DFT-Based Channel Estimation for Holographic MIMO

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Abstract—Holographic MIMO (hMIMO) systems with a massive number N of individually controlled antennas make minimum mean square error (MMSE) channel estimation particularly challenging, due to its computational complexity that scales as N^3 . This paper investigates uniform linear arrays and proposes a low-complexity method based on the discrete Fourier transform approximation, which follows from replacing the covariance matrix by a suitable circulant matrix. Numerical results show that, already for arrays with moderate size (in the order of tens of wavelengths), it achieves the same performance of the optimal MMSE, but with a significant lower computational load that scales as $N \log N$. Interestingly, the proposed method provides also increased robustness in case of imperfect knowledge of the covariance matrix.

Index Terms—Holographic MIMO, channel estimation, circulant matrix, uniform linear arrays, covariance matrix estimation.

I. INTRODUCTION AND MOTIVATION

Communication theorists are always on the lookout for new technologies to improve the speed and reliability of wireless communications. One such technology that has shown significant progress is multiple antenna technology, with the latest version being massive multiple-input multiple-output (MIMO) [1], which was introduced with the advent of 5G [2]. Researchers are now exploring ways to deploy massive MIMO with more antennas and optimized signal processing, given the potential benefits of numerous antennas. This technology evolution was named massive MIMO 2.0 [3], and new research directions are being pursued under different names, such as holographic MIMO (hMIMO) [2], [4], extremely large-scale MIMO [5], and large intelligent surfaces [6].

The capacity of such technology evolution is theoretically unlimited [3], but is practically limited, when the number of antennas increases, by the high computational complexity and the ability to learn the spatial channel correlation matrices. In a hMIMO system with thousands of antennas, it is challenging to both acquire the spatial correlation matrix and implement the minimum mean square error (MMSE) estimator [7]. The channel sparsity in the angular domain was exploited in [8] to perform channel estimation while reducing the pilot overhead, and [9] exploited the polar-domain sparsity when the angulardomain one is not applicable. To reduce the complexity, [7] derives a subspace-based channel estimation approach based on the rank deficiency of the spatial correlation matrix caused by the hMIMO geometry. In this case, the knowledge of the channel statistics is not required, and the complexity is reduced by considering isotropic scattering, which includes all possible angular spreads.

Unlike the aforementioned literature, we propose a different low-complexity channel estimation scheme, based on the discrete Fourier transform (DFT), and derived from a suitable circulant approximation of the channel covariance matrix [10]-[12]. Unlike [7], the estimation of the channel covariance matrix is required. To this aim, we also propose an improved, low-complexity algorithm to estimate the channel correlation matrix. Numerical results show that the proposed method provides almost the same accuracy of the optimal MMSE estimator, while significantly reducing the complexity thanks to the DFT processing. This holds true for arrays of moderate size (order of tens of wavelenghts). Furthermore, when considering imperfect knowledge of the channel covariance matrix, the DFT-based approach guarantees higher robustness and stability compared to the MMSE method, thanks to a simpler eigenvalue structure.

II. SYSTEM AND CHANNEL MODEL

We consider a hMIMO system where the base station (BS) is equipped with a vertical uniform linear array (ULA) located in the yz plane, and consisting of N antennas, with interelement spacing d [13, Fig. 1]. The location of the *n*th antenna with respect to the origin is $\mathbf{u}_n = [0, 0, nd]^{\mathsf{T}}$, with $n = 0, \ldots, N - 1$.¹ If a planar wave is impinging on the ULA from the azimuth angle φ and elevation angle θ , the array response vector is [1, Sect. 7.3]

$$\mathbf{a}(\varphi,\theta) = \left[e^{\mathbf{j}\mathbf{k}(\varphi,\theta)^{\mathsf{T}}\mathbf{u}_{1}}, \dots, e^{\mathbf{j}\mathbf{k}(\varphi,\theta)^{\mathsf{T}}\mathbf{u}_{N}}\right]^{\mathsf{T}}$$
(1)

