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Centralized Multi-Cell Resource and Power Allocation for Multiuser OFDMA Networks

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***Abstract*—Multiuser Orthogonal Frequency Division Multiple Access (OFDMA) networks, such as Long Term Evolution networks, use the frequency reuse-1 model to face the tremendous increase of mobile traffic demands, and to increase network capacity. However, inter-cell interference problems are generated, and they have a negative impact on cell-edge users performance. Resource and power allocation should be managed in a manner that alleviates the negative impact of inter-cell interference on system performance. In this paper, we formulate a novel centralized multi-cell resource and power allocation problem for multiuser OFDMA networks. The objective is to maximize system throughput while guaranteeing a proportional fair rate for all the users. We decompose the joint problem into two independent problems: a resource allocation problem and a power allocation problem. We prove that each of these problems is a convex optimization problem, and that their optimal solution is also an optimal solution to the original joint problem. Lagrange duality theory and subgradient projection method are used to solve the centralized power allocation problem. We study the convergence of our centralized approach, and we find out that it reduces inter-cell interference, and increases system throughput and spectral efficiency in comparison with the frequency reuse-1 model, reuse-3 model, fractional frequency reuse, and soft frequency reuse techniques.**

***Index Terms*—Convex optimization, resource and power allocation, inter-cell interference, ICIC, OFDMA.**

I. Introduction

Multiuser Orthogonal Frequency Division Multiple Access (OFDMA) networks, such as the Third Generation Partnership Project (3GPP) Long Term Evolution (LTE) [1] and LTE-Advanced (LTE-A) [2] networks, are able to avoid the negative impact of multipath fading and intra-cell interference, by virtue of the orthogonality between subcarrier frequencies. Nevertheless, Inter-Cell Interference (ICI) problems arise in dense frequency reuse networks due to simultaneous transmissions on the same frequency resources. System performance is interference-limited, since the achievable throughput is reduced due to ICI.

Fractional Frequency Reuse (FFR) [3] and Soft Frequency Reuse (SFR) [4] were introduced to avoid the harmful impact of ICI on system performance, by applying static rules on Resource Block (RB) usage and power allocation between cell zones. Heuristic Inter-Cell Interference Coordination (ICIC) techniques are proposed to achieve ICI mitigation without severe degradation of the overall system throughput. In [5], a heuristic power allocation algorithm is introduced to reduce energy consumption and to improve cell-edge UEs throughput. It has been proven that the proposed algorithm reduces power consumption without reducing the achievable throughput. Moreover, it mitigates ICI and increases the achievable throughput for cell-edge UEs.

Beside heuristic resource and power allocation algorithms [6], convex optimization is used to improve the performance of multiuser OFDMA networks, and to alleviate the negative impact of ICI on UE throughput. Resource and power allocation problems are usually formulated as nonlinear optimization problems, where the objective consists in maximizing system throughput, spectral efficiency, or energy efficiency, with constraints on the minimum throughput per UE or other Quality of Service (QoS) parameters [7].

The majority of state-of-the-art contributions formulate the resource and power allocation problem for a single cell network [8–10]. Moreover, low-complexity suboptimal algorithms are proposed to perform resource and power allocation [10]. Therefore, the optimal solution is not always guaranteed.

In this paper, we formulate the joint resource and power allocation problem for multiuser OFDMA networks, as a centralized optimization problem. We demonstrate that the original problem is separable into two independent optimization problems: a resource allocation problem and a power allocation problem. Our objective is to maximize system throughput while guaranteeing proportional fair rate among the UEs, under constraints related to resource usage, Signal-to-Interference and Noise Ratio (SINR), and power allocation. Our major contributions are summarized as follows:

- Propose an original formulation of the centralized joint resource and power allocation problem: instead of considering a single cell OFDMA network, we formulate our problem for a multi-cell OFDMA network. Moreover, ICI problems are taken into account.
- Maximize the mean rate per UE, and ensure a proportional fair rate for all the active UEs.
- Prove the convexity of our centralized problem by applying an adequate variable change.
- Decompose the joint resource and power allocation problem into two independent problems.
- Solve the centralized power allocation problem using Lagrange duality theory and subgradient projection method.
- Validate the convergence of our proposed approach and evaluate its performance in comparison with the fre-

quency reuse-1 model, reuse-3 model, FFR, and SFR techniques.

The remainder of this paper is organized as follows. In section II, we describe the limitations of the existing state-of-the-art approaches. In section III, system model is presented followed by our joint resource and power allocation problem formulation. The joint problem is decomposed into two independent problems in section IV: a resource allocation problem and a power allocation problem. We also demonstrate the convexity of the formulated problems. In section V, we solve both resource and power allocation problems. Then we investigate the convergence of the centralized approach in section VI, where we also provide comparisons with state-of-the-art ICIC approaches. Section VII concludes this paper and summarizes our main contributions.

II. Related Work

For a given multiuser OFDMA network, resource and power allocation problem is formulated as a centralized optimization problem. Centralized inter-cell coordination is therefore required to find the optimal solution, where the necessary information about SINR, power allocation, and resource usage are sent to a centralized coordination entity.

In [11], the multi-cell optimization problem is decomposed into two distributed optimization problems. The objective of the first problem is to minimize the transmission power allocated for cell-edge UEs, while guaranteeing a minimum throughput for each UE. RB and power are allocated to cell-edge UEs so that they satisfy their minimum required throughput. The remaining RBs and the remaining transmission power are uniformly allocated to cell-center UEs. At this stage, the second problem finds the resource allocation strategy that maximizes cell-center zone throughput. An improved version of this adaptive ICIC technique is proposed in [12], where resource allocation for cell-edge UEs is performed depending on their individual channel conditions. However, the main disadvantage of this adaptive ICIC technique and the proposed improvement is that they do not consider the impact of ICI between adjacent cells when power allocation is performed.

Resource and power allocation for a cluster of coordinated OFDMA cells are studied in [13]. Energy efficiency is maximized under constraints related to the downlink transmission power. However, noise-limited regime is considered, and ICI is neglected. Moreover, energy-efficient resource allocation for OFDMA systems is investigated in [14], where generalized and individual energy efficiencies are defined for the downlink and the uplink of the OFDMA system, respectively. Properties of the energy efficiency objective function are studied, then a low-complexity suboptimal algorithm is introduced to reduce the computational burden of the optimal solution. Subcarrier assignment is made easier using heuristic algorithms. Authors of [15] consider the joint resource allocation, power allocation, and Modulation and Coding Scheme (MCS) selection problem. The joint optimization problem is separated into resource allocation and power allocation problems, and suboptimal algorithms are proposed. Another low complexity suboptimal resource allocation algorithm is proposed in [16]. The objective consists in maximizing the achievable throughput, under constraints related to resource usage in the different cells. Cooperation between adjacent cells is needed.

