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# ESTIMATION OF CRYSTAL ORIENTATION FABRIC FROM AIRBORNE POLARIMETRIC ICE SOUNDING RADAR DATA

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## ABSTRACT

Information on ice sheet crystal orientation fabrics (COF) is valuable, as it tells about past deformation and current flow properties, which can potentially improve ice flow modelling. Due to the birefringence of ice sheets, ice sounding radars can provide the desired COF information if multiple polarizations are employed. Until now such data have mostly been acquired with ground-based single-polarization radars by rotating the antenna between multiple data takes. In this paper, a novel method based on polarimetric radar data, polarization synthesis, and complex coherence is proposed and tested with data from an airborne, polarimetric ice sounding radar. The results are consistent with ice core analyses, ice sheet physics, and existing radar results.

***Index Terms*****—**Radar sounding, polarimetry, coherence, birefringence, crystal orientation fabric (COF), ice sheets.

## 1. INTRODUCTION

A single ice crystal is birefringent, i.e. its refractive index depends on the polarization of the electromagnetic field [1]. An ice sheet is also birefringent in presence of an anisotropic crystal orientation fabric (COF). A COF describes the distribution of ice crystal orientations, for instance, a vertical girdle COF is observed at certain depths in ice cores from ice divides, as the c-axis of ice crystals tend to rotate away from the tension axis perpendicular to the ice divide [2], [3]. Due to the drilling process, the azimuthal orientation of ice cores is generally lost [3]. Radar can provide the COF orientation, which is valuable, as it tells about the deformation history and anisotropic ice viscosity, of relevance to ice sheet flow modelling [1].

Complex coherence has previously been proposed as a means of quantifying birefringence (the degree of horizontal alignment of the c-axes) [4]. Subsequently, the coherence method was used to also estimate the horizontal orientation of the COF from data acquired with a rotating single-polarized radar antenna [5]. In this paper the method is extended to polarimetric radar data, which implies that COF can be mapped with airborne radar along long flight tracks.

The method is proposed in Section 2, and in Section 3 an ice sheet model is presented and used to test the method. Data measured with airborne radar are introduced in Section 4, and the application of the method to these data is addressed in Section 5, followed by a discussion of the outcome and a validation. Finally, the conclusions are listed in Section 6.

## 2. METHODOLOGY

The proposed COF estimation method is based on the complex coherence

$$c_{12} = \frac{\langle s_1 s_2^* \rangle}{\sqrt{\langle |s_1|^2 \rangle \langle |s_2|^2 \rangle}} . \qquad (1)$$

Here $s_1$ and $s_2$ are two radar signals, $*$ and <…> denote complex conjugation and expectation value, respectively. The amplitude of the coherence is a real number between 0 and 1 quantifying the correlation of the two signals, and the phase of the coherence is a statistically efficient estimator of the phase difference between the two signals. For a signal with a polarization parallel to one of the principal axes of the COF (the horizontal orientation of the greatest and smallest concentration of the ice crystal c-axes) the polarization does not change, when the signal propagates through the ice. Assume that 1) $s_1$ and $s_2$ are chosen as two orthogonal colinearly polarized signals, aligned with two horizontal principal axes of a biaxial birefringent volume, 2) the birefringence does not change with depth, and 3) the phase of the radar signals depends exclusively on the anisotropic propagation (isotropic scattering), then the $c_{12}$ phase is proportional to the depth $z$ and the difference between the refractive indices at the two polarizations, $\Delta n$ [7][4]

$$\delta = -\frac{4\pi\Delta n}{\lambda} z \qquad (2)$$

where $\lambda$ is the wavelength. This relationship has been used to estimate the birefringence from airborne radar data acquired at two locations on the ice divide in Greenland [4].

Power-based methods have often been used for COF characterization (e.g. [1]), but phase-based methods like interferometry and the coherence method are often more sensitive than power-based methods. In this paper the coherence method is used to estimate the principal axes of COF. The coherence method is applied to quad-pol data and combined with polarization synthesis. When applied to single-pol data, the orientation cannot be estimated unless co-pol data are acquired at multiple azimuthal angles, e.g. by rotating a single-pol antenna over at least 180° [5].