where $\mathbf{k}(\varphi, \theta) = \frac{2\pi}{\lambda} [\cos(\theta) \cos(\varphi), \cos(\theta) \sin(\varphi), \sin(\theta)]^{\mathsf{T}}$ is the wave vector for a planar wave at wavelength λ . We call $\mathbf{h}_k \in \mathbb{C}^N$ the channel vector between the single-antenna user equipment (UE) k and the BS, and assume that it consists of a superposition of multipath components that can be expanded as a continuum of plane waves [14]. Hence, we have

$$\mathbf{h}_{k} = \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} g_{k}(\varphi, \theta) \mathbf{a}(\varphi, \theta) d\theta d\varphi$$
(2)

where the angular spreading function $g_k(\varphi, \theta)$ specifies gain and phase-shift from each direction (φ, θ) , and depends on UE k's position (i.e., angles φ_k and θ_k with respect to the ULA).

¹The analysis is valid for any orientation of the ULA with respect to the reference system, and can be extended to an horizontal ULA straightforwardly.

We consider the conventional block fading model, where the channel \mathbf{h}_k is constant within one time-frequency block and takes independent realizations across blocks from a stationary stochastic distribution. In accordance with [14], we model $g_k(\varphi, \theta)$ as a spatially uncorrelated circularly symmetric Gaussian stochastic process with cross-correlation

$$\mathbb{E}\{g_k(\varphi,\theta)g_k^*(\varphi',\theta')\} = \beta_k f_k(\varphi,\theta)\delta(\varphi-\varphi')\delta(\theta-\theta') \quad (3)$$

where β_k is the average channel gain and $f_k(\varphi, \theta)$ is the normalized *spatial scattering function* [14] such that $\iint f_k(\varphi, \theta) d\theta d\varphi = 1$. By using (1), the elements of $\mathbf{R}_k = \mathbb{E}\{\mathbf{h}_k \mathbf{h}_k^H\}$ are computed as [1, Sect. 7.3.2]

$$[\mathbf{R}_k]_{m,l} = \beta_k \iint e^{j\mathbf{k}(\varphi,\theta)^{\mathsf{T}}(\mathbf{u}_m - \mathbf{u}_l)} f_k(\varphi,\theta) d\varphi d\theta \qquad (4)$$

where the integration is over all angles. If a vertical ULA is used, the expression simplifies as

$$[\mathbf{R}_k]_{m,l} = \beta_k \int e^{j\frac{2\pi}{\lambda}d(m-l)\sin(\theta)} f_k(\theta)d\theta$$
 (5)

where $f_k(\theta) = \int f_k(\varphi, \theta) d\varphi$.

III. CHANNEL ESTIMATION WITH PERFECT KNOWLEDGE

We assume that channel estimation is performed by using orthogonal pilot sequences of length τ_p . We call $\phi_k \in \mathbb{C}^{\tau_p}$ the pilot sequence used by UE k and assume that $|[\phi_k]_i|^2 = 1$ and $\phi_k^{\mathsf{T}} \phi_k^* = \tau_p$. In the absence of pilot contamination and with perfect knowledge of channel statistics, the linear MMSE estimate of \mathbf{h}_k based on the observation vector $\mathbf{y}_k = \tau_p \sqrt{\rho} \mathbf{h}_k + \mathbf{w}$, where ρ is the transmit power, and $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \tau_p \sigma^2 \mathbf{I}_N)$, is [1, Sect. 3]

$$\widehat{\mathbf{h}}_{k}^{\mathsf{MMSE}} = \mathbf{A}_{k}^{\mathsf{MMSE}} \mathbf{y}_{k} \tag{6}$$

where

$$\mathbf{A}_{k}^{\mathsf{MMSE}} = \frac{1}{\tau_{p}\sqrt{\rho}}\mathbf{R}_{k}\mathbf{Q}_{k}^{-1}$$
(7)

with $\mathbf{Q}_k = \mathbf{R}_k + \frac{1}{\gamma} \mathbf{I}_N$ and $\gamma = \tau_p \rho / \sigma^2$ denoting the signalto-noise ratio (SNR). The MMSE channel estimator in (6) is optimal, but it requires an intense computational effort when N grows large. Indeed, once \mathbf{R}_k is computed, $\mathcal{O}(N^3)$ operations are needed for the pre-computation of $\mathbf{A}_k^{\text{MMSE}}$. The computation of (6) requires a matrix-vector product evaluation whose complexity is $\mathcal{O}(N^2)$. Its overall complexity is reported in Table I. Note also that the MMSE estimator relies on the perfect knowledge of \mathbf{R}_k , which needs to be estimated.

