WCCI 2012 IEEE World Congress on Computational Intelligence
June, 10-15, 2012 - Brisbane, Australia IJCNN

Rotor Associative Memory with a Periodic Activation Function

Yozo Suzuki
University of Yamanashi,
Interdisciplinary Graduate School of
Medicine and Engineering,
4-3-11 Takeda, Kofu,
Yamanashi 400-8511, Japan

Michimasa Kitahara
University of Yamanashi,
Interdisciplinary Graduate School of
Medicine and Engineering,
4-3-11 Takeda, Kofu,
Yamanashi 400-8511, Japan

Masaki Kobayashi
University of Yamanashi,
Interdisciplinary Graduate School of
Medicine and Engineering,
4-3-11 Takeda, Kofu,
Yamanashi 400-8511, Japan
Email: k-masaki@yamanashi.ac.jp

***Abstract*—Complex-valued Associative Memory (CAM) can store multi-state patterns unlike Hopfield Associative Memory (HAM). CAM stores not only given training patterns but also many spurious patterns, such as their rotated patterns, at the same time. Rotor Associative Memory (RAM) can make the most of rotated patterns unstable but the reversed patterns remain stable. In the present work, we propose RAM with a Periodic Activation Function (PAF) to make the reversed patterns unstable. PAF is an activation function that Aizenberg introduced to CAM. We prove that RAM with a PAF has far fewer spurious patterns by using dynamic associative memories which can search the stored patterns.**

***Index Terms*—complex-valued neural networks, associative memory, rotor associative memory, periodic activation function**

I. Introduction

Several advanced associative memory models have been proposed since Hopfield Associative Memory (HAM) was proposed by Hopfield [1]. Complex-valued Associative Memory (CAM) is one of them (Aizenberg et al. [2], Noest [3], Jankowski et al. [4]). CAM is often applied to storing gray scale images (Aoki and Kosugi [5], Aoki et al. [6], Muezzinoglu et al. [7]). However, CAM stores not only training patterns but also their rotated patterns. This is referred to as rotation invariance (Zemel et al. [8]). Kitahara and Kobayashi [9], and Kitahara et al. [10], [11] proposed Rotor Associative Memory (RAM) to reduce spurious patterns. RAM can make the most rotated patterns unstable but the reversed patterns remain stable.

Aizenberg proposed universal binary neurons, a flexible binary neuron model (Aizenberg et al. [2]). Moreover, he extended them to complex-valued neurons with a Periodic Activation Function (PAF) (Aizenberg [12], [13]). In the present work, we propose rotor neurons with a PAF to remove the stable reversed patterns. It is impossible, however, to make only the training patterns stable. So it is desirable that only the training patterns are strong attractors and the other stable states are weaker ones.

Dynamic associative memories can search patterns stored in the associative memories. The time the recalled pattern, which has minimal energy, holds depends on the depth of energy. We use a dynamic associative memory model based on Nagumo and Sato [14] to investigate stable states (Kitahara et al. [10], [11]). Our computer simulations show that our proposed model makes all the rotated patterns, including the reversed patterns, unstable and only the training patterns are strong attractors.

II. Complex-valued Associative Memory

A. Complex-valued neurons

A complex-valued neuron has a complex number. And the state of complex-valued neuron is K-valued on the complex unit circle, where K is an integer greater than two. It divides the complex unit circle into K sectors. Let a real number θ_K and complex numbers s_k $(k = 0, \cdots, K-1)$be as follows:

$$\theta_K = \frac{\pi}{K}, \tag{1}$$

$$s_k = \exp(\sqrt{-1}(2k+1)\theta_K), \tag{2}$$

The states of complex-valued neurons belong to the set $\{s_k\}$.

A complex-valued neuron receives the weighted sum input from all the other neurons. Then it selects a new state for the weighted sum input by following the activation function. In the present work, we use the following activation function $f(\cdot)$:

$$f(z) = \begin{cases} s_0 & 0 \le \arg(z) < 2\theta_K \\ s_1 & 2\theta_K \le \arg(z) < 4\theta_K \\ s_2 & 4\theta_K \le \arg(z) < 6\theta_K \\ \vdots & \\ s_{K-1} & 2(K-1)\theta_K \le \arg(z) < 2K\theta_K \end{cases} \tag{3}$$

where $\arg(z)$ is the argument of the complex number z. Therefore, $f(z)$ maximizes $\mathrm{Re}(\bar{s}_k z)$, where $\mathrm{Re}(z)$ and $\bar{z}$ are the real part and the complex conjugate of z respectively.