The majority of state-of-the-art contributions that formulate spectral efficiency or energy efficiency problems as centralized optimization problems, neglect the impact of ICI on system performance [8–10], or introduce suboptimal approaches to solve resource and power allocation problems [17–19]. Moreover, performance comparisons are not made with other distributed heuristic ICIC algorithms with a lower complexity. In the next section, we formulate our multi-cell resource and power allocation problem that takes inter-cell interference into account.

III. System Model and Problem Formulation

A. System Model

We consider the downlink of a multiuser OFDMA system that consists of I adjacent cells and K active UEs. Let $\mathcal{I} = \{1, 2, ..., I\}$ denote the the set of cells, and $\mathcal{K} = \{1, 2, ..., K\}$ the total set of active UEs. We also define $K(i)$ as the number of UEs served by cell i. Thus, we have $\sum_{i=1}^{I} K(i) = K$. The set of available RBs in each cell is denoted by $\mathcal{N} = \{1, 2, ..., N\}$.

In OFDMA networks, system spectrum is divided into several channels, where each channel consists of a number of consecutive orthogonal OFDM subcarriers [20]. An RB is the smallest scheduling unit. It consists of 12 consecutive subcarriers in the frequency domain, and seven OFDM symbols with normal cyclic prefix in the time domain [21] (or six OFDM symbols with extended cyclic prefix). Resources are allocated to UEs each Transmit Time Interval (TTI), which is equal to 1 ms. When the frequency reuse-1 model is applied along with homogeneous power allocation, each RB is allocated the same downlink transmission power $\frac{P_{max}}{N}$, where P_{max} denotes the maximum downlink transmission power per cell. The signal to interference and noise ratio for a UE k attached to cell i and allocated RB n is given by:

$$\sigma_{k,i,n} = \frac{\pi_{i,n} G_{k,i,n}}{N_0 + \sum_{i' \neq i} \pi_{i',n} G_{k,i',n}}, \tag{1}$$

where $\pi_{i,n}$ is the downlink transmission power allocated by cell i to RB n, $G_{k,i,n}$ denotes channel gain for UE k attached to cell i and allocated RB n, and N_0 is the thermal noise power. Indexes i and i' refer to useful and interfering signals respectively. Notations, symbols, parameters, and variables used within this paper are reported in Table I.

B. Problem Formulation

1) Centralized Multi-Cell Optimization Problem: We define $\theta_{k,n}$ as the percentage of time during which UE k is associated with RB n. $\theta_{k,n}$ and $\pi_{i,n}$ are the optimization variables of the joint resource and power allocation problem. Our objective is to manage resource and power allocation in a manner that maximizes system throughput and guarantees

TABLE I: Sets, parameters and variables in the paper

i	Index of cell
k	Index of UE
n	Index of RB
$\mathcal{I}$	Set of cells
$\mathcal{K}$	Total set of UEs
$\mathcal{K}(i)$	Set of UEs associated to cell i
$\mathcal{N}$	Set of RBs
$\rho_{k,i,n}$	Rate of UE k associated with RB n on cell i
$\pi_{i,n}$	Transmit power of cell i on RB n
$G_{k,i,n}$	Channel gain for UE k over RB n on cell i
N_0	Thermal noise density
$\theta_{k,n}$	Percentage of time RB n is allocated to UE k
$\sigma_{k,i,n}$	SINR for UE k over RB n on cell i
P_{max}	Maximum DL transmission power per cell
π_{min}	Minimum DL transmission power per RB
$\mathcal{I}'(i)$	Set of neighboring cells for cell i

throughput fairness between the different UEs. The peak rate of UE k when associated with RB n on cell i is given by:

$$\rho_{k,i,n} = \log\left(1 + \frac{\pi_{i,n}G_{k,i,n}}{N_0 + \sum_{i' \neq i} \pi_{i',n}G_{k,i',n}}\right). \quad (2)$$

Then, the mean rate of UE k is given by:

$$\sum_{n\in\mathcal{N}} \left(\theta_{k,n}.\rho_{k,i,n}\right). \quad (3)$$

Our centralized resource and power allocation problem seeks rate maximization. We make use of the logarithmic function that is intimately associated with the concept of proportional fairness [22]. Our problem is formulated as follows:

$$\underset{\boldsymbol{\theta},\boldsymbol{\pi}}{\text{maximize}} \quad \eta = \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}(i)} \log\left(\sum_{n\in\mathcal{N}} \theta_{k,n}.\log\left(1 + \frac{\pi_{i,n}G_{k,i,n}}{N_0 + \sum_{i' \neq i} \pi_{i',n}G_{k,i',n}}\right)\right) \quad (4a)$$

$$\text{subject to} \quad \sum_{k\in\mathcal{K}(i)} \theta_{k,n} \leq 1, \ \forall n \in \mathcal{N}, \quad (4b)$$

$$\sum_{n\in\mathcal{N}} \theta_{k,n} \leq 1, \ \forall k \in \mathcal{K}(i), \quad (4c)$$

$$\sum_{n\in\mathcal{N}} \pi_{i,n} \leq P_{max}, \ \forall i \in \mathcal{I}, \quad (4d)$$

$$\pi_{i,n} \geq \pi_{min}, \ \forall i \in \mathcal{I}, \forall n \in \mathcal{N}, \quad (4e)$$

$$0 \leq \theta_{k,n} \leq 1, \ \forall k \in \mathcal{K}(i), \forall n \in \mathcal{N}. \quad (4f)$$

The objective function η ensures a proportional fair rate for all UEs in the network. Constraints (4b) ensure that an RB is used at most 100% of the time, and constraints (4c) ensure that a UE shares its time on the available RBs. Constraints (4d) guarantee that the total downlink transmission power allocated to the available RBs does not exceed the maximum transmission power P_{max} for each cell i, and constraints (4e) represent the minimum power constraint of the transmit power allocated to each RB. $\theta_{k,n}, \forall k \in \mathcal{K}, \forall n \in \mathcal{N}$, and $\pi_{i,n}, \forall i \in \mathcal{I}, \forall n \in \mathcal{N}$ are the optimization variables of the joint resource and power allocation problem.

