Parallel Interference Cancellation for Cell-Free C-RANs

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Abstract—This paper formulates a linear receiver for the uplink of centralized, possibly cloud-based, radio access networks operating in a cell-free fashion. This receiver, which draws on the notion of soft parallel interference cancellation, exhibits substantial interference rejection abilities, yet it does not involve any matrix inversions. Its computational cost is hence decidedly inferior to that of an MMSE receiver, but its performance is markedly superior to that of matched-filter beamforming. And, with an adequate sparsification of the channel matrix that it derives from, the proposed receiver can be rendered scalable in the sense of its computational cost per access point not growing with size of the network.

I. Introduction

In cellular networks, every user is served by a single access point (AP) while being regarded as interference by other APs. Capitalizing on the invariance with the cell size of the signal-to-interference ratio (SIR), such networks have sustainedly increased their area capacity by means of densification. However, a point is bound to be reached where the SIR invariance breaks down because of line-of-sight propagation across cells and an ensuing surge in interference [1]. Along with considerations of software definition, elasticity, and flexibility, this motivates the new paradigm of centralized, possibly cloud-based, radio access networks (C-RANs) [2].

C-RANs are compatible with a cellular structure, but they also open the door to cell-free operation. Influenced by massive MIMO, much of the cell-free literature posits matched-filter (MF) beamformers [3], yet the true potential of C-RANs emerges when their centralized nature is exploited to feature more sophisticated transceivers. In the uplink specifically, an MMSE receiver would perform decidedly better than an MF [4]–[6], but at the cost of having to invert a large-dimensional matrix per fading coherence block. The performance and computational disparity between the MF and the MMSE receiver invites exploring alternatives that can outperform the former without the computational cost of the latter.

Since the MF is optimum against noise, improving upon it necessarily entails dealing with the interference. And, since the computational cost of an MMSE filter is dominated by the matrix inversion, reducing that cost calls for a receiver whose computation is devoid of inverses. Our goal is hence that of an interference-aware inverse-free receiver, and a promising path towards this goal lies in the realm of interference cancellation.

The most established such technique in MIMO communication is successive interference cancellation (SIC). However, SIC is out of the question for a C-RAN with hundreds or even thousands of users, as a huge number of iterations would

be needed to peel off all those users. Parallel interference cancellation (PIC) is much more appealing, as the number of iterations then decouples from the number of users. There is extensive research on PIC receivers that can be built upon, both without [7]–[14] and with [15] the decoders in the cancellation loops. Although having the decoders in the PIC loops promises superior performance, the large number of users—again—and their potential asynchronicity strongly suggests applying soft PIC before decoding, and this is the option we focus on.

II. NETWORK AND CHANNEL MODELS

The networks under consideration feature N APs and K users, all equipped with a single omnidirectional antenna.

The local-average channel gain between the kth user and the nth AP, denoted by $G_{n,k}$, subsumes distance-dependent pathloss and shadowing. For the pathloss, we consider the commonplace single-slope model, characterized by its exponent η , as well as a dual-slope model whereby the pathloss decays quadratically up to some breakpoint distance, and with exponent η thereafter. This entails the additional parametrization of this breakpoint distance, but it better reflects the idiosyncrasy of radio propagation in ultradense networks.

Letting P and σ^2 denote the user's transmit power and the noise power, respectively, the local-average SNR of user k at the nth AP is $\mathsf{SNR}_{n,k} = G_{n,k} \frac{P}{\sigma^2}$.

Besides $G_{n,k}$, the channel between the kth user and the nth AP features a small-scale fading coefficient $h_{n,k} \sim \mathcal{N}_{\mathbb{C}}(0,1)$, independent across users and APs.

