# Device Detection and Channel Estimation in MTC with Correlated Activity Pattern

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Abstract—This paper provides a solution for the activity detection and channel estimation problem in grant-free access with correlated device activity patterns. In particular, we consider a machine-type communications (MTC) network operating in event-triggered traffic mode, where the devices are distributed over clusters with an activity behaviour that exhibits both intra-cluster and inner-cluster sparsity patterns. Furthermore, to model the network's intra-cluster and inner-cluster sparsity, we propose a structured sparsity-inducing spike-and-slab prior which provides a flexible approach to encode the prior information about the correlated sparse activity pattern. Furthermore, we drive a Bayesian inference scheme based on the expectation propagation (EP) framework to solve the JUICE problem. Numerical results highlight the significant gains obtained by the proposed structured sparsity-inducing spike-and-slab prior in terms of both user identification accuracy and channel estimation performance.

Index Terms—Bayesian inference, grant-free MTC, EP, structured sparsity

#### I. INTRODUCTION

Sparse signal recovery techniques have become prevalent in the development of solutions for machine-type communications (MTC) with grant-free access protocols. One of the main challenges in grant-free access is joint user identification and channel estimation (JUICE). Motivated by the sporadic nature of the activity pattern of the MTC devices, namely user equipments (UEs), JUICE has been approached as a problem of sparse recovery and addressed through several algorithms, including approximate message passing (AMP), sparse Bayesian learning (SBL), and mixed-norm minimization. Most of the prior work on the JUICE considers MTC networks with a random UE activity pattern [1]–[4]. This could model, e.g., a scenario where UEs monitor independent random processes and thus activate randomly based on certain application criteria.

This paper makes the following distinction from the prior works: we consider an MTC network where the UEs are clustered in groups around the epicentre of alarm-event, thus, rendering their activity highly correlated. For instance, this models a network where the UEs form clusters based on their geographical locations and each cluster is associated with a monitoring task. Here, an event could trigger a small subset of UEs belonging to a cluster to activate concurrently, leading to clustered UE activity system-wise.

This paper addresses the JUICE in MTC under correlated user activity. More precisely, We propose a solution based on a variational Bayesian inference framework that utilizes a structured spike-and-slab model [5] to account for such correlation of the UEs sparse activity. Moreover, we derived an expectation propagation-based (EP) algorithm [6] to solve the Bayesian inference problem under the structured activity pattern. Numerical results demonstrate the clear advantages of the proposed solution over state-of-the-art sparse recovery algorithms.

#### **II. SYSTEM MODEL AND PROBLEM FORMULATION**

We consider a single-cell uplink network consisting of a set  $\mathcal{N}$  of N UEs served by a single BS equipped with a uniform linear array (ULA) of M antennas. The UEs are geographically distributed so that they form  $N_c$  clusters. For simplicity, we assume that each cluster contains L UEs such that  $N = LN_c$ , but the extension to a more general case is conceptually straighforward. A cluster containing a subset of UE indices is denoted by  $C_l \subseteq \{1, 2, \ldots, N\}$ . We consider a block Rayleigh fading channel response  $\mathbf{h}_i \sim (\mathbf{0}, \beta_i \mathbf{I}_M) \in \mathbb{C}^M$ , where  $\beta_i$  represents the unknown path-loss and shadowing component. In addition, the BS assigns to each UE  $i \in \mathcal{N}$ a unique unit-norm pilot sequence  $\phi_i \in \mathbb{C}^{\tau_p}$ . Accordingly, the received signal associated with the transmitted pilots at the BS,  $\mathbf{Y} \in \mathbb{C}^{\tau_p \times M}$ , is given by

$$\mathbf{Y} = \sum_{i=1}^{N} \gamma_i \phi_i \mathbf{h}_i^{\mathrm{T}} + \mathbf{W} = \mathbf{\Phi} \mathbf{X}^{\mathrm{T}} + \mathbf{W}, \qquad (1)$$

where  $\gamma_i = 0$  when the *i*th is active and  $\gamma_i = 0$  when *i*th UE is inactive,  $\mathbf{W} \in \mathbb{C}^{\tau_p \times M}$  is an additive white Gaussian noise with independent and identically distributed (i.i.d.) elements as  $\mathcal{CN}(0, \sigma^2)$ ,  $\mathbf{\Phi} = [\boldsymbol{\phi}_1, \dots, \boldsymbol{\phi}_N] \in \mathbb{C}^{\tau_p \times N}$ , and  $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_N] \in \mathbb{C}^{M \times N}$ , with  $\mathbf{x}_i = \gamma_i \mathbf{h}_i$ .