2) Upper Bound of the Objective Functions Difference: In order to reduce the complexity of the joint resource and power allocation problem (4), we prove that this problem is separable into two independent problems: a resource allocation problem and a power allocation problem. Given Jensen's inequality and the concavity of the log function, we have:

$$\log\left(\frac{\sum_{n\in\mathcal{N}} \theta_{k,n}.\rho_{k,i,n}}{|\mathcal{N}|}\right) \geq \frac{\sum_{n\in\mathcal{N}} \log\left(\theta_{k,n}.\rho_{k,i,n}\right)}{|\mathcal{N}|} \quad (5a)$$

$$\Rightarrow \log\left(\sum_{n\in\mathcal{N}} \theta_{k,n}.\rho_{k,i,n}\right) \geq \frac{\sum_{n\in\mathcal{N}} \log\left(\theta_{k,n}.\rho_{k,i,n}\right)}{|\mathcal{N}|} + \log\left(|\mathcal{N}|\right), \quad (5b)$$

Since $\frac{1}{|\mathcal{N}|}$ and $|\mathcal{K}|.\log\left(|\mathcal{N}|\right)$ are constant terms, maximizing the objective function of problem (4) is achieved by maximizing the following term:

$$\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}(i)}\sum_{n\in\mathcal{N}} \left(\log\left(\theta_{k,n}\right) + \log\left(\rho_{k,i,n}\right)\right). \quad (6)$$

IV. Problem Decomposition

We tackle ICIC as an optimization problem, where we intend to maximize the mean rate of UEs in a multiuser OFDMA system. We consider a system of I cells, having $K(i)$ UEs per cell i. According to (6), and due to the absence of binding constraints, the optimization problem (4) is linearly separable into two independent problems: a power allocation problem and a resource allocation problem.

A. Centralized Multi-Cell Power Allocation Problem

In the first problem, the optimization variable $\boldsymbol{\pi}$ is considered, and the problem is formulated as follows:

$$\underset{\boldsymbol{\pi}}{\text{maximize}} \quad \eta_1 = \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}(i)}\sum_{n\in\mathcal{N}} \log\left(\log\left(1 + \frac{\pi_{i,n}G_{k,i,n}}{N_0 + \sum_{i' \neq i} \pi_{i',n}G_{k,i',n}}\right)\right) \quad (7a)$$

$$\text{subject to} \quad \sum_{n\in\mathcal{N}} \pi_{i,n} \leq P_{max}, \ \forall i \in \mathcal{I}, \quad (7b)$$

$$\pi_{i,n} \geq \pi_{min}, \ \forall i \in \mathcal{I}, \forall n \in \mathcal{N}. \quad (7c)$$

Problem (7) consists in finding the optimal power allocation. In the following, we introduce a variable change that allows to formulate problem (7) as a convex optimization problem.

The power allocation problem (7) can be written as follows:

$$\underset{\boldsymbol{\rho}}{\text{maximize}} \quad \eta_1 = \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}(i)}\sum_{n\in\mathcal{N}} \log\left(\rho_{k,i,n}\right) \tag{8a}$$

$$\text{subject to} \quad \rho_{k,i,n} \leq \log\left(1+\frac{\pi_{i,n}G_{k,i,n}}{N_0+\sum_{i'\neq i}\pi_{i',n}G_{k,i',n}}\right), \quad \forall i\in\mathcal{I}, \forall k\in\mathcal{K}(i), \forall n\in\mathcal{N}, \tag{8b}$$

$$\sum_{n\in\mathcal{N}}\pi_{i,n} \leq P_{max},\ \forall i\in\mathcal{I}, \tag{8c}$$

$$\pi_{i,n} \geq \pi_{min},\ \forall i\in\mathcal{I}, \forall n\in\mathcal{N}. \tag{8d}$$

Let us consider the following variable change:

$$\widehat{\rho}_{k,i,n} = \log\left(\exp\left(\rho_{k,i,n}\right)-1\right),\ \forall\, i\in\mathcal{I}, \forall\, k\in\mathcal{K}(i), \forall n\ \in\mathcal{N}, \tag{9a}$$

$$\widehat{\pi}_{i,n} = \log(\pi_{i,n}),\ \forall\, i\in\mathcal{I}, \forall\, n\in\mathcal{N}. \tag{9b}$$

To show that the optimization problem (8) is a convex optimization problem, we need to show that the objective function is concave and the inequality constraint functions define a convex set. After applying the variable change on UE peak rate constraints (8b), these constraints can be written as follows:

$$\log(\exp\left(\widehat{\rho}_{k.i.n}-\widehat{\pi}_{i,n}\right)\frac{N_0}{G_{k,i,n}}+ \sum_{i'\neq i}\exp\left(\widehat{\rho}_{k.i.n}+\widehat{\pi}_{i',n}-\widehat{\pi}_{i,n}\right)\frac{G_{k,i',n}}{G_{k,i,n}}) \leq 0,$$

which are the logarithmic of the sum of exponential functions. Therefore, they are convex functions [23]. When we apply the variable change on power constraints (8c), we get the following:

$$\sum_{n\in\mathcal{N}}\pi_{i,n} \leq P_{max},\ \forall i\in\mathcal{I} \Rightarrow \log\left(\sum_{n\in\mathcal{N}}\exp\left(\widehat{\pi}_{i,n}\right)\right)-\log\left(P_{max}\right)\leq 0,\ \forall i\in\mathcal{I}.$$

Since $\log(\sum\exp)$ is convex [23], the constraints at hand are therefore convex. Using the variable change, the power allocation problem (8) can be written as follows:

$$\underset{\widehat{\boldsymbol{\rho}}}{\text{maximize}} \quad \eta_1 = \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}(i)}\sum_{n\in\mathcal{N}} \log\left(\log\left(\exp\left(\widehat{\rho}_{k,i,n}\right)+1\right)\right) \tag{10a}$$

$$\text{subject to} \quad \log(\exp\left(\widehat{\rho}_{k.i.n}-\widehat{\pi}_{i,n}\right)\frac{N_0}{G_{k,i,n}}+ \sum_{i'\neq i}\exp\left(\widehat{\rho}_{k.i.n}+\widehat{\pi}_{i',n}-\widehat{\pi}_{i,n}\right)\frac{G_{k,i',n}}{G_{k,i,n}}) \leq 0, \quad \forall i\in\mathcal{I}, \forall k\in\mathcal{K}(i), \forall n\in\mathcal{N}, \tag{10b}$$

$$\log\left(\sum_{n\in\mathcal{N}}\exp\left(\widehat{\pi}_{i,n}\right)\right)-\log\left(P_{max}\right)\leq 0,\ \forall i\in\mathcal{I}, \tag{10c}$$

$$\widehat{\pi}_{i,n} \geq \log\left(\pi_{min}\right),\ \forall i\in\mathcal{I}, \forall n\in\mathcal{N}. \tag{10d}$$

The objective function of problem (10) is concave in $\widehat{\boldsymbol{\rho}}$, and constraints (10b), (10c), and (10d) are convex functions. Thus, the power allocation problem is a convex optimization problem.