An alternative estimation scheme is the least-squares (LS) estimator $\hat{\mathbf{h}}_{k}^{\text{LS}} = \mathbf{A}_{k}^{\text{LS}} \mathbf{y}_{k}$, with

$$\mathbf{A}_{k}^{\mathsf{LS}} = \frac{1}{\tau_{p}\sqrt{\rho}}\mathbf{I}_{N} \tag{8}$$

which utilizes no prior information on the channel statistics and array geometry. Unlike the MMSE channel estimator, it does not require any pre-computation phase and has a complexity of $\mathcal{O}(N)$, due to the product between the diagonal matrix $\mathbf{A}_{k}^{\text{LS}}$ and \mathbf{y}_{k} . The price to pay is a reduced accuracy.

 TABLE I

 COMPLEXITY OF CHANNEL ESTIMATION SCHEMES.

scheme	pre-computation of \mathbf{A}_k	computation of $\mathbf{A}_k \mathbf{y}_k$
MMSE	$\mathcal{O}(N^3)$	$\mathcal{O}(N^2)$
LS	-	$\mathcal{O}(N)$
LoS	-	$\mathcal{O}(N)$
ISO	-	$\mathcal{O}(N^2)$
DFT	$\mathcal{O}(N \log N)$	$\mathcal{O}(N \log N)$

Two other alternatives are described next and can be applied in specific propagation conditions. If propagation is assumed to take place in a line-of-sight (LoS) scenario with a single planewave arriving from θ_k and φ_k , then $\mathbf{h}_k = g_k(\varphi_k, \theta_k) \mathbf{a}(\varphi_k, \theta_k)$ and $\mathbf{R}_k^{\text{LoS}} = \beta_k \mathbf{a}(\varphi_k, \theta_k) \mathbf{a}(\varphi_k, \theta_k)^{\text{H}}$. Replacing \mathbf{R}_k with $\mathbf{R}_k^{\text{LoS}}$ into (6) yields $\hat{\mathbf{h}}_k^{\text{LoS}} = \mathbf{A}_k^{\text{LoS}} \mathbf{y}_k$, where

$$\mathbf{A}_{k}^{\text{LoS}} = \frac{1}{\tau_{p}\sqrt{\rho}} \frac{\beta_{k}\gamma}{1+N\beta_{k}\gamma} \mathbf{a}(\varphi_{k},\theta_{k})\mathbf{a}(\varphi_{k},\theta_{k})^{\mathsf{H}}$$
(9)

whose complexity is $C_{\text{LoS}} = \mathcal{O}(N)$, due to the evaluation of the product between $\mathbf{A}_k^{\text{LoS}}$ in (9) and \mathbf{y}_k (no pre-computation phase is required). However, the LoS-based estimator works well only when the channel vector is generated by a single plane-wave arriving from (φ_k, θ_k) , whose knowledge must be perfectly known at the BS. When the propagation scenario is highly scattered, and plane waves arrive uniformly within the angular domain in front of the ULA, we can make use of the isotropic (ISO) approximation proposed in [15]. According to [15], $\mathbf{R}_k^{\text{ISO}} = \overline{\mathbf{UAU}}^{\text{H}}$ where $\overline{\mathbf{U}}$ and $\overline{\mathbf{A}}$ are the (reduced-order) eigenvector and eigenvalue matrices, obtained through the *compact* eigenvalue decomposition of a matrix whose (m, l)th entry is sinc $[2 (m - l) d/\lambda]$, with $\operatorname{sinc}(x) = \frac{\sin(\pi x)}{(\pi x)}$. Note that the rank of $\mathbf{R}_k^{\text{ISO}}$ is approximately $2Nd/\lambda$, given by the degrees of freedom observed in the ISO propagation conditions [16]. Replacing \mathbf{R}_k with $\mathbf{R}_k^{\text{ISO}}$ into (6) yields $\widehat{\mathbf{h}}_k^{\text{ISO}} = \mathbf{A}_k^{\text{ISO}} \mathbf{y}_k$, with