B. Complex-valued Associative Memory (CAM)

Let a complex number w_{ji} be the connection weight from the neuron i to the neuron j. Then the connection weight w_{ji} needs to satisfy the following requirement:

$$w_{ji} = \bar{w}_{ij}. \tag{4}$$

This requirement ensures that CAM reaches a stable state. In section III-D, it is proven that CAM is a special case of RAM due to (2). In section III-C, it is proven that RAM always

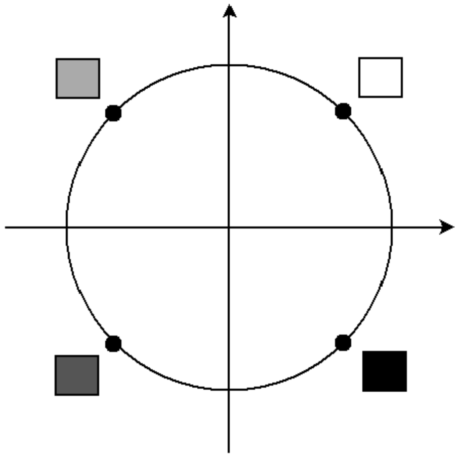

Fig. 1. The correspondence of the state of neuron and gray scale

Fig. 2. A training pattern: This pattern consists of 20×20 pixels, so CAM also consists of 400 neurons.

reaches a stable state. Therefore, CAM always reaches a stable state, too.

Let z_i be the state of the neuron i. The weighted sum input S_j that the neuron j receives from all the other neurons is defined as follows:

$$S_j = \sum_i w_{ji} z_i. \tag{5}$$

Assume that $\mathbf{z}^p$ is the pth training pattern and $\mathbf{z}^p = (z_1^p, \cdots, z_N^p)$ $(p = 1, 2, \cdots, P)$, where P and N are the numbers of the training patterns and the neurons. Then the complex-valued hebbian learning rule is as follows:

$$w_{ji} = \sum_p z_j^p \bar{z}_i^p. \tag{6}$$

The connection weights w_{ji} obtained by (6) clearly satisfy (4). Given a training pattern z^q to CAM, the weighted sum input S_j to the neuron j is as follows:

$$S_j = \sum_p \sum_{i \neq j} z_j^p \cdot \bar{z}_i^p \cdot z_i^q \tag{7}$$

$$= (N-1) z_j^p + \sum_{p \neq q} \sum_{i \neq j} z_i^p \cdot \bar{z}_i^p \cdot z_i^q. \tag{8}$$

If the second term, which we call the crosstalk term, is enough small, the training pattern z^q is stable.

C. Rotated patterns in CAM

In the present work, in order to visualize the behavior of associative memories, the states of neurons and the gray scale levels correspond. Figure 1 shows correspondence between the states of complex-valued neurons and the gray scale levels in case of $K = 4$. The corresponding gray scale becomes darker as the suffix k of s_k increases. Then a training pattern stands for a gray scale image and each neuron corresponds to a pixel. Figure 2 shows an example of training pattern. This pattern consists of 20×20 pixels. Since each pixel corresponds to a neuron of CAM, this CAM has 400 neurons.

Fig. 3. Rotated patterns of Fig. 2

For a training pattern $\mathbf{z} = (z_1, z_2, \cdots, z_N)$, the patterns $s_k\mathbf{z} = (s_k z_1, \cdots, s_k z_N)$ $(k = 1, 2, \cdots, K-1)$ are referred to as its rotated patterns. Therefore, the rotated patterns are obtained by rotating the states of all neurons by $2k\theta_K$. The rotated patterns of Fig. 2 are the three patterns shown in Fig.3.

Suppose that a training pattern $\mathbf{z}$ is stable. Then the following equation holds for each j:

$$f(\sum_{i \neq j} w_{ji} z_i) = z_j. \tag{9}$$

For a rotated pattern $s_k\mathbf{z}$, the following equation holds:

$$f(\sum_{i \neq j} w_{ji} s_k z_i) = s_k f(\sum_{i \neq j} w_{ji} z_i) = s_k z_j. \tag{10}$$

This implies that the rotated patterns $s_k\mathbf{z}$ are also stable. Therefore, a training pattern has $K-1$ stable rotated patterns.

D. Dynamic Complex-valued Associative Memory (DCAM)

Once CAM recalls a stable state, it keeps the stable state. Dynamic associative memories can get out of stable state and search patterns stored in the associative memories. In the recall process of dynamic associative memory, The time the recalled pattern, which has minimal energy, holds depends on the depth of energy. The strong attractors hold for a long time and the weak ones do for a short time.

Let $z(t)$ and $S(t)$ be the neuron output and weighted sum input at time t. The dynamics of dynamic complex-valued neuron is defined as follows:

$$z(t+1) = f(S(t+1) - \alpha \sum_{d=0}^{t} k^d z(t-d)). \tag{11}$$

The coefficients α and k are the scaling factor and damping factor respectively. The second term of (11) is the historical term. In short, it is accumulation of old values of neuron. When CAM remains a stable state for a while, the historical terms become stronger. When the historical terms become enough large, CAM can get out of the stable state and reach another stable state. Therefore, DCAM can get out of the stable states and recall all the training patterns by the historical terms.

III. Rotor Associative Memory

A. Rotor neurons

The states of rotor neurons are represented by two-dimensional vectors. Let K be an integer greater than two. We define $\mathbf{h}_k$ as follows:

$$\mathbf{h}_k = \begin{pmatrix} \cos(2k+1)\theta_K \\ \sin(2k+1)\theta_K \end{pmatrix}. \tag{12}$$

Therefore, $\{\mathbf{h}_k\}$ divides the unit circle of the $x-y$ plane into K sectors.