B. Centralized Resource Allocation Problem

The optimization variable $\boldsymbol{\theta}$ is considered in the second optimization problem that is given in the following:

$$\underset{\boldsymbol{\theta}}{\text{maximize}} \quad \eta_2 = \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}(i)}\sum_{n\in\mathcal{N}} \log\left(\theta_{k,n}\right) \tag{11a}$$

$$\text{subject to} \quad \sum_{k\in\mathcal{K}(i)}\theta_{k,n} \leq 1,\ \forall n\in\mathcal{N}, \tag{11b}$$

$$\sum_{n\in\mathcal{N}}\theta_{k,n} \leq 1,\ \forall k\in\mathcal{K}(i), \tag{11c}$$

$$0 \leq \theta_{k,n} \leq 1,\ \forall k\in\mathcal{K}(i), \forall n\in\mathcal{N}. \tag{11d}$$

As demonstrated for the power allocation problem (7), we prove that problem (11) is indeed a convex optimization problem in $\boldsymbol{\theta}$. The objective function (11a) of the resource allocation problem (11) is concave in $\boldsymbol{\theta}$, since the $\log$ function is concave for $\boldsymbol{\theta}\in]0;1]$. Moreover, constraints (11b), (11c), and (11d) are linear and separable constraints. Hence, the resource allocation problem (11) is a convex optimization problem, and it is separable into $\mathcal{I}$ subproblems. For each cell i, the ith optimization problem is written as follows:

$$\underset{\boldsymbol{\theta}}{\text{maximize}} \quad \left(\eta_2\right)_i = \sum_{k\in\mathcal{K}(i)}\sum_{n\in\mathcal{N}} \log\left(\theta_{k,n}\right) \tag{12a}$$

$$\text{subject to} \quad \sum_{k\in\mathcal{K}(i)}\theta_{k,n} \leq 1,\ \forall n\in\mathcal{N}, \tag{12b}$$

$$\sum_{n\in\mathcal{N}}\theta_{k,n} \leq 1,\ \forall k\in\mathcal{K}(i), \tag{12c}$$

$$0 \leq \theta_{k,n} \leq 1,\ \forall k\in\mathcal{K}(i), \forall n\in\mathcal{N}. \tag{12d}$$

V. Centralized Multi-Cell Resource and Power Allocation

As proven in the previous section, the joint resource and power allocation problem (4) is separable into two independent convex optimization problems: a power allocation problem, and a resource allocation problem. In this section, we solve the resource and power allocation problems using Lagrange duality theory and subgradient projection method.

A. Solving the Centralized Power Allocation Problem

1) Lagrange-Based Method: Since the power allocation problem (10) is a convex optimization problem, we can make use of Lagrange duality properties, which also lead to decomposability structures [24]. Lagrange duality theory links the original problem, or *primal problem*, with a dual maximization problem. The primal problem (10) is relaxed by transferring the constraints to the objective in the form of weighted sum. The Lagrangian is formed by relaxing the

coupling constraints (10b) and (10c) in problem (10):

$$L\left(\widehat{\boldsymbol{\rho}},\widehat{\boldsymbol{\pi}},\boldsymbol{\lambda},\boldsymbol{\nu}\right)=\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}(i)}\sum_{n\in\mathcal{N}}\log\left(\log\left(\exp\left(\widehat{\rho}_{k,i,n}\right)+1\right)\right)$$
$$-\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}(i)}\sum_{n\in\mathcal{N}}\lambda_{k,i,n}(\log(\exp(\widehat{\rho}_{k.i.n}-\widehat{\pi}_{i,n})\frac{N_0}{G_{k,i,n}}$$
$$+\sum_{\substack{i'\in\mathcal{N}\\ i'\neq i}}\exp(\widehat{\rho}_{k.i.n}+\widehat{\pi}_{i',n}-\widehat{\pi}_{i,n})\frac{G_{k,i',n}}{G_{k,i,n}}))$$
$$-\sum_{i\in\mathcal{I}}\nu_i\left(\log\left(\sum_{n\in\mathcal{N}}\exp\left(\widehat{\pi}_{i,n}\right)\right)-\log\left(P_{max}\right)\right). \tag{13}$$

The optimization variables $\widehat{\boldsymbol{\rho}}$ and $\widehat{\boldsymbol{\pi}}$ are called the primal variables. $\lambda_{k,i,n}$ and ν_i are the *Lagrange multipliers* or *prices* associated with the (k,i,n)th inequality constraint (10b) and with the ith inequality constraint (10c), respectively. $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ are also termed the *dual variables*.

After relaxing the coupling constraints, the optimization problem separates into two levels of optimization: lower level and higher level. At the lower level, $L(\widehat{\boldsymbol{\rho}},\widehat{\boldsymbol{\pi}},\boldsymbol{\lambda},\boldsymbol{\nu})$ is the objective function to be maximized. $\widehat{\rho}_{k,i,n}$ and $\widehat{\pi}_{i,n}$ are the optimization variables to be found, and the primal problem is given by:

$$\underset{\widehat{\boldsymbol{\rho}},\widehat{\boldsymbol{\pi}}}{\text{maximize}} \quad L\left(\widehat{\boldsymbol{\rho}},\widehat{\boldsymbol{\pi}},\boldsymbol{\lambda},\boldsymbol{\nu}\right) \tag{14a}$$
$$\text{subject to} \quad \widehat{\pi}_{i,n}\geq\log(\pi_{min}),\ \forall i\in\mathcal{I},\forall n\in\mathcal{N}. \tag{14b}$$

In order to solve the primal optimization problem (14), we use the subgradient projection method. It starts with some initial feasible values of $\widehat{\rho}_{k,i,n}$ and $\widehat{\pi}_{i,n}$ that satisfy the constraints (14b). Then, the next iteration is generated by taking a step along the subgradient direction of $\widehat{\rho}_{k,i,n}$ and $\widehat{\pi}_{i,n}$. For the primal optimization variables, iterations of the subgradient projection are given by:

$$\widehat{\rho}_{k,i,n}(t+1)=\widehat{\rho}_{k,i,n}(t)+\delta(t)\times\frac{\partial L}{\partial\widehat{\rho}_{k,i,n}},$$
$$\forall k\in\mathcal{K}(i),\forall i\in\mathcal{I},\forall n\in\mathcal{N}, \tag{15a}$$
$$\widehat{\pi}_{i,n}(t+1)=\widehat{\pi}_{i,n}(t)+\delta(t)\times\frac{\partial L}{\partial\widehat{\pi}_{i,n}},\forall i\in\mathcal{I},\forall n\in\mathcal{N}. \tag{15b}$$

The scalar $\delta(t)$ is a step size that guarantees the convergence of the optimization problem (14). At the higher level, we have the master dual problem in charge of updating the dual variables $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ by solving the dual problem:

$$\underset{\boldsymbol{\lambda},\boldsymbol{\nu}}{\text{minimize}} \quad \max_{\widehat{\boldsymbol{\rho}},\widehat{\boldsymbol{\pi}}}\left(L\left(\widehat{\boldsymbol{\rho}},\widehat{\boldsymbol{\pi}},\boldsymbol{\lambda},\boldsymbol{\nu}\right)\right) \tag{16a}$$
$$\text{subject to} \quad \boldsymbol{\lambda}\geq 0, \tag{16b}$$
$$\boldsymbol{\nu}\geq 0. \tag{16c}$$