$$\mathbf{A}_{k}^{\mathsf{ISO}} = \frac{1}{\tau_{p}\sqrt{\rho}} \overline{\mathbf{U}} \overline{\mathbf{\Lambda}} \left(\overline{\mathbf{\Lambda}} + \frac{1}{\gamma} \mathbf{I}_{N} \right)^{-1} \overline{\mathbf{U}}^{\mathsf{H}}.$$
 (10)

Compared to MMSE, the main advantage of using the ISO estimator derives from the fact that it does not require any matrix estimation and inversion, since all the quantities in $\mathbf{A}_k^{\text{ISO}}$ are known. Accordingly, its complexity is only due to the matrix-vector product computation between $\mathbf{A}_k^{\text{ISO}}$ and \mathbf{y}_k and is $C_{\text{ISO}} = \mathcal{O}(N^2)$. Note also that the ISO estimator does not require any prior knowledge of the channel statistics and can be applied to any propagation conditions, since the eigenspace of $\mathbf{R}_k^{\text{ISO}}$ covers the eigenspace of any spatial correlation matrix \mathbf{R}_k [15], and exploits the array geometry only.

A. Discrete Fourier transform approximation

We now develop a channel estimation scheme that exploits the correlation induced by the array geometry and propagation conditions to approach MMSE performance, while having a computational complexity that scales log-linearly with N. To this aim, we proceed as follows.



Fig. 1. NMSEE as a function of the ratio L/λ .

If an ULA is used, the covariance matrix \mathbf{R}_k is Hermitian Toeplitz, and it can be approximated with a suitable circulant matrix \mathbf{C}_k [10]–[12], whose first row $\mathbf{c}_k = [c_k(0), c_k(1), \cdots, c_k(N-1)]$ is related to the first row $\mathbf{r}_k = [r_k(0), r_k(1), \cdots, r_k(N-1)]$ of \mathbf{R}_k by [12]

$$c_k(n) = \begin{cases} r_k(0) & n = 0, \\ \frac{(N-n)r_k(n) + nr_k^*(N-n)}{N} & n = 1, \dots, N-1. \end{cases}$$
(11)

Any circulant matrix can be unitarily diagonalized using the DFT matrix, i.e., $\mathbf{C}_k = \mathbf{F} \mathbf{\Lambda}_k \mathbf{F}^{\mathsf{H}}$ where $\mathbf{F} = [\mathbf{f}_0 \mathbf{f}_1 \cdots \mathbf{f}_{N-1}]$ is the inverse DFT matrix, with $[\mathbf{f}_n]_m = e^{j2\pi mn/N} / \sqrt{N}$ for $0 \le m, n \le N-1$, and $\mathbf{\Lambda}_k$ is the diagonal matrix containing the eigenvalues of \mathbf{C}_k , i.e.,

$$[\mathbf{\Lambda}_k]_{n,n} = \sum_{m=0}^{N-1} c_k(m) e^{-j2\pi m n/N}$$
(12)

which are obtained by taking the DFT of the first row of \mathbf{C}_k . Replacing \mathbf{R}_k with \mathbf{C}_k into (6) yields $\hat{\mathbf{h}}_k^{\mathsf{DFT}} = \mathbf{A}_k^{\mathsf{DFT}} \mathbf{y}_k$, with

$$\mathbf{A}_{k}^{\mathsf{DFT}} = \frac{1}{\tau_{p}\sqrt{\rho}} \mathbf{F} \, \mathbf{\Lambda}_{k} \left(\mathbf{\Lambda}_{k} + \frac{1}{\gamma} \mathbf{I}_{N}\right)^{-1} \mathbf{F}^{\mathsf{H}}.$$
 (13)