The activation function of rotor neurons is a mapping from two-dimensional vectors to two-dimensional vectors. Let ϕ be the angle of a two-dimensional vector $\mathbf{z}$ to a two-dimensional vector $(1,0)^{\mathrm{T}}$, where superscript T stands for the transpose. Then the two-dimensional vector $(x,y)^{\mathbf{T}}$ is written as follows:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \sqrt{x^2+y^2}\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}. \tag{13}$$

We define the activation function $\mathbf{g}(\cdot)$ of rotor neurons as follows:

$$\mathbf{g}(\mathbf{z}) = \begin{cases} \mathbf{h_0} & 0 \le \phi < 2\theta_K \\ \mathbf{h_1} & 2\theta_K \le \phi < 4\theta_K \\ \mathbf{h_2} & 4\theta_K \le \phi < 6\theta_K \\ \vdots & \\ \mathbf{h_{K-1}} & 2(K-1)\theta_K \le \phi < 2K\theta_K \end{cases} \tag{14}$$

From ${\mathbf{h}_k}^{\mathrm{T}}\mathbf{z} = |\mathbf{z}|\cdot\cos(\phi-(2k+1)\theta_K)$, $\mathbf{g}(\mathbf{z})$ maximizes ${\mathbf{h}_k}^{\mathrm{T}}\mathbf{z}$. Rotor neurons are equivalent to complex-valued neurons if we regard complex numbers as two-dimensional vectors.

B. Rotor Associative Memory (RAM)

First, we define connection weights of RAM. The connection weights of RAM are expressed by 2×2 matrices. Let $\mathbf{W}_{ji}$ be the connection weight from the neuron i to the neuron j. The connection weights $\mathbf{W}_{ji}$ must satisfy the following relation.

$$\mathbf{W}_{ji} = \mathbf{W}_{ij}^{\mathrm{T}} \tag{15}$$

Next, we define the weighted sum input of RAM. Let a two-dimensional vector $\mathbf{z}_i$ be the state of the neuron i. Then we define the weighted sum input $\mathbf{S}_j$ to the neuron j as follows:

$$\mathbf{S}_j = \sum_{i\neq j}\mathbf{W}_{ji}\mathbf{z}_i. \tag{16}$$

Finally, we describe rotor hebbian learning rule. Assume that $\mathbf{A}^p$ is the pth training pattern and $\mathbf{A}^p = (\mathbf{a}_1^p,\cdots,\mathbf{a}_N^p)$ $(p = 1,2,\cdots,P)$. Then, the rotor hebbian learning rule is as follows:

$$\mathbf{W}_{ji} = \sum_p \mathbf{a}_j^p\mathbf{a}_i^{p\mathrm{T}}. \tag{17}$$

Then $\mathbf{W}_{ji}$ is a 2×2 matrix. It is clear that (17) satisfies (15). Given a training pattern $\mathbf{A}^q$ to CAM, the weighted sum input $\mathbf{S}_j$ to the neuron j is as follows:

$$\mathbf{S}_j = \sum_p\sum_{i\neq j}\mathbf{a}_j^p\mathbf{a}_i^{p\mathrm{T}}\mathbf{a}_i^q \tag{18}$$

$$= (N-1)\mathbf{a}_j^p + \sum_{p\neq q}\sum_{i\neq j}\mathbf{a}_i^p\mathbf{a}_i^{p\mathrm{T}}\mathbf{a}_i^q. \tag{19}$$

If the second term, which we call the crosstalk term, is enough small, the training pattern is stable.

C. Energy function

The energy function E of RAM is defined as follows:

$$E = -\frac{1}{2}\sum_j\sum_{i\neq j}\mathbf{z}_j^{\mathrm{T}}\mathbf{W}_{ji}\mathbf{z}_i. \tag{20}$$

Next, we prove that the energy function decreases monotonously. Suppose that the state $\mathbf{z}_k$ of the neuron k changes to the state $\mathbf{z'}_k$. Then the energy gap ΔE is as follows:

$$\Delta E = -\frac{1}{2}\sum_{i\neq k}{\mathbf{z}'}_k^{\mathrm{T}}\mathbf{W}_{ki}\mathbf{z}_i - \frac{1}{2}\sum_{j\neq k}\mathbf{z}_j^{\mathrm{T}}\mathbf{W}_{jk}\mathbf{z}'_k + \frac{1}{2}\sum_{i\neq k}\mathbf{z}_k^{\mathrm{T}}\mathbf{W}_{ki}\mathbf{z}_i + \frac{1}{2}\sum_{j\neq k}\mathbf{z}_j^{\mathrm{T}}\mathbf{W}_{jk}\mathbf{z}_k \tag{21}$$

$$= -\frac{1}{2}\sum_{i\neq k}{\mathbf{z}'}_k^{\mathrm{T}}\mathbf{W}_{ki}\mathbf{z}_i - \frac{1}{2}\sum_{j\neq k}\mathbf{z}_j^{\mathrm{T}}\mathbf{W}_{kj}^{\mathrm{T}}\mathbf{z}'_k + \frac{1}{2}\sum_{i\neq k}\mathbf{z}_k^{\mathrm{T}}\mathbf{W}_{ki}\mathbf{z}_i + \frac{1}{2}\sum_{j\neq k}\mathbf{z}_j^{\mathrm{T}}\mathbf{W}_{kj}^{\mathrm{T}}\mathbf{z}_k \tag{22}$$

