Operational Aircraft Routing Problem: Some Insights into the Capacitated Maintenance Resources

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Abstract - **In aviation industry, airlines solve operational aircraft routing problems (OARP) to assign maintenance-feasible routes to a fleet of aircrafts. Note that the scheduled aircrafts must be provided with sufficient maintenance resources to prevent costly recovery. Most existing OARP models assume that airlines are given fixed amount of maintenance resources. However, in the case of a capacitated maintenance stations serving more than one airline, resources available to an airline may be highly susceptible to the MP's resource allocation scheme.**

This paper aims to capture the uncertain supply of maintenance resources by modelling the market-based resource allocation schemes of MPs. Specifically, two possible scenarios in maintenance stations are explored: (1) each airline have no information about other airlines that shares the same maintenance station; (2) each airline have complete information about other airlines. This paper discusses game-theoretic models that can be used in these scenarios. It is shown that the equilibrium strategies of airlines in such games contribute to make proper OARP decisions by reducing maintenance misalignments.

Keywords - **Aircraft routing, maintenance resources, uncertainty, game-theoretic model**

I. INTRODUCTION

In aviation industry, airlines solve large logistical problems to optimize their operations. After flight scheduling and fleet assignment, a specific fleet is assigned to a set of scheduled flight legs. Then the aircraft routing problem is solved to generate and select a sequence of flight legs for each aircraft in the fleet [1]. It is required that each flight leg is covered exactly once while aircraft operational constraints are satisfied. At this stage, aircraft maintenance requirements are of great concern. On the one hand, it is necessary to guarantee all aircrafts are properly maintained to prevent aviation accidents [2]. On the other hand, airlines are striving to cut down maintenance and operational costs in today's highly competitive airline market [3]. Therefore, the operational aircraft maintenance routing problem (OARP) is proposed to assign short-term routes to each individual aircraft based on its unique maintenance requirements [4].

OARP are usually modelled with the objective of minimizing the maintenance cost of the fleet, while considering maintenance requirements imposed by aviation administration [5]. Aircraft requires various types of maintenance. Most existing studies adopt the traditional categorization--type A, B, C and D checks, but only consider type A check, which requires visual inspection of major systems in aircrafts [6]. Recently, both line maintenance and heavy maintenance are considered in OARP [7]. while the latter maintenance task requires hanger facilities. More than 50 different maintenance checks are generalized according to various maintenance workload due of each aircraft [8]. Generally, aircraft maintenance requires various combinations of resources, and these requirements are tailored for each aircraft based on their unique needs.

When making OARP decisions, airlines should consider the amount of maintenance resources available in the candidate stations. In current OARP models, to make full use of the aircraft, airlines usually target at minimizing the unused flying time of aircrafts [4]. Most aircrafts will not be maintained until the maintenance tasks become very "urgent". If the maintenance resources are insufficient, the aircrafts must be rescheduled to make sure that they are properly maintained elsewhere. To tackle these misalignments, expensive recovery operations are necessary [9]. Therefore, matching demand to supply is a great concern of airlines.

MP at a maintenance station serves aircrafts from more than one airline at the same time [10]. Maintenance resources are shared by these airlines. In the planning stage, given maintenance requirements required by airlines, MPs make maintenance schedules with the objective of profit maximization [11]. The maintenance schedule is characterized by flexibility due to complicated environments at the day of operation [12]. Usually, airlines sign maintenance service contracts with MPs, which are typically performance-based [13]. The price of this kind of contract depends on service level that an MP can provide. Therefore, it is reasonable to assume that MPs, with capacitated maintenance resources, employ market-based resource allocation scheme, which explicitly defines a price for the performance of each maintenance task. In fact, market-based resource allocation scheme is widely used in aviation industry (e.g., en-route airspace [14], airport slots [15], repairable aircraft components [16], etc.).