We call it the *DFT*-based channel estimator. Its complexity derives from the pre-computation phase, which is $\mathcal{O}(N \log N)$ due to the computation of Λ_k through (12), and from the computation of the matrix-vector product, which is again $\mathcal{O}(N \log N)$, since the DFT matrix **F** and its inverse are involved. Hence, the complexity of the DFT-based estimator is $C_{\text{DFT}} = \mathcal{O}(N \log N)$. Unlike the ISO estimator, the DFTbased estimator depends on the true covariance matrix \mathbf{R}_k , which must be estimated as with the MMSE estimator.

B. Performance analysis

Fig. 1 shows the normalized mean square estimation error (NMSEE), given by [1, Sect. 3]

$$\frac{\operatorname{tr}(\mathbf{R}_{k}) - 2\sqrt{\rho}\tau_{p}\Re\left(\operatorname{tr}(\mathbf{R}_{k}\mathbf{A}_{k})\right) + \rho\tau_{p}^{2}\operatorname{tr}\left(\mathbf{A}_{k}\mathbf{Q}_{k}\mathbf{A}_{k}^{\mathsf{H}}\right)}{\operatorname{tr}(\mathbf{R}_{k})}$$
(14)



Fig. 2. NMSEE as a function of σ_{θ} .

where A_k depends on the estimation strategies defined above (e.g., (7) for MMSE, and (8) for LS), as a function of the array size L, normalized with respect to the wavelength λ . We consider an ULA characterized by $d = \lambda/4$ at 3 GHz (and hence $\lambda = 10$ cm). Three estimation schemes are considered: MMSE, LS, and DFT. We evaluate the average performance for a UE randomly placed in the sector $\varphi \in [-\pi/3, +\pi/3]$ of a cell with minimum and maximum distances from the ULA of 5 and 100 meters, respectively. The ULA is elevated by $b = 10 \,\mathrm{m}$ with respect to the UE plane (and thus, considering the distance range, $\theta \in [-63.4^\circ, -5.7^\circ]$, with negative elevations due to the fact that the UE plane is below the ULA), and the received SNR is $\beta_k \tau_p \rho / \sigma^2$, where: β_k is computed following [1, Sect. 2] assuming a reference distance of 1 km, a path loss exponent $\alpha = 3.76$, and a channel gain at 1 km equal to $-148.1 \,\mathrm{dB}$; $\tau_p = 10$; $\rho = 20 \,\mathrm{dBm}$; and $\sigma^2 = -87 \, \text{dBm}$, obtained considering a communication bandwidth B = 100 MHz. We assume a local scattering model with a Laplacian distribution characterized by angular scattering spread $\sigma_{\theta} = 10^{\circ}$. We see that the accuracy of the DFT-based estimator is comparable with the optimal MMSE one, and the gap decreases as L/λ increases. This is due to the fact that the circulant approximation C_k of the covariance matrix \mathbf{R}_k improves as the number of antennas N (or, equivalently, the ratio L/λ) grows large. Interestingly, the circulant approximation is already quite tight for $L/\lambda = 16$ (L = 1.6 m and N = 64). More importantly, this is obtained with a complexity of $\mathcal{O}(N \log N)$, instead of $\mathcal{O}(N^3)$. If N = 64, this corresponds to two orders of magnitude of computational saving compared to MMSE.

To evaluate the impact of the angular spread, Fig. 2 plots the NMSEE as a function of σ_{θ} in the same simulation setup of Fig. 1. The results show that the DFT-based estimator significantly outperforms (with a gap that increases with L/λ) both the LoS and the ISO-based estimators for values of σ_{θ} in the range (5°, 20°) and attains good performance compared to the (optimal) MMSE. As expected, the LoS estimator is close to optimal only for very low values of σ_{θ} .