$$= -\frac{1}{2}\sum_{i\neq k}{\mathbf{z}'}_k^{\mathrm{T}}\mathbf{W}_{ki}\mathbf{z}_i - \frac{1}{2}\sum_{j\neq k}{\mathbf{z}'}_k^{\mathrm{T}}\mathbf{W}_{kj}\mathbf{z}_j + \frac{1}{2}\sum_{i\neq k}\mathbf{z}_k^{\mathrm{T}}\mathbf{W}_{ki}\mathbf{z}_i + \frac{1}{2}\sum_{j\neq k}\mathbf{z}_k^{\mathrm{T}}\mathbf{W}_{kj}\mathbf{z}_j \tag{23}$$

$$= -{\mathbf{z}'}_k^{\mathrm{T}}\sum_{i\neq k}\mathbf{W}_{ki}\mathbf{z}_i + \mathbf{z}_k^{\mathrm{T}}\sum_{i\neq k}\mathbf{W}_{ki}\mathbf{z}_i. \tag{24}$$

In (23), since the term $\mathbf{z}_k^{\mathrm{T}}\mathbf{W}_{kj}\mathbf{z}_j$ is a real number, the following equation holds:

$$\mathbf{z}_j^{\mathrm{T}}\mathbf{W}_{kj}^{\mathrm{T}}\mathbf{z}_k = \left(\mathbf{z}_j^{\mathrm{T}}\mathbf{W}_{kj}^{\mathrm{T}}\mathbf{z}_k\right)^{\mathrm{T}} \tag{25}$$

$$= \mathbf{z}_k^{\mathrm{T}}\mathbf{W}_{kj}\mathbf{z}_j. \tag{26}$$

Since $\mathbf{z}'_k$ maximizes $\mathbf{z}_k^{\mathrm{T}}\sum_{i\neq k}\mathbf{W}_{ki}\mathbf{z}_i$, where $\sum_{i\neq k}\mathbf{W}_{ki}\mathbf{z}_i$ is the weighted sum input to the neuron j, the energy E does not increase. The number of states of RAM is finite, so RAM always reaches a stable state.

D. Relationship with CAM

Let the state of the complex-valued neuron i and the connection weight from the neuron i to the neuron j be $z_i = x_i + y_i\sqrt{-1}$ and $w_{ji} = u_{ji} + v_{ji}\sqrt{-1}$ respectively. Then, the input from the complex-valued neuron i to the complex-valued neuron j is as follows:

$$w_{ji}z_i = (u_{ji}x_i - v_{ji}y_i) + (v_{ji}x_i + u_{ji}y_i)\sqrt{-1}. \tag{27}$$

We define the connection weights $\mathbf{W}_{ji}$ and the states $\mathbf{z}_i$ of the neuron of RAM corresponding CAM as follows:

$$\mathbf{W}_{ji} = \begin{pmatrix} u_{ji} & -v_{ji} \\ v_{ji} & u_{ji} \end{pmatrix}, \tag{28}$$

$$\mathbf{z}_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}. \tag{29}$$

Then connection weights $\mathbf{W}_{ji}$ satisfy (15) from $u_{ji} = u_{ij}$ and $v_{ji} = -v_{ij}$. The behavior of CAM is equivalent to that of the corresponding RAM. Therefore, we can regard CAM as a special case of RAM. However, complex-valued hebbian learning rule is different from rotor hebbian learning rule. RAM does not store the rotated patterns by learning rule of RAM, but stores the reversed patterns in case that K is even.

E. Projection Property of Rotor Hebbian Learning Rule

Suppose that the number of training patterns is one. Let $\mathbf{A} = (\mathbf{a}_1, \cdots, \mathbf{a}_N)$ be a training pattern. Then the connection weight is $\mathbf{W}_{ji} = \mathbf{a}_j \mathbf{a}_i^{\mathrm{T}}$ The state of the neuron i rotated by $2k\theta_K$ is expressed by $\mathbf{R}(k)\mathbf{z}_i$, where $\mathbf{R}(k)$ is as follows:

$$\mathbf{R}(k) = \begin{pmatrix} \cos 2k\theta_K & -\sin 2k\theta_K \\ \sin 2k\theta_K & \cos 2k\theta_K \end{pmatrix}. \tag{30}$$

Then $\mathbf{R}(k)\mathbf{A}$ is a rotated pattern. Suppose $\mathbf{R}(k)\mathbf{A}$ is given to RAM. Then the weighted sum input from the neuron i to the neuron j is as follows:

$$\mathbf{W}_{ji}\mathbf{R}(k)\mathbf{a}_i = \mathbf{a}_j \mathbf{a}_i^{\mathrm{T}} \mathbf{R}(k)\mathbf{a}_i \tag{31}$$

$$= \cos 2k\theta_K \mathbf{a}_j. \tag{32}$$

This equation means the projection from the two-dimensional vector $\mathbf{R}(k)\mathbf{a}_i$ to the two-dimensional vector $\mathbf{a}_j$. The weighted sum input $\mathbf{S}_j$ to the neuron j is as follows:

$$\mathbf{S}_j = (N-1)\cos 2k\theta_K \mathbf{a}_j. \tag{33}$$

Therefore, RAM recalls $\mathbf{A}$ in case of $\cos 2k\theta_K > 0$ and $-\mathbf{A}$ in case of $\cos 2k\theta_K < 0$. RAM does not recall the rotated patterns. Moreover, the larger $|\cos 2k\theta_K|$ is, the more strongly RAM recalls the training pattern $\mathbf{A}$ or the inverse pattern $-\mathbf{A}$.