This paper identifies a research gap that most current OARP models assume the maintenance resources are given and fixed to each airline in the planning stage, which is infeasible due to the variable maintenance schedule of MP. To capture the uncertainties in of maintenance resources available to each airline, possible scenarios under market-based resource allocation schemes of MPs are discussed. Game-theoretic models are employed to describe the relationship between airlines and MPs, and competition among airlines. By adding considerations in uncertainties of maintenance resources, the OARP would yields more robust results which reduce costly maintenance misalignments. In Section.2, the research gap is discussed. Existing OARP model is briefly described in Section.3. We propose the game-theoretic models in Section.4. Section 5 concludes this paper with discussion of future research.

II. RESEARCH GAP

Many existing OARP models have already been constrained by the capacitated maintenance resources. An important assumption is that airlines know exactly how many resources are available for them at each maintenance station on the day of operation (see Table.1). However, this assumption is not realistic. There is no guarantee that airlines are provided with pre-determined and fixed amount of resources for their scheduled aircrafts at each maintenance station.

TABLE I
ASSUMPTION ON MAINTENANCE REQUIREMENTSY

Paper	Assumption in maintenance capacity	Resource type
[9]; [17]; [18]	No restriction in maintenance station	N/A
[1]	Only one balance-check per night	Slots
[2]; [19]; [20]; [21]; [22]	Given fixed maintenance capacity in each maintenance station	
[23]	Given suitable overestimate of the actual maintenance station capacity	
[4]	Given fixed maintenance capacities in each maintenance station	Slots; Man-hours
[8]	Given fixed number of maintenance opportunities	Generalized maintenance constraints

In the planning stage, airlines contracts with MPs about their maintenance requirements, which is forecasted based on their aircraft routing decisions. MPs make schedules of their resources according to theses maintenance requirements. However, these maintenance schedules are highly susceptible to disruptions during operations. On the one hand, the maintenance demand of aircrafts, sporadic in nature, gets even more uncertain under disrupted flight schedules. On the other hand, capacity of resources in the maintenance station may be uncertain (e.g. absence of workforce, breakdown of equipment, etc.). However, once fixed in the planning stage, the original maintenance schedules are not supposed to change. Note that when the schedule is not flexible enough to cope with these uncertainties, it would be disrupted. There would be some airlines suffer from undersupply of maintenance resources.

Uncertain supply of maintenance resources is a source of disruption to the aircraft routing decisions, which results in costly maintenance misalignments. Therefore, the effectiveness of an OARP model dependents largely on how correct the resources available at each station are described. In this regard, existing OARP models may not produce satisfactory results with the assumption on fixed and given maintenance capacity. A better modelling method is needed.

III. EXISTING OARP MODELS

Various models are used to formulate OARP (string-based model [4], multi-commodity network flow model [22], rotation tour network model [2], compact optimization models [21], etc). Most existing OARP models assume maintenance resource capacity is given and fixed, which is used to constrain the selection of candidate aircraft routes. We build string-based OARP models as an example.

TABLE II
NOTATIONS IN OARP MODEL

Parameter	Description
N	Set of flights, indexed by i;
K	Set of aircrafts, indexed by k;
S	Set of maintenance stations, indexed by s;
C_s	Capacity of maintenance resources at station s;
R_k	Set of feasible routes by aircraft k, indexed by j;
t_k	Remaining time of aircraft k;
q_k	Maintenance requirements of aircraft k;
d_i	Duration of flight i;
c_j^k	Cost of route j covered by aircraft k;
a_{ij}^k	= 1 if route j of aircraft k contains flight i; 0 otherwise;
m_{js}^k	= 1 if route j of aircraft k ends at station s; 0 otherwise;
y_j^k	= 1 if route j is selected for aircraft k; 0 otherwise.