In contrast to the majority of the literature on grant-free access MTC that consider random UE activation, we consider herein the following technical observations on MTC under the event-triggered traffic model: i) the UEs activity is triggered by event concentrated around a very small subset of *active* clusters, thus, giving rise to an *inner cluster* sparsity structure. ii) An active cluster refers to any cluster with at least one active UE, while containing at most  $L_c \leq L$  active UEs, thus, inducing a correlation between the UEs activity in the form of *intra-cluster* sparsity structure.

Therefore, in order to encode the prior knowledge on both the intra and inner-cluster sparsity of the network, we introduce first the following parameters:

- The binary indicator variable c<sub>l</sub>, l = 1,..., N<sub>c</sub>, that controls the intra-cluster sparsity, defined as c<sub>l</sub> = 1 if the *l*th cluster is active, and c<sub>l</sub> = 0 otherwise. Thus, we can statistically model c<sub>l</sub> as a Bernoulli random variable with p(c<sub>l</sub> = 1) = ε and p(c<sub>l</sub> = 0) = 1 ε.
- 2) The hyper-parameter  $\bar{\gamma}_i \in \mathbb{R}^+$ ,  $i \in \mathcal{N}$  that controls model the intra-cluster sparsity. Ideally, we aim to estimate  $\bar{\gamma}_i = \gamma_i \beta_i$ .

Subsequently, we can model the effective channel  $\mathbf{x}_i$ ,  $\mathcal{N}$ , using the the structured spike-and-slab prior as

$$p(\mathbf{x}_i|c_l,\bar{\gamma}_i) = (1-c_l)\delta(\mathbf{x}_i) + c_l \mathcal{CN}(\mathbf{x}_i;\mathbf{0},\bar{\gamma}_i\mathbf{I}_M).$$
 (2)

The main idea in (2) can be summarized as follow

- If  $c_i = 0$ , the vector  $\mathbf{x}_i$  would have only the spike component, delta function, from (2), thus estimated as  $\mathbf{x}_i = \mathbf{0}$ .
- If c<sub>l</sub> = 1, x<sub>i</sub> would have only the slab component from (2) in the form be a Gaussian random vector with covariance \(\overline{\gamma\_i}\) I<sub>M</sub>. Therefore, if \(\overline{\gamma\_i}\) \(\pi\) o, the variance of the slab component in (2) would be very small that we could safely estimate that x<sub>i</sub> ≈ 0, whereas if \(\overline{\gamma\_i}\) > 0, x<sub>i</sub> would be a non-zero Gaussian random vector.

#### III. A BAYESIAN INFERENCE SOLUTION VIA EP

The JUICE problem can be formulated from a Bayesian perspective as maximum *a posteriori* probability (MAP) problem as follows

$$\begin{aligned} \{\hat{\mathbf{X}}, \hat{\mathbf{c}}, \hat{\boldsymbol{\gamma}}\} &= \max_{\mathbf{X}, \mathbf{c}, \bar{\boldsymbol{\gamma}}} p(\mathbf{X}, \mathbf{c}, \hat{\boldsymbol{\gamma}} | \mathbf{Y}) = \max_{\mathbf{X}, \mathbf{c}, \bar{\boldsymbol{\gamma}}} \frac{1}{p(\mathbf{Y})} p(\mathbf{Y} | \mathbf{X}) p(\mathbf{X} | \bar{\boldsymbol{\gamma}}, \mathbf{c}) p(\mathbf{c}) \\ &= \max_{\mathbf{X}, \mathbf{c}, \bar{\boldsymbol{\gamma}}} \frac{1}{p(\mathbf{Y})} f_1(\mathbf{X}) f_2(\mathbf{X}, \mathbf{c}, \bar{\boldsymbol{\gamma}}) f_3(\mathbf{c}), \end{aligned}$$

where

$$f_{1}(\mathbf{X}) = p(\mathbf{Y}|\mathbf{X}) = C\mathcal{N}(\mathbf{Y}; \mathbf{\Phi}\mathbf{X}, \sigma^{2}\mathbf{I}),$$

$$f_{2}(\mathbf{X}, \mathbf{c}, \bar{\boldsymbol{\gamma}}) = p(\mathbf{X}|\boldsymbol{\gamma}, \mathbf{c}) = \prod_{l=1}^{N_{c}} f_{2}(\mathbf{X}_{\mathcal{C}_{l}}, c_{l}, \bar{\boldsymbol{\gamma}}_{\mathcal{C}_{l}})$$

$$= \prod_{l=1}^{N_{c}} \left[ (1 - c_{l})\delta(\mathbf{X}_{\mathcal{C}_{l}}) + c_{l} \prod_{i \in \mathcal{C}_{l}} C\mathcal{N}(\mathbf{x}_{i}; \mathbf{0}, \gamma_{i}\mathbf{I}_{M}) \right],$$

$$f_{3}(\mathbf{c}) = p(\mathbf{c}) = \prod_{l=1}^{C} \mathcal{B}(\mathbf{c}_{l}|\epsilon).$$
(4)