The method proposed in the following uses polarization synthesis to estimate a pair of orthogonal linear polarizations that maximize the absolute value of the coherence phase. In the simplest case, where the COF orientation is depth independent, these polarizations are the ordinary and the extraordinary polarizations.

Any polarization can be synthesized as [5]

$$s = \mathbf{p}_r^{\mathrm{H}} \mathbf{S} \mathbf{p}_t \, . \tag{3}$$

It is noted that polarization synthesis is not feasible unless quad-pol data are available. In the following the ice sounding radar is assumed to be linearly polarized, but other types of quad-pol data can also be used, e.g. circular right- and left-handed polarizations. For linear polarization the scattering matrix is defined as

$$\mathbf{S} = \begin{bmatrix} s_{\mathrm{hh}} & s_{\mathrm{vh}} \\ s_{\mathrm{hv}} & s_{\mathrm{vv}} \end{bmatrix} \tag{4}$$

where, for instance, $s_{\mathrm{hv}}$ is transmitted at h polarization and received at v polarization, which are defined as along track polarization and across track polarization, respectively. $\mathbf{p}_t$ and $\mathbf{p}_r$ are unit vectors defining the synthesis of transmit and receive polarizations, respectively. The elements of these vectors have the same phase for colinear polarization, while circular transmit polarization, for instance, is synthesized with $\mathbf{p}_t = [1,\ j]^{\mathrm{T}}/\sqrt{2}$ where $j$ is the imaginary unit, and T denotes matrix transpose.

The expectation value $\langle s_1 s_2^* \rangle$ can be expressed as [6]

$$\langle s_1 s_2^* \rangle = \mathbf{B}_1 \mathbf{C} \mathbf{B}_2^{\mathrm{H}} \tag{5}$$

where the $\mathbf{B}_1$ and $\mathbf{B}_2$ matrices are defined as

$$\mathbf{B}_1 = \begin{bmatrix} \mathrm{p}_{\mathrm{r}1,0} & \mathrm{p}_{\mathrm{t}1,0} \\ \mathrm{p}_{\mathrm{r}1,1} & \mathrm{p}_{\mathrm{t}1,0} \\ \mathrm{p}_{\mathrm{r}1,0} & \mathrm{p}_{\mathrm{t}1,1} \\ \mathrm{p}_{\mathrm{r}1,1} & \mathrm{p}_{\mathrm{t}1,1} \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} \mathrm{p}_{\mathrm{r}2,0} & \mathrm{p}_{\mathrm{t}2,0} \\ \mathrm{p}_{\mathrm{r}2,1} & \mathrm{p}_{\mathrm{t}2,0} \\ \mathrm{p}_{\mathrm{r}2,0} & \mathrm{p}_{\mathrm{t}2,1} \\ \mathrm{p}_{\mathrm{r}2,1} & \mathrm{p}_{\mathrm{t}2,1} \end{bmatrix}. \tag{6}$$

The 4×4 covariance matrix is defined as [6]

$$\mathbf{C} = \langle \mathbf{s}\, \mathbf{s}^{\mathrm{H}} \rangle \tag{7}$$

where in turn the lexicographic scattering vector is

$$\mathbf{s} = [s_{\mathrm{vv}}\ s_{\mathrm{vh}}\ s_{\mathrm{hv}}\ s_{\mathrm{hh}}\ ]^{\mathrm{T}} \, . \tag{8}$$

In practice the expectation values in (7) is estimated by averaging over small range-azimuth patches.