A string is a maintenance-feasible route for aircrafts. The OARP model aims to generate and assign strings to aircrafts in a fleet, with the objective of minimizing the maintenance costs of the fleet (1). Constraint (2) ensures that each aircraft is assigned exactly one route. Constraint (3) ensures that every flight is covered exactly once. Constraint (4) ensures that the maintenance requirements at each station will be satisfied given fixed capacity C_s at station s.

$$\text{mininimize} \sum_{k\in K} \sum_{j\in R_k} c_j^k y_j^k \tag{1}$$

subject to:

$$\sum_{j\in R_k} y_j^k = 1, \qquad \forall k \in K, \tag{2}$$

$$\sum_{k\in K}\sum_{j\in R_k} a_{ij}^k y_j^k = 1, \qquad \forall i \in N, \tag{3}$$

$$\sum_{k\in K}\sum_{j\in R_k} q_k m_{js}^k y_j^k = C_s, \qquad \forall s \in S, \tag{4}$$

$$y_j^k \in \{0,1\} \qquad \forall k \in K, j \in R_k. \tag{5}$$

Usually, the remaining hours of aircrafts (c_j^k) are used as substitute for the maintenance cost (6). However, as aforementioned, undersupply of maintenance resource results in maintenance misalignment, leading to costly delay and/or recovery. In this paper, we conceptualize the "maintenance risk" (b_s) as the cost of guaranteeing sufficient maintenance resources provided to an airline by a MP in station s. b_s should be part of maintenance costs, which is used to evaluate the candidate aircraft routes in OARP models. One way is to add the weighted maintenance risk in maintenance cost (7).

$$c_j^k = t_k - \sum_{i\in N} a_{ij}^k d_i \tag{6}$$

$$\overline{c_j^k} = c_j^k + \alpha b_s \tag{7}$$

This remainder of this paper proposes to quantify the maintenance risk using game-theoretic models, which can successfully capture the relationships among different airlines and MPs.

IV. GAME-THEORETIC MODELS

As mentioned in Section.1, a MP holds capacitated maintenance resources at a maintenance station, which are required by maintenance tasks from airlines. With some objective (usually, profit-maximization), MP signs performance-based contracts with airlines. In other words, price of maintenance resources with certain performance level are stipulated by MPs. From the scope of an airline, it shares the maintenance resources with other airlines at the station. As each airline aims to secure sufficient maintenance resources to meet the requirements of its scheduled aircrafts, it is reasonable to assume that airlines compete non-cooperatively for maintenance resources at the station.

To model the risk of uncertain maintenance resources supply to an airline, both the competition among airlines and the relationship between airlines and MPs should be analyzed. There are several methods in game theory for modelling these interrelations [24]. This paper identifies the all-pay auction models [25] as one of the suitable models. In such game, players (airlines) bids for multiple prizes (maintenance resources), the player with the highest bid wins the first prize (resources with highest service level), the player with the second-highest bid wins the second prize (resources with the second-highest level), and so on until all the prizes are allocated (all resources are scheduled).

This paper discusses the applicability of such games in scenarios that an airline has incomplete / complete information about other competing airlines. The "information" means the features of airlines which have influence on their decisions about maintenance resources obtainment. For example, airlines with higher route recoverability level would suffer less from undersupply of maintenance resources, leading to lower necessity of guaranteeing timely provision of sufficient resources. Other features include the contract type between the airline and MPs, the market power of an airline, etc.

A. *Incomplete Information Among Airlines*

In situation where an airline does not know any information about other airlines that share the same maintenance resources, an all-pay auction with incomplete information can be applied. We use the model proposed by [26] as an example.

Given m resources (indexed by i) governed by a profit-maximizing MP, each resource has a stipulated price V_i (where $V_1 \geq V_2 \geq \cdots \geq V_m$). There are k airlines competing for these maintenance resources (where $k \geq m$). The "information" used in this example is route recoverability level (e), which is drawn (independently by each airline) from an interval $[e_0, 1]$ according to the distribution function F that is commonly-known among these airlines. Airline k only knows his own route recoverability, and its strategy depends on this feature. There exists a pure equilibrium, where the bids of airline with route recoverability e, denoted by $b^*(e)$, can be described in closed-form (8).

$$b^*(e) = \sum_{i=1}^{m} V_i \int_e^1 -\frac{1}{a} F_i'(\mathrm{a})\,\mathrm{da}$$