F. Periodic activation function of rotor neurons

As described above, The RAM would recall the training patterns or the reversed patterns. To avoid recalling the reversed patterns, we introduce a Periodic Activation Function (PAF) to RAM. A PAF proposed by Aizenberg is a flexible activation function for complex-valued neurons. Let l be a positive integer. A PAF produces K-valued repeated periodically with periodicity coefficient l. Consider a rotor neuron with lK states. A PAF identifies $\mathbf{h}_k$ and $\mathbf{h}_{k'}$ if $k \equiv k' \bmod K$. Then l separated sectors are identified by a PAF.

We apply a PAF with $l = 2$. Then the number of states or sectors is $2K$ and the opposite sectors are identified. The two states $\mathbf{h}_k$ and $\mathbf{h}_{K+k}$, where $0 \leq k < K$, are different values but are identical. The behavior of RAM with a PAF is equivalent to that of ordinary RAM which has double number of neuron's state and only the interpretations of the neuron's states are different. Figure 4 illustrates the rotor neuron with a PAF in case of $K = 4$.

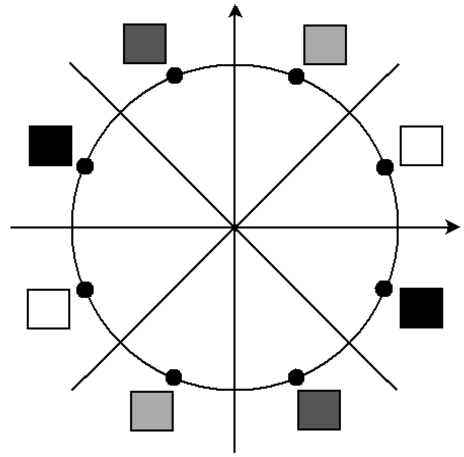

Fig. 4. Rotor neuron with a PAF

G. Dynamic Rotor Associative Memory (DRAM)

This model can get out of stable states and search the stable patterns. We construct the DRAM based on the DCAM. Let $\mathbf{z}(t)$ and $\mathbf{S}(t)$ be the neuron output and the neuron input at time t. The dynamics of dynamic rotor neurons is as follow:

$$\mathbf{z}(t+1) = \mathbf{f}(\mathbf{S}(t+1) - \alpha \sum_{d=0}^{t} k^d \mathbf{z}(t-d)). \tag{34}$$

The coefficients α and k are the scaling factor and damping factor respectively. The second term of (34) is the historical term. The bigger historical terms are, the stronger the effect of getting out of the current stable state is. In case of DRAM with a PAF, the dynamics is unchanged and only the interpretation changes.

IV. Computer Simulation

In this section, we confirm the behaviors of DCAM, DRAM and DRAM with a PAF. Dynamic associative memories can search patterns stored in associative memories. Moreover, we can estimate how strong attractors are by how long they hold.

A. Computer simulation for DCAM

Figure 5 shows the training patterns and their rotated patterns. The patterns A0, B0 and C0 are the training patterns. The patterns A1-3, B1-3 and C1-3 are their rotated patterns. Then CAM stores all these patterns.

In computer simulation, the parameters were as follows:

$$N = 400, k = 0.98, \alpha = 7. \tag{35}$$

The initial pattern was A0. Figure 6 shows the result of this computer simulation during $t < 200$, where t is the time. All neuron states are updated sequentially one by one, but in Fig.6, each pattern is shown after all neurons are updated. The number below the patterns is the time when the patterns appeared. In the simulation result, the training patterns A0 appeared during $t = 84 - 123$. The rotated patterns C1 and A2 appeared during $t = 26 - 61$ and $t = 129 - 150$