Copolarization is ensured by requiring

$$\mathbf{p}_{\mathrm{r}1} = \mathbf{p}_{\mathrm{t}1}^*, \quad \mathbf{p}_{\mathrm{r}2} = \mathbf{p}_{\mathrm{t}2}^* \, . \tag{9}$$

Orthogonal polarization is ensured by requiring

$$\mathbf{p}_{\mathrm{t}2} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{p}_{\mathrm{t}1}^*, \quad \mathbf{p}_{\mathrm{r}2} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{p}_{\mathrm{r}1}^* \, . \tag{10}$$

## 3. MODELLING

A backscattering and propagation model has been implemented and used to generate synthetic scattering matrix data for a range of depths. The model is similar to that presented in [7], e.g. the COF has three orthogonal principal axes, a vertical one and two horizontal ones. In this paper the model is simplified by assuming isotropic scattering. Also, it is assumed that the attenuation coefficient is isotropic, and the principal axes have the same orientation at all depths.

The radar signals propagate in the vertical direction, and the signal components that are polarized along the two horizontal axes propagate without changing their polarizations. The absolute amplitude and propagation phase of the two polarizations are of no interest, but the phase difference changes linearly with depth, as expressed by the one-way propagation matrix [7], [5]

$$\mathbf{P}(\delta/2) = \begin{bmatrix} \exp(j\delta/2) & 0 \\ 0 & 1 \end{bmatrix} \tag{11}$$

where $\delta$ is the two-way propagation phase difference. The model exploits the simple propagation in an $(x, y)$ coordinate system that is aligned with the principal axes of the COF. First the transmit polarization vector is converted from the $(h, v)$ coordinate system of the radar to the $(x, y)$ system, and subsequently the receive polarization is converted back to the $(h, v)$ system. This is accomplished with the rotation matrix $\mathbf{R}$ and its inverse, where

$$\mathbf{R}(\alpha) = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}. \tag{12}$$

$\alpha$ is the angle between the $(h, v)$ system and the $(x, y)$ system. Since $\mathbf{R}$ is a rotation matrix its inverse is $\mathbf{R}' = \mathbf{R}^{\mathrm{T}}$, and the modelled scattering matrix can be expressed as

$$\mathbf{S} \ = \ \mathbf{R}(\alpha)\mathbf{P}(\delta/2)\boldsymbol{\Gamma}(z)\mathbf{P}(\delta/2)\mathbf{R}'(\alpha) \tag{13}$$

where $\boldsymbol{\Gamma}$ is the reflection matrix and $z$ is the depth under the ice surface. The coherence phase in (1) is computed from the modelled scattering matrix data by synthesizing pairs of orthogonal colinear polarizations cf. (9) and (10), e.g. the transmit polarizations are

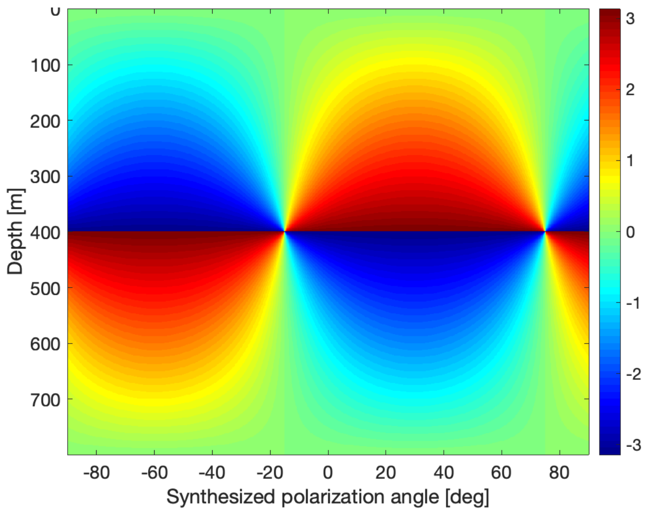


Fig. 1 Modelled coherence phase. The COF is rotated by $\alpha = -60°$ with respect to the $(h, v)$ system of the radar, and the difference between the refractive indices along the principal axes in the horizontal plane is $\lambda/1600$.

$$\mathbf{p}_{t1} = \begin{bmatrix} \cos(\varphi) \\ \sin(\varphi) \end{bmatrix} \tag{14}$$

where $\varphi$ is the orientation angle for which $-90° < \varphi < 90°$.

One modelling result is shown in Fig. 1. If a larger range of depths had been covered, the plot would repeat in the vertical direction. As expected, the phase is seen to vary linearly with depth when the synthesized polarizations are aligned with the principal axes of the COF, i.e. when $\varphi = \alpha = -60°$ and $\varphi = \alpha + 90° = 30°$.