III. CHANNEL ESTIMATION AND DATA TRANSMISSION

Disregarding pilot contamination, which can be kept at bay with procedures such as the ones propounded in [3, sec. IV] or in [16], [17], the MMSE fading estimate $\hat{h}_{n,k}$ gathered by the network upon observation at the nth AP of a pilot transmission from user k satisfies $h_{n,k} = \hat{h}_{n,k} + \tilde{h}_{n,k}$ where

$$\mathbb{E}[|\hat{h}_{n,k}|^2] = \frac{\mathsf{SNR}_{n,k}}{1 + \mathsf{SNR}_{n,k}} \tag{1}$$

while $\tilde{h}_{n,k} \sim \mathcal{N}_{\mathbb{C}} \left(0, \frac{1}{1 + \mathsf{SNR}_{n,k}}\right)$ is uncorrelated error. Generalizing it to multiple pilot transmissions per user and per fading coherence block entails a mere scaling of $\mathsf{SNR}_{n,k}$ within.

Subsequently, upon data transmission, on a given time-frequency resource unit the nth AP observes

$$y_n = \sum_{k=0}^{K-1} \sqrt{G_{n,k}} h_{n,k} x_k + v_n \tag{2}$$

1

$$= \sum_{k=0}^{K-1} \sqrt{G_{n,k}} \hat{h}_{n,k} x_k + \underbrace{\sum_{k=0}^{K-1} \sqrt{G_{n,k}} \tilde{h}_{n,k} x_k + v_n,}_{Z_n}$$
(3)

with x_k the signal from user k, satisfying $\mathbb{E}[|x_k|^2] = P$, and with $v_n \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$. The transmit-receive relationship between users and APs can be vectorized into

$$\mathbf{y} = \begin{bmatrix} y_0 \\ \vdots \\ y_{N-1} \end{bmatrix} = \sum_{k=0}^{K-1} \hat{\mathbf{c}}_k x_k + \mathbf{z}$$
 (4)

$$=\hat{C}x+z,\tag{5}$$

where $\hat{\boldsymbol{C}} = [\hat{\boldsymbol{c}}_0 \cdots \hat{\boldsymbol{c}}_{K-1}]$ with

$$\hat{c}_{k} = \begin{bmatrix} \sqrt{G_{0,k}} \, \hat{h}_{0,k} \\ \vdots \\ \sqrt{G_{N-1,k}} \, \hat{h}_{N-1,k} \end{bmatrix}$$
 (6)

while $\boldsymbol{x} = [x_0 \cdots x_{K-1}]^T$ and $\boldsymbol{z} = [z_0 \cdots z_{N-1}]^T$. In turn, $\mathbb{E}[\boldsymbol{z}\boldsymbol{z}^*] = \sigma^2 \boldsymbol{D}$ with \boldsymbol{D} the diagonal matrix

$$[D]_{n,n} = 1 + \sum_{k=0}^{K-1} \frac{\mathsf{SNR}_{n,k}}{1 + \mathsf{SNR}_{n,k}}.$$
 (7)

IV. NETWORK-WIDE RECEIVER BENCHMARKS

The MF receiver for all K users, or more precisely the whitening MF, is given by $D^{-1}\hat{C}$ and, for given fading realizations,

$$\operatorname{sinr}_{k}^{\mathsf{MF}} = \frac{\left| \hat{c}_{k}^{*} \boldsymbol{D}^{-1} \hat{c}_{k} \right|^{2}}{\sum_{\ell \neq k} \left| \hat{c}_{k}^{*} \boldsymbol{D}^{-1} \hat{c}_{\ell} \right|^{2} + \frac{\sigma^{2}}{P} \hat{c}_{k}^{*} \boldsymbol{D}^{-1} \hat{c}_{k}}.$$
 (8)

In turn, the MMSE receiver for all K users is [18, sec. 6.4]

$$\boldsymbol{W}^{\text{MMSE}} = \left(\hat{\boldsymbol{C}}\hat{\boldsymbol{C}}^* + \frac{\sigma^2}{P}\boldsymbol{D}\right)^{-1}\hat{\boldsymbol{C}} \tag{9}$$

$$= D^{-1}\hat{C}\left(\hat{C}^*D^{-1}\hat{C} + \frac{\sigma^2}{P}I\right)^{-1}, \qquad (10)$$