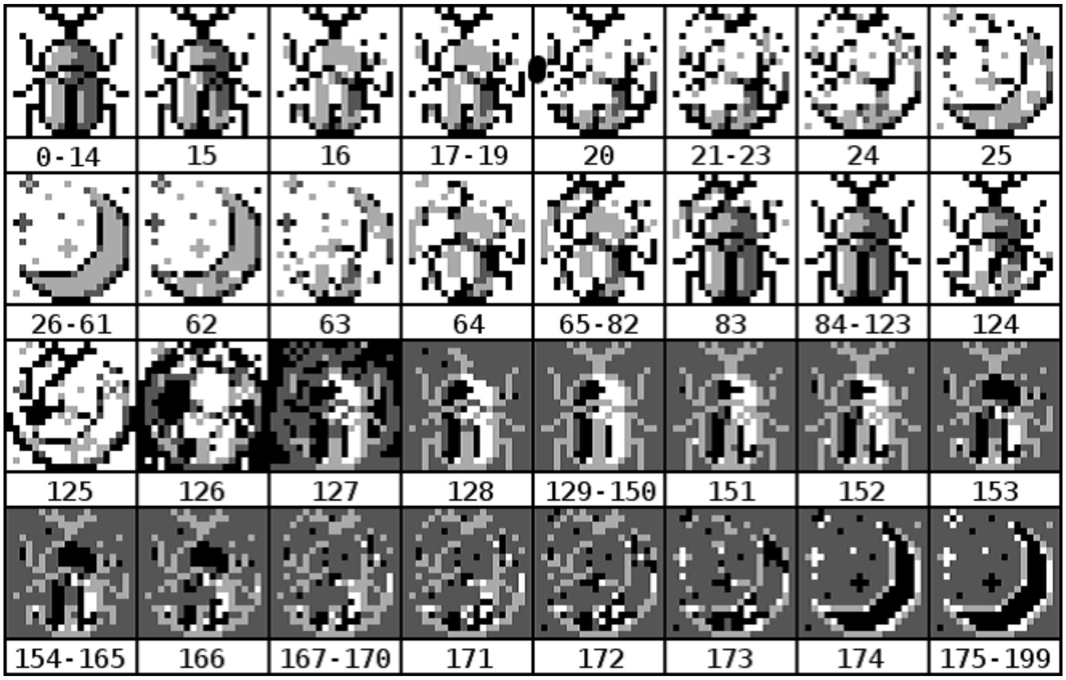

Fig. 6. The result of computer simulation for DCAM

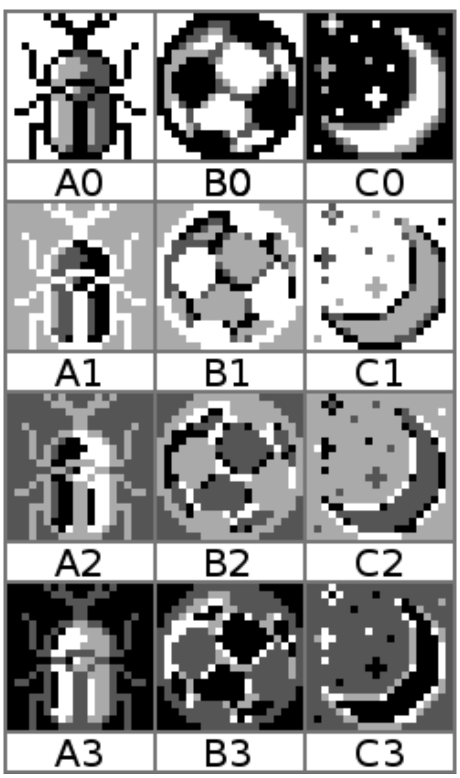

Fig. 5. Training and rotated patterns

respectively. Patterns except training and rotated patterns held during $t = 17 - 19, 21 - 23$ and others. Several mixture patterns, the secondary typical spurious patterns, also seem to have appeared. For example, the pattern appearing during $t = 21 - 23$ seems to be the mixture pattern of A0 and B0.

B. *Computer simulation for DRAM*

The DRAM would store the training patterns A0, B0 and C0, and the reversed patterns A2, B2 and C2. The parameters were as follows:

$$N = 400, k = 0.95, \alpha = 19. \tag{36}$$

The initial state was A0. Figure 7 shows the result of this computer simulation during $t < 200$. All the training and reversed patterns appeared while any rotated patterns except the reversed patterns did not appear. Another spurious pattern appeared during $t = 146 - 161$.

C. *Computer simulation for DRAM with a PAF*

The DRAM with a PAF would store only the training patterns A0, B0 and C0. There are two states $\mathbf{h}_k$ and $\mathbf{h}_{k+K}$ $(0 \leq k < K)$ corresponding to level k. We assigned $\mathbf{h}_k$ on upper half plane to level k. The parameters were as follows:

$$N = 400, k = 0.96, \alpha = 17. \tag{37}$$

The initial pattern was A0.

Figure 8 shows the result of this computer simulation during $t < 200$. All the training patterns appeared while any rotated and reversed patterns did not appear. Some mixture patterns seem to have appeared for a really short time.

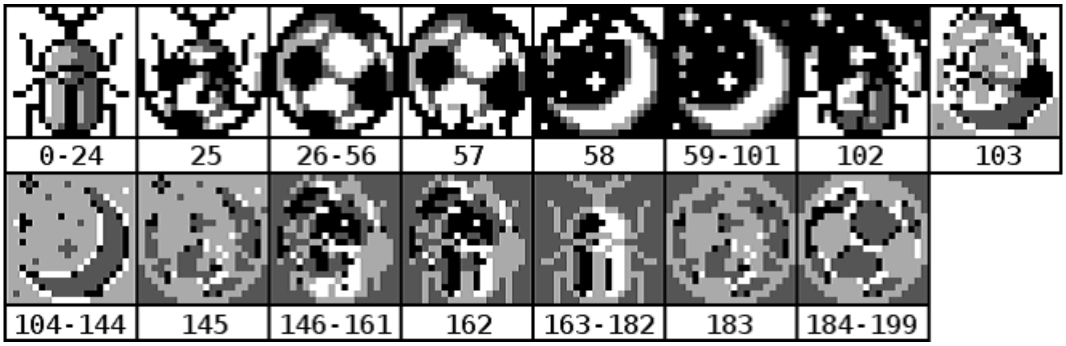

Fig. 7. The result of computer simulation for DRAM

TABLE I
APPEARENCE FREQUENCY UNTIL T=10000

Patterns	Frequency		
	DCAM	DRAM	DRAM with a PAF
A0	29	32	42
A1	25	0	0
A2	29	32	0
A3	25	0	0
B0	6	32	84
B1	6	0	0
B2	8	32	0
B3	6	0	0
C0	14	64	107
C1	16	0	0
C2	14	64	0
C3	16	0	0