The orientation of the principal axes can be estimated for each depth as the angle $\varphi$ where the absolute value of the coherence phase is maximum. In practice a more accurate result can be obtained by estimating a pair of orthogonal polarizations for which the coherence phase is zero, because the absolute value of the phase has a narrow notch at these polarizations, while the phase maxima are broad. To find the angle of the girdle, 45° is added, and a 90° ambiguity is resolved by comparing the depth derivative of the phase at the maxima of the absolute coherence phase, i.e. at –60° and 30°. The derivative is negative for a polarization parallel to the girdle because an ice crystal is positive birefringent (minimum refractive index for polarizations perpendicular to the c-axis and hence perpendicular to the girdle). Consequently, in Fig. 1 the girdle is oriented at –60°.

The COF orientation angle cannot be estimated at depths where the coherence phase does not depend on the synthesized polarization angle $\varphi$. In Fig. 1 this is the case at depths around 0 m and 800 m.

Once the orientation of the principal axes has been determined, (2) can be used to quantify the birefringence in terms of $\Delta n$ [4]. Without polarimetric data and the option to synthesize polarizations the birefringence can still be estimated [5], provided the COF orientation is determined by other means. When the orthogonal radar polarizations are not aligned with the principal axes, the coherence phase does however not change linearly with phase as seen from Fig. 1.

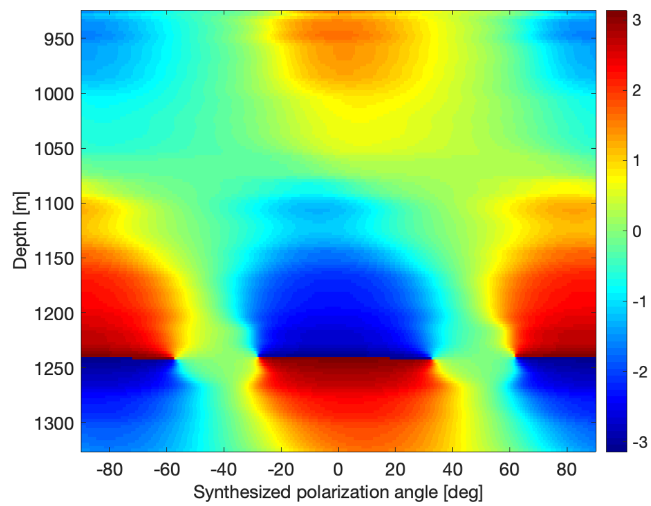


Fig. 2 Coherence phase computed from POLARIS data acquired at NEEM from a flight track parallel to the ice divide. The synthesized polarization angle is defined with respect to the $(h, v)$ system.

## 4. RADAR DATA

The coherence method has been applied to airborne quad-pol ice sounding radar data acquired at two locations on the ice divide in the dry snow zone of Greenland. These data were acquired with ESA's Polarimetric Airborne Radar Ice Sounder (POLARIS), developed and operated by the Technical University of Denmark [9]. POLARIS is a P-band radar, i.e. the center frequency is 435 MHz. The quad-pol data used in this paper were acquired over the NEEM ice core drilling site from a flight track parallel to the ice divide, i.e. the h polarization is almost parallel to the ice divide (no yaw correction applied). Quad-pol data were also acquired from a perpendicular flight track.

## 5. RESULTS

The coherence phase shown in Fig. 2 is based on measured radar data. The complex coherence is computed from the covariance matrix in (7), where the expectation value is estimated by averaging over patches with a dimension of 5 pixels in range, whereas the entire data extent in the along track dimension was exploited. For the stratified ice sheet, $\mathbf{s}\cdot\mathbf{s}^H$ changes very little in the along track direction.

Largely, the measured result in Fig. 2 is consistent with the modelled result in Fig. 1, though a slight skew is seen in the former. This could be due to a slight change of COF orientation with depth.