IV. CHANNEL ESTIMATION WITH IMPERFECT KNOWLEDGE

So far, we have assumed perfect knowledge of \mathbf{R}_k . This may not be the case in practical scenarios since \mathbf{R}_k changes due to different reasons [3]. Measurements suggest roughly two orders of magnitude slower variations compared to the fast variations of channel vectors. Therefore, we may reasonably assume that they do not change over τ_s coherence blocks, where τ_s can be in the order of thousands [3]. Suppose that the BS has received the pilot matrix \mathbf{y}_k in $M \leq \tau_s$ coherence blocks. We denote these M observations by $\mathbf{y}_k[1], \ldots, \mathbf{y}_k[M]$. An estimate of \mathbf{Q}_k can be obtained by computing the sample correlation matrix given by

$$\widehat{\mathbf{Q}}_{k}^{\mathsf{sample}} = \frac{1}{M} \sum_{m=1}^{M} \mathbf{y}_{k}[m] \mathbf{y}_{k}^{\mathsf{H}}[m].$$
(15)

A better estimator is typically obtained through matrix regularization by computing the convex combination [3]:

$$\widehat{\mathbf{Q}}_{k}(\eta) = \eta \widehat{\mathbf{Q}}_{k}^{\mathsf{sample}} + (1 - \eta) \widehat{\mathbf{Q}}_{k}^{\mathsf{diag}} \quad \eta \in [0, 1]$$
(16)

where $\widehat{\mathbf{Q}}_{k}^{\text{diag}}$ contains the main diagonal of $\widehat{\mathbf{Q}}_{k}^{\text{sample}}$. Once $\widehat{\mathbf{Q}}_{k}(\eta)$ is computed, an estimate of \mathbf{R}_{k} follows:

$$\widehat{\mathbf{R}}_{k}(\eta) = \widehat{\mathbf{Q}}_{k}(\eta) - \frac{1}{\gamma}\mathbf{I}_{N}$$
(17)

which requires only knowledge of γ , i.e., the SNR during the pilot transmission phase.

A. Improved estimation of the channel correlation matrix

We now develop an improved estimation scheme of the correlation matrix \mathbf{Q}_k that can be used with ULAs. In this case, \mathbf{Q}_k is Hermitian Toeplitz, i.e.,

$$[\mathbf{Q}_k]_{1,j} = [\mathbf{Q}_k]_{1+m,j+m} \tag{18}$$

for $j = 1, \ldots, N-1$ and $m = 1, \ldots, N-j$, and $[\mathbf{Q}_k]_{i,j} = [\mathbf{Q}_k]_{j,i}^*$. To proceed, we denote by $\widehat{\mathbf{Q}}_k^{\text{toe}}$ the estimate of \mathbf{Q}_k obtained by taking the Toeplitz structure (18) into account. The first row of $\widehat{\mathbf{Q}}_k^{\text{toe}}$ is computed by simply averaging the entries of $\widehat{\mathbf{Q}}_k^{\text{sample}}$ in (15) over the diagonals, i.e.,

$$[\widehat{\mathbf{Q}}_{k}^{\text{toe}}]_{1,j} = \frac{1}{N-j+1} \sum_{m=1}^{N-j+1} [\widehat{\mathbf{Q}}_{k}^{\text{sample}}]_{m,j+m-1}.$$
 (19)

Once the first row is computed, the other elements are easily found. In particular, from (18) we have that

$$[\widehat{\mathbf{Q}}_{k}^{\text{toe}}]_{1+m,j+m} = [\widehat{\mathbf{Q}}_{k}^{\text{toe}}]_{1,j}$$
(20)

for j = 1, ..., N - 1 and m = 1, ..., N - j, and

$$[\widehat{\mathbf{Q}}_{k}^{\text{toe}}]_{j,i} = [\widehat{\mathbf{Q}}_{k}^{\text{toe}}]_{i,j}^{*} \quad \text{for } j > i$$
(21)

because of the Hermitian symmetry of the covariance matrix. An estimate of \mathbf{R}_k is finally obtained as

$$\widehat{\mathbf{R}}_{k}^{\text{toe}} = \widehat{\mathbf{Q}}_{k}^{\text{toe}} - \frac{1}{\gamma} \mathbf{I}_{N}.$$
(22)





Fig. 3. Box plot of the NMSEE as a function of the elevation θ with imperfect statistical knowledge (N = 64).