which gives

$$\operatorname{sinr}_{k}^{\operatorname{MMSE}} = \frac{|\boldsymbol{w}_{k}^{\operatorname{MMSE}*} \hat{\boldsymbol{c}}_{k}|^{2}}{\sum_{\ell \neq k} |\boldsymbol{w}_{k}^{\operatorname{MMSE}*} \hat{\boldsymbol{c}}_{\ell}|^{2} + \frac{\sigma^{2}}{P} \boldsymbol{w}_{k}^{\operatorname{MMSE}*} \boldsymbol{D} \boldsymbol{w}_{k}^{\operatorname{MMSE}}}, \quad (11)$$

where

$$\boldsymbol{w}_{k}^{\text{MMSE}} = [\boldsymbol{W}^{\text{MMSE}}]_{:,k} = \left(\hat{\boldsymbol{C}}\hat{\boldsymbol{C}}^* + \frac{\sigma^2}{P}\boldsymbol{D}\right)^{-1}\hat{\boldsymbol{c}}_{k}$$
 (12)

is the kth column of $\boldsymbol{W}^{\text{MMSE}}$.

V. LINEAR RECEIVER BASED ON SOFT PIC

From the standpoint of user k, (4) can be rewritten as

$$y = \hat{c}_k x_k + \sum_{\ell \neq k} \hat{c}_\ell x_\ell + z. \tag{13}$$

The PIC proposition is to subtract a soft estimate of the interference from \boldsymbol{y} so as to obtain

$$\boldsymbol{y}_k = \boldsymbol{y} - \sum_{\ell \neq k} \hat{\boldsymbol{c}}_{\ell} \hat{\boldsymbol{x}}_{\ell} \tag{14}$$

where \hat{x}_{ℓ} is a soft estimate of x_{ℓ} . From y_k , a better decision statistic can be derived for user k. Any estimation strategy, linear or nonlinear, can be applied to obtain $\hat{x}_0, \ldots, \hat{x}_{K-1}$.

A. Linear Interference Estimation

Linear estimators do not depend on the signal distributions, endowing the receiver with broader generality. And, when the interference estimators are linear, the overall receiver becomes itself linear [8]–[11], [19]. With linear interference estimation based on y, $\hat{x}_{\ell} = a_{\ell}^* y$ such that (14) can be rewritten as

$$\mathbf{y}_k = \mathbf{y} - \sum_{\ell \neq k} \hat{\mathbf{c}}_{\ell} \mathbf{a}_{\ell}^* \mathbf{y} \tag{15}$$

$$= \left(I - \sum_{\ell \neq k} \hat{\boldsymbol{c}}_{\ell} \boldsymbol{a}_{\ell}^* \right) \boldsymbol{y} \tag{16}$$

and, further applying a whitening MF to y_k , the decision statistic for user k becomes

$$\hat{\boldsymbol{c}}_{k}^{*} \boldsymbol{D}^{-1} \boldsymbol{y}_{k} = \underbrace{\left(\hat{\boldsymbol{c}}_{k}^{*} \boldsymbol{D}^{-1} - \sum_{\ell \neq k} \hat{\boldsymbol{c}}_{k}^{*} \boldsymbol{D}^{-1} \hat{\boldsymbol{c}}_{\ell} \, \boldsymbol{a}_{\ell}^{*}\right)}_{\boldsymbol{w}_{k}^{\mathsf{PIC}*}} \boldsymbol{y}. \tag{17}$$

The soft PIC architecture is therefore equivalent to the oneshot linear receiver

$$w_k^{\text{PIC}} = D^{-1} \hat{c}_k - \sum_{\ell \neq k} a_\ell \, \hat{c}_\ell^* D^{-1} \hat{c}_k,$$
 (18)

whose performance and cost depend on a_0, \ldots, a_{K-1} . An inviting choice for a_ℓ is the linear MMSE estimator

$$oldsymbol{a}_\ell = \left(\mathbb{E}[oldsymbol{y}oldsymbol{y}^*\,|\,\hat{oldsymbol{c}}_0,\ldots,\hat{oldsymbol{c}}_{K-1}]
ight)^{-1}\mathbb{E}[oldsymbol{y}oldsymbol{x}_\ell^*\,|\,\hat{oldsymbol{c}}_0,\ldots,\hat{oldsymbol{c}}_{K-1}],$$

which, unsurprisingly, returns $a_\ell = w_\ell^{\text{MMSE}}$. The interference estimates coincide with the MMSE receiver outputs, with SINRs that cannot be improved upon by any linear receiver. Indeed, plugging (12) into (18), what emerges after some algebra is $w_k^{\text{PIC}} \propto w_k^{\text{MMSE}}$ with the scaling factor not affecting the SINR. This confirms that the one-shot linear receiver that results from applying PIC with MMSE interference estimation is itself equivalent to a linear MMSE receiver. Let us next explore alternative forms for the interference estimation.