The dual function $g\left(\boldsymbol{\lambda},\boldsymbol{\nu}\right)=\max_{\widehat{\boldsymbol{\rho}},\widehat{\boldsymbol{\pi}}}\left(L\left(\widehat{\boldsymbol{\rho}},\widehat{\boldsymbol{\pi}},\boldsymbol{\lambda},\boldsymbol{\nu}\right)\right)$ is differentiable. Thus, the master dual problem (16) can be solved using the following gradient method:

$$\lambda_{k,i,n}(t+1)=\lambda_{k,i,n}(t)+\delta(t)\times\frac{\partial L}{\partial\lambda_{k,i,n}},$$
$$\forall k\in\mathcal{K}(i),\forall i\in\mathcal{I},\forall n\in\mathcal{N}, \tag{17a}$$
$$\nu_i(t+1)=\nu_i(t)+\delta(t)\times\frac{\partial L}{\partial\nu_i},\forall i\in\mathcal{I},\forall n\in\mathcal{N}, \tag{17b}$$

where t is the iteration index, and $\delta(t)$ is the step size at iteration t. Appropriate choice of the step size [25] leads to convergence of the dual algorithm. $\widehat{\pi}^{\star}_{i,n}$ and $\widehat{\rho}^{\star}_{k,i,n}$ denote the solution to the primal optimization problem (14). When $t\rightarrow\infty$ the dual variables $\boldsymbol{\lambda}(t)$ and $\boldsymbol{\nu}(t)$ converge to the dual optimal $\boldsymbol{\lambda}^*$ and $\boldsymbol{\nu}^*$, respectively. The difference between the optimal primal objective and the optimal dual objective, called *duality gap*, reduces to zero at optimality, since the problem (10) is convex and the KKT conditions are satisfied. We define $\Delta\widehat{\boldsymbol{\rho}},\Delta\widehat{\boldsymbol{\pi}},\Delta\boldsymbol{\lambda}$, and $\Delta\boldsymbol{\nu}$ as the differences between the optimization variables obtained at the current iteration and their values at the previous iteration. They are given by:

$$\Delta\widehat{\boldsymbol{\rho}}(t+1)=\|\widehat{\boldsymbol{\rho}}(t+1)-\widehat{\boldsymbol{\rho}}(t)\|, \tag{18a}$$
$$\Delta\widehat{\boldsymbol{\pi}}(t+1)=\|\widehat{\boldsymbol{\pi}}(t+1)-\widehat{\boldsymbol{\pi}}(t)\|, \tag{18b}$$
$$\Delta\boldsymbol{\lambda}(t+1)=\|\boldsymbol{\lambda}(t+1)-\boldsymbol{\lambda}(t)\|, \tag{18c}$$
$$\Delta\boldsymbol{\nu}(t+1)=\|\boldsymbol{\nu}(t+1)-\boldsymbol{\nu}(t)\|. \tag{18d}$$

2) Iterative Power Allocation Algorithm: The procedure for solving the centralized power allocation problem is described in Algorithm 1. Initially, the primal optimization variables $\widehat{\rho}_{k,i,n}$ and $\widehat{\pi}_{i,n}$ as well as the dual variables $\lambda_{k,i,n}$ and ν_i start with some initial feasible values. t, t_{primal}, and t_{dual} denote the number of rounds required for the centralized power allocation problem to converge, the number of iterations for the primal problem, and the number of iterations for the dual problem, respectively. At each round t, we start by updating the primal optimization variables, using the PRIMALPROBLEM function given in Algorithm 2. The solution to the primal optimization problem at the current round t is denoted by $\widehat{\pi}^{\star}_{i,n}(t+1)$ and $\widehat{\rho}^{\star}_{k,i,n}(t+1)$. The PRIMALPROBLEM function updates $\widehat{\pi}_{i,n}(t_{primal}+1)$ and $\widehat{\rho}_{k,i,n}(t_{primal}+1)$, and increments t_{primal} until $\Delta\widehat{\boldsymbol{\pi}}(t_{primal}+1)$ and $\Delta\widehat{\boldsymbol{\rho}}(t_{primal}+1)$ become less than ϵ.

Then, the solution to the dual optimization problem at the current round t, denoted by $\nu^{\star}_i(t+1)$ and $\lambda^{\star}_{k,i,n}(t+1)$ is calculated using the DUALPROBLEM function given in Algorithm 3. ν_i and $\lambda_{k,i,n}$ are updated using the primal solution $\widehat{\pi}^{\star}_{i,n}(t+1)$ and $\widehat{\rho}^{\star}_{k,i,n}(t+1)$, until $\Delta\boldsymbol{\nu}(t_{dual}+1)$ and $\Delta\boldsymbol{\lambda}(t_{dual}+1)$ become less than ϵ. An additional round of calculations is performed, and t is incremented as long as $\Delta\widehat{\boldsymbol{\pi}}^{\star}(t+1)$ or $\Delta\widehat{\boldsymbol{\pi}}^{\star}(t+1)$ or $\Delta\boldsymbol{\nu}^{\star}(t+1)$ or $\Delta\boldsymbol{\lambda}^{\star}(t+1)$ is greater than ϵ. Otherwise, the current solution is the optimal solution to the centralized power allocation problem.

B. Solving the Resource Allocation Problem

In this subsection, we search for the optimal solution to the resource allocation problem (12). For each cell i, the problem (12) is a convex optimization problem.

Algorithm 1 Dual algorithm for centralized power allocation

1: Parameters: $L(\widehat{\boldsymbol{\rho}}, \widehat{\boldsymbol{\pi}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$, P_{max}, and π_{min}.
2: Initialization: set $t = t_{primal} = t_{dual} = 0$, and $\pi_{i,n} \geq \pi_{min}, \forall i \in \mathcal{I}, \forall n \in \mathcal{N}$, such as $\sum_{n \in \mathbb{N}} \pi_{i,n} \leq P_{max}, \forall i \in \mathcal{I}$. Calculate $\widehat{\pi}_{i,n}(0)$ and $\widehat{\rho}_{k,i,n}(0)$ accordingly.
3: Set $\lambda_{k,i,n}(0)$ and $\nu_i(0)$ equal to some non negative value.
4: $(\widehat{\boldsymbol{\pi}}^{\star}(t+1), \widehat{\boldsymbol{\rho}}^{\star}(t+1)) \leftarrow$ PRIMALPROBLEM$(\boldsymbol{\nu}^{\star}(t), \boldsymbol{\lambda}^{\star}(t))$
5: $(\boldsymbol{\nu}^{\star}(t+1), \boldsymbol{\lambda}^{\star}(t+1)) \leftarrow$ DUALPROBLEM$(\widehat{\boldsymbol{\pi}}^{\star}(t+1), \widehat{\boldsymbol{\rho}}^{\star}(t+1))$
6: **if** $(\Delta\widehat{\boldsymbol{\pi}}^{\star}(t+1) > \epsilon)$ **or** $(\Delta\widehat{\boldsymbol{\rho}}^{\star}(t+1) > \epsilon)$ **or** $(\Delta\boldsymbol{\nu}^{\star}(t+1) > \epsilon)$ **or** $(\Delta\boldsymbol{\lambda}^{\star}(t+1) > \epsilon)$ **then**
7: $t \leftarrow t+1$
8: **go to** 4
9: **end if**