When $i = 1$,

$$F_i'(e) = -(k-1)\left(1 - F(e)\right)^{k-2} F'(e), \tag{8}$$

When $i > 1$,

$$F_i'(e) = \frac{(k-1)!}{(p-1)!(k-p)!} \times \left(1 - F(e)\right)^{k-i-1} \left(F(e)\right)^{i-2} F'(e).$$

Note that the equilibrium strategy depicts the best bid make by an airline in the such situation. Any bid deviates from $b^*(e)$ cannot achieve as high maintenance performance level as this equilibrium bid can make. Therefore, $b^*(e)$ can be used as the cost of guaranteeing highest maintenance performance level of an airline, which is the "maintenance risk" of an airline.

B. Complete Information Among Airlines

Compared to the incomplete-information scenario described in Section 3.1, there is another scenario that airlines know much more information about other competitors. In this case the assumption of incomplete information in all-pay auction is relaxed. We use the model proposed by [27] as an example.

Note that features of airlines should be treated carefully, because all-pay auctions with different asymmetricity among players may have different equilibrium results. In our example, route recoverability of an airline can be modeled as asymmetric cost coefficient in [27]. However, it would be more reasonable to model different contract types and market power of airlines as asymmetric head-start as in [28], because these features may have direct impact on the MP's preference among competing airlines.

Given m resources with homogeneous price V (i.e. the MP does not differentiate the performance level among airlines who win resources). The set of airlines competing for these resources are denoted by K (where $|K| = m + 1$). The route recoverability level of airline k is γ_k, and V_k is the valuation of wining for airline k. In the mixed strategy equilibrium of such game, the probability of airline k chooses a bid lower or equal to b is $\mathrm{G}_k^*(\mathrm{b})$, which can be showed in closed-form (9).

$$\mathrm{G}_k^*(\mathrm{b}) = 1 - \left(\frac{1}{\mathrm{a}_{m+1}} - \mathrm{b}\right)^{\frac{1}{m+1-j}} \frac{\prod_{k=j}^{m+1}(\mathrm{a}_k)^{\frac{1}{m+1-j}}}{\mathrm{a}_k}$$

Where $\mathrm{a}_k = \frac{\gamma_k}{v_k}$,

$$s_k^l = \left(\frac{1}{\mathrm{a}_{m+1}} - \frac{\mathrm{a}_i^{m-i}}{\prod_{k=j}^{m+1}\mathrm{a}_k}\right)^{-1}, \tag{9}$$

$\mathrm{j} \in \{1, \dots, \mathrm{m}\}$ such that b is in $\left[s_j^l, s_{j-1}^l\right)$.

Similar to the model in Section 3.1, the equilibrium strategy obtained from such game can be used to described how much effort an airline has to pay to guarantee satisfactory maintenance performance level based on its own features. As the equilibrium obtained in the example is a mixed strategy equilibrium, if an airline as a specified budget of how much they can pay for maintenance contracts, results of this model can used to quantify the risk of signing such contracts.

V. CONCLUSION

This paper identifies a research gap in the area of operational aircraft routing. Specifically, most current studies assume the maintenance resources are given and fixed to each airline in the planning stage, which is infeasible due to the variable maintenance schedule of MP with uncertain maintenance demand and supply during operations. Since most current OARP models make very "tight" aircraft routings, the undersupply of maintenance resources is likely to results in costly delay and/or recovery. Therefore, it is necessary to model the uncertainties of maintenance resources in OARP models.

This paper conceptualizes these uncertainties as "maintenance risk" and propose game-theoretic models as quantification methods. In situation where the MP employs market-based resource allocation scheme, the applicability of all-pay contest models is discussed under two possible scenarios (either incomplete or complete information among airlines). The quantified "maintenance risk" contributes to calculate the maintenance costs of each candidate aircraft routes. In future research, it is highly recommended to solve the OARP model with the revised maintenance costs in the objective, the resulting aircraft routes would be less likely to have maintenance misalignment, and thus more robust in nature.

ACKNOWLEDGMENT

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