TABLE II
LONGEST DURATION UNTIL T=10000

Patterns	Longest duration		
	DCAM	DRAM	DRAM with a PAF
A0	40	25	20
B0	47	31	20
C0	44	43	30
others	26	16	8

V. DISCUSSION

We summarize these computer simulations until t=10000 about the appearance frequencies and the longest stable durations. Table I shows the frequency of stable patterns. Dynamic associative memories can search the patterns stored in associative memories. However, they often recalled patterns but the training patterns as well. They are required not to produce spurious patterns as possible.

DCAM recalled all the training and rotated patterns but the frequency of the pattern B0 is extremely low. We cannot distinguish the rotated patterns obtained by DCAM from the training patterns because the rotated patterns hold as long as the training patterns.

DRAM recalled all the training and reversed patterns. As with DCAM, we cannot distinguish the reversed patterns obtained by DRAM from the training patterns.

DRAM with a PAF recalled all the training patterns. DRAM with a PAF can avoid recalling any rotated patterns.

Table II shows the longest durations of the training patterns and spurious patterns except the rotated patterns. We can estimate how strongly a pattern attracts by how long it holds. In the simulation results for DCAM, DRAM and DRAM with a PAF, the longest durations of the training patterns are much longer than those of the other spurious patterns. It implies that the training and reversed patterns are stronger attractors of DRAM and the training patterns are stronger attractors of DRAM with a PAF than any other attractors.

VI. CONCLUSION

Complex-valued Associative Memory (CAM) stores not only training patterns but also their rotated patterns. Rotor Associative Memory (RAM) can avoid storing the rotated patterns but stores the reversed patterns. In the present work, we introduce a Periodic Activation Function (PAF), which is a flexible activation function proposed by Aizenberg, to RAM in order to avoid storing the reversed patterns. The result of computer simulations shows that RAM with a PAF can avoid storing the rotated and reversed patterns.

In the present work, we used hebbian learning rule, so the storage capacity is very low. Thus CAM and RAM with a PAF can store no more patterns. For experiences with a large number of patterns, we have to apply advanced learning methods, such as gradient descent learning rule (Kitahara and Kobayashi [15]).

Our computer simulations imply the following results:

1) DCAM recalled all the training and rotated patterns. Moreover, several other strong attractors appeared.
2) DRAM recalled all the training and reversed patterns. It did not recall any rotated patterns except the reversed patterns. In addition, no other strong attractors appeared.
3) DRAM with a PAF recalled all the training patterns. It did not recall any rotated patterns. In addition, no other strong attractors appeared.

RAM with a PAF remains a problem. The area for each state of RAM with a PAF is narrower than that of ordinary RAM. Thus the storage capacity would decreas. To improve this problem, we should develop more advanced learning algorithms.