Algorithm 2 Primal problem function

1: **function** PRIMALPROBLEM$(\boldsymbol{\nu}^{\star}(t), \boldsymbol{\lambda}^{\star}(t))$
2: **for** $i = 1$ to $|\mathcal{I}|$ **do**
3: **for** $n = 1$ to $|\mathcal{N}|$ **do**
4: $\widehat{\pi}_{i,n}(t_{primal}+1) \leftarrow \max\left(\log(\pi_{min}); \widehat{\pi}_{i,n}(t_{primal}) + \delta(t) \times \frac{\partial L}{\partial \widehat{\pi}_{i,n}}\right)$
5: **for** $k = 1$ to $|\mathcal{K}(i)|$ **do**
6: $\widehat{\rho}_{k,i,n}(t_{primal}+1) \leftarrow \widehat{\rho}_{k,i,n}(t_{primal}) + \delta(t) \times \frac{\partial L}{\partial \widehat{\rho}_{k,i,n}}$
7: **end for**
8: **end for**
9: **end for**
10: **if** $(\Delta\widehat{\boldsymbol{\pi}}(t_{primal}+1) > \epsilon)$ **or** $(\Delta\widehat{\boldsymbol{\rho}}(t_{primal}+1) > \epsilon)$ **then**
11: $t_{primal} \leftarrow t_{primal}+1$
12: **go to** 2
13: **end if**
14: **return** $\widehat{\boldsymbol{\pi}}(t_{primal}+1), \widehat{\boldsymbol{\rho}}(t_{primal}+1)$
15: **end function**

Algorithm 3 Dual problem function

1: **function** DUALPROBLEM$(\widehat{\boldsymbol{\pi}}^{\star}(t+1), \widehat{\boldsymbol{\rho}}^{\star}(t+1))$
2: **for** $i = 1$ to $|\mathcal{I}|$ **do**
3: $\nu_i(t_{dual}+1) \leftarrow \max(0; \nu_i(t_{dual}) + \delta(t) \times \frac{\partial L}{\partial \nu_i})$
4: **for** $n = 1$ to $|\mathcal{N}|$ **do**
5: **for** $k = 1$ to $|\mathcal{K}(i)|$ **do**
6: $\lambda_{k,i,n}(t_{dual}+1) \leftarrow \max(0; \lambda_{k,i,n}(t_{dual}) + \delta(t) \times \frac{\partial L}{\partial \lambda_{k,i,n}})$
7: **end for**
8: **end for**
9: **end for**
10: **if** $(\Delta\boldsymbol{\nu}(t_{dual}+1) > \epsilon)$ **or** $(\Delta\boldsymbol{\lambda}(t_{dual}+1) > \epsilon)$ **then**
11: $t_{dual} \leftarrow t_{dual}+1$
12: **go to** 2
13: **end if**
14: **return** $\boldsymbol{\nu}(t_{dual}+1), \boldsymbol{\lambda}(t_{dual}+1)$
15: **end function**

Theorem 5.1: For each cell i, the optimal solution to the resource allocation problem (12) is given by: $\theta_{k,n} = \frac{1}{\max(|\mathcal{K}(i)|, |\mathcal{N}|)}, \forall k \in \mathcal{K}(i), \forall n \in \mathcal{N}$.

The proof of Theorem 5.1 is given in Appendix I. When the number of active UEs is less than the number of available resources, $\theta_{k,n} = \frac{1}{|\mathcal{N}|}, \forall k \in \mathcal{K}(i), \forall n \in \mathcal{N}$. Thus, the available resources are not fully used over time, and each UE is permanently served. Otherwise, when $|\mathcal{K}(i)| > |\mathcal{N}|$, the optimal solution is: $\theta_{k,n} = \frac{1}{|\mathcal{K}(i)|}, \forall k \in \mathcal{K}(i), \forall n \in \mathcal{N}$. In this case, each RB is fully used over time, while UEs are not permanently served over time.

VI. PERFORMANCE EVALUATION

In this section, we evaluate the convergence and the performance of our proposed centralized joint resource and power allocation problem.

A. Centralized Resource and Power Allocation

To verify the convergence of the centralized solution, we consider a multi-user OFDMA network, such as LTE/LTE-A networks, that consists of seven adjacent hexagonal cells, with one UE served by each cell. UE positions and radio conditions are randomly generated, and the initial power allocation for each RB equals π_{min}. System bandwidth equals 5 MHz. Thus, 25 RBs are available in each cell. The maximum transmission power per cell P_{max} is set to 43 dBm or 20 W. At the first iteration, the dual variables $\lambda_{k,i,n}(0), \forall k \in \mathcal{K}(i), \forall i \in \mathcal{I}, \forall n \in \mathcal{N}$, and $\nu_i(0), \forall i \in \mathcal{I}$, are assigned initial positive values. The evolution of $\widehat{\pi}_{i,1}$ along with the number of iterations is shown in Fig. 1, where $\widehat{\pi}_{i,1}$ is the logarithm of the transmission power allocated by the cell i to the RB 1. In addition, the number of primal iterations and the number of dual iterations per round are shown in Fig. 2.