At each depth the COF orientation can be estimated as the orientation angle of the synthesized polarization that maximizes the absolute coherence phase. As mentioned, this is done via the zero crossings. At depths where the coherence phase is close to a multiple of $2\pi$ for all polarization angles,

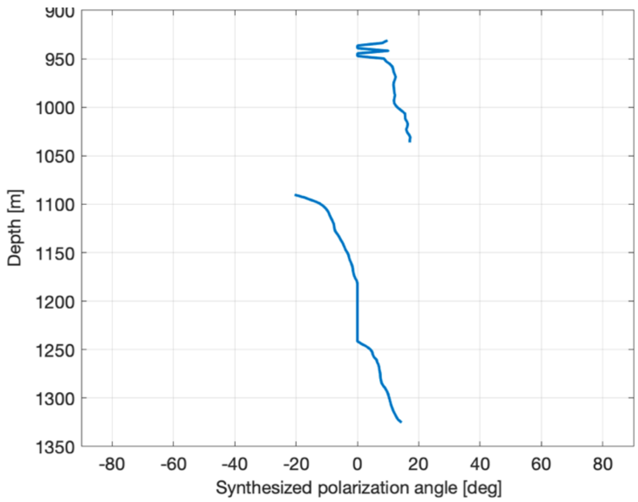


Fig. 3 Estimated COF orientation angle (girdle direction) at NEEM. For $\varphi = 0°$ the synthesized polarization is parallel to the aircraft center line, i.e. approximately parallel to the ice divide.

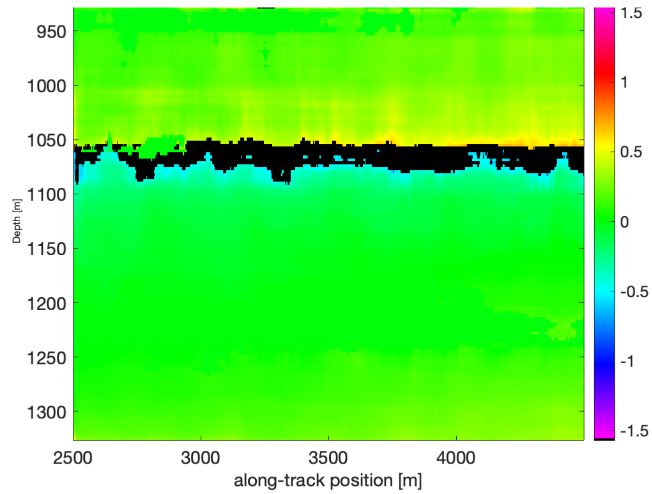


Fig. 4. Estimated COF orientation angle at NEEM. For the black pixels, no COF orientation could be estimated, because at this depth the coherence phase has little sensitivity to the polarization.

the method is unable to estimate the COF orientation. This is seen as a gap in the plot at 1050 m depth in Fig. 3.

The estimated COF orientation is close to 0° (i.e. along the center line of the aircraft), and hence almost parallel to the ice divide. This is consistent with the vertical girdle COF observed in the NEEM ice core and its theoretical orientation parallel to the ice divide [2]. Also, it is consistent with the orientation estimated by rotating a single-pol antenna of a ground-based radar [5].

In Fig. 4 the patch size is still 5 pixels in range but only 100 pixels in the along track direction, which enables an estimation of the COF orientation versus both range and along track position. The result is quite constant across both dimensions, and it is not very noisy. The corresponding result obtained from the perpendicular flight track is approximately 90° as expected.

## 6. CONCLUSIONS

The primary findings are:

- A method for estimation of COF orientation angle from polarimetric ice sounding radar data has been presented.
- The method can be applied to airborne data, thereby enabling area-extensive COF orientation mapping.
- The results obtained from measured radar data are consistent with model results.
- The estimated COF orientation is consistent with a vertical girdle fabric as observed in the NEEM ice core and consistent with the orientation of this girdle as expected from theory.
- Once the COF orientation has been estimated polarizations parallel and perpendicular to the COF can be synthesized.
- Thereby the birefringence can be computed directly from the derivative of the coherence phase [4].

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