The complexity of the estimator above is mainly due to the computation of $\widehat{\mathbf{Q}}_{k}^{\text{sample}}$ in (19) and thus is comparable to the one not exploiting the Toeplitz structure. We can then replace the elements of \mathbf{R}_{k} with the ones in $\widehat{\mathbf{R}}_{k}^{\text{toe}}$ to implement (11), when considering the proposed DFT-based estimator with imperfect knowledge of channel statistics.

B. Performance analysis

We now evaluate the accuracy of the estimators when the covariance matrix is estimated using (22). Figs. 3(a) and (b) report the box charts for the MMSE and the DFTbased estimators, respectively, showing median, lower and upper quartiles, minimum and maximum non-outlier values, and outliers (the latter depicted by circular markers), computed over 200,000 independent realizations per box. Blue, red, and yellow boxes correspond to the different values of M considered for the estimation of the covariance matrix: M = 20, 50 and 100, respectively. The system setup is the same considered in Sect. III-B. For comparison, we also



Fig. 4. NMSEE as a function of the ratio L/λ with imperfect statistical knowledge (DFT-based estimator).

report the NMSEE with perfect knowledge of \mathbf{R}_k (dashed line). By inspecting Fig. 3, the following considerations can be drawn: the average estimation accuracy improves as the (absolute) elevation increases, thanks to a reduced distance (which is related to the elevation angle through $b/|\sin\theta|$), and thus to an increased SNR. However, especially for the MMSE case, the robustness of the estimator decreases as the (absolute) elevation increases, owing to the reduced array directivity at large elevations. This is confirmed by the huge presence of outliers, which highly affect the reliability of the MMSE estimation, especially when M decreases. This result is somewhat expected, as we are using a reduced set of snapshots compared to the degrees of freedom offered by the N-sized ULA, which prevents from a stable and accurate estimation of the channel statistics.

Most interestingly, although the same trends apply to both Figs. 3(a) and (b), a significant difference can be observed when focusing on the DFT-based estimator. As can be seen, not only the average performance is close to the one with perfect knowledge, but also the standard deviation is orders of magnitude lower than the MMSE counterpart, and so does the number of outliers, already at M = 20. This is due to a simpler estimation scheme, which requires a reduced number of independent realizations, and thus exhibits larger robustness. To provide further insights, Fig. 4 reports the average NMSEE (which also includes the outliers) as a function of the ratio L/λ , obtained by averaging over all possible UE positions in the range [5, 100] m and $\varphi \in [-\pi/3, +\pi/3]$, and using the same system parameters considered for Fig. 3. As can be seen, an estimation accuracy comparable to that obtained with perfect knowledge of \mathbf{R}_k is already achieved with M = 20. Similar conclusions can be drawn by considering different simulation setups (not reported for space limitations), in which different array sizes and/or scattering scenarios are considered.

V. CONCLUSION

We proposed a low-complexity scheme, based on the circulant approximation of the channel covariance matrix, to perform channel estimation in hMIMO systems equipped with ULAs. The estimation accuracy was evaluated with perfect and imperfect knowledge of the channel covariance matrix. Comparisons were made against the optimal MMSE estimator and other alternatives with lower complexity. The proposed scheme achieves close to optimal estimation accuracy for ULAs of moderate size (in the order of tens of wavelength), while considerably reducing the estimation complexity by a factor that scales with the square of array size. Moreover, it is more robust to the imperfect knowledge of channel statistics. This makes it more suited for applications in which the statistics changes rapidly over time and must be estimated frequently using a limited number of coherence blocks. Future work is needed to extend the proposed scheme to uniform planar arrays.

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