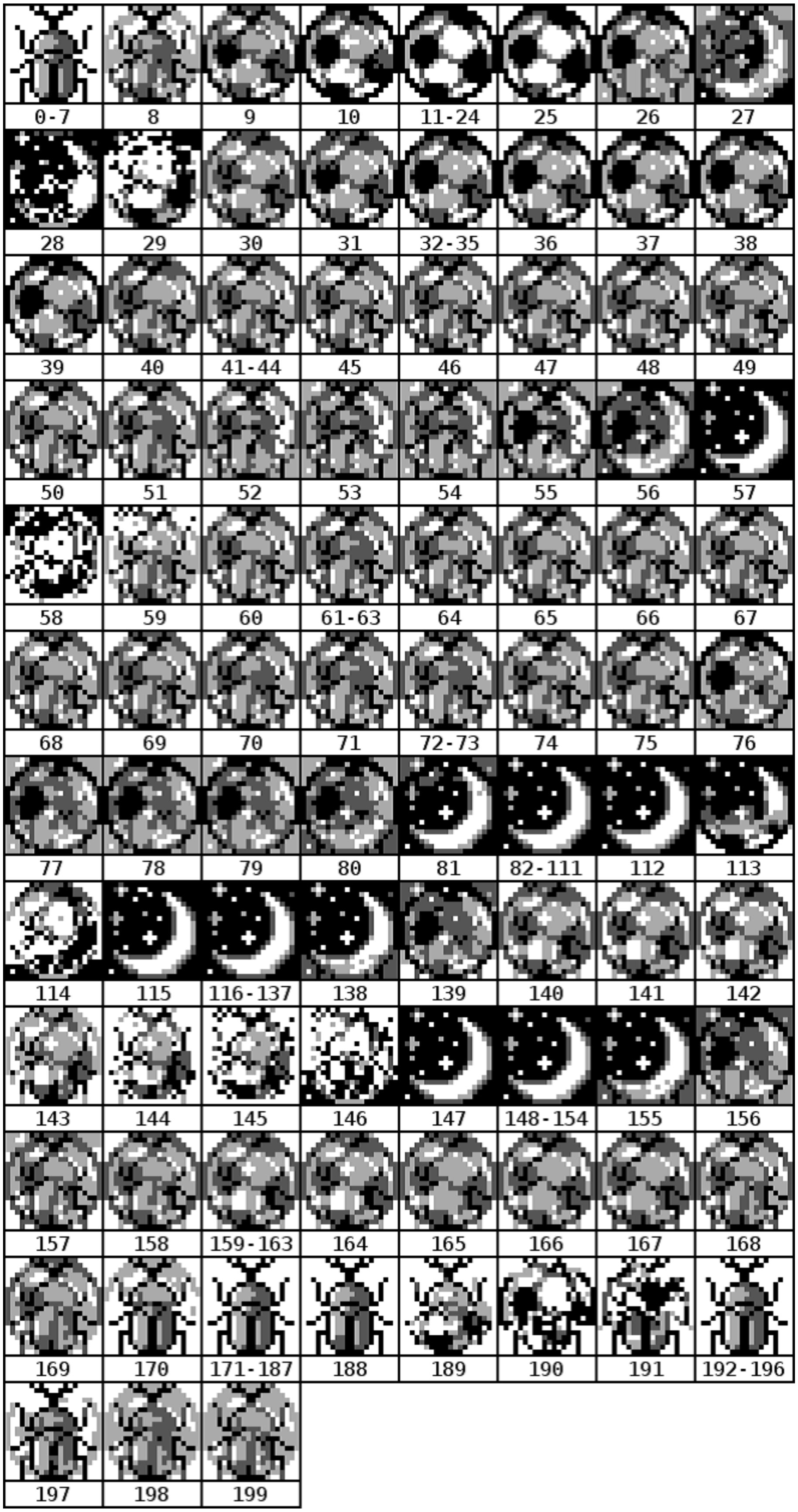

Fig. 8. The result of computer simulation for DRAM with a PAF

REFERENCES

[1] J. J. Hopfield: "Neural networks and physical systems with emergent collective computational abilities," *Proc. National Academy Sciences*, vol.79, no.8, pp.2554-2558(1982)

[2] I. N. Aizenberg, N. N. Aizenberg and J. Vandewalle, "Multi-valued and universal binary neurons - theory, learning and application," *Kluwer Academic Pubhishers*, Boston, 2000.

[3] A. J. Noest: "Phasor neural networks," *Neural Information Processing Systems*, ed. D. Z. Anderson, pp.584-591, AIP, New York (1988)

[4] S. Jnakowski, A. Lozowski and J. M. Zurada: "Complex-valued multistate neural associative memory," *IEEE Trans. Neural Networks*, Vol.7, No.6, pp.1491-1496 (1996)

[5] H. Aoki and Y. Kosugi, "An image storage system using complex-valued associative memory," *Proceedings of the International Conference on Pattern Recognition*, vol.2, pp.626-629, 2000.

[6] H. Aoki, M. R. Azimi-Sadjadi and Y. Kosugi, "Image association using a complex-valued associative memory model," *IEICE TRANSACTIONS on Fundamentals of Electronics, Communications and Computer Sciences*, vol.E83-A, pp.1824-1832, 2000.

[7] M. K. Muezzinoglu, C. S. Guzelis and J. M. Zurada, "A new design method for the complex-valued multistate Hopfield associative memory," *IEEE Transaction on Neural Networks*, vol.14, no.4, pp.891-899, 2003.

[8] R. S. Zemel, C. K. I. Williams and M. C. Mozer, "Lending direction to neural networks," *Neural Networks*, vol.8, no.4, pp.503-512, 1995.

[9] M. Kitahara and M. Kobayashi, "Fundamental abilities of rotor associative memory," *Proceedings of the 9th IEEE/ACIS International Conference on Computer and Information Science*, pp.497-502, 2010.

[10] M. Kitahara, M. Kobayashi and M. Hattori, "Chaotic rotor associative memory," *Proceedings of the International Symposium on Nonlinear Theory and its Applications*, pp.399-402, 2009.

[11] M. Kitahara, M. Kobayashi and M. Hattori, "Reducing spurious states by rotor associative memory," (in Japanese), *IEEJ Transactions on Electronics, Information and Systems*, vol.131, no.1, pp.109-115, 2011.

[12] I. Aizenberg: "A Multi-Valued Neuron with a Periodic Activation Function," *Proceedings of the International Joint Conference on Computational Intelligence*, vol.21, no.12, pp.347-354 (2009)

[13] I. Aizenberg: "Periodic Activation Function and a Modified Learning Algorithm for the Multi-Valued Neuron," *IEEE Trans. Neural Networks*, vol.21, no.12, pp.1939-1949 (2010)

[14] J. Nagumo and S. Sato, "On a response characteristic of a mathmatical neural model," *Kybernetik*, vol.10, no.3, pp.155-164, March 1972.

[15] M. Kitahara and M. Kobayashi, "Gradient descent learning for rotor associative memory," (in Japanese), *IEEJ Transactions on Electronics, Information and Systems*, vol.131, no.1, pp.116-121, 2011.