Unfortunately, the optimization problem (3) is intractable for large N due to the presence of the delta function. Thus, we settle for an approximated solution to (3). In particular, we invoke the expectation propagation (EP) framework of [7]. In EP, the main objective is to approximate iteratively the probability distributions in the true posterior  $p(\mathbf{X}, \mathbf{c}, \bar{\boldsymbol{\gamma}} | \mathbf{Y})$  by a simpler distribution  $Q(\mathbf{X}, \boldsymbol{\gamma}, \mathbf{c})$  that belongs to an exponential family. More precisely, EP aims is to approximate the factors  $f_1(\cdot), f_2(\cdot), f_3(\cdot)$  by  $q_1(\cdot), q_2(\cdot), q_3(\cdot)$ , respectively, such that

$$p(\mathbf{X}, \mathbf{c}, \bar{\boldsymbol{\gamma}} | \mathbf{Y}) \approx Q(\mathbf{X}, \boldsymbol{\gamma}, \mathbf{c}) = \frac{1}{K^{\text{EP}}} q_1(\mathbf{X}) q_2(\mathbf{X}, \mathbf{c}) q_3(\mathbf{c}).$$
(5)

In the EP framework, each factor  $q_k(\cdot)$ , k = 1, 2, 3, of the joint variations approximation  $Q(\mathbf{X}, \gamma, \mathbf{c})$  is obtained by minimizing iteratively the Kullback-Leibler divergence [8] as

$$q_k^* = \min_{q_k} \operatorname{KL}\left(f_k(\cdot)Q^{\setminus k}(\cdot)||q_k(\cdot)Q^{\setminus k}(\cdot)\right)$$
(6)

where  $Q^{\setminus k}(\cdot) = \frac{Q(\cdot)}{q_k(\cdot)}$ .

### IV. NUMERICAL RESULTS

Fig. 1 compares the performance of *the proposed EP* solution in terms of normalized mean square error (NMSE) and support recovery rate (SRR) against two sparse recovery algorithms: iterative reweighted  $\ell_{2,1}$ -norm minimization (IRW- $\ell_{2,1}$ ) [4], and M-SBL [9] as well as an oracle minimum mean square error (MMSE) estimator that is given both the set of true active UEs and the exact values of  $\beta_i$ , i = 1, ..., N.

Fig. 1 shows two main features for the proposed solution. 1) The proposed algorithm, which considers the activity correlation, provides a significant gain over M-SBL and IRW- $\ell_{2,1}$  in terms of both channel estimation quality and activity detection accuracy. In fact, the proposed EP algorithm provides nearoptimal NMSE performance by approaching the performance provided by the oracle MMSE denoiser which is computed with the set of true active UEs given by the oracle. 2) Although the proposed algorithm performance degrades when the activity correlation is not taken into consideration by setting each cluster to contain only one UE, i.e.,  $N_c = N$ , it still outperforms IRW- $\ell_{2,1}$  and matches the performance of M-SBL. The obtained results highlight clearly: 1) the importance of using the structured spike-and-slab prior, 2) the gains obtained by using the EP framework to solve the MAP problem.

#### V. CONCLUSIONS AND EXTENSIONS IN THE FINAL PAPER

We provided a solution for activity detection and channel estimation in grant-free MTC under correlated activity patterns. First, we introduced the structured spike-and-slab model, which allows for incorporating the prior knowledge of the network traffic pattern. Second, we derived an EPbased approximation to solve the JUICE formulation under the variational Bayesian framework.

In the final paper, we will provide in detail the derivations for the proposed EP algorithm. Furthermore, we will discuss in more detail the computational complexity of the algorithms and propose a few modifications aiming to reduce the computational costs while maintaining the same performance. Finally, we will provide more simulation results to quantify



Fig. 1: Performance evaluation of the proposed algorithm with 2 active clusters each containing 8 active UEs, N = 200,  $N_C = 20$ , M = 10.

the effect of system parameters, such as the number of BS antennas, transmission power, etc.

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