] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, pp. 1458–1461, Oct. 1999.
- [9] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 461–471, Feb. 2004.
- [10] C. Peel, B. Hochwald, and A. Swindlehurst, "A vector-perturbation technique for near-capacity multi-antenna multiuser communication-Part I: Channel inversion and regularization," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 195–202, Jan. 2005.
- [11] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2646–2660, June 2007.
- [12] M. Medra and T. N. Davidson, "Low-complexity robust MISO downlink precoder design with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 66, no. 2, pp. 515–527, Jan. 2018.
- [13] C. Dessel and B. Debaillie, "Massive MIMO for energy-efficient communications," in *46th European Microwave Conf.*, Oct. 2016, pp. 138–141.
- [14] C. Dessel, B. Debaillie, and F. Louagie, "Modeling the hardware power consumption of large scale antenna systems," in *IEEE Online Conf. Green Commun.*, Nov. 2014, pp. 1–6.
- [15] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1436–1449, April 2013.
- [16] J. Hoydis, S. ten Brink, and M. Debbah, "Massive MIMO in the UL/DL of cellular networks: How many antennas do we need?" *IEEE Journal Sel. Areas Commun.*, vol. 31, no. 2, pp. 160–171, February 2013.
- [17] D. Ha, K. Lee, and J. Kang, "Energy efficiency analysis with circuit power consumption in massive MIMO systems," in *IEEE 24th Annual Int. Symposium on Personal, Indoor, and Mobile Radio Commun.*, Sep. 2013, pp. 938–942.
- [18] E. J. Candes and T. Tao, "Decoding by linear programming," *IEEE Trans. Info. Theory*, vol. 51, no. 12, pp. 4203–4215, Dec. 2005.
- [19] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Info. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.
- [20] A. F. Molisch and M. Z. Win, "MIMO systems with antenna selection," *IEEE Microwave Mag.*, vol. 5, no. 1, pp. 46–56, March 2004.
- [21] N. P. Le, F. Safaei, and L. C. Tran, "Antenna selection strategies for MIMO-OFDM wireless systems: An energy efficiency perspective," *IEEE Trans. Vehicular Tech.*, vol. 65, no. 4, pp. 2048–2062, April 2016.
- [22] K. Senel, E. Björnson, and E. G. Larsson, "Joint transmit and circuit power minimization in massive MIMO with downlink SINR constraints: When to turn on massive MIMO?" *IEEE Trans. Wireless Commun.*, vol. 18, no. 3, pp. 1834–1846, March 2019.
- [23] M. Yukawa and I. Yamada, "Minimal antenna-subset selection under capacity constraint for power-efficient MIMO systems: A relaxed ℓ_1 minimization approach," in *IEEE Int. Conf. Acoustics, Speech and Signal Process.*, March 2010, pp. 3058–3061.
- [24] S. K. Mohammed, "Impact of transceiver power consumption on the energy efficiency of zero-forcing detector in massive MIMO systems," *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 3874–3890, Nov. 2014.
- [25] Y. Pei, T.-H. Pham, and Y. Liang, "How many RF chains are optimal for large-scale MIMO systems when circuit power is considered?" in *IEEE Global Commun. Conf.*, Dec. 2012, pp. 3868–3873.
- [26] Z. Jiang, S. Zhou, and Z. Niu, "Minimum power consumption of a base station with large-scale antenna array," in *19th Asia-Pacific Conf. Commun.*, Aug. 2013, pp. 262–266.
- [27] M. Grant, S. Boyd, and Y. Ye, "CVX: Matlab software for disciplined convex programming," 2008.
- [28] M. Medra, Y. Huang, and T. N. Davidson, "Offset-based beamforming: A new approach to robust downlink transmission," *IEEE Trans. Signal Process.*, vol. 67, no. 1, pp. 70–82, Jan. 2019.
- [29] X. Zhibin, W. Jinkuan, W. Yun, and M. Yan, "Fast algorithm for antenna selection in MIMO systems based on minimum error rate," in *Int. Symposium Intelligent Signal Process. Commun. Systems*, Nov. 2007, pp. 510–513.