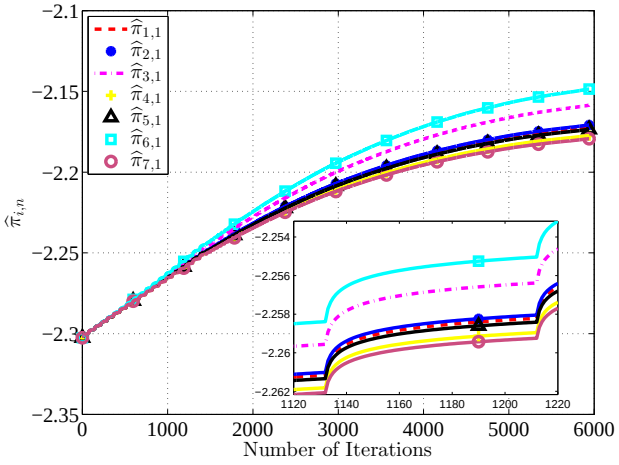


Fig. 1: Primal variables $\widehat{\pi}_{i,n}$

We notice that for the centralized power allocation approach, the primal problem requires approximately 6000 iterations to converge. As shown in Fig. 2, 1100 rounds are required to reach the optimal values of the primal and the dual variables. The zoomed box within Fig. 1 shows $\widehat{\pi}_{i,n}$ versus the number of primal iterations for a given round t. The values of $\widehat{\pi}_{i,n}$ are calculated using the dual variables obtained at the round $(t-$

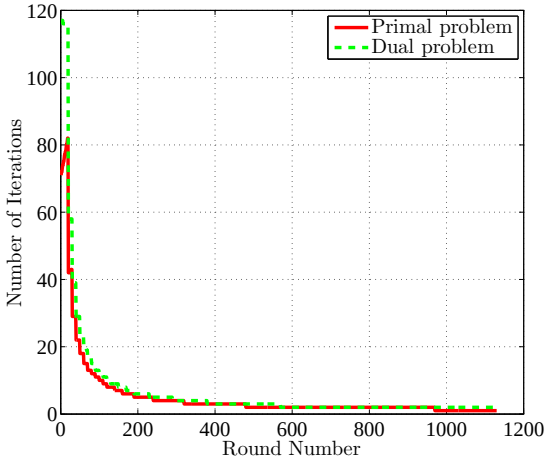


Fig. 2: Primal and dual iterations per round

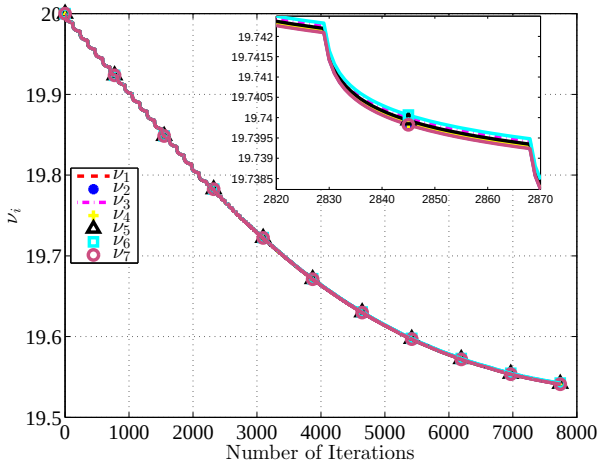


Fig. 4: Lagrange prices ν_i

1). We also notice that the number of primal iterations and the number of dual iterations decreases with the number of rounds. When t increases, the impact of Lagrange prices $\lambda_{k,i,n}(t)$ and $\nu_i(t)$ on the primal variables calculation is reduced, and the number of primal iterations required for convergence becomes lower. The same behavior is noticed for the number of dual iterations when the number of rounds increases.

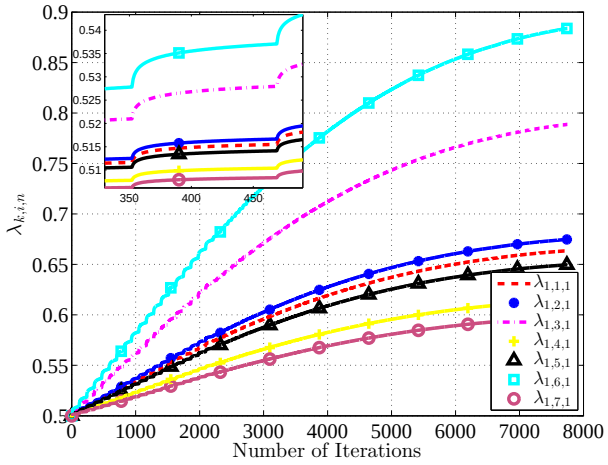


Fig. 3: Lagrange prices $\lambda_{k,i,n}$

For the same simulated scenario, we also show the dual variables $\lambda_{k,i,n}$ and ν_i versus the number of dual iterations in Fig. 3 and Fig. 4, respectively. We notice that approximately 8000 iterations are required for the dual problem to converge. At a given round t, the Lagrange prices $\lambda_{k,i,n}$ and ν_i are updated using the most recent values of the primal variables. The zoomed boxes within Fig. 3 and Fig. 4 show the evolution of $\lambda_{k,i,n}$ and ν_i versus the number of iterations, respectively. These values are updated until $\Delta\lambda_{k,i,n}$ and $\Delta\nu_i$ become less than ϵ. Convergence of the centralized power allocation problem occurs when two conditions are satisfied: first, the difference between the updated primal variables at round t and their values at round $(t-1)$ is less than ϵ. Second, the difference between the updated primal variables at round t and their values at round $(t-1)$ is less than ϵ.

B. Comparison with State-of-the-Art Approaches

We also compare the performance of our proposed centralized resource and power allocation approach with that of state-of-the-art resource and power allocation approaches such as the frequency reuse-1 model, the frequency reuse-3 model, FFR, and SFR techniques [26]. The frequency reuse-1 model allows the usage of the same frequency spectrum simultaneously in all the network cells. Moreover, homogeneous power allocation is performed.

In the frequency reuse-3 model, one third of the available spectrum is used in each cell in a cluster of three adjacent cells. Interference problems are eliminated, but the spectral efficiency is reduced. FFR and SFR techniques divide each cell into a cell-center and a cell-edge zones, and set restrictions on resource usage and power allocation in each zone. For all the compared techniques, resource allocation is performed according to Theorem 5.1.

1) System Throughput: For several simulation runs, we show the total system throughput for our proposed centralized resource and power allocation approach, for the frequency reuse-1 model, reuse-3 model, FFR, and SFR techniques under the same simulation scenarios. Simulation results, including the 95% confidence interval, are illustrated in Fig. 5.

It is shown that the centralized resource allocation approach offers the highest system throughput among all the compared techniques. In fact, it searches for the optimal resource and power allocation while taking into account restrictions on resource usage between the active UEs and on the downlink transmission power allocation. The achievable throughput is greater than that of the frequency reuse-3 model, FFR, and SFR techniques. Although the restrictions made on resource usage by these techniques mitigate ICI, the achievable throughput is reduced since the available spectrum in each cell or in each cell zone, is reduced.