B. Scalar Linear Interference Estimation

A way to circumvent the need for matrix inversions is to estimate each interference term on the basis of a scalar input, rather than from \boldsymbol{y} , and a prime candidate is the output of the respective MF. Then, $\hat{x}_\ell = a_\ell^* \cdot \hat{\boldsymbol{c}}_\ell^* \boldsymbol{D}^{-1} \boldsymbol{y}$ with the MMSE estimation coefficient being

$$a_{\ell} = \frac{\mathbb{E}[\hat{\boldsymbol{c}}_{\ell}^{*} \boldsymbol{D}^{-1} \boldsymbol{y} \cdot \boldsymbol{x}_{\ell}^{*} \mid \hat{\boldsymbol{c}}_{0}, \dots, \hat{\boldsymbol{c}}_{K-1}]}{\mathbb{E}[\hat{\boldsymbol{c}}_{\ell}^{*} \boldsymbol{D}^{-1} \boldsymbol{y} \cdot \boldsymbol{y}^{*} \boldsymbol{D}^{-1} \hat{\boldsymbol{c}}_{\ell} \mid \hat{\boldsymbol{c}}_{0}, \dots, \hat{\boldsymbol{c}}_{K-1}]}$$
(19)

$$= \frac{\hat{c}_{\ell}^* D^{-1} \hat{c}_{\ell}}{\sum_{i=0}^{K-1} \left| \hat{c}_{\ell}^* D^{-1} \hat{c}_{i} \right|^2 + \frac{\sigma^2}{P} \hat{c}_{\ell}^* D^{-1} \hat{c}_{\ell}}.$$
 (20)

Recalling (8), the above can be seen to equal

$$a_{\ell} = \frac{\operatorname{sinr}_{\ell}^{\mathsf{MF}}}{1 + \operatorname{sinr}_{\ell}^{\mathsf{MF}}} \frac{1}{\hat{\boldsymbol{c}}_{\ell}^{*} \boldsymbol{D}^{-1} \hat{\boldsymbol{c}}_{\ell}}$$
(21)

such that the linear PIC receiver in (18) morphs into

$$\boldsymbol{w}_{k}^{\text{PIC}} = \boldsymbol{D}^{-1} \hat{\boldsymbol{c}}_{k} - \sum_{\ell \neq k} \frac{\mathsf{sinr}_{\ell}^{\text{MF}}}{1 + \mathsf{sinr}_{\ell}^{\text{MF}}} \frac{\hat{\boldsymbol{c}}_{\ell}^{*} \boldsymbol{D}^{-1} \hat{\boldsymbol{c}}_{k}}{\hat{\boldsymbol{c}}_{\ell}^{*} \boldsymbol{D}^{-1} \hat{\boldsymbol{c}}_{\ell}} \boldsymbol{D}^{-1} \hat{\boldsymbol{c}}_{\ell}$$
(22)

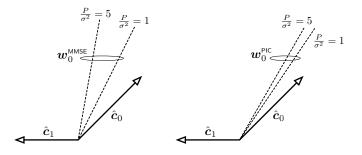


Fig. 1. On the left-hand side, directions taken by $\boldsymbol{w}_0^{\text{MMSE}}$ for $\hat{\boldsymbol{c}}_0 = [1 \ 1]^{\text{T}}$ and $\hat{\boldsymbol{c}}_1 = [-1 \ 0]^{\text{T}}$ with $P/\sigma^2 = 1$ and $P/\sigma^2 = 5$. On the right-hand side, respective directions for $\boldsymbol{w}_0^{\text{PIC}}$. In all cases, $\boldsymbol{D} = \boldsymbol{I}$.

and the SINR of user k at the output of $\boldsymbol{w}_k^{\text{PIC}}$ is

$$\operatorname{sinr}_{k}^{\operatorname{PIC}} = \frac{|\boldsymbol{w}_{k}^{\operatorname{PIC}} \hat{\boldsymbol{c}}_{k}|^{2}}{\sum_{\ell \neq k} |\boldsymbol{w}_{k}^{\operatorname{PIC}} \hat{\boldsymbol{c}}_{\ell}|^{2} + \frac{\sigma^{2}}{P} \boldsymbol{w}_{k}^{\operatorname{PIC}} \boldsymbol{D} \boldsymbol{w}_{k}^{\operatorname{PIC}}}.$$
 (23)

The expression in (22) offers considerable intuition on how the soft PIC modifies the original MF for user k:

- As $\sin r_\ell^{\rm MF}$ grows, the confidence on \hat{x}_ℓ increases and a larger share of the ensuing interference is cancelled. Equivalently, $\boldsymbol{w}_k^{\rm PIC}$ deviates progressively from its MF direction so as to avoid the interference from the ℓ th user.
- As the projection of \hat{c}_{ℓ} onto \hat{c}_{k} grows strong, the interference from the ℓ th user further afflicts the kth user and the cancellation of the former from the latter intensifies accordingly. Equivalently, $\boldsymbol{w}_{k}^{\text{PIC}}$ again deviates from its MF direction, away from the ℓ th user.

Altogether, each initial MF vector is modified so as to reduce its projection on the rest, a procedure that can be illustrated in a toy setting with K=N=2. Consider

$$\hat{\boldsymbol{c}}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \hat{\boldsymbol{c}}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \tag{24}$$

with D=I. The MF receiver for user 0 would align with \hat{c}_0 whereas, as shown in Fig. 1, the corresponding MMSE receiver tilts to be more orthogonal to \hat{c}_1 ; moderately for $P/\sigma^2=1$, and more pronouncedly for $P/\sigma^2=5$. The respective PIC receiver directions for user 0 are also depicted in the figure; they also lean to increase their orthogonality to \hat{c}_1 , but somewhat short of optimally. This reflects the different amount of information based on which either receiver aligns its vectors: finding the SINR-maximizing directions requires processing \hat{C} jointly, and inevitably a matrix inversion, whereas the PIC approximation to those directions only necessitates the pairwise projection of its columns, $\hat{c}_0, \ldots, \hat{c}_{K-1}$.

C. Performance Evaluation

To evaluate the performance, we resort to a wrapped-around universe with N=200 APs. The AP and user positions are drawn uniformly at random for every network snapshot, which makes unnecessary the explicit inclusion of shadowing [20]. We set P/σ^2 such that $\mathsf{SNR}_{n,k}=10$ dB at a distance d, where d would be the inter-AP spacing if the network were arranged as a hexagonal grid with the same spatial density. Under reasonable values for the transmit power, bandwidth,

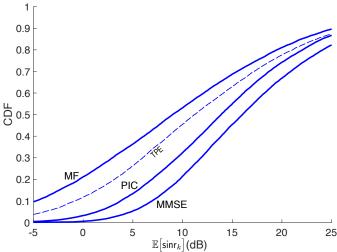


Fig. 2. CDF of the user SINR for MF, linear PIC and MMSE receivers with K/N=0.25 and $\eta=4$. Also included is the curve corresponding to a linear TPE of the MMSE receiver.

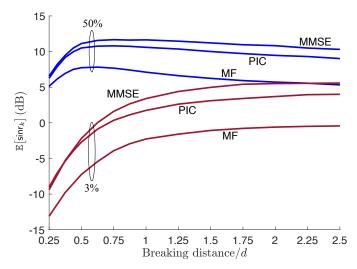


Fig. 3. 3% and 50% user SINR levels as a function of the pathloss breakpoint distance normalized by d, with K/N=0.25.

and pathloss intercept, this is compatible with ultradense deployments ($d \approx$ 5–20 m).