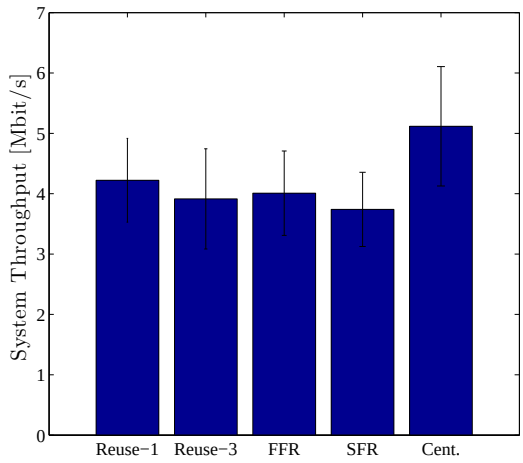


Fig. 5: System throughput

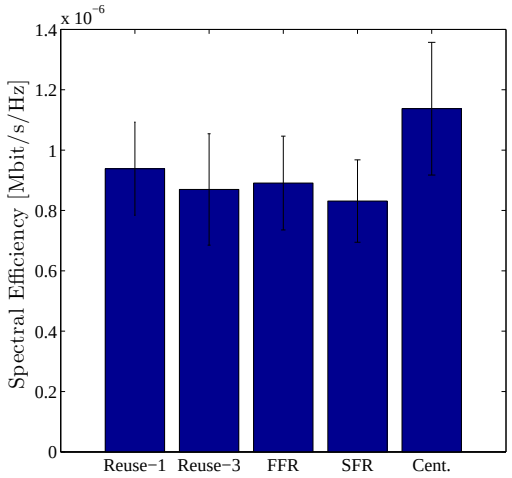


Fig. 6: Spectral efficiency

2) Spectral Efficiency: We also investigate the impact of the compared techniques on the spectral efficiency. Simulation results are shown in Fig. 6. Our proposed centralized resource allocation approach offers the highest spectral efficiency, since the optimal resource and power allocation is guaranteed. In fact, the static resource allocation between cell zones, and the quantified transmission power levels do not allow to perform flexible resource allocation in a manner that satisfies UE needs in each cell.

VII. Conclusion

Resource and power allocation problem is a challenging problem for present and future wireless networks. Several state-of-the-art techniques consider the joint resource and power allocation problem, and formulate it as nonlinear optimization problems. However, the main disadvantage of these techniques is that they do not consider the impact of inter-cell interference. Indeed, each cell solves its own resource and power allocation problem without taking into account resource usage and power allocation in the neighboring cells.

In this paper, we formulated the multi-cell joint resource and power allocation problem for multiuser OFDMA networks as a centralized optimization problem, where the objective is to maximize system throughput while guaranteeing throughput fairness between UEs. The joint problem is then decomposed into two independent problems: a resource allocation problem and a power allocation problem. Contrarily to the majority of the state-of-the-art approaches, inter-cell interference is not neglected, and the impact of the simultaneous transmissions in the neighboring cells is taken into account when managing the resource and power allocation. Simulation results prove the convergence of the optimization variables, and show the positive impact of our proposed centralized resource and power allocation approach on system performance.

Appendix I

Proof of Theorem 5.1

The objective function (12a), can be written as follows:

$$(\eta_2)_i \quad = \log\left(\prod_{\substack{k\in\mathcal{K}(i)\\ n\in\mathcal{N}}} \theta_{k,n}\right). \tag{19}$$

Since the logarithmic function is monotonically increasing, the maximization of $(\eta_2)_i$ is equivalent to the maximization of the term $\prod_{\substack{k\in\mathcal{K}(i)\\ n\in\mathcal{N}}} \theta_{k,n}$. We consider the following cases:

1) Let us assume that:

$$\sum_{k\in\mathcal{K}(i)} \theta_{k,n} < \sum_{n\in\mathcal{N}} \theta_{k,n},\ \forall\, k\ \in \mathcal{K}(i),\ \forall\, n \in\ \mathcal{N}. \tag{20}$$

We suppose that $\theta_{k,n}, \forall\ k \in \mathcal{K}(i), \forall\ n \in \mathcal{N}$ is an optimal solution to the resource allocation problem (12) $i.e.$, this solution maximizes the objective function (12a). For this solution, we assume that:

$$\exists\ k\ \in\ \mathcal{K}(i)\ /\ \sum_{n\in\mathcal{N}} \theta_{k,n} < 1. \tag{21}$$

We define $\epsilon > 0$ as follows:

$$\epsilon = 1 - \sum_{n\in\mathcal{N}} \theta_{k,n},$$

and we demonstrate that this solution is not an optimal solution to problem (12) using the proof by contradiction. In fact, we define a set of $\theta'_{k,n}$ variables as follows:

$$\theta'_{k,n} = \begin{cases} \theta_{k,n}, & \forall\ n \in \mathcal{N},\ n \neq n_1, \forall\ k \in \mathcal{K}(i), \\ \theta_{k,n} + \epsilon, & \text{if } n = n_1, \forall\ k \in \mathcal{K}(i). \end{cases}$$

Therefore, we have:

$$\prod_{\substack{k\in\mathcal{K}(i)\\ n\in\mathcal{N}}} \theta'_{k,n} = \prod_{\substack{k\in\mathcal{K}(i)\\ n\in\mathcal{N}}} \theta_{k,n} + \epsilon \cdot \prod_{\substack{k\in\mathcal{K}(i)\\ n\in\mathcal{N}}} \theta_{k,n} > \prod_{\substack{k\in\mathcal{K}(i)\\ n\in\mathcal{N}}} \theta_{k,n},$$

and the assumption made in (21) is false, since it does not maximize the objective function (12a). Con-

sequently, we have:

$$\sum_{n\in\mathcal{N}} \theta_{k,n} = 1, \forall\ k\ \in\ \mathcal{K}(i),$$
$$\Rightarrow \quad \sum_{k\in\mathcal{K}(i)} \sum_{n\in\mathcal{N}} \theta_{k,n} = |\mathcal{K}(i)|.$$

Since the sum of all the $\theta_{k,n}$ variables is constant, the term $\prod_{\substack{k\in\mathcal{K}(i)\\ n\in\mathcal{N}}} \theta_{k,n}$ reaches its maximum when all the variables $\theta_{k,n}$ are equal $i.e.$,

$$\theta_{k,n} = \frac{|\mathcal{K}(i)|}{|\mathcal{K}(i)|\cdot|\mathcal{N}|} = \frac{1}{|\mathcal{N}|}, \forall\ k \in \mathcal{K}(i), \forall\ n \in \mathcal{N},$$

which is an optimal solution to the resource allocation problem (12).

2) Similarly, when:

$$\sum_{n\in\mathcal{N}} \theta_{k,n} < \sum_{k\in\mathcal{K}(i)} \theta_{k,n},\ \forall\ k\ \in \mathcal{K}(i),\ \forall\ n \in\ \mathcal{N}. \quad (22)$$

In this case, the optimal solution is given by:

$$\theta_{k,n} = \frac{|\mathcal{N}|}{|\mathcal{K}(i)|\cdot|\mathcal{N}|} = \frac{1}{|\mathcal{K}(i)|}, \forall\ k \in \mathcal{K}(i), \forall\ n \in \mathcal{N}.$$

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