Fig. 2 shows the distribution over the AP and user locations of the local-average SINR, $\mathbb{E}[\sin r_k]$ expected over the small-scale fading, with K/N=0.25 and a single-slope pathloss. The PIC receiver is uniformly superior to the MF and within 3 dB of the MMSE benchmark. Also shown is how the PIC receiver outperforms the SINR-optimized first-order truncated polynomial expansion (TPE) of the MMSE receiver [21], a competing alternative to bridge the MF and MMSE extremes.

Under a dual-slope pathloss model, a similar behavior is observed over a wide range of breaking distances (see Fig. 3).

VI. SCALABILITY

A network-wide receiver spanning the entire C-RAN is not scalable. Moreover, it is an overkill because, due to pathloss and shadowing, only a small subset of APs capture substantial power from user k and only a small subset of other users

cause substantial interference to user k. This suggests that the vast majority of entries of C should be disregarded and the estimation of those entries should be foregone altogether.

Let us consider scalability in terms of those aspects that are inherent to a C-RAN, namely (i) receiver computational cost, and (ii) channel estimation. To measure the former, we denote by M the number of complex multiply-and-accumulate (MA) operations accrued computing and applying the receiver coefficients. To measure the latter, L is the number of channel coefficients to be estimated. For growing N and K, we want $M/N = \mathcal{O}(1)$ and $L/N = \mathcal{O}(1)$ as in a cellular network.

Measured in MA operations, the cost of $N \times N$ matrix inversions is $\mathcal{O}(N^3)$ while the multiplication of $N \times K$ and $K \times N$ matrices costs $\mathcal{O}(KN^2)$. Our goal here is not to present a detailed complexity analysis, but rather to establish scalability. With this in mind, these measures suffice and simpler operations such as additions can be neglected, leading to the following considerations:

- The whitening MF is directly available from the channel estimates, and its application to the observation y incurs a cost per AP of $M^{\text{MF}}/N = \mathcal{O}(K)$.
- The MMSE cost is dominated by the computation of (10), which satisfies $M^{\text{MMSE}}/N = \mathcal{O}(K^2)$.
- For PIC, $\sin r_0^{\text{MF}}, \ldots, \sin r_{K-1}^{\text{MF}}$ can be measured directly and a_0, \ldots, a_{K-1} thus require computing $\hat{c}_\ell^* \boldsymbol{D}^{-1} \hat{c}_\ell$ for $\ell = 0, \ldots, K-1$, with a cost of $\mathcal{O}(KN)$. Then, producing $\hat{x}_0, \ldots, \hat{x}_{K-1}$ again incurs a cost of $\mathcal{O}(KN)$. Likewise, (14) and the subsequent MF application both have a cost of $\mathcal{O}(KN)$. All in all, $M^{\text{PIC}}/N = \mathcal{O}(K)$.
- For all the receiver types, $L/N = \mathcal{O}(K)$.

Although the PIC receiver does involve additional operations, its computational cost per AP is $\mathcal{O}(K)$ as for the MF, in contrast with $\mathcal{O}(K^2)$ in the MMSE case. The PIC receiver thus goes a long way towards reconciling the MMSE performance with the MF cost. But the challenge remains to render the PIC receiver truly scalable, meaning $M^{\text{PIC}}/N = \mathcal{O}(1)$ and $L^{\text{PIC}}/N = \mathcal{O}(1)$, without compromising its performance.

VII. SPARSE PIC

The path to scalability lies in recognizing and exploiting the nature of C, which, as mentioned, has most of its mass concentrated on a small share of its entries.

An intuitive idea could be to zero out all but the dominant entries of C, thereby obtaining a sparse matrix C to be plugged into the various expressions in lieu of C itself. Unfortunately, such a direct sparsification might yield a channel matrix that is sparse, but unbalanced, with some users heavily favored by many connections while others are outright disconnected from the network. Likewise, some APs might be essentially taken out of service. The desideratum is thus a sparse channel matrix that is balanced across rows and columns.

Let us restrict to a subset K_n the users whose channels are estimated by the nth AP; users not in K_n are treated as noise by such AP. Then, (3) can be rewritten as

$$y_n = \sum_{k \in \mathcal{K}_n} \hat{c}_{n,k} x_k + \underbrace{\sum_{k \notin \mathcal{K}_n} \hat{c}_{n,k} x_k + z_n}_{\mathbf{Z}_n}$$
 (25)

from which (4) changes to

$$\mathbf{y} = \begin{bmatrix} y_0 \\ \vdots \\ y_{N-1} \end{bmatrix} = \hat{\mathbf{C}}\mathbf{x} + \mathbf{z}$$
 (26)

with $\mathbf{z} = [\mathbf{z}_0 \cdots \mathbf{z}_{N-1}]^{\mathrm{T}}$,

$$[\hat{\mathbf{C}}]_{n,k} = \begin{cases} [\hat{\mathbf{C}}]_{n,k} & k \in \mathcal{K}_n \\ 0 & \text{otherwise,} \end{cases}$$
 (27)

and $\mathbb{E}[\mathbf{z}\mathbf{z}^*] = \sigma^2\mathbf{D}$ where

$$[\mathbf{D}]_{n,n} = \sum_{k \notin \mathcal{K}_n} \frac{\mathsf{SNR}_{n,k}^2}{1 + \mathsf{SNR}_{n,k}} + [\mathbf{D}]_{n,n}$$
(28)

$$=1+\sum_{k\in\mathcal{K}_n}\frac{\mathsf{SNR}_{n,k}}{1+\mathsf{SNR}_{n,k}}+\sum_{k\notin\mathcal{K}_n}\mathsf{SNR}_{n,k}.\tag{29}$$

The signals being borne by the zeroed-out entries of $\hat{\mathbf{C}}$ have been moved to the augmented noise \mathbf{z} and, with a proper choice of $\mathcal{K}_0, \dots, \mathcal{K}_{N-1}$, the rows of $\hat{\mathbf{C}}$ have a balanced number of nonzero entries.

Next, let us curb to a subset \mathcal{N}_k the APs that participate in the reception of user k. This can be effected by a zeroing process dual to that of (27), namely

$$[\hat{\mathbf{C}}]_{n,k} = \begin{cases} [\hat{\mathbf{C}}]_{n,k} & n \in \mathcal{N}_k \\ 0 & \text{otherwise,} \end{cases}$$
(30)

and, with a proper choice of \mathcal{N}_k , the columns of the final $\hat{\mathbf{C}}$ are also balanced for every k.

Based on the sparsified channel, the PIC for user k is applied as

$$y - \sum_{\ell \neq l} \hat{\mathbf{c}}_{\ell} \hat{x}_{\ell}, \tag{31}$$

where $\hat{x}_{\ell} = \mathsf{a}_{\ell} \, \hat{\mathsf{c}}_{\ell}^* \mathsf{D}^{-1} \boldsymbol{y}$ and

$$\mathbf{a}_{\ell} = \frac{\mathsf{sinr}_{\ell}^{\mathsf{MF}}}{1 + \mathsf{sinr}_{\ell}^{\mathsf{MF}}} \frac{1}{\hat{\mathbf{c}}_{\ell}^* \mathbf{D}^{-1} \hat{\mathbf{c}}_{\ell}}$$
(32)

with $\operatorname{sinr}_{\ell}^{\operatorname{MF}}$ the value obtained when user ℓ is MF-received by the APs that constitute \mathcal{N}_{ℓ} . Altogether, the linear PIC receiver for user k is now

$$\mathbf{w}_{k}^{\mathsf{PIC}} = \mathbf{D}^{-1} \hat{\mathbf{c}}_{k} - \sum_{\ell \neq k} \frac{\mathsf{sinr}_{\ell}^{\mathsf{MF}}}{1 + \mathsf{sinr}_{\ell}^{\mathsf{MF}}} \frac{\hat{\mathbf{c}}_{\ell}^{*} \mathbf{D}^{-1} \hat{\mathbf{c}}_{k}}{\hat{\mathbf{c}}_{\ell}^{*} \mathbf{D}^{-1} \hat{\mathbf{c}}_{\ell}} \mathbf{D}^{-1} \hat{\mathbf{c}}_{\ell}$$
(33)

and

$$\operatorname{sinr}_{k}^{\operatorname{PIC}} = \frac{|\mathbf{w}_{k}^{\operatorname{PIC}*} \hat{\mathbf{c}}_{k}|^{2}}{\sum_{\ell \neq k} |\mathbf{w}_{k}^{\operatorname{PIC}*} \hat{\mathbf{c}}_{\ell}|^{2} + \frac{\sigma^{2}}{P} \mathbf{w}_{k}^{\operatorname{PIC}*} \mathbf{D} \mathbf{w}_{k}^{\operatorname{PIC}}}.$$
 (34)

What remains is to determine the composition of \mathcal{N}_k and \mathcal{K}_n to ensure that $\hat{\mathbf{C}}$ is sparse, but balanced, and that the performance of the network-wide PIC receiver is approached. These subsets should be determined from large-scale quantities only, such that they are relatively stable in time and frequency.

A. Subset Selection

For user k, the number of AP subsets of size $|\mathcal{N}_k|$ is

$$\frac{N!}{(|\mathcal{N}_k|)! (N - |\mathcal{N}_k|)!},\tag{35}$$

hence an exhaustive inspection is out of the question. Rather, to select $\mathcal{N}_0, \ldots, \mathcal{N}_{K-1}$ we embrace the policy propounded in [22] for MF and in [4], [6] for MMSE reception, whereby \mathcal{N}_k contains the $|\mathcal{N}_k|$ APs whose $G_{n,k}$ is largest.

In terms of the selection of \mathcal{K}_n , a dual policy of the one adopted for \mathcal{N}_k seems sensible. However, an added requirement exists: the nth AP should always estimate the channels of users in whose reception it participates. We hence propose that \mathcal{K}_n contain the union of:

- A fixed number of the users whose $G_{n,k}$ are largest.
- All users for which $n \in \mathcal{N}_k$, i.e., being received by AP n.

Provided the subset cardinalities are fixed, scalability is guaranteed. Precisely, computing $\mathbf{a}_0,\dots,\mathbf{a}_{K-1}$ requires $\hat{\mathbf{c}}_\ell^*\mathbf{D}^{-1}\hat{\mathbf{c}}_\ell$ for $\ell=0,\dots,K-1$, with a cost of $\mathcal{O}\big(\sum_{\ell=0}^{K-1}|\mathcal{N}_k|\big)$, and subsequently producing $\hat{x}_0,\dots,\hat{x}_{K-1}$ has a cost of the same order. Likewise, (31) and the necessary MF applications have a cost of that same order. All in all, the cost of obtaining and applying $\mathbf{w}_0^{\mathrm{pic}},\dots,\mathbf{w}_{K-1}^{\mathrm{pic}}$ satisfies

$$\frac{M^{\text{PIC}}}{N} = \mathcal{O}\left(\frac{\sum_{k=0}^{K-1} |\mathcal{N}_k|}{N}\right),\tag{36}$$

which is $\mathcal{O}(1)$ for fixed K/N. In turn,

$$\frac{L^{\text{PIC}}}{N} = \frac{\sum_{n=0}^{N-1} |\mathcal{K}_n|}{N} = \mathcal{O}(1).$$
 (37)

B. Performance Evaluation

Let the AP subset size $|\mathcal{N}|$ be identical for all users, while $\mathcal{K}_0,\ldots,\mathcal{K}_{N-1}$ contain the $\frac{K}{N}|\mathcal{N}|$ users with strongest channels to the respective APs in union with the users being received by each such AP. Presented in Fig. 4 is the distribution of the user SINR with sparse PIC reception, parameterized by $|\mathcal{N}|$. Remarkably, very small subset sizes suffice to essentially match the performance of network-wide PIC reception.

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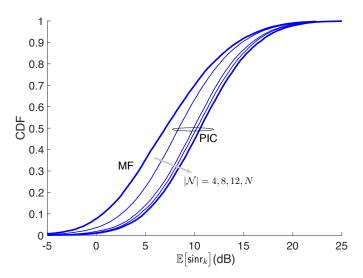


Fig. 4. CDF of the user SINR for MF and linear PIC receivers with K/N=0.25 and a dual-slope pathloss with breaking distance equal to d. The PIC performance is parameterized by $|\mathcal{N}|$, with $|\mathcal{K}|/|\mathcal{N}|=K